$$
\begin{aligned}
\varphi_1 \,\mathtt{S}\, \varphi_2 &\overset{\texttt{dsc}}{=} (\mathit{sing} \wedge \mathtt{X}\,(\varphi_1 \,\mathtt{U}\, \varphi_2)) \vee (\neg \mathit{sing} \wedge (\varphi_1 \,\mathtt{U}\, \varphi_2)) \\
&\overset{\texttt{t}}{=} (\mathit{sing} \wedge \mathtt{X}\,((\varphi_1 \mathtt{U}\, \varphi_2) \wedge \mathit{alive})) \vee (\neg \mathit{sing} \wedge (\varphi_1 \mathtt{U}\, \varphi_2)) \\
&\overset{\texttt{dsc,t}}{=} (\mathit{sing} \wedge \mathtt{X}\,((\boxed{\varphi_1}\,\mathtt{U}\,(\boxed{\varphi_2} \wedge (\boxed{\varphi_1} \vee \mathit{sing}) \wedge \mathit{alive})) \wedge \mathit{alive})) \vee \\
&\qquad (\neg \mathit{sing} \wedge (\boxed{\varphi_1}\,\mathtt{U}\,(\boxed{\varphi_2} \wedge (\boxed{\varphi_1} \vee \mathit{sing}) \wedge \mathit{alive})))
\end{aligned}
$$