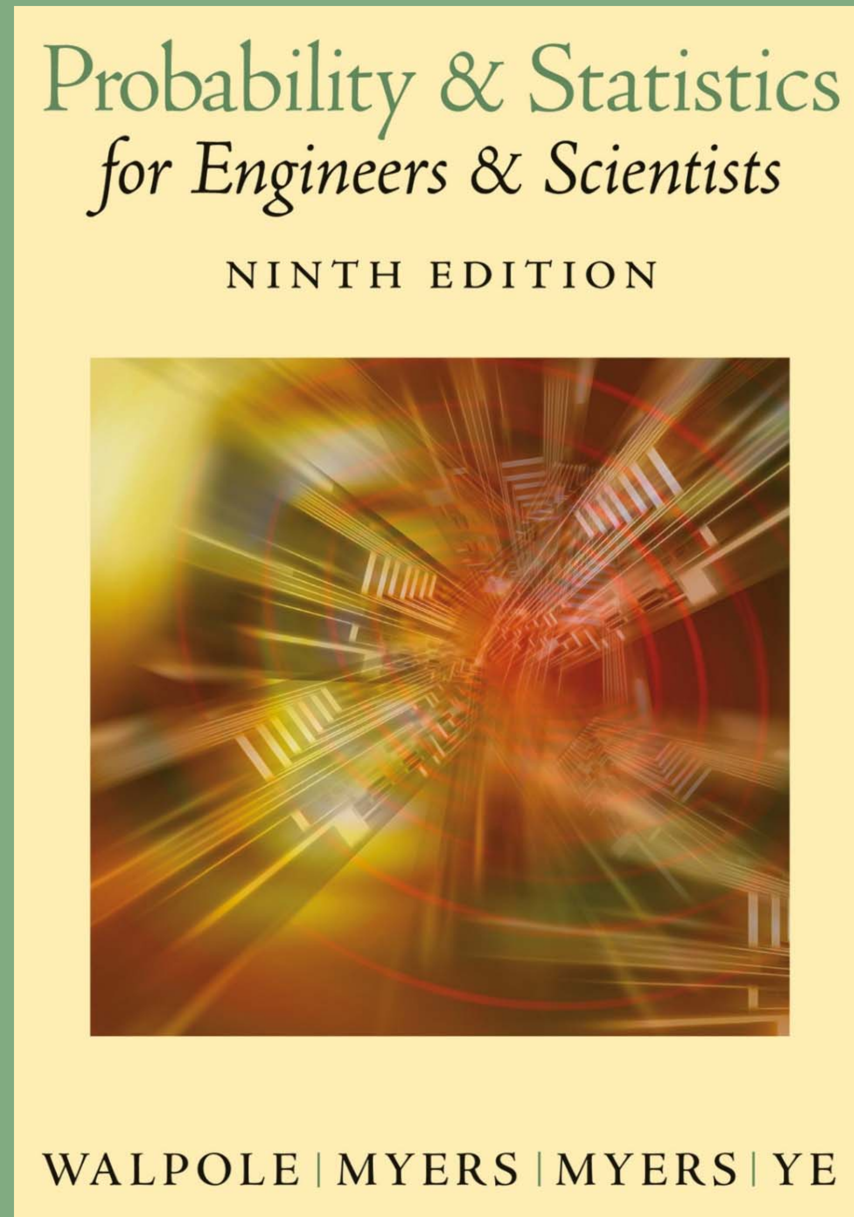
Chapter 5

Some Discrete Probability Distributions



PEARSON
Addison Wesley

Discrete distributions

Four important discrete distributions:

1. **The Uniform distribution (discrete)**

2. **The Binomial distribution**

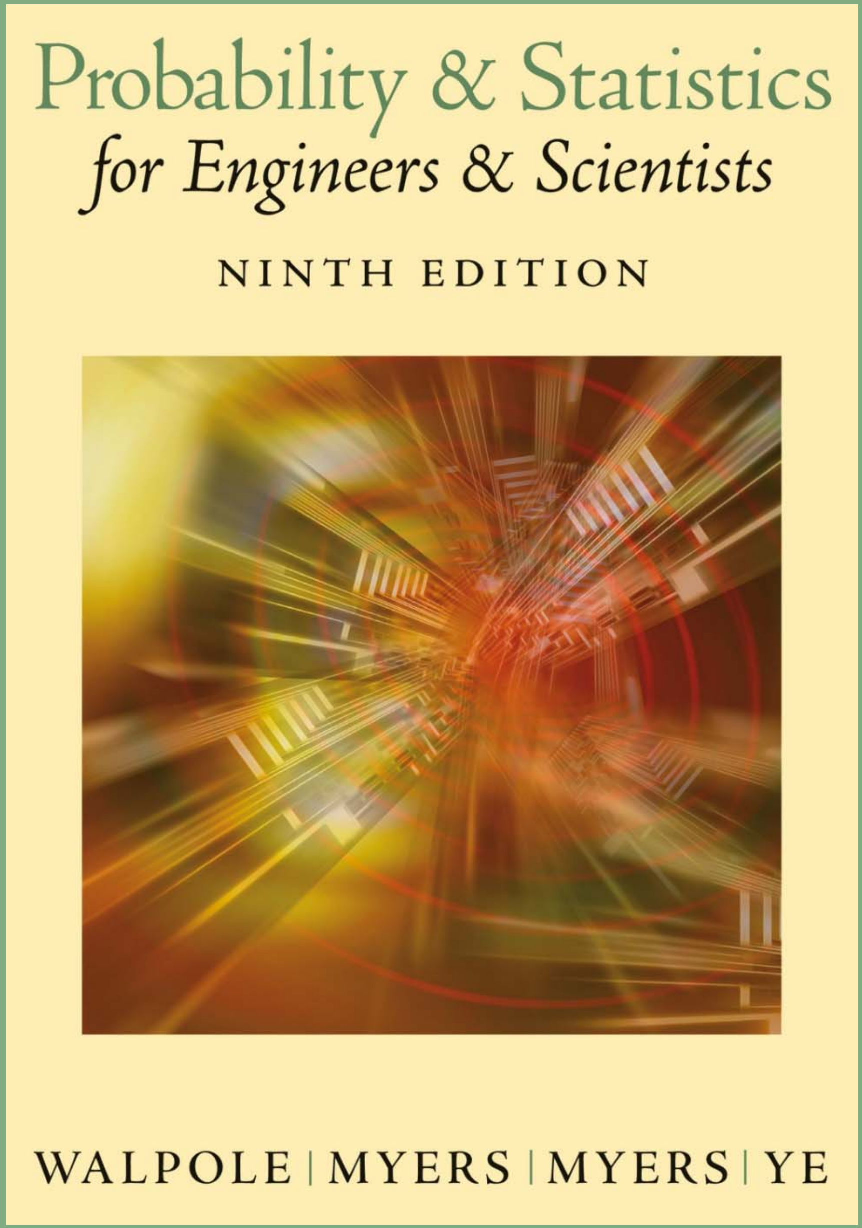
3. **The Hyper-geometric distribution**

4. **The Poisson distribution**

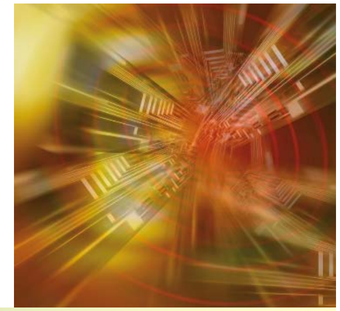
Section 5.2

Uninform

Distribution



Uniform distribution
Definition



Experiment with k equally likely outcomes.

Definition:

If the discrete random variable X assumes the values x_1, x_2, …, x_k with equal probabilities, then X has the discrete uniform distribution given by:

$$f(x) = P(X = x) = f(x;k) = \begin{cases} \dfrac{1}{k} \; ; \; x = x_1, x_2, \cdots, x_k \\ 0 \; ; \; elsewhere \end{cases}$$

Note:

$f(x)=f(x;k)=P(X=x)$

k is called the parameter of the distribution.

Uniform distribution
Definition

Let $\mathbf{X}: \mathbf{S} \to \mathbf{R}$ be a discrete random variable. If

$$P(X_1 = x_1) = P(X_2 = x_2) = \cdots P(X_k = x_k) = \frac{1}{k}$$

then the distribution of X is the (discrete) uniform distribution.

