MODULE - 2

MODERN PHYSICS & QUANTUM MECHANICS

MODERN PHYSICS

Blackbody:

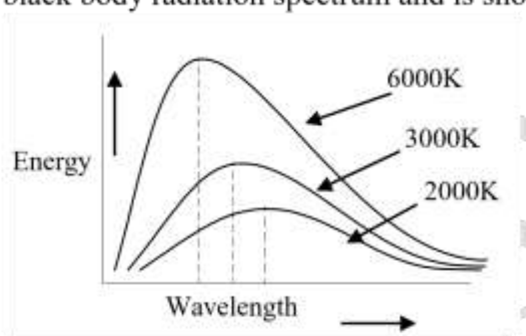
A Blackbody is one which absorbs the entire radiation incident on it and emits all the absorbed radiation when it is heated to a suitable high temperature. A true blackbody does not exist practically.

Ex: Lamp black body can be considered to be nearest natural black body
Ferry's Black body can be considered to be nearest man made black body

Blackbody Radiation spectrum

Since a perfect black body does not exist in nature, in 1895, Lumer and Preingshiem designed a special type of black body which has features very close to the black body. They studied the spectrum of black body.

A graph plotted energy radiated (Intensity) versus wavelength of emitted radiation is as called black body radiation spectrum and is shown below.



Salient features of the spectrum are

1. There are different curves for different temperatures of the black body.
2. At a given temperature the distribution of energy is not uniform over all the wave length.
3. At a given temperature intensity of radiation increases with increase in wavelength and reaches a maximum for a particular temperature, beyond which the intensity decreases.
4. With increase in temperature of the body the maximum intensity increases and the wavelength corresponding to maximum intensity λ_m shifts towards lower wavelength side.

5. The area under the curve gives the energy emitted per unit area of cross section of the black body.

The laws accounts for the black body radiation spectrum are as follows:

Wein's Displacement Law:

The law states that "the wavelength corresponds to maximum intensity is inversely proportional to the absolute temperature of the black body, because of which the peaks of the energy curves for different temperatures get displaced towards the lower wavelength side".

i.e. $\lambda_m \propto \left(\frac{1}{T}\right)$ or $\lambda_m T = \text{constant} = 2.898\times10^{-3}$ mK

Wein showed that the maximum energy of the peak emission is directly proportional to the fifth power of absolute temperature.

$E_m \; \alpha \; T^5$ or $E_m = \text{constant} \times T^5$

Wein, Rayleigh-Jeans and Planck have given their explanations to account these observed experimental facts as follows:

1. **Wein's law:**

In 1896, Wein obtained the relation between the wavelength of emission and the temperature of the source as

$$U_\lambda d\lambda = C_1 \lambda^{-5} e^{\left(-\frac{C_2}{\lambda T}\right)} d\lambda$$

Where $U_\lambda d\lambda$ is the energy / unit volume in the range of wavelength λ and $\lambda+d\lambda$, C_1 and C_2 are constants. This is called Wein's law of energy distribution in the black body radiation spectrum.

Drawbacks of Wein's law:

Wein's law holds good for the shorter wavelength region and high temperature of the source. It is failed to explain gradual drop in intensity of radiation corresponding to longer wavelength greater than the peak value.

2. Rayleigh-Jeans Law:

In 1900, Rayleigh derived an equation for the blackbody radiation on the basis of principle of equipartition of energy. The principle of equipartition of energy suggests that an average energy kT is assigned to each mode of vibration. The number of modes vibrations/unit volume whose wavelength is in the range of λ and $\lambda+d\lambda$ is given by $8\pi\lambda^{-4}d\lambda$. The energy/unit volume in the wavelength range λ and $\lambda + d\lambda$ is

$$U_\lambda d\lambda = 8\pi kT\lambda^{-4}d\lambda$$ Where

k is Boltzmann constant= 1.38×10^{-23}J/K.

This is Rayleigh-Jeans equation. Accordingly, energy radiated by the blackbody decreases with increasing wavelength.

Drawbacks of Rayleigh-Jeans Law: (or Ultra Violet Catastrophe)

Rayleigh-Jeans Law holds good only for longer wavelength region but failed to explain shorter wavelength region. According to Rayleigh-Jeans Law the energy increases with decrease in wavelength and is very large in the ultra-violet region. But such a large increase in energy emission does not occur experimentally. This discrepancy is known as ultra-violet catastrophe.

