Linear Momentum States for Translational Joints in Momentum State Dynamics

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Background

- This work will make a lot more sense if you read these slides first:
  - Kane_NBody.pdf
  - MSD_NBody.pdf
  - Translational Joints.pdf

Prismatic Joints in MSD

- We draw heavily on analogies to angular momentum states, which have already been developed
- Translational joint degrees of freedom appear in ${}^{N}v^{R}$, leading to:

$$\{H_k\} = (S[J] + R[\tilde{c}])\Omega_u u - (S[\tilde{c}]^T + R[m])(V_u u + V_s s)$$
$$\{P_k\} = \Sigma\left[[m](V_u u + V_s s) - [\tilde{c}]\Omega_u u\right]$$

Compare with MSD_NBody.pdf, slide 16

Linear Momentum States

Let $p_{ik} = \Delta_{ik}^T P_k$, $p_{ok} = \Delta_{ok}^T P_k$, so:

$$\begin{Bmatrix} p_0 \\ p_{i1} \\ p_{o1} \\ p_{i2} \\ p_{o2} \\ p_{i3} \\ p_{o3} \\ p_{i4} \\ p_{o4} \end{Bmatrix} = \begin{Bmatrix} P_0 \\ \Delta_{i1}^T P_1 \\ \Delta_{o1}^T P_1 \\ \Delta_{i2}^T P_2 \\ \Delta_{o2}^T P_2 \\ \Delta_{i3}^T P_3 \\ \Delta_{o3}^T P_3 \\ \Delta_{i4}^T P_4 \\ \Delta_{o4}^T P_4 \end{Bmatrix}$$

Analogous to h_k, this is a minimum-length vector of linear momentum states

Abbreviating, we write

$$\{p_k\} = [\Delta_k]^T \{P_k\}$$

so

$$\{\dot{p}_k\} = [\Delta_k]^T \{F_k\}$$

to replace $\dot{P}_0 = F_0$ on slide 18 of MSD_NBody.pdf

F_k is the sum of all contributing forces applied to all bodies outboard of joint G_k. It must account for inter-body contributing forces, such as springs and dampers.

MSD Equations of Motion

Integrate:

$$\begin{Bmatrix} \{\dot{p}_k\} \\ \{\dot{h}_k\} \end{Bmatrix} = \begin{Bmatrix} [\Delta_k]^T \{F_k\} \\ [\Gamma_k]^T \left\{ T_k - v_k \times P_k - \sum_i \omega_i \times {}^{G_k}H^{B_i} \right\} \end{Bmatrix}$$

(where F_k is the total resultant force, and T_k is the resultant torque about G_k of all forces and moments acting on bodies outboard of G_k)

Solve for u, s:

$$\begin{Bmatrix} \{p_k\} \\ \{h_k\} \end{Bmatrix} = \begin{bmatrix} \Lambda_u & \Lambda_s \\ [\Gamma_k]^T A & 0 \end{bmatrix} \begin{Bmatrix} u \\ s \end{Bmatrix}$$

Construct $\{\omega_i\}, \{v_k\}$:

$$\{\omega_i\} = \Omega_u u$$
$$\{v_k\} = V_u u + V_s s$$

Repeat

That's all.