Probability function: $f(x:k) = \dfrac{1}{k} \quad for\ x = x_1, x_2, \ldots, x_k$

(Cumulative) distribution function:

$$F(x;k) = \frac{x}{k} \quad for\ x = x_1, x_2, \ldots, x_k$$

Example

Example 5.2:

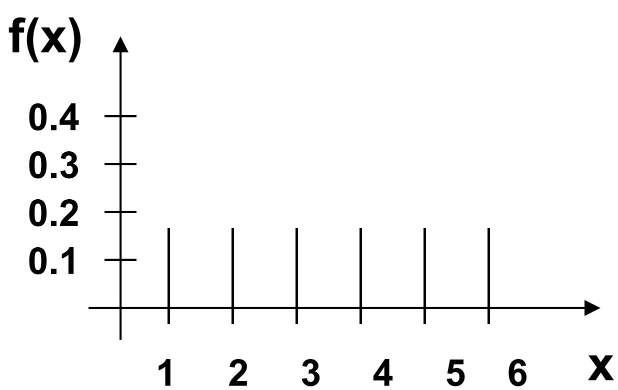
- Experiment: tossing a balanced die.
- Sample space: S={1,2,3,4,5,6}
- Each sample point of S occurs with the same probability 1/6.
- Let X= the number observed when tossing a balanced die.

The probability distribution of X is:

Uniform distribution
Example

Example: Rolling a dice

X: # eyes



Probability function: $$f(x\,;k)=\frac{1}{6}\quad for\ x=1,2,\ldots,6$$

Distribution function: $$F(x\,;6)=\frac{x}{6}\quad for\ x=1,2,\ldots,6$$

Theorem 5.1:

If the discrete random variable X has a discrete uniform distribution with parameter k, then the mean and the variance of X are:

$$E(X) = \mu = \frac{\sum_{i=1}^{k} x_i}{k}$$

$$Var(X) = \sigma^2 = \frac{\sum_{i=1}^{k} (x_i - \mu)^2}{k}$$

Example 5.3:

Find E(X) and Var(X) in Example 5.2.

Solution:

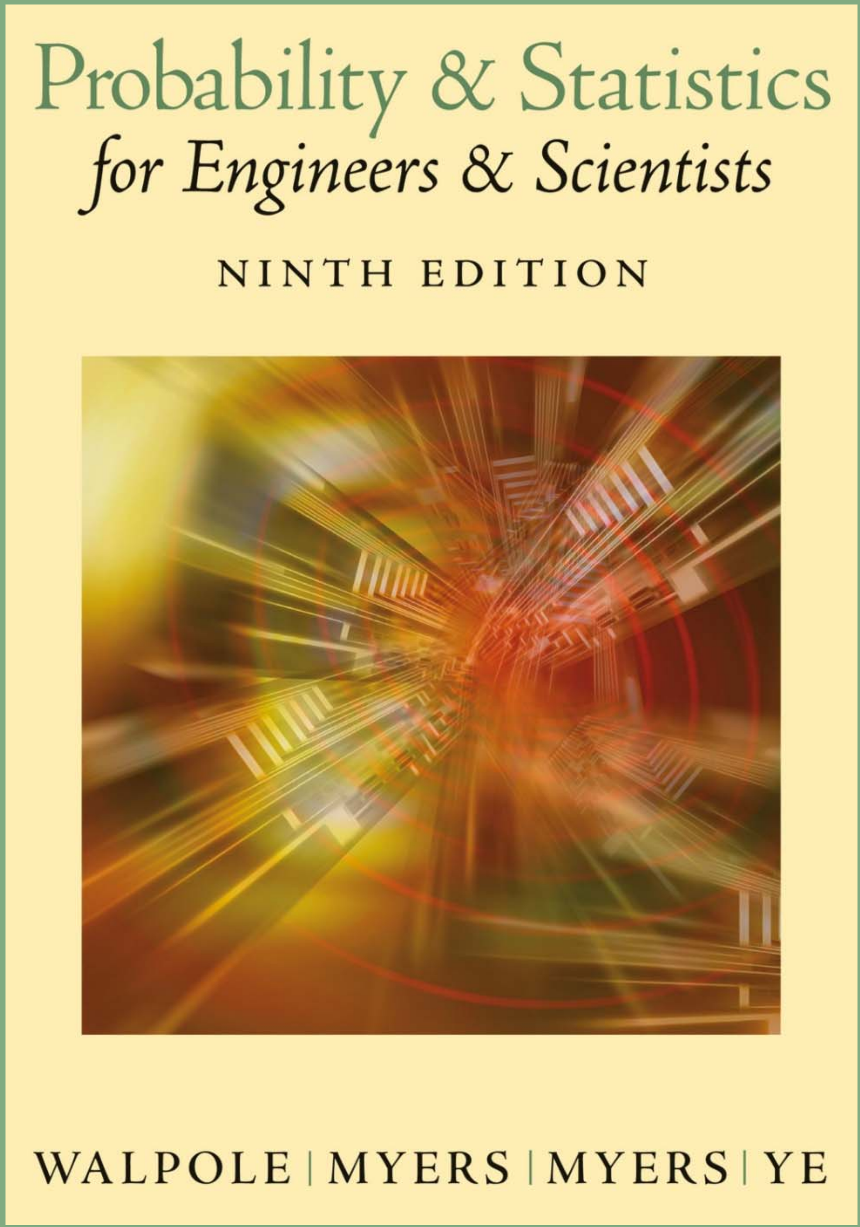
$$E(X) = \mu = \frac{\sum_{i=1}^{k} x_i}{k} = \frac{1+2+3+4+5+6}{6} = 3.5$$

$$Var(X) = \sigma^2 = \frac{\sum_{i=1}^{k} (x_i - \mu)^2}{k} = \frac{\sum_{i=1}^{k} (x_i - 3.5)^2}{6}$$

$$= \frac{(1-3.5)^2 + (2-3.5)^2 + \cdots + (6-3.5)^2}{6} = \frac{35}{12}$$

Section 5.2

Binomial Distribution



Bernoulli Trial:

- Bernoulli trial is an experiment with only two possible outcomes.
- The two possible outcomes are labeled:

  success (s) and failure (f)
- The probability of success is $P(s)=p$ and the probability of failure is $P(f)= q = 1-p$.

Examples:

1. Tossing a coin (success=H, failure=T, and p=P(H))
2. Inspecting an item (success=defective, failure=non-defective, and p=P(defective))

Bernoulli Process:

Bernoulli process is an experiment that must satisfy the following properties:

1. The experiment consists of ***n*** **repeated Bernoulli trials**.

2. The probability of success, $P(s)=p$, remains constant from trial to trial.

3. The repeated trials are **independent**; that is the outcome of one trial has no effect on the outcome of any other trial

Binomial distribution
Bernoulli process

Repeating an experiment with two possible outcomes.

Bernoulli process:

1. The experiment consists in repeating the same trail n times.

2. Each trail has two possible outcomes: “success” or “failure”, also known as Bernoulli trail.

3. P(”success”) = p is the same for all trails.

4. The trails are independent.

Binomial Random Variable:

Consider the random variable :

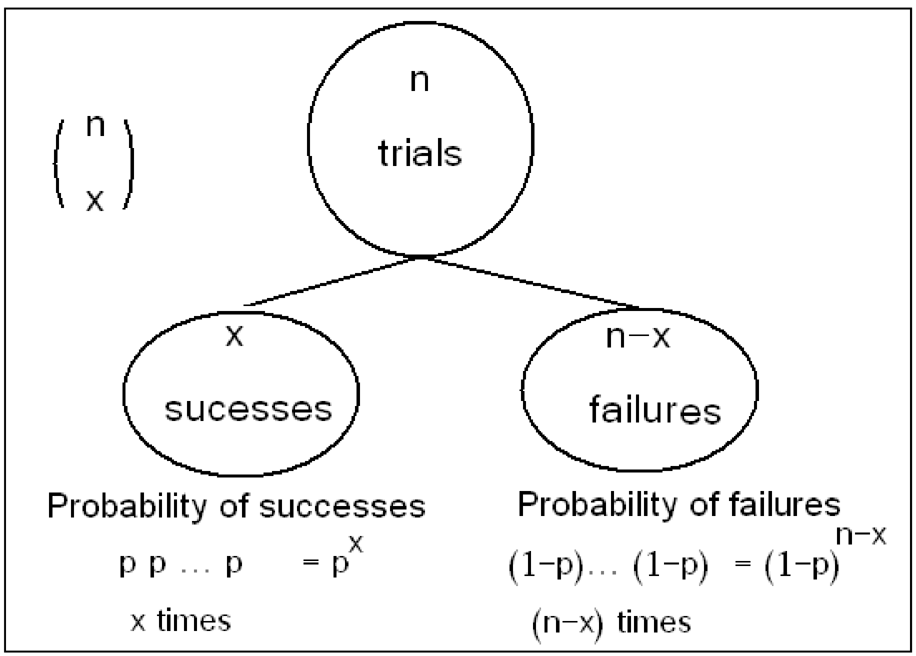
X = The number of successes in the n trials in a Bernoulli process

The random variable X has a binomial distribution with parameters n (number of trials) and p (probability of success), and we write:

$$X \sim \text{Binomial}(n,p)$$

The probability distribution of *X* is given by:

$$f(x) = P(X = x) = b(x; n, p) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} \ ; & x = 0, 1, 2, \ldots, n \\ 0 \ ; & otherwise \end{cases}$$



We can write the probability distribution of X as a table as follows.

x	f(x)=P(X=x)=b(x;n,p)
0	$\binom{n}{0} p^0 (1-p)^{n-0} = (1-p)^n$
1	$\binom{n}{1} p^1 (1-p)^{n-1}$
2	$\binom{n}{2} p^2 (1-p)^{n-2}$
$\vdots$	$\vdots$
$n-1$	$\binom{n}{n-1} p^{n-1} (1-p)^1$
n	$\binom{n}{n} p^n (1-p)^0 = p^n$
Total	1.00

Example:

Suppose that 25% of the products of a manufacturing process are defective. Three items are selected at random, inspected, and classified as defective (D) or non-defective (N). Find the probability distribution of the number of defective items.