3.Stefan Boltzmann's law

Stefan's law states that the energy radiated by a perfect black body is directly proportional to fourth power of absolute temperature.

$$E=\sigma T^4$$

Where **σ = Stefan Boltzmann's constant** = 5.67 x 10-8 W m^{-2} K^{-4}

4. Planck's Law:

i) Planck assumed that walls of the experimental blackbody consists very large number of electrical oscillators. Each oscillator vibrates with its own frequency.

ii) Each oscillator has an energy given by integral multiple of hυ where h is Planck's constant & υ is the frequency of vibration.

$E = nh\upsilon$ where $n = 1, 2, 3 \ldots$ etc.

iii) An oscillator may lose or gain energy by emitting or absorbing respectively a radiation of frequency υ where $\upsilon = \Delta E/h$, ΔE is difference in energies of the oscillator before and after the emission or absorption take place.

Planck derived the law which holds good for the entire spectrum of the blackbody radiation as

$$U_\lambda\, d\lambda = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{e^{hc/\lambda kT}-1}\right) d\lambda \quad (\text{since } \upsilon = c/\lambda) \longrightarrow (1)$$

This is Planck's Radiation Law.

Reduction of Planck's law to Wein's law:

For shorter wavelengths, $\upsilon = c/\lambda$ is large.

When υ is large, $e^{h\upsilon/kT}$ is very large.

∴ $e^{h\upsilon/kT} >> 1$

∴ $(e^{h\upsilon/kT}-1) \approx e^{h\upsilon/kT} = e^{hc/\lambda kT}$ Substituting in eqn 1:

$$U_\lambda\, d\lambda = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{e^{hc/\lambda kT}}\right) d\lambda = C_1 \lambda^{-5} e^{\left(-\frac{C_2}{\lambda T}\right)} d\lambda$$

Where $C_1 = 8\pi hc$ and $C_2 = hc/k$ This is the Wein's law of radiation.

Reduction of Planck's law to Rayleigh Jeans law:

For longer wavelengths $\upsilon = c/\lambda$ is small.

When υ is small $h\upsilon/kT$ is very small.
Expanding $e^{h\upsilon/kT}$ as power series:

$$e^{h\upsilon/kT} = 1 + h\upsilon/kT + (h\upsilon/kT)^2 + \ldots$$
$$\approx 1 + h\upsilon/kT.$$

$\because$ If $h\upsilon/kT$ is small, its higher powers are neglected.

$$\therefore\ e^{h\upsilon/kT} - 1 = \frac{h\upsilon}{kT} = \frac{hc}{\lambda kT}$$

Substituting in eqn 1:

$$U_\lambda\, d\lambda = \left[\frac{8\pi hc}{\lambda^5\left(\frac{hc}{\lambda kT}\right)}\right] d\lambda$$

$$= \left[\frac{8\pi kT}{\lambda^4}\right] d\lambda$$

This is Rayleigh Jeans Law of Radiation.

Dual nature of matter (de-Broglie Hypothesis)

Dual nature of light:

The concept of photoelectric effect and Compton Effect gives the evidence for particle nature of light. Where as in physical optics the phenomenon like interference, diffraction, superposition was explained by considering wave nature of light. This is wave particle duality of light.

Dual nature of matter:

On the basis of above concept (dual nature of light), in 1923, Louis de Broglie gave a hypothesis

"Since nature loves symmetry, if the radiation behaves as particles under certain conditions and as waves under certain conditions, then one can expect that, the entities which ordinarily behaves as particles (ex. Like electrons, protons, neutrons) must also exhibit properties attributable to waves under appropriate circumstances" This is known as **deBroglie hypothesis**

Matter is made up of discrete constituent particles like atoms, molecules, protons, neutrons and electrons, hence matter has particle nature. Wave nature of matter is explained by Davisson and Germer experiment. Hence matter also exhibit wave particle duality.

The waves associated with the moving particles are called de Broglie waves or matter waves or pilot waves.

Characteristics of matter waves:

1. Waves associated with moving particles are called matter waves. The wavelength 'λ' of a deBroglie wave associated with particle of mass 'm' moving with velocity 'v' is $\lambda = h/(mv)$
2. Matter waves are not electromagnetic waves because the de Broglie wavelength is independent of charge of the moving particle.
3. The amplitude of the matter wave depends on the probability of finding the particle in that position.
4. The speed of matter waves depends on the mass and velocity of the particle associated with the wave.

