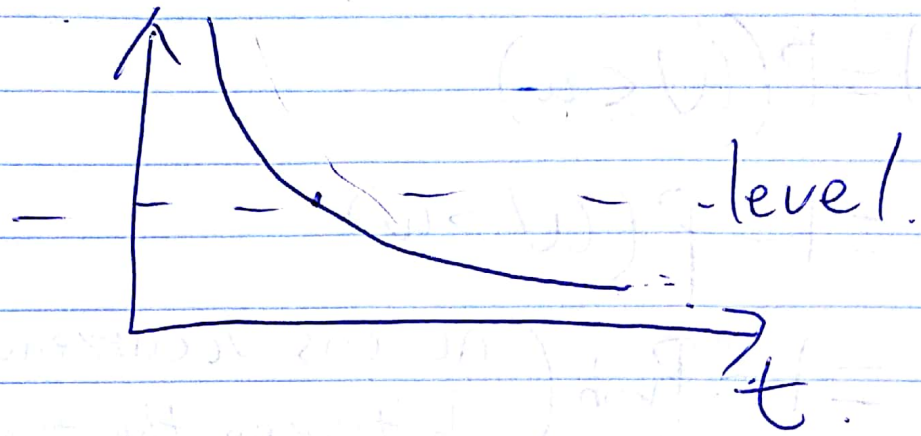
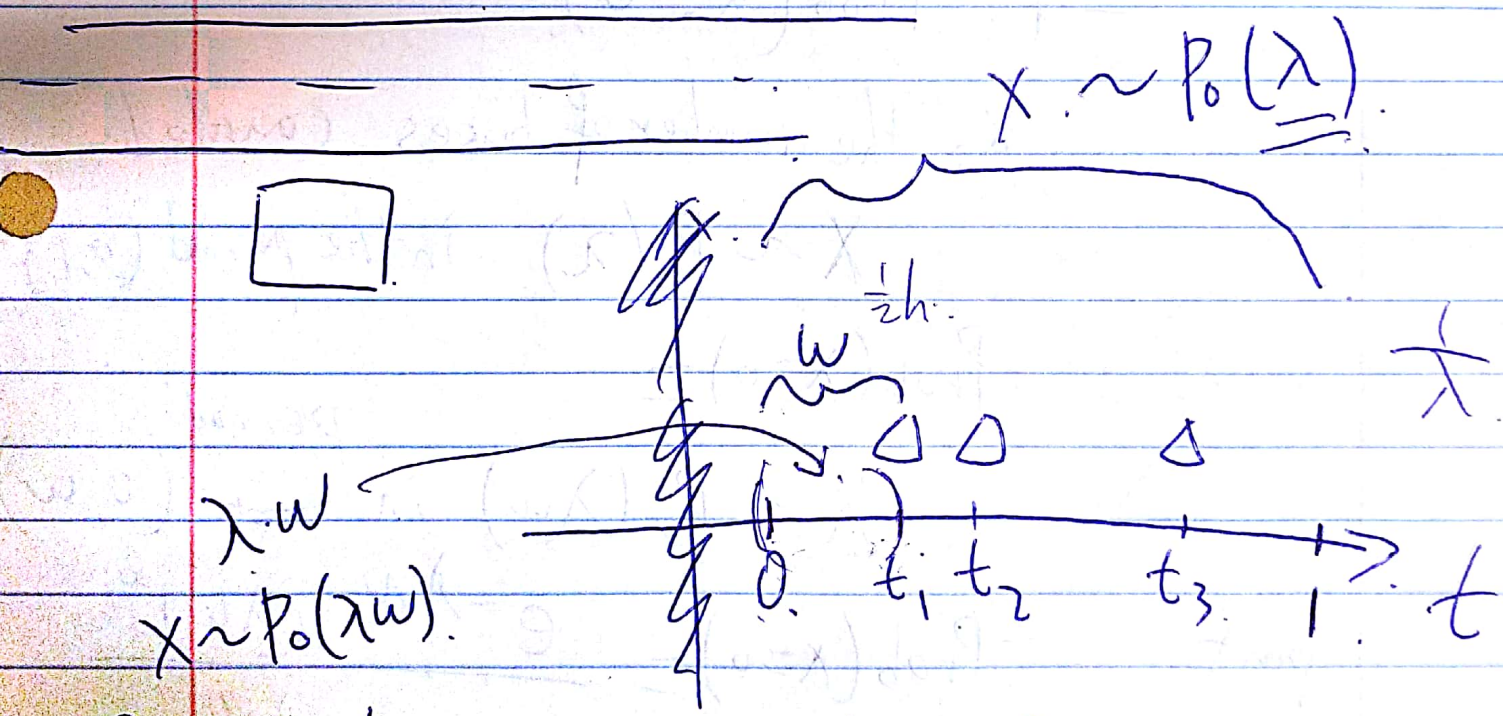
11/8/17