Solution:

· Experiment: selecting 3 items at random, inspected, and classified as (D) or (N).

· The sample space is

S={DDD,DDN,DND,DNN,NDD,NDN,NND,NNN}

· Let X = the number of defective items in the sample

· We need to find the probability distribution of X.

(1) First Solution:

Outcome	Probability	X
NNN	$\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}=\frac{27}{64}$	0
NND	$\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}=\frac{9}{64}$	1
NDN	$\frac{3}{4}\times\frac{1}{4}\times\frac{3}{4}=\frac{9}{64}$	1
NDD	$\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4}=\frac{3}{64}$	2
DNN	$\frac{1}{4}\times\frac{3}{4}\times\frac{3}{4}=\frac{9}{64}$	1
DND	$\frac{1}{4}\times\frac{3}{4}\times\frac{1}{4}=\frac{3}{64}$	2
DDN	$\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4}=\frac{3}{64}$	2
DDD	$\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}=\frac{1}{64}$	3

The probability distribution of X is

x	f(x)=P(X=x)
0	$\frac{27}{64}$
1	$\frac{9}{64}+\frac{9}{64}+\frac{9}{64}=\frac{27}{64}$
2	$\frac{3}{64}+\frac{3}{64}+\frac{3}{64}=\frac{9}{64}$
3	$\frac{1}{64}$

(2) Second Solution:

Bernoulli trial is the process of inspecting the item. The results are success=D or failure=N, with probability of success $P(s)=25/100=1/4=0.25$.

The experiments is a Bernoulli process with:

- number of trials: $n=3$
- Probability of success: $p=1/4=0.25$
- $X \sim \text{Binomial}(n,p)=\text{Binomial}(3,1/4)$

The probability distribution of *X* is given by:

$$f(x) = P(X = x) = b(x;3,\frac{1}{4}) = \begin{cases} \binom{3}{x} (\frac{1}{4})^{x} \ (\frac{3}{4})^{3-x} ; & x = 0, 1, 2, 3 \\ 0 \quad ; & otherwise \end{cases}$$

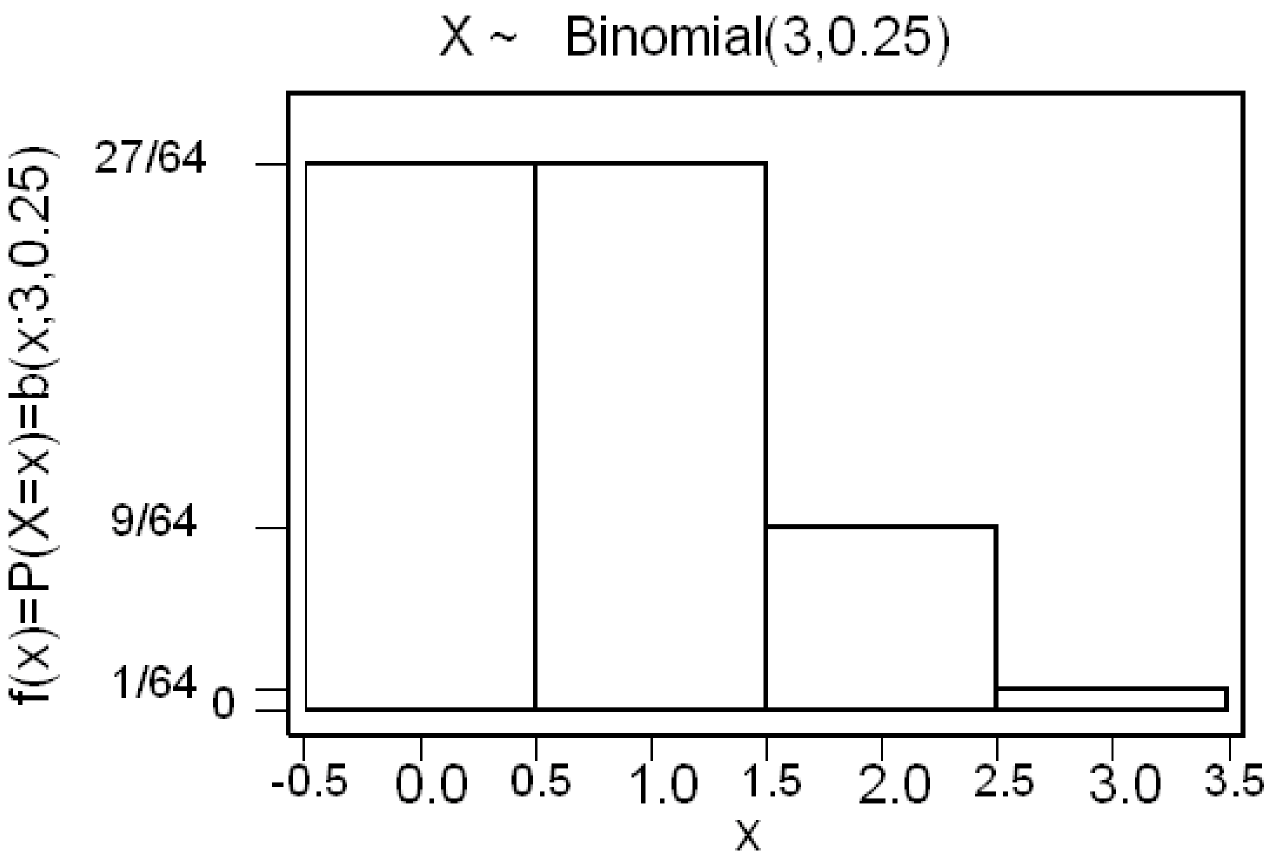
$$f(0) = P(X = 0) = b(0;3,\frac{1}{4}) = \binom{3}{0}(\frac{1}{4})^{0} (\frac{3}{4})^{3} = \frac{27}{64}$$

$$f(2) = P(X = 2) = b(2;3,\frac{1}{4}) = \binom{3}{2}(\frac{1}{4})^{2} (\frac{3}{4})^{1} = \frac{9}{64}$$

$$f(3) = P(X = 3) = b(3;3,\frac{1}{4}) = \binom{3}{3}(\frac{1}{4})^{3} (\frac{3}{4})^{0} = \frac{1}{64}$$

The probability distribution of *X* is

x	f(x)=P(X=x) =b(x;3,1/4)
0	27/64
1	27/64
2	9/64
3	1/64


X ~ Binomial(3,0.25)
f(x)=P(X=x)=b(x;3,0.25)
27/64
9/64
1/64
0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
X

Theorem 5.2:

The mean and the variance of the binomial distribution b(x;n,p) are:

$$\mu = n\,p$$
$$\sigma^2 = n\,p\,(1 - p)$$

Example:

In the previous example, find the expected value (mean) and the variance of the number of defective items.

Example:

In the previous example, find the expected value (mean) and the variance of the number of defective items.

Solution:

- X = number of defective items
- We need to find $E(X)=\mu$ and $Var(X)=\sigma^2$
- We found that $X \sim$ Binomial(n,p)=Binomial(3,1/4)
- .n=3 and p=1/4

The expected number of defective items is

$$E(X)=\mu = n\,p = (3)\,(1/4) = 3/4 = 0.75$$

The variance of the number of defective items is

$$Var(X)=\sigma^2 = n\,p\,(1-p) = (3)\,(1/4)\,(3/4) = 9/16 = 0.5625$$

Example:

In the previous example, find the following probabilities:

(1) The probability of getting at least two defective items.

(2) The probability of getting at most two defective items.

