① CLT.

② $Z=\frac{X-\mu}{\sigma}$ $\quad X\sim N(\mu,\sigma^2)$, $\quad Z\sim N(0,1)$.

Thm. CLT.

If $X_1, X_2, \dots, X_n$ are n i.i.d. ~~ds~~ random variables

* statistical inference.

with $E(X_i)=\mu$.

$Var(X_i)=\sigma^2$

$$-2<t=\frac{\bar{X}-\mu}{s/\sqrt{n}}<2 . \quad \bar{X}-\mu<2\cdot\frac{s}{\sqrt{n}}$$

? μ. $\quad \mu>\bar{X}-2\cdot\frac{s}{\sqrt{n}}$. $\quad$ ~~$\mu<$~~

then $\sum X_i \sim N(n\mu, n\sigma^2)$.

$$\bar{X}=\frac{\sum X_i}{n}$$

$$\to \bar{X}\sim N\left(\mu, \frac{\sigma^2}{n}\right).$$

$$Z=\frac{\bar{X}-\mu}{\sqrt{\sigma^2/n}}\sim N(0,1).$$

recall. $t=\frac{Z}{\sqrt{V/n}}$

$V\sim\chi^2(n)$.

Q what is V?

let $V=\frac{(n-1)s^2}{\sigma^2}$ $\quad$ s^2 is the sample variance to estimate σ^2.

fact: $V\sim\chi^2(n-1)$.

$$t=\frac{Z}{\sqrt{V/(n-1)}}=\frac{\frac{\bar{X}-\mu}{\sqrt{\sigma^2/n}}}{\sqrt{\frac{(n-1)s^2}{\sigma^2}/(n-1)}}=\frac{\bar{X}-\mu}{s/\sqrt{n}}$$

$$\sigma^2=E\left([X-E(X)]^2\right)$$

$$s^2\overset{def}{=}\frac{\sum(X_i-\bar{X})^2}{n-1}.$$