$$\begin{array}{c} \varphi_1 \, \mathrm{U} \, \varphi_2 \, \stackrel{\mathrm{dsc}}{=} \, \varphi_1 \, \mathrm{U} \, (\varphi_2 \, \wedge \, (\varphi_1 \, \vee \, sing)) \\ \\ \stackrel{\mathrm{t}}{=} \, \boxed{\varphi_1} \, \mathrm{U} \, (\boxed{\varphi_2} \, \wedge \, (\boxed{\varphi_1} \, \vee \, sing) \, \wedge \, alive)) \end{array}$$

$$\begin{split} \mathbf{F}\,\varphi &= true\,\mathbf{U}\,\varphi \\ &\stackrel{\mathrm{dsc}}{=} true\,\mathbf{U}\,(\varphi\,\wedge\,(true\,\vee\,sing)) \\ &= true\,\mathbf{U}\,\varphi \\ &\stackrel{\mathrm{t}}{=} true\,\mathbf{U}\,(\,\varphi\,\wedge\,alive) \\ &= \mathbf{F}\,(\boxed{\varphi}\,\wedge\,alive) \end{split}$$

$$\begin{split} \mathbf{G}\,\varphi &= \neg \mathbf{F} \neg \varphi \\ &\stackrel{\mathtt{dsc}}{=} \neg \mathbf{F} \left(\neg \varphi \, \wedge \, alive \right) \\ &= \mathbf{G} (\boxed{\varphi} \, \lor \, \neg alive) \end{split}$$

$$\begin{vmatrix} \varphi_1 \, \mathbb{W} \, \varphi_2 \, = \, (\varphi_1 \, \mathbb{U} \, \varphi_2) \, \vee \, \mathbb{G} \, \varphi_1 \\ \\ \underset{=}{\operatorname{dsc}}, \underset{=}{\operatorname{t}} \underbrace{ \left[\varphi_1 \right] } \mathbb{U} \left(\underbrace{ \left[\varphi_2 \right] } \wedge \left(\underbrace{ \left[\varphi_1 \right] } \vee \, sing \right) \, \wedge \, alive \right) \right) \vee \\ \\ \overset{=}{\operatorname{g}} \left(\underbrace{ \left[\varphi_1 \right] } \vee \, \neg alive \right)$$

$$\begin{array}{c|c} \varphi_1 \, \mathbf{R} \, \varphi_2 & \stackrel{\mathtt{n}}{=} \neg (\neg \varphi_1 \, \mathbf{U} \, \neg \varphi_2) \\ & \stackrel{\mathtt{dsc}}{=} \neg (\neg \varphi_1 \, \mathbf{U} \, (\neg \varphi_2 \, \wedge \, (\neg \varphi_1 \, \vee \, sing))) \\ & \stackrel{\mathtt{t}}{=} \neg (\neg \, \varphi_1 \, \mathbf{U} \, (\neg \, \varphi_2 \, \wedge \, (\neg \, \varphi_1 \, \vee \, sing) \, \wedge \, alive)) \\ & = \boxed{\varphi_1} \, \mathbf{R} \, (\boxed{\varphi_2} \, \vee \, (\boxed{\varphi_1} \, \wedge \, \neg sing) \, \vee \, \neg alive) \end{array}$$

$$\begin{array}{c|c} \varphi_1 \, \mathbf{M} \, \varphi_2 \, = \, \varphi_1 \, \mathbf{R} \, \varphi_2 \, \wedge \, \mathbf{F}(\varphi_1 \, \wedge \, \varphi_2) \\ & \underset{=}{\operatorname{dsc,t}} \, \left(\begin{array}{|c|c|c} \varphi_1 \, \mathbf{R} \, (\begin{array}{|c|c|c} \varphi_2 \end{array} \vee \, (\begin{array}{|c|c|c} \varphi_1 \, \wedge \, \neg sing) \, \vee \, \neg alive) \right) \wedge \\ & \stackrel{=}{=} \, \mathbf{F} \, (\begin{array}{|c|c|c} \varphi_1 \, \wedge \, (\begin{array}{|c|c|c} \varphi_2 \end{array} \wedge \, alive) \end{array} \right) \end{array}$$

$$\begin{split} \varphi_1 \, \mathbf{S} \, \varphi_2 &\stackrel{\mathsf{dsc}}{=} (sing \, \wedge \, \mathbf{X} \, (\varphi_1 \, \mathbf{U} \, \varphi_2)) \vee (\neg sing \, \wedge \, (\varphi_1 \, \mathbf{U} \, \varphi_2)) \\ &\stackrel{\mathsf{t}}{=} (sing \, \wedge \, \mathbf{X} \, ((\varphi_1 \, \mathbf{U} \, \varphi_2) \wedge alive)) \vee (\neg sing \, \wedge \, (\varphi_1 \, \mathbf{U} \, \varphi_2)) \\ &\stackrel{\mathsf{dsc}, \, \mathsf{t}}{=} (sing \, \wedge \, \mathbf{X} \, ((\varphi_1 \, \mathbf{U} \, (\varphi_2) \, \wedge \, (\varphi_1 \, \vee \, sing) \, \wedge \, alive)) \wedge alive)) \vee \\ & (\neg sing \, \wedge \, (\varphi_1 \, \mathbf{U} \, (\varphi_2) \, \wedge \, (\varphi_1 \, \vee \, sing) \, \wedge \, alive))) \end{split}$$