Solution:

X ~ Binomial(3,1/4)

$$f(x) = P(X = x) = b(x;3,\frac{1}{4}) = \begin{cases} \binom{3}{x} (\frac{1}{4})^{x} (\frac{3}{4})^{3-x} & \textit{for} \;\; x = 0,\, 1,\, 2, 3 \\ 0 & \textit{otherwise} \end{cases}$$

x	.f(x)=P(X=x)=b(x;3,1/4)
0	27/64
1	27/64
2	9/64
3	1/64

(1) The probability of getting at least two defective items:

$$P(X \geq 2) = P(X=2) + P(X=3) = f(2) + f(3) = \frac{9}{64} + \frac{1}{64} = \frac{10}{64}$$

(2) The probability of getting at most two defective item:

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

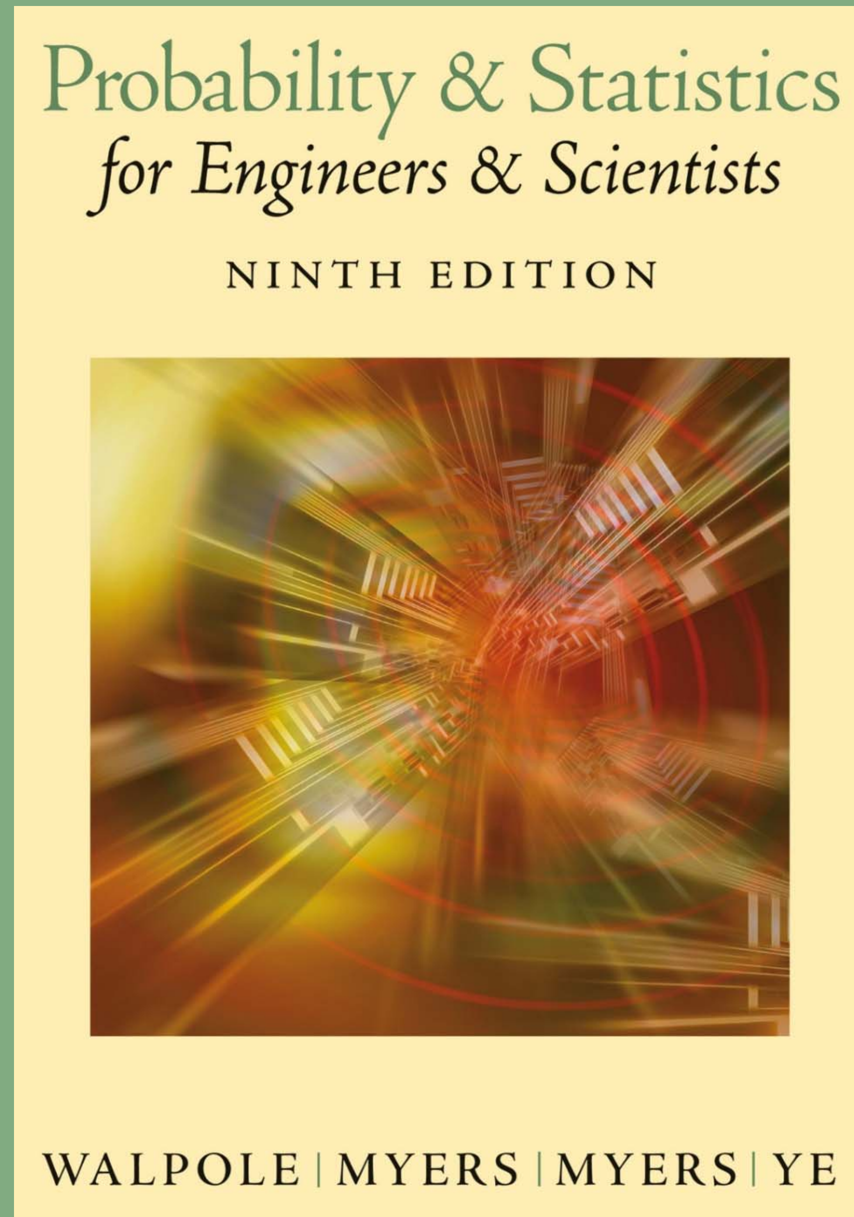
$$= f(0) + f(1) + f(2) = \frac{27}{64} + \frac{27}{64} + \frac{9}{64} = \frac{63}{64}$$

or

$$P(X \leq 2) = 1 - P(X > 2) = 1 - P(X=3) = 1 - f(3) = 1 - \frac{1}{64} = \frac{63}{64}$$

Section 5.3

Hypergeometric Distribution



Hyper-geometric distribution
Hyper-geometric experiment

Hyper-geometric experiment:

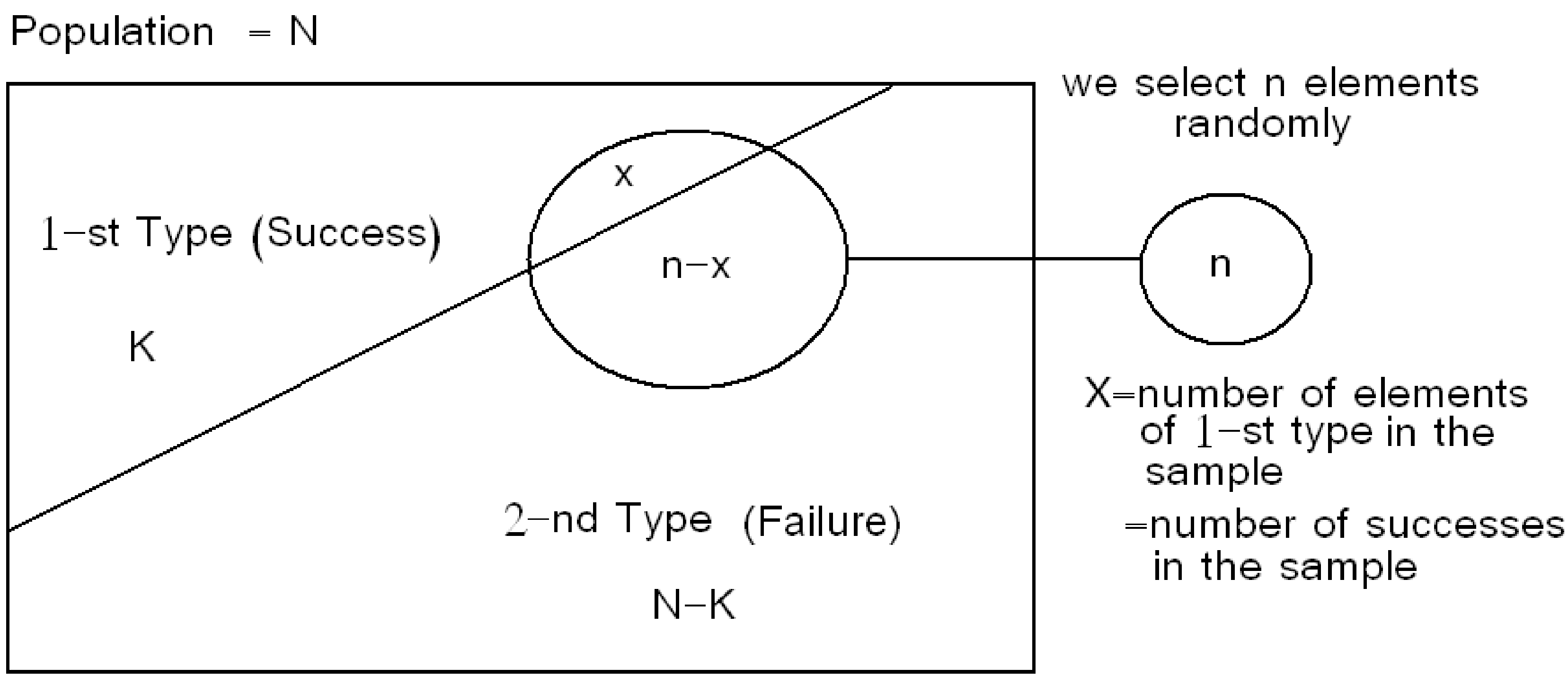
1. n elements chosen from N elements without replacement.

2. k of these N elements are ”successes” and N-k are ”failures”

Notice!! Unlike the binomial distribution the selection is done without replacement and the experiments and not independent.

Often used in quality control.

5.4 Hypergeometric Distribution :



- Suppose there is a population with 2 types of elements:

  1-st Type = success

  2-nd Type = failure
- *N*= population size
- *K*= number of elements of the 1-st type
- *N* –*K* = number of elements of the 2-nd type

- We select a sample of n elements at random from the population
- Let X = number of elements of 1-st type (number of successes) in the sample
- We need to find the probability distribution of X.

