STAT 4705:

Q 1: Continuous uniform distribution.

$$f(x;A,B) = \begin{cases} \dfrac{1}{B-A}, & A \leq x \leq B \\ 0, & \text{elsewhere.} \end{cases}$$

$$* E(X) = \int_A^B x \cdot f(x)\,dx = \int_A^B x \cdot \frac{1}{B-A}\,dx = \frac{1}{B-A} \cdot \frac{x^2}{2} \Big|_A^B$$

$$= \frac{1}{B-A}\left(\frac{B^2-A^2}{2}\right) = \frac{B+A}{2}.$$

$$* Var(X) = E[X^2] - \mu^2$$

$$E[X^2] = \int_A^B x^2 \cdot f(x)\,dx = \int_A^B x^2 \cdot \frac{1}{B-A}\,dx = \frac{1}{B-A} \cdot \frac{x^3}{3} \Big|_A^B$$

$$= \frac{1}{B-A} \cdot \frac{1}{3} \cdot (B^3 - A^3) = \frac{1}{3} \cdot \frac{1}{B-A} \cdot (B-A)(B^2+AB+A^2)$$

$$= \frac{1}{3} \cdot (B^2 + AB + A^2).$$

$$Var(X) = \frac{1}{3}(B^2+AB+A^2) - \frac{1}{4}(B+A)^2$$

$$= \frac{1}{3}(B^2+AB+A^2) - \frac{1}{4}(B^2 + 2AB + A^2).$$

$$= \frac{1}{12}(4B^2 + 4AB + 4A^2 - 3B^2 - 6AB - 3A^2)$$

$$= \frac{1}{12}(B^2 - 2AB + A^2) = \frac{1}{12}(B-A)^2$$

Question 3:

$Var(X) = \sigma^2$

$Z = \dfrac{X - \mu}{\sigma}$

$X \sim N(\mu, \sigma^2)$ $\quad N(0,1)$

- $E(Z) = E\left(\dfrac{X-\mu}{\sigma}\right) = \dfrac{1}{\sigma} E(X) - \dfrac{\mu}{\sigma} = \dfrac{\mu}{\sigma} - \dfrac{\mu}{\sigma} = 0$

- $Var(Z) = Var\left(\dfrac{X-\mu}{\sigma}\right) = \left(\dfrac{1}{\sigma}\right)^2 \cdot Var(X) = \dfrac{1}{\sigma^2} \cdot Var(X) = 1.$

⊛ density function of transformed variable:

$f(z) = \dfrac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2} z^2}$