$$\begin{split} \mathbf{X}\,\varphi \, &\stackrel{\mathsf{dsc}}{\equiv} \left(sing \, \wedge \, \mathbf{X}\,(\varphi)\right) \,\vee\, \left(\neg sing \, \wedge\, (\varphi)\right) \\ &\stackrel{\mathsf{t}}{\equiv} \left(sing \, \wedge \, \mathbf{X}\,\big(\overline{\varphi}\, \wedge\, alive\big)\right) \,\vee\, \left(\neg sing \, \wedge\, \overline{\varphi}\,\right) \end{split}$$

$$\begin{split} \varphi_1 \, \mathbf{U} \, \varphi_2 &\overset{\mathrm{dsc}}{\equiv} \, \varphi_1 \, \mathbf{U} \, (\varphi_2 \, \wedge \, (\varphi_1 \, \vee \, sing)) \\ &\overset{\mathrm{t}}{\equiv} \, \boxed{\varphi_1} \, \mathbf{U} \, (\boxed{\varphi_2} \, \wedge \, (\boxed{\varphi_1} \, \vee \, sing) \, \wedge \, alive)) \end{split}$$

$$\begin{split} \mathbf{F}\,\varphi &\equiv true\,\mathbf{U}\,\varphi \\ &\stackrel{\mathrm{dsc}}{\equiv} true\,\mathbf{U}\,(\varphi\,\wedge\,(true\,\vee\,sing)) \\ &\equiv true\,\mathbf{U}\,\varphi \\ &\stackrel{\mathrm{t}}{\equiv} true\,\mathbf{U}\,(\,\varphi\,\wedge\,alive) \\ &\equiv \mathbf{F}\,(\!\![\,\varphi\,]\!\!]\,\wedge\,alive) \end{split}$$

$$\begin{array}{c} \mathsf{G}\,\varphi \equiv \neg \mathsf{F} \neg \varphi \\ &\stackrel{\mathsf{dsc}}{\equiv} \neg \mathsf{F} \left(\neg \varphi \, \wedge \, alive \right) \\ &\equiv \mathsf{G} \big(\boxed{\varphi} \, \lor \, \neg alive \big) \end{array}$$

$$\begin{array}{c} \varphi_1 \, \mathbb{W} \, \varphi_2 \, \equiv (\varphi_1 \, \mathbb{U} \, \varphi_2) \, \vee \, \mathbb{G} \, \varphi_1 \\ & \stackrel{\mathtt{dsc}, \mathtt{t}}{\equiv} \underbrace{ \begin{array}{c} [\varphi_1] \, \mathbb{U} \, (\left[\varphi_2 \right] \, \wedge \, (\left[\varphi_1 \right] \, \vee \, sing) \, \wedge \, alive)) \, \vee \\ & \stackrel{\mathtt{dsc}, \mathtt{t}}{\equiv} \underbrace{ \begin{array}{c} [\varphi_1] \, \mathbb{U} \, (\left[\varphi_2 \right] \, \wedge \, (\left[\varphi_1 \right] \, \vee \, sing) \, \wedge \, alive) \end{array} }_{\mathbf{G} \, (\left[\varphi_1 \right] \, \vee \, \neg alive) }$$

$$\begin{array}{c|c} \varphi_1 \ \mathbf{R} \ \varphi_2 \ \stackrel{\mathbf{n}}{\equiv} \neg (\neg \varphi_1 \ \mathbf{U} \ \neg \varphi_2) \\ \stackrel{\mathrm{dsc}}{\equiv} \neg (\neg \varphi_1 \ \mathbf{U} \ (\neg \varphi_2 \ \wedge \ (\neg \varphi_1 \ \vee \ sing))) \\ \stackrel{\mathbf{t}}{\equiv} \neg (\neg \varphi_1 \ \mathbf{U} \ (\neg \varphi_2 \ \wedge \ (\neg \varphi_1 \ \vee \ sing) \ \wedge \ alive)) \\ \equiv \boxed{\varphi_1} \ \mathbf{R} \ (\boxed{\varphi_2} \ \vee \ (\boxed{\varphi_1} \ \wedge \ \neg sing) \ \vee \ \neg alive) \end{array}$$

$$\begin{array}{c} \varphi_1 \, \mathrm{M} \, \varphi_2 \, \equiv \varphi_1 \, \mathrm{R} \, \varphi_2 \, \wedge \, \mathrm{F}(\varphi_1 \, \wedge \, \varphi_2) \\ \\ \overset{\mathrm{dsc,t}}{\equiv} \, \underbrace{ \begin{pmatrix} \varphi_1 \, \mathrm{R} \, (\varphi_2 \, \vee \, (\varphi_1 \, \wedge \, \neg sing) \, \vee \, \neg alive) \end{pmatrix} \wedge }_{\mathrm{F} \, (\varphi_1 \, \wedge \, (\varphi_2 \, \wedge \, alive))} \\ \end{array}$$

Non necessario

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\begin{array}{l} \varphi_1 \, \mathbb{S} \, \varphi_2 \, \stackrel{\mathrm{dsc}}{\equiv} \big( sing \, \wedge \, \mathbb{X} \, \big( \varphi_1 \, \mathbb{U} \, \varphi_2 \big) \big) \, \vee \big( \neg sing \, \wedge \, \big( \varphi_1 \, \mathbb{U} \, \varphi_2 \big) \big) \\ & \stackrel{\mathtt{t}}{\equiv} \big( sing \, \wedge \, \mathbb{X} \, \big( \big( \varphi_1 \mathbb{U} \, \varphi_2 \big) \, \wedge \, alive \big) \big) \, \vee \big( \neg sing \, \wedge \, \big( \varphi_1 \mathbb{U} \, \varphi_2 \big) \big) \\ & \stackrel{\mathrm{dsc}, \, \mathtt{t}}{\equiv} \big( sing \, \wedge \, \mathbb{X} \, \big( \big( \varphi_1 \, \mathbb{U} \, \big( \varphi_2 \, \big) \, \wedge \, \big( \varphi_1 \, \mathbb{U} \, \vee \, sing \big) \, \wedge \, alive \big) \big) \, \wedge \, alive \big) \big) \, \vee \\ & \qquad \qquad \big( \neg sing \, \wedge \, \big( \varphi_1 \, \mathbb{U} \, \big( \varphi_2 \, \big) \, \wedge \, \big( \varphi_1 \, \mathbb{U} \, \vee \, sing \big) \, \wedge \, alive \big) \big) \big) \end{array}
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