There are to two methods of selection:

1. selection with replacement

(1) If we select the elements of the sample at random and with replacement, then

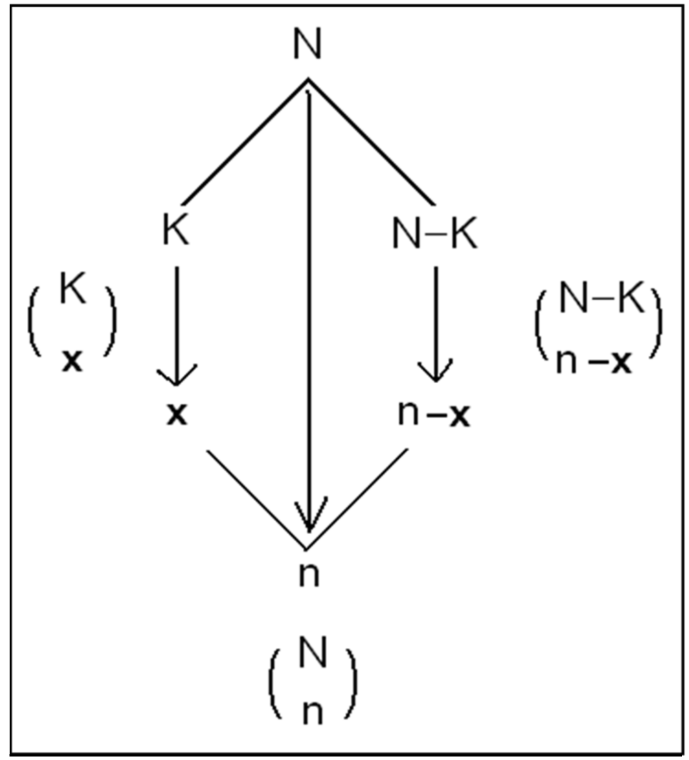
X ~ Binomial(n,p); where $$p=\frac{K}{N}$$

2. selection without replacement

(2) Now, suppose we select the elements of the sample at random and without replacement. When the selection is made without replacement, the random variable X has a hyper geometric distribution with parameters N, n, and K. and we write X~h(x;N,n,K).

$$f(x) = P(X = x) = h(x; N, n, K)$$

$$= \begin{cases} \dfrac{\dbinom{K}{x} \times \dbinom{N-K}{n-x}}{\dbinom{N}{n}}; & x = 0, 1, 2, \cdots, n \\ 0; \; otherwise \end{cases}$$



Note that the values of X must satisfy:

$0 \le x \le K$ and $0 \le n - x \le N - K$

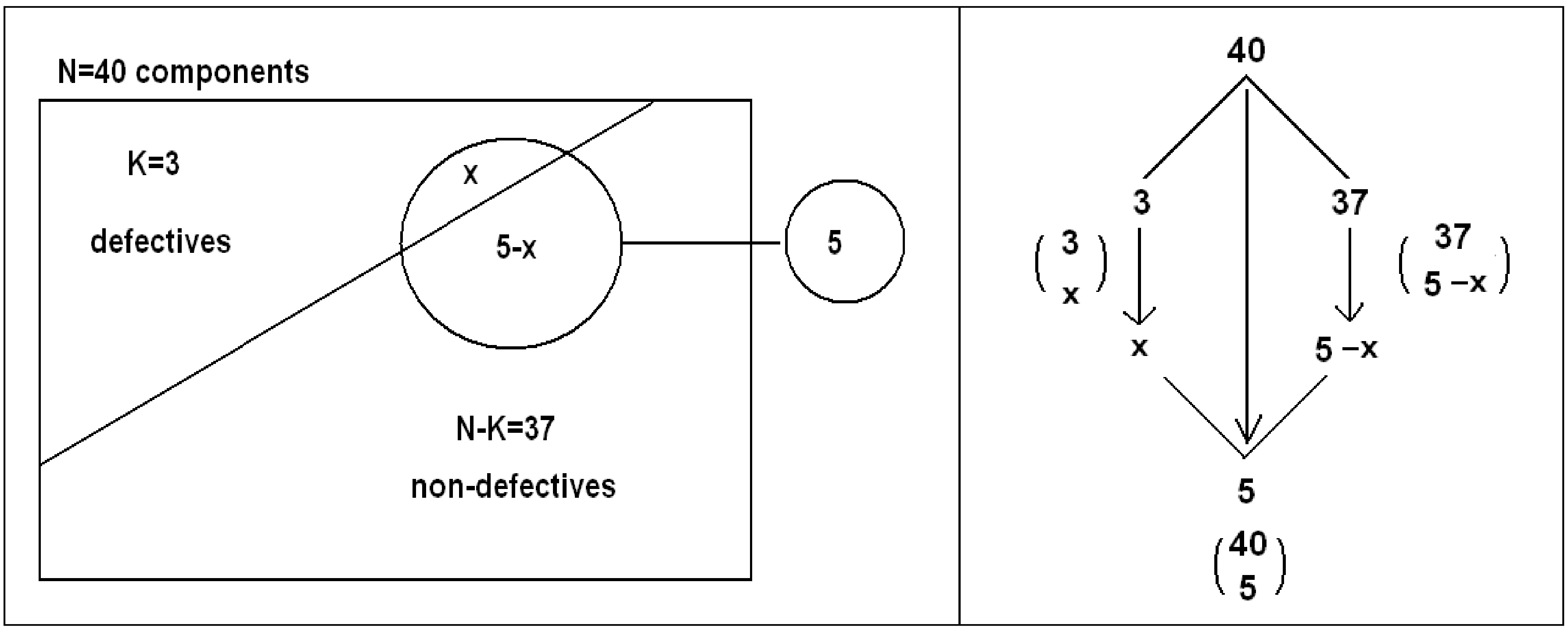
$\Leftrightarrow$

$0 \le x \le K$ and $n - N + K \le x \le n$

Example 5.9:

Lots of 40 components each are called acceptable if they contain no more than 3 defectives. The procedure for sampling the lot is to select 5 components at random (without replacement) and to reject the lot if a defective is found. What is the probability that exactly one defective is found in the sample if there are 3 defectives in the entire lot.

Solution:



N=40 components
K=3
defectives
x
5-x
5
N-K=37
non-defectives
40
3
37
$\binom{3}{x}$
$\binom{37}{5-x}$
x
5 −x
5
$\binom{40}{5}$

- Let X= number of defectives in the sample
- *N*=40, *K*=3, and *n*=5
- X has a hypergeometric distribution with parameters *N*=40, *n*=5, and *K*=3.
- X~h(x;*N*,*n*,*K*)=h(x;40,5,3).
- The probability distribution of *X* is given by:

$$f(x) = P(X = x) = h(x;40,5,3) = \begin{cases} \dfrac{\dbinom{3}{x} \times \dbinom{37}{5-x}}{\dbinom{40}{5}}; & x = 0, 1, 2, \cdots, 5 \\ 0; \ otherwise \end{cases}$$

But the values of X must satisfy:

$0 \leq x \leq K$ and $n - N + K \leq x \leq n \Leftrightarrow 0 \leq x \leq 3$ and $-42 \leq x \leq 5$

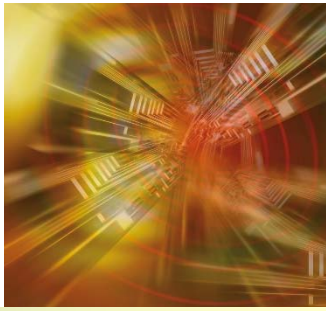
Therefore, the probability distribution of X is given by:

$$f(x) = P(X = x) = h(x;40,5,3) = \begin{cases} \dfrac{\dbinom{3}{x} \times \dbinom{37}{5-x}}{\dbinom{40}{5}}; & x = 0,\ 1,\ 2,3 \\ 0;\ otherwise & \end{cases}$$

Now, the probability that exactly one defective is found in the sample is

$$f(1)=P(X=1)=h(1;40,5,3)= \frac{\binom{3}{1}\times\binom{37}{5-1}}{\binom{40}{5}} = \frac{\binom{3}{1}\times\binom{37}{4}}{\binom{40}{5}} = 0.3011$$

Hyper-geometric distribution
Mean & variance



Theorem:
If X ~ hg(N,n,k), then

- **mean of X:** $$E(X) = \frac{n\,k}{N}$$
- **variance of X:** $$Var(X) = \frac{N-n}{N-1}\, n\, \frac{k}{N}\left(1 - \frac{k}{N}\right)$$

Example 5.10:
In Example 5.9, find the expected value (mean) and the variance of the number of defectives in the sample.

Solution:

- **X = number of defectives in the sample**
- **We need to find $E(X)=\mu$ and $Var(X)=\sigma^2$**
- **We found that $X \sim h(x;40,5,3)$**
- **N=40, n=5, and K=3**

The expected number of defective items is

$$E(X)=\mu = n\frac{K}{N} = 5\times\frac{3}{40} = 0.375$$

The variance of the number of defective items is

$$Var(X)=\sigma^2 = n\frac{K}{N}\left(1-\frac{K}{N}\right)\frac{N-n}{N-1} = 5\times\frac{3}{40}\left(1-\frac{3}{40}\right)\frac{40-5}{40-1} = 0.311298$$

Relationship to the binomial distribution:

*** Binomial distribution:**

$$b(x;n,p) = \binom{n}{x} p^{x}(1-p)^{n-x};\ x = 0, 1, \ldots, n$$

*** Hypergeometric distribution:**

$$h(x;N,n,K) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}; x = 0, 1, \cdots, n$$

