Computational Fluid Dynamics

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The notation and process mostly follow Reference [1].

Mass conservation

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where the **.**

Momentum conservation

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

Energy conservation

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

where are velocity components, is density, is specific energy, is pressure, is absolute temperature, represent body forces and other source terms, is thermal conductivity, and are the deviatoric stress components.

# Relating pressure and density

The equation of state for a perfect gas is a popular choice. A perfect gas is defined as a gas in which the intermolecular forces are neglected [2]. The following is the equation of state for a perfect gas

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

where is the specific gas constant, which has a different value for different gasses.

# Compressibility

Reference [2] Figure 7.3 illustrates the definition of compressibility.

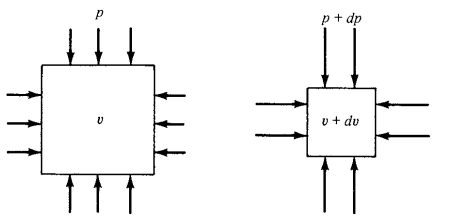


Figure 1. Definition of compressibility [2].

A pressure change results in a volume change (positive results in negative with this sign convention).

# Bernoulli from momentum conservation

Assume steady, inviscid, and no body forces.

Chain rule

Assume 1D (this may imply incompressibility)

|  |
| --- |
| 1D conservation of mass  Steady |

Apply conservation of mass to 1D momentum equation to simplify

Substitute

Move inside derivative

So

Integrate

# Characteristic-based split form A

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

Separate contributions with and without pressure terms

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

With

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

From mass conservation

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

Mass conservation algebra

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

|  |  |  |
| --- | --- | --- |
|  |  | (12) |

|  |  |  |
| --- | --- | --- |
|  |  | (13) |

and are not independent. Something like is enforced, where is the speed of sound. So

This seems to agree with the discretized form shown in Zienkiewicz Equation 3.57. Integration by parts is used to weaken the shape function derivative requirements.

# Pressure-density equation discretization

From (11)

Simplify

Integrate weighted residual

Integrate by parts

Substitutions

Rearrange

Substitute

Matrix form

Where

~~(diagonals wrong?)~~

# Expanded form of momentum conservation

|  |  |  |
| --- | --- | --- |
|  |  | (14) |

Deviatoric stress components

# Momentum Equation Discretization

## Convection term

Iteration-constant terms:

## Stress term

Deviatoric stresses

Strain rate in three dimensions

Strain rate in two dimensions

Volumetric strain rate is

With

Deviatoric strain rate

So

Cauchy stress vector

Deviatoric stress vector

Where the normal stresses are related to the deviatoric stresses (; recall that positive-compression sign convention is used for pressure). Deviatoric stresses are proportional to the deviatoric strain rates

# References

1. O. Zienkiewicz, R. L. Taylor, P. Nithiarasu. “The Finite Element Method for Fluid Dynamics.” Butterworth-Heinemann, Seventh Edition, 2013.
2. J. D. Anderson Jr. “Fundamentals of Aerodynamics.” McGraw-Hill, Fourth Edition, 2007.