$$
\begin{aligned}
\varphi_1 \,\mathtt{S}\, \varphi_2 &\overset{\mathtt{dsc}}{=} (\mathit{sing} \wedge \mathtt{X}\,(\varphi_1 \,\mathtt{U}\, \varphi_2)) \vee (\neg \mathit{sing} \wedge (\varphi_1 \,\mathtt{U}\, \varphi_2)) \\
&\overset{\mathtt{t}}{=} (\mathit{sing} \wedge \mathtt{X}\,((\boxed{\varphi_1 \,\mathtt{U}\, \varphi_2}) \wedge \mathit{alive})) \vee (\neg \mathit{sing} \wedge (\boxed{\varphi_1 \,\mathtt{U}\, \varphi_2})) \\
&\overset{\mathtt{dsc}}{=} (\mathit{sing} \wedge \mathtt{X}\,((\boxed{\varphi_1}\,\mathtt{U}\,(\boxed{\varphi_2} \wedge (\boxed{\varphi_1} \vee \mathit{sing}) \wedge \mathit{alive})) \wedge \mathit{alive})) \vee \\
&\quad (\neg \mathit{sing} \wedge (\boxed{\varphi_1}\,\mathtt{U}\,(\boxed{\varphi_2} \wedge (\boxed{\varphi_1} \vee \mathit{sing}) \wedge \mathit{alive})))
\end{aligned}
$$