1st.



2nd.



Q What is the distribution of w?

w is the waiting time for the first occurrence of bus

$F(w) = P(W \le w)$

$= 1 - P(W > w)$ *

$= 1 - \text{Prob}$ (no bus occurrence between the time period $(0, w)$)

$= 1 - \text{Prob}(X = 0)$

X: the number of buses counted in the period $(0, 1)$

$X \sim Po(\lambda)$

$\text{Prob}(X=0) =$

$X \sim Po(\lambda w)$ in the period $(0, w)$

pmf $\text{Prob}(X=0) = \dfrac{e^{-\lambda w} \cdot (\lambda w)^0}{0!}$

$= e^{-\lambda w}$ $w > 0$

$F(w) = 1 - e^{-\lambda w}$, $w > 0$.

pdf $f(w) = F'(w) = \lambda \cdot e^{-\lambda w}$, $w > 0$. **

for the exponential dist.

$$f(x) = \lambda \cdot e^{-\lambda x}, \quad x > 0$$

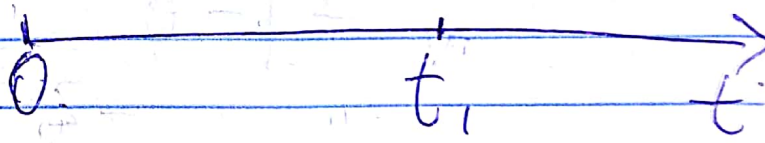
→ λ is called the rate parameter.

$$f(x) = \frac{1}{\beta} \cdot e^{-\frac{1}{\beta}x} \quad x > 0$$

$\beta = \frac{1}{\lambda}$, β is called scale parameter...

$$E(X) = \frac{1}{\lambda}$$

$$Var(X) = \left(\frac{1}{\lambda}\right)^2$$



Ex 1. webpage, starting from time 0, on average there are 2 hits/minute.

Q1. What is the value of rate parameter λ?

$$\lambda = 2$$

Q2. the waiting time for the first hit follows an exponential dist, $Y \sim exp(\lambda=2)$. $E(Y) = ?$

Q3. What is the prob that we have to wait 40 secs to observe the first hit?

$Y \sim \exp(\lambda = 2)$

λ is 2 hit/min

$40 \text{ secs} = \frac{40}{60} \text{ min} = \frac{2}{3} \text{ min}$

$\text{prob} = P(Y \leq \frac{2}{3})$

$= 1 - e^{-\lambda y}$

$= 1 - e^{-2 \cdot \frac{2}{3}}$

$= 1 - e^{-\frac{4}{3}}$

$= \ldots$

$f(y) = \lambda \cdot e^{-\lambda y}, \; y \geq 0$

$F(y) = 1 - e^{-\lambda y}, \; y \geq 0$

Q4. How long do we have to wait, to observe the first hit with a prob 0.9?

$\text{prob} = P(Y \leq y) = 0.9$

y

t

$$P(Y \le y) = 1 - e^{-\lambda y}$$

$$= 1 - e^{-2y}, \quad \lambda = 2.$$

$$= 0.9$$

Q5. Are they going to have same results for Q1, Q2, Q3, Q4 ?

Conclusion:

for the exponential dist, the waiting time doesn't relate to the starting time point. the memoryless property.

Q6. What is the waiting time for the second hit after you see the first hit ?