If n is small compared to N and K, then the hypergeometric distribution h(x;N,n,K) can be approximated by the binomial distribution b(x;n,p), where $p = \frac{K}{N}$; i.e., for large N and K and small n, we have:

$$h(x;N,n,K) \approx b\left(x;n,\frac{K}{N}\right)$$

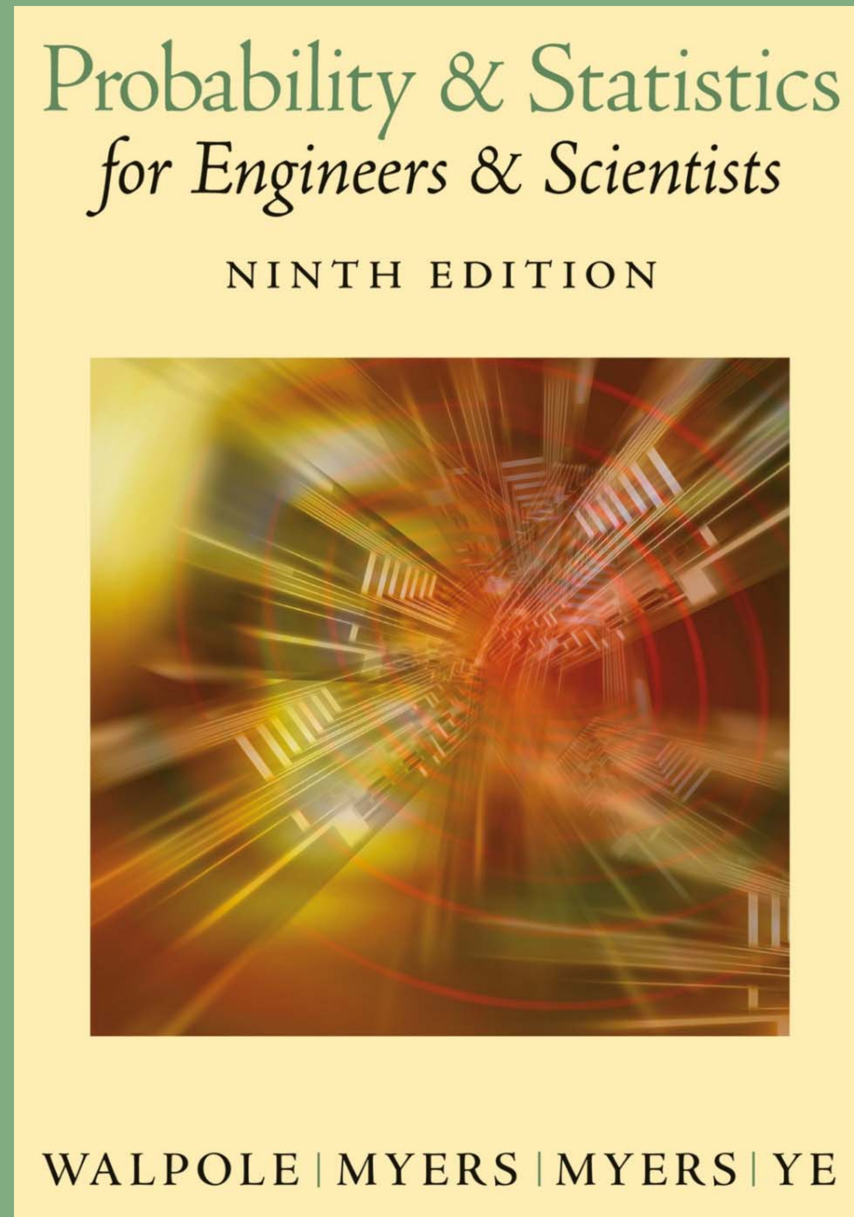
$$\frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} \approx \binom{n}{x}\left(\frac{K}{N}\right)^{x}\left(1-\frac{K}{N}\right)^{n-x} ;x = 0,1,\cdots,n$$

Note:

If *n* is small compared to *N* and *K*, then there will be almost no difference between selection without replacement and selection with replacement $\left(\frac{K}{N} \approx \frac{K-1}{N-1} \approx \cdots \approx \frac{K-n+1}{N-n+1}\right)$.

Section 5.5

Poisson Distribution and the Poisson Process



Poisson experiment is an experiment yielding numerical values of a random variable that count the number of outcomes occurring in a given time interval or a specified region denoted by *t*.

X = The number of outcomes occurring in a given time interval or a specified region denoted by *t*.

Example:

1. X = number of field mice per acre (*t*= 1 acre)
2. X= number of typing errors per page (*t*=1 page)
3. X=number of telephone calls received every day (*t*=1 day)
4. X=number of telephone calls received every 5 days (*t*=5 days)

Let λ be the average (mean) number of outcomes per unit time or unit region (*t*=1).

Poisson distribution
Definition

Definition:
Let the random variable X be the number of events in a time interval of length t from a Poisson process, which has on average λ events pr. unit time.

The distribution of X is called the Poisson distribution with parameter $\mu = \lambda t$.

Notation: $X \sim \text{Pois}(\mu)$, where $\mu = \lambda t$

Poisson distribution

The average (mean) number of outcomes (mean of X) in the time interval or region *t* is:

$$\mu = \lambda t$$

- The random variable X is called a Poisson random variable with parameter μ ($\mu=\lambda t$), and we write X~Poisson(μ), if its probability distribution is given by:

$$f(x) = P(X = x) = p(x;\mu) = \begin{cases} \dfrac{e^{-\mu}\mu^{x}}{x!} & ; \quad x = 0, 1, 2, 3, \ldots \\ 0 & ; \quad otherwise \end{cases}$$

Theorem 5.5:

The mean and the variance of the Poisson distribution Poisson($x;\mu$) are:

$$\mu = \lambda t$$

$$\sigma^2 = \mu = \lambda t$$

Note:

- **λ is the average (mean) of the distribution in the unit time (t=1).**
- **If X=The number of calls received in a month (unit time t=1 month) and X~Poisson(λ), then:**

(i) Y = number of calls received in a year.

Y ~ Poisson (μ); μ=12λ (t=12)

(ii) W = number of calls received in a day.

W ~ Poisson (μ); μ=λ/30 (t=1/30)

Example 5.16: Reading Assignment

Example 5.17: Reading Assignment

Theorem 5.5:

The mean and the variance of the Poisson distribution Poisson(x,μ) are:

$$\mu = \lambda t$$
$$\sigma^2 = \mu = \lambda t$$

Note:

λ is the average (mean) of the distribution in the unit time (t=1).

If X=The number of calls received in a month (unit time t=1 month) and X~Poisson(λ), then:

(i) Y = number of calls received in a year.

$$Y \sim \text{Poisson}(\mu); \quad \mu = 12\lambda \quad (t=12)$$

(ii) W = number of calls received in a day.

$$W \sim \text{Poisson}(\mu); \quad \mu = \lambda/30 \quad (t=1/30)$$

Example 5.16: Reading Assignment

Example 5.17: Reading Assignment

Example:

Suppose that the number of typing errors per page has a Poisson distribution with average 6 typing errors.

(1) What is the probability that in a given page:

(i) The number of typing errors will be 7?

(ii) The number of typing errors will at least 2?

(2) What is the probability that in 2 pages there will be 10 typing errors?

(3) What is the probability that in a half page there will be no typing errors?

Solution:

(1) X = number of typing errors per page.

X ~ Poisson (6) **(t=1, λ=6, $\mu=\lambda t$=6)**

$$f(x) = P(X = x) = p(x;6) = \frac{e^{-6}6^x}{x!}\ ; \quad x = 0, 1, 2, \ldots$$

(i) $$f(7) = P(X = 7) = p(7;6) = \frac{e^{-6}6^7}{7!} = 0.13768$$

(ii) **P(X≥2) = P(X=2)+ P(X=3)+ . . . =** $\sum_{x=2}^{\infty} P(X = x)$

P(X≥2) = 1– P(X<2) = 1 – [P(X=0)+ P(X=1)]

=1 – [f(0) + f(1)] = 1 – [$\frac{e^{-6}6^0}{0!} + \frac{e^{-6}6^1}{1!}$ **]**

= 1 – [0.00248+0.01487]

= 1 – 0.01735 = 0.982650

(2) X = number of typing errors in 2 pages

X ~ Poisson(12) **(t=2, λ=6, $\mu=\lambda t$=12)**

$$f(x) = P(X = x) = p(x;12) = \frac{e^{-12}12^{x}}{x!} : \quad x = 0\ ,1\ ,2\ldots$$

$$f(10) = P(X = 10) = \frac{e^{-12}12^{10}}{10} = 0.1048$$

(3) X = number of typing errors in a half page.

X ~ Poisson (3)   (*t*=1/2, λ=6, μ=λ*t*=6/2=3)

$$f(x) = P(X = x) = p(x;3) = \frac{e^{-3}\ 3^{x}}{x!} : \qquad x = 0\ ,1\ ,2\ldots$$

$$\mathrm{P(X = 0)} = \frac{\mathrm{e}^{-3}(3)^{0}}{0!} = 0.0497871$$

Theorem 5.6: (Poisson approximation for binomial distribution:

Let X be a binomial random variable with probability distribution b(x;n,p). If $n \to \infty$, $p \to 0$, and $\mu = np$ remains constant, then the binomial distribution b(x;n,p) can approximated by Poisson distribution $p(x;\mu)$.

