

Def

sample mean.

population: it consists of the totality of the observations

sample: a fraction of population.

sample sampling: each observation is made independently and randomly

statistic: a function of the random sample X_1, X_2, \dots, X_n .

Sample mean:

✓
$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$$

Sample Variance:

✓
$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$$
$$= \frac{\sum_{i=1}^n X_i^2 - n \cdot \bar{X}^2}{n-1}$$

sampling distribution:

the probability distribution of statistic.

△ \bar{X} :
CLT.

if X_1, X_2, \dots, X_n are n i.i.d.

$$E(X_i) = \mu.$$

$$\text{Var}(X_i) = \sigma^2.$$

then.

$$\sum_{i=1}^n X_i \sim N(\text{mean}, \text{var.})$$

$$\text{mean} = \sum \mu = n\mu.$$

$$\text{Var} = n\sigma^2$$

Sample mean.

$$\bar{X} = \frac{1}{n} \cdot \sum X_i$$

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \quad \text{①}$$

$$E(\bar{X}) = \frac{1}{n} \cdot E(\sum X_i) = \frac{1}{n} \cdot n\mu = \mu.$$

$$\text{Var}(\bar{X}) = \frac{1}{n^2} \text{Var}(\sum X_i) = \frac{1}{n^2} \cdot n \cdot \sigma^2 = \frac{\sigma^2}{n}.$$

3 areas in Statistical Inference.

→ ① parameter estimation. ☐

② hypothesis testing ☐

③ prediction on the future data. ☐

Q. How do we find the estimators?

Maximum Likelihood Estimation.
(MLE)

How does estimation ~~sample~~ ^{population} mean?

~~sample~~ population variance?

①

Sample mean \longrightarrow population mean.

Sample variance \longrightarrow population variance.

CI.

Q. How well is your estimator?