



$$\vec{D} = D \hat{D}$$

$$\begin{aligned}\vec{r} &= \vec{R} + \vec{S} \\ \vec{r}_1 &= \vec{R}_1 + \vec{S} \\ \vec{r}_2 &= \vec{R}_2 + \vec{S}\end{aligned}$$

$$\begin{aligned}\ddot{\vec{R}} &= -\frac{\mu_1}{R_1^3} \vec{R}_1 - \frac{\mu_2}{R_2^3} \vec{R}_2 \\ \ddot{\vec{r}} &= -\frac{\mu_1}{r^3} \vec{r} - \frac{\mu_2}{R_2^3} \vec{r}_2\end{aligned}$$

Accelerations due to gravity

② $\ddot{\vec{R}}_1 = \ddot{\vec{R}} - \rho \ddot{\vec{D}} \hat{D} - 2\rho \dot{\vec{D}} (\Omega \hat{V}) + \rho D \Omega^2 \hat{D}$ Moved origin of reference orbit to μ_1 instead of \oplus

③ $\ddot{\vec{r}}_1 = \ddot{\vec{r}} - \rho \ddot{\vec{D}} \hat{D} - 2\rho \dot{\vec{D}} (\Omega \hat{V}) + \rho D \Omega^2 \hat{D}$ Same derivation as ②

$$\ddot{\vec{r}}_1 = -\frac{\mu_1}{r_1^3} \vec{r}_1 - \frac{\mu_2}{r_2^3} \vec{r}_2 - \rho \ddot{\vec{D}} \hat{D} - 2\rho \dot{\vec{D}} \Omega \hat{V} + \rho D \Omega^2 \hat{D}$$

$$\ddot{\vec{S}} = \ddot{\vec{r}}_1 - \ddot{\vec{R}}_1 = \left[-\frac{\mu_1}{r_1^3} \vec{r}_1 - \frac{\mu_2}{r_2^3} \vec{r}_2 \right] - \left[-\frac{\mu_1}{R_1^3} \vec{R}_1 - \frac{\mu_2}{R_2^3} \vec{R}_2 \right] + (\text{Rotating Frame}) - (\text{Rotating Frame})$$

④ $\ddot{\vec{S}} = \mu_1 \left[-\frac{\vec{r}_1}{r_1^3} + \frac{\vec{R}_1}{R_1^3} \right] + \mu_2 \left[-\frac{\vec{r}_2}{r_2^3} + \frac{\vec{R}_2}{R_2^3} \right]$ Substitute ① into ③ - ②
Rotating frame terms cancel

Roy p.202: $\frac{\vec{R}_k}{R_k^3} = \frac{\vec{r}_k}{r_k^3} = \frac{\vec{R}_k}{R_k^3} - \frac{(\vec{R}_k + \vec{S})}{r_k^3}$ $k=1,2$

$$= \vec{R}_k \left(\frac{1}{R_k^3} - \frac{1}{r_k^3} \right) - \frac{\vec{S}}{r_k^3}$$

$$\frac{1}{R_k^3} - \frac{1}{r_k^3} = \frac{r_k^3 - R_k^3}{R_k^3 r_k^3} = \frac{(r_k - R_k)(r_k^2 + r_k R_k + R_k^2)}{R_k^3 r_k^3} = \frac{(r_k^2 - R_k^2)}{R_k^3 r_k^3} \frac{(r_k^2 + r_k R_k + R_k^2)}{(r_k + R_k)}$$

$$\vec{S} \cdot (\vec{r}_k + \vec{R}_k) = (\vec{r}_k - \vec{R}_k) \cdot (\vec{r}_k + \vec{R}_k) = r_k^2 - R_k^2 = \vec{S} \cdot (\vec{R}_k + \vec{S} + \vec{R}_k)$$

$$\text{so } r_k^2 - R_k^2 = S^2 + 2\vec{S} \cdot \vec{R}_k$$

$$\text{Let } Q_k = \frac{S^2 + 2\vec{S} \cdot \vec{R}_k}{R_k^3 r_k^3} \frac{(r_k^2 + r_k R_k + R_k^2)}{r_k + R_k}$$

$$\ddot{\vec{S}} = \sum_{k=1}^2 \mu_k \left[Q_k \vec{R}_k - \frac{\vec{S}}{r_k^3} \right]$$

④ rewritten to avoid subtracting two nearly equal numbers.