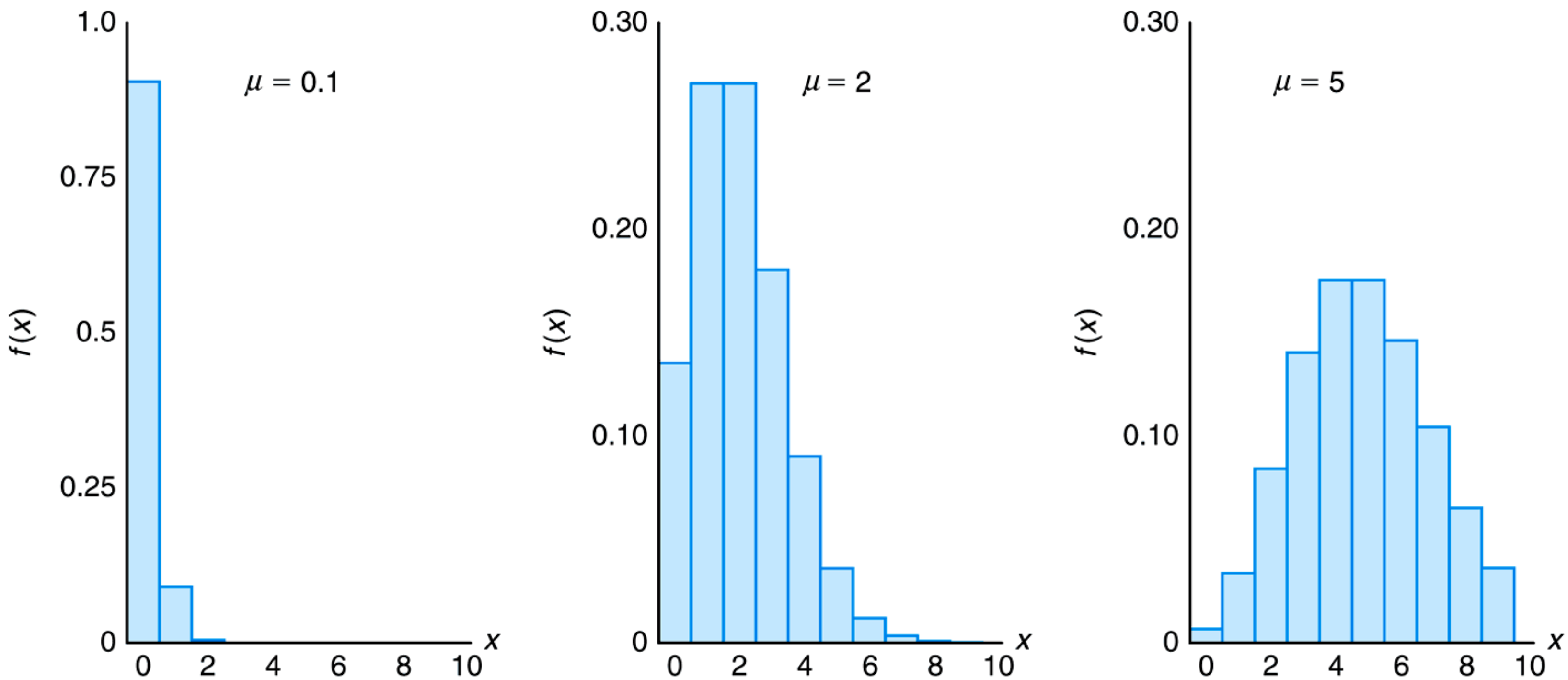
- For large n and small p we have:

$$b(x;n,p) \approx \text{Poisson}(\mu) \quad (\mu = np)$$

$$\binom{n}{x} p^x (1-p)^{n-x} \approx \frac{e^{-\mu}\mu^x}{x!};\, x = 0,1,\cdots,n; \quad (\mu = np)$$

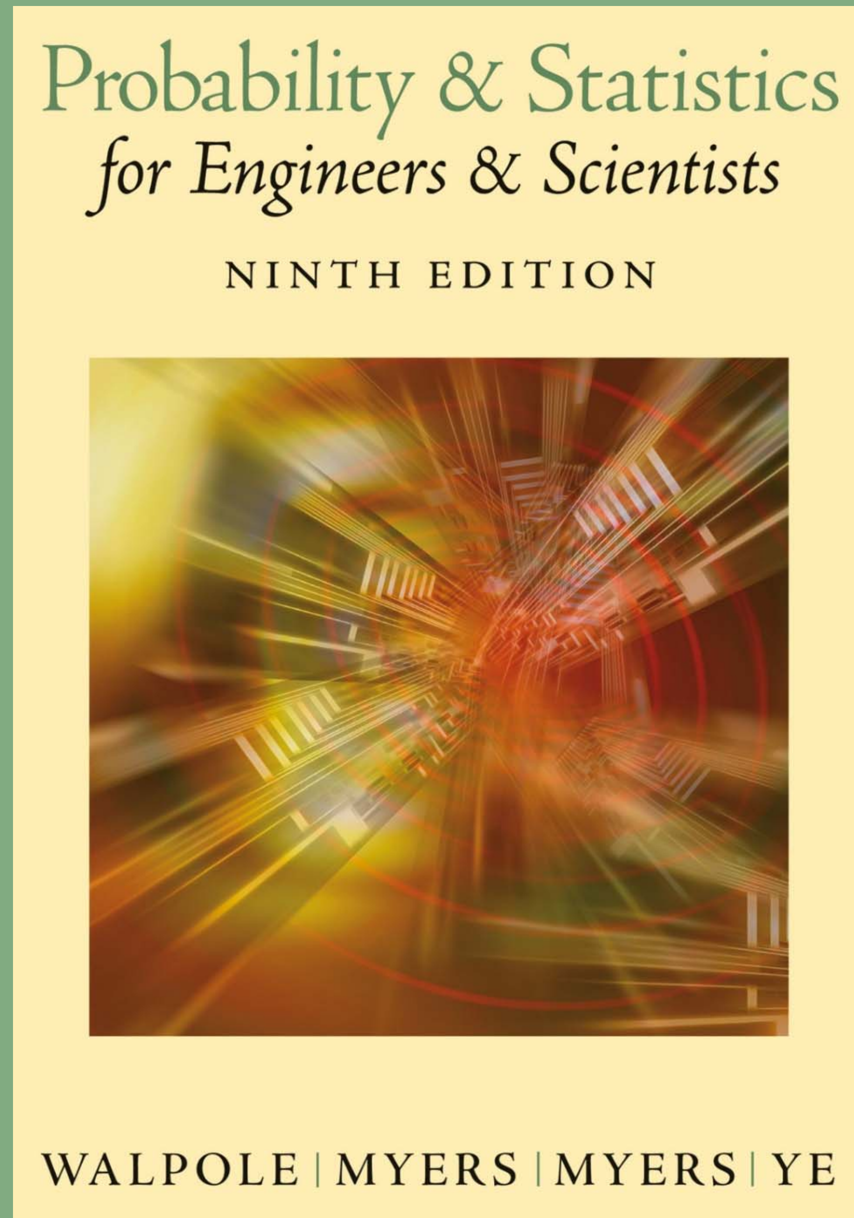
Example 5.18: **Reading Assignment**

Figure 5.1 Poisson density functions for different means



Section 5.5

Geometric distribution



Geometric distribution

- The Geometric Distribution
  - Has parameter p.
  - Models the number of independent trials to obtain a "success".
  - p is the probability of success on any given trial.

- The PMF:

$$f(x) = (1-p)^{x-1}p \qquad 0 < p < 1;\ x=1,2,\ldots$$

$$E(X) = \frac{1}{p} \qquad Var(X) = \frac{1-p}{p^2}$$

P(X > n)

The probability that it takes *more* than n trials to see the first success is

$$P(X > n) = (1-p)^n$$

Examples of Geometric

- First car arriving at a service station that needs brake work
- Flipping a coin until the first tail is observed
- First plane arriving at an airport that needs repair
- Number of house showings before a sale is concluded
- Length of time(in days) between sales of a large computer system

Example

The drilling records for an oil company suggest that the probability the company will hit oil in productive quantities at a certain offshore location is 0.2 . Suppose the company plans to drill a series of wells.

$P(X) = 0.2$ $P(x) = p(1 - p)^{x-1}$

c) Is it likely that x could be as large as 15?

