

Task-3

Primality Tests

To solve programming problems which includes the concept of prime numbers and its related properties.

- a. Sieve of Eratosthenes with example
- b. Fermat's Primality Testing with example
- c. Miller-Rabin Primality Testing with example.

a. Sieve of Eratosthenes with example

The sieve of Eratosthenes is an efficient algorithm to find all prime numbers up to given number n .

Steps:

1. Create a list of numbers from 2 to n .
2. Start with the first prime (2)
3. Eliminate all multiples of 2 (except 2 itself)
4. Move to the next unmarked number (3) → mark its multiples
5. Repeat until you reach \sqrt{n}
6. The remaining unmarked numbers are all primes

```
#include <stdio.h>
#include <stdbool.h>
void sieveOfEratosthenes(int n) {
    bool prime[n+1]; // Boolean array to track primes
    for (int i = 0; i <= n; i++)
        prime[i] = true; // Assume all numbers are prime initially
    prime[0] = prime[1] = false; // 0 and 1 are not prime
    for (int p = 2; p * p <= n; p++) {
        if (prime[p] == true) {
            // Mark all multiples of p as not prime
            for (int i = p * p; i <= n; i += p)
                prime[i] = false;
        }
    }
} // Print prime numbers
printf("Prime numbers up to %d are:\n", n);
for (int i = 2; i <= n; i++) {
    if (prime[i])
        printf("%d ", i);
}
printf("\n");
}

int main() {
    int n;
    printf("Enter the limit: ");
    scanf("%d", &n);
    sieveOfEratosthenes(n);

    return 0;
}
```

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Task - 7B

Fermat's Primality Testing with example

Fermat's Little theorem:

If p is prime and a is any integer such that $1 \leq a < p$,
then $a^{p-1} \equiv 1 \pmod{p}$ or $a^{p-1} \equiv 1 \pmod{p}$

Idea:

1. Pick a random number a in range $(2, p-2)$.
2. Compute $a^{p-1} \pmod{p}$ or $a^{p-1} \pmod{p}$.
3. If result $\neq 1 \rightarrow p$ is composite.
4. If result = 1 for several random values of $a \rightarrow p$ is probably prime.

```

#include <stdio.h>
#include <stdlib.h>
#include <time.h>

// Function for modular exponentiation (a^b % mod)
long long power(long long a, long long b, long long mod) {
    long long result = 1;
    a = a % mod;
    while (b > 0) {
        if (b & 1)
            result = (result * a) % mod;
        b = b >> 1; // Divide b by 2
        a = (a * a) % mod;
    }
    return result;
}

```

```

// Fermat Primality Test
int isPrimeFermat(int n, int k) {
    if (n <= 1 || n == 4) return 0;
    if (n <= 3) return 1;

    // Try k times
    for (int i = 0; i < k; i++) {
        int a = 2 + rand() % (n - 4); // random number in [2, n-2]
    }
}

```

```
    if (power(a, n - 1, n) != 1)
        return 0; // composite
    }
    return 1; // probably prime
}

int main() {
    srand(time(0)); // Seed for random numbers

    int n, k;
    printf("Enter number to test: ");
    scanf("%d", &n);
    printf("Enter number of iterations: ");
    scanf("%d", &k);

    if (isPrimeFermat(n, k))
        printf("%d is probably prime.\n", n);
    else
        printf("%d is composite.\n", n);

    return 0;
}
```

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Task-7c

Miller-Rabin Primality Testing with example

It is a probabilistic test but much stronger than Fermat's test.

Algorithm

1. Write $n-1 = 2^s \cdot d$, where d is odd
2. Pick a random number a in $(2, n-2)$.
3. Compute $x = a^d \mod n$
4. If $x = 1$ or $x = n-1$, then continue with next iteration.
5. Otherwise, square x repeatedly:
 - a. If you ever get $n-1$, then continue.
 - b. If you never get $n-1$, then n is composite
6. Repeat the test multiple times with different random a .
7. If it passes all, n is probably prime.

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

// Function for modular exponentiation (a^b % mod)
long long power(long long a, long long b, long long mod) {
    long long result = 1;
    a = a % mod;
    while (b > 0) {
        if (b & 1)
            result = (result * a) % mod;
        b = b >> 1;
        a = (a * a) % mod;
    }
    return result;
}
```

```
// Miller test for a single base 'a'
int millerTest(long long d, long long n) {
    long long a = 2 + rand() % (n - 4);
    long long x = power(a, d, n);

    if (x == 1 || x == n - 1)
        return 1;
```

```

    }
    return 1;
}

int main() {
    srand(time(0));
    long long n;
    int k;

    printf("Enter number to test: ");
    scanf("%lld", &n);
    printf("Enter number of iterations: ");
    scanf("%d", &k);

    if (isPrimeMillerRabin(n, k))
        printf("%lld is probably prime.\n", n);
    else
        printf("%lld is composite.\n", n);

    return 0;
}

```

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VIVA VOCE (3)	2
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TOTAL (15)	15
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Result :-
Thus the program was executed and verified successfully.