

$$\begin{aligned}
\varphi_1 \mathbf{R} \varphi_2 &\stackrel{\mathbf{n}}{=} \neg(\neg\varphi_1 \mathbf{U} \neg\varphi_2) \\
&\stackrel{\mathbf{dsc}}{=} \neg(\neg\varphi_1 \mathbf{U} (\neg\varphi_2 \wedge (\neg\varphi_1 \vee \textit{sing}))) \\
&\stackrel{\mathbf{t}}{=} \neg(\neg\varphi_1 \mathbf{U} (\neg\varphi_2 \wedge (\neg\varphi_1 \vee \textit{sing}) \wedge \textit{alive})) \\
&= \boxed{\varphi_1} \mathbf{R} (\boxed{\varphi_2} \vee (\boxed{\varphi_1} \wedge \neg\textit{sing}) \vee \neg\textit{alive})
\end{aligned}$$