$P(x=15) = p(1 - p)^{x-1} = (0.2)(0.8)^{15-1} = (0.2)(0.8)^{14} = 0.008796$

$P(x \geq 15) = 1 - P(x \leq 14) = 1 - 0.95602 = 0.04398$

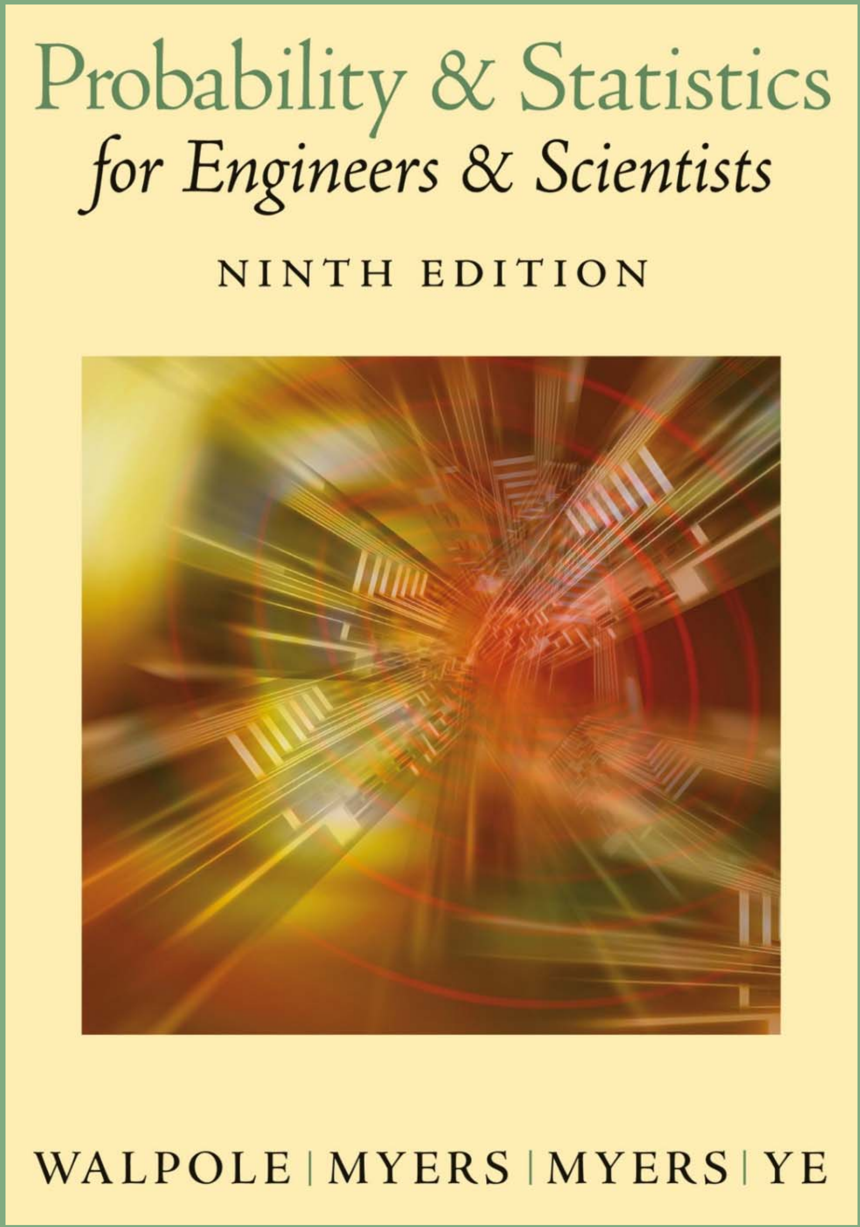
d) Find the mean and standard deviation of the number of wells that must be drilled before the company hits its first productive well.

Mean: $\mu_x = 1/p = 1/0.2 = 5$ (drills before a success)

Standard Deviation : $\sigma_x = (\sqrt{1-p})/p) = \sqrt{(.8)}/(.2) = \sqrt{4} = 2$

Section 5.6

Multinomial Distribution



Multinomial Distribution

- Extension of Binomial Distribution to experiments where each trial can end in exactly one of k categories
- n independent trials
- Probability a trial results in category i is p_{i}
- Y_{i} is the number of trials resulting in category I
- $p_1 + \ldots + p_{\mathrm{k}} = 1$
- $Y_1 + \ldots + Y_{\mathrm{k}} = n$

Multinomial Distribution

$$p(y_1,\ldots,y_k) = P(Y_1 = y_1,\ldots,Y_k = y_k) =$$

$$= \frac{n!}{y_1!\ldots y_k!} p_1^{y_1}\ldots p_k^{y_k}$$

$$Where \sum_{i=1}^{k} y_i = n, \sum_{i=1}^{k} p_i = 1, y_i \geq 0, p_i \geq 0$$

$$\Rightarrow E(Y_i) = np_i \qquad V(Y_i) = np_i(1-p_i)$$

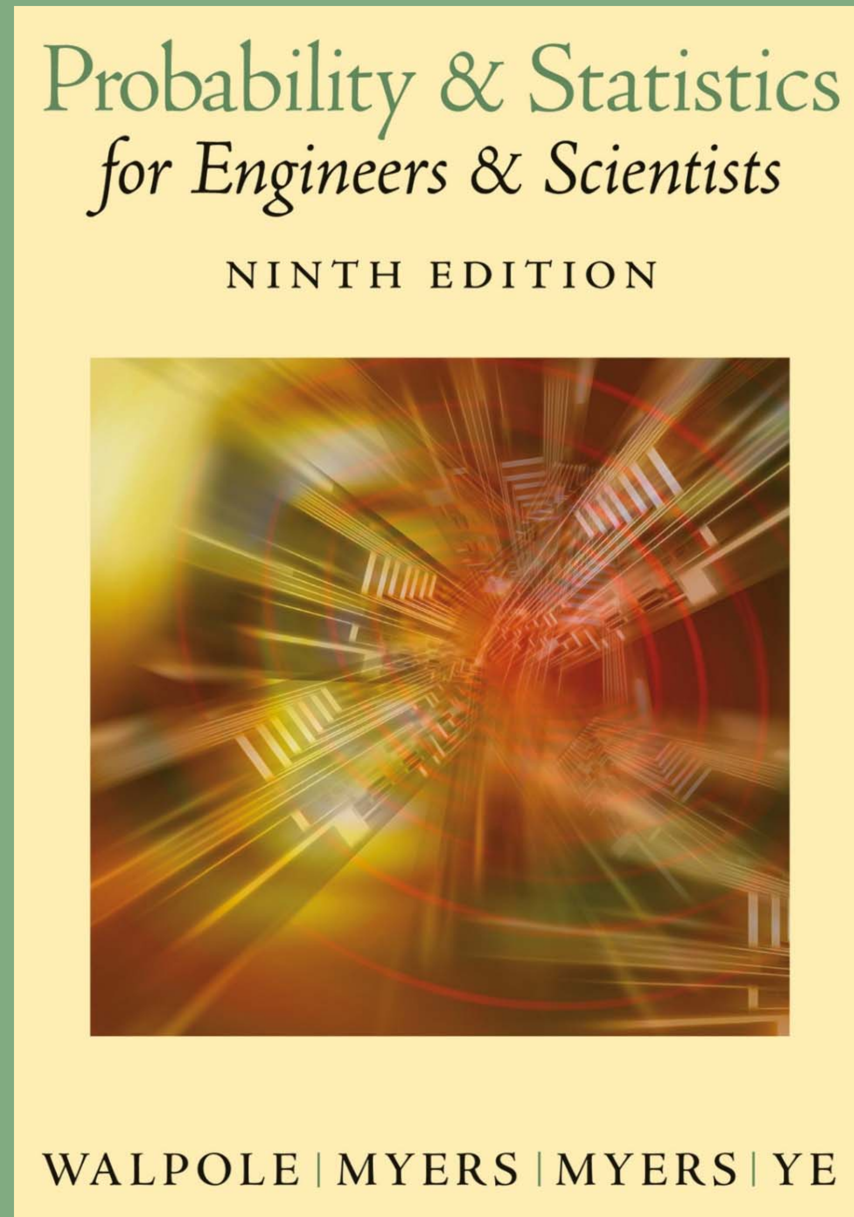
Example

Specific Example: if you are randomly choosing 8 people from an audience that contains 50% democrats, 30% republicans, and 20% green party, what's the probability of choosing exactly 4 democrats, 3 republicans, and 1 green party member?

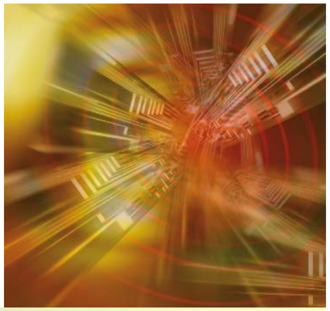
$$\boxed{P(D=4, R=3, G=1) = \frac{8!}{4!\,3!\,1!}(.5)^4(.3)^3(.2)^1}$$

Section 5.7

Review



Theorem 5.2



The mean and variance of the hypergeometric distribution $h(x; N, n, k)$ are

$$\mu = \frac{nk}{N} \text{ and } \sigma^2 = \frac{N-n}{N-1} \cdot n \cdot \frac{k}{N}\left(1 - \frac{k}{N}\right).$$

Theorem 5.5

Let X be a binomial random variable with probability distribution $b(x; n, p)$. When $n \to \infty$, $p \to 0$, and $np \overset{n \to \infty}{\longrightarrow} \mu$ remains constant,

$$b(x; n, p) \overset{n \to \infty}{\longrightarrow} p(x; \mu).$$