POLYGON Project: Session 3

Mathematics 1 - Logic and Proofs

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Introduction

Definition

Logic *is the basis of all mathematical reasoning.*

How to understand mathematics?

- We must understand **what makes up** a correct mathematical argument. That is **proof** (chứng minh).
- We prove a mathematical statement is true $\longrightarrow$ **theorem** (định lí).
- To learn a mathematica topic, you need to **construct** mathematical arguments on this topic, not just read exposition.

What you should expect after this session?

- Explain **what makes up** a correct mathematical argument.
- Introduce **tools** to construct these arguments.

Propositions (Mệnh đề)

Definition

Proposition *is a declarative sentence (fact) that is either true or false, but not both.*

Propositions (Mệnh đề)

- Denote: use letters like p, q, r, s, t,... to denote propositions.
- **Truth value** of a proposition: *true* denoted by T or *false* denoted by F.
- **Compound propositions** (mệnh đề phức hợp) are formed by combining propositions using **logical operators** (toán tử logic).

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Negation (Phủ định)

Let p be a proposition. The **negation** of p ($\neg p$ - not p), is the statement: *"It is not the case that p"*.
The truth value of $\neg p$ is the opposite of the truth value of p.

Truth Table (Bảng chân trị)

p	$\neg p$
T	F
F	T

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Conjunction (Kết hợp)

Let p and q be propositions. The **conjunction** of p and q ($p \wedge q$ - p and q) is the proposition "p and q". The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

Truth Table (Bảng chân trị)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

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Disjunction (Phân hợp)

Let p and q be propositions. The **disjunction** of p and q ($p \vee q$ - p or q) is the proposition "p or q". The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

Truth Table (Bảng chân trị)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

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Exclusive OR - XOR

Let p and q be propositions. The **exclusive OR** of p and q ($p \oplus q$ - p xor q) is the proposition "p xor q". The exclusive OR $p \oplus q$ is true when exactly one of p and q is true and is false otherwise.

Truth Table (Bảng chân trị)

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

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Implication (Mệnh đề kéo theo)

Let p and q be propositions. The **conditional statement** of p and q ($p \rightarrow q$ - p implies q) is the proposition "p implies q". The conditional statement $p \rightarrow q$ í false when p is true and q is false, and true otherwise.
p is called **hypothesis**. q is called **conclusion**.

Truth Table (Bảng chân trị)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Ways to express

$$p \rightarrow q$$

- "if p, then q"
- "if p, q"
- "p is sufficient for q"
- "q if p"
- "q when p"
- "a necessary condition for p is q"
- "q unless $\neg p$

Ways to express

- "p implies q"
- "p only if q"
- "a sufficient condition for q is p"
- "q whenever p"
- "q is necessary for p"
- "q follows from p"
- "q provided that p"

Exercise 1

Suppose that Smartphone A has 256MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

Exercise 1

1. Smartphone B has the most RAM of these three smartphones.
2. Smartphone C has more ROM or a higher resolution camera than Smartphone B.
3. Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
4. If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
5. Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

Exercise 2

Let p and q be the propositions.

- p: You drive over 65 miles per hour.
- q: You get a speeding ticket.

Write these propositions using p and q and logical connectives (including negations).

Exercise 2

1. You do not driver over 65 miles per hour.
2. You drive over 65 miles per hour, but you do not get a speeding ticket.
3. You will get a speeding ticket if you drive over 65 miles per hour.
4. If you do not driver over 65 miles per hour, then you will not get a speeding ticket.
5. Driving over 65 miles per hour is sufficient for getting a speeding ticket.
6. You get a speeding ticket, but you do not drive over 65 miles per hour.
7. Whenever you get a speeding ticket, you are driving over 65 miles per hour.

Exercise 3

Determine whether each of these conditional statements is true or false.

1. If $1 + 1 = 2$, then $2 = 2 = 5$.
2. If $1 + 1 = 3$, then $2 + 2 = 4$.
3. If $1 + 1 = 3$, then $2 + 2 = 5$.
4. If monkeys can fly, then $1 + 1 = 3$.

Exercise 4

Construct a truth table for each of these compound propositions.

1. $p \wedge \neg q$
2. $p \vee \neg p$
3. $(p \vee \neg q) \rightarrow q$