Debroglie's Wavelength:

A particle of mass 'm' moving with velocity 'c' possess energy given by

$E = mc^2$ $\rightarrow$ **(**Einstein's Equation) (1)

According to Planck's quantum theory the energy of quantum of frequency 'υ' is

$E = h\upsilon \rightarrow (2)$

From (1) & (2)

$mc^2 = h\upsilon = hc/\lambda$ since $\upsilon = c/\lambda$

$\lambda = hc/mc^2 = h/mc$ $\lambda =$

h/mv since $v \approx c$ **de-**

Broglie wavelength of a

free particle in terms of

its kinetic energy

Consider a particle, since the particle is free, the total energy is same as

$$E = \frac{1}{2} mv^2 = \frac{p^2}{2m}$$

Where 'm' is the mass, 'v' is the velocity and 'p' is the momentum of the particle.

$$p = \sqrt{2mE}$$

The expression for de-Broglie wavelength is given by

h h

Debroglie Wavelength of an Accelerated Electron: $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

If an electron accelerated with potential difference 'V' the work done on the 'eV', which is converted to kinetic energy.

Then $eV = \frac{1}{2}mv^2$ → (1)

If 'p' is the momentum of the electron, then p=mv

Squaring on both sides, we have

$p^2 = m^2v^2$

$mv^2 = p^2/m$

Using in equation (1) we have

$eV = p^2/(2m)$ or

$p = \sqrt{2meV}$

According to de-Broglie λ = h/p

Therefore $\lambda = \frac{h}{\sqrt{2meV}}$

$\lambda = \frac{1}{\sqrt{V}} \left[\frac{6.626\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.602\times10^{-19}}} \right] = \frac{1.226\times10^{-9}}{\sqrt{V}}\ m$, $\lambda = \frac{1.226}{\sqrt{V}}$

QUANTUM MECHANICS

Heisenberg's Uncertainty Principle:

According to classical mechanics a particle occupies a definite place in space and possesses a definite momentum. If the position and momentum of a particle is known at any instant of time, it is possible to calculate its position and momentum at any later instant of time. The path of the particle could be traced. This concept breaks down in quantum mechanics leading to Heisenberg's Uncertainty Principle.

Heisenberg's Uncertainty Principle states that "It is impossible to measure simultaneously both the position and momentum of a particle accurately. If we make an effort to measure very accurately the position of a particle, it leads to large uncertainty in the measurement of momentum and vice versa".

If Δx and ΔP_x are the uncertainties in the measurement of position and momentum of the particle then the uncertainty can be written as

$$\Delta x \,.\Delta P_x \geq (h/4\pi)$$

In any simultaneous determination of the position and momentum of the particle, the product of the corresponding uncertainties inherently present in the measurement is equal to or greater than $h/4\pi$.

Similarly 1) $\Delta E.\Delta t \geq h/4\pi$ 2) $\Delta L.\Delta\theta \geq h/4\pi$

Significance of Heisenberg's Uncertainty Principle:

Heisenberg's Uncertainty Principle asserts that it is impossible to measure simultaneously both the position and momentum of a particle accurately. If we make an effort to measure very accurately the position of a particle, it leads to large uncertainty in the measurement of momentum and vice versa. Therefore one should think only of the probability of finding the particle at a certain position or of the probable value for the momentum of the particle.

Application of Uncertainty Principle:

Impossibility of existence of electrons in the atomic nucleus:

According to the theory of relativity, the energy E of a particle is:

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1 - (v^2/c^2)}}$$

Where ‘m_o’ is the rest mass of the particle and ‘m’ is the mass when its velocity is ‘v’.

i.e. $$E^2 = \frac{m_o^2 c^4}{1 - (v^2/c^2)} = \frac{m_o^2 c^6}{c^2 - v^2} \rightarrow (1)$$

If ‘p’ is the momentum of the particle:

i.e. $$p = mv = \frac{m_o v}{\sqrt{1 - (v^2/c^2)}}$$

$$p^2 = \frac{m_o^2 v^2 c^2}{c^2 - v^2}$$

Multiply by c^2

$$p^2c^2 = \frac{m_o^2 v^2 c^4}{c^2 - v^2} \rightarrow (2)$$

Subtracting (2) by (1) we have

$$E^2 - p^2c^2 = \frac{m_o^2 c^4 (c^2 - v^2)}{c^2 - v^2}$$

$E^2 = p^2c^2 + m_o^2c^4 \rightarrow (3)$

Heisenberg's uncertainty principle states that

$$\Delta x \, . \Delta P_x \geq \frac{h}{4\pi} \rightarrow (4)$$

The diameter of the nucleus is of the order 10^{-14}m. If an electron is to exist inside the nucleus, the uncertainty in its position Δx must not exceed 10^{-14}m.

i.e. $\Delta x \leq 10^{-14}$m

The minimum uncertainty in the momentum

$$(\Delta P_x)_{min} \geq \frac{h}{4\pi (\Delta x)_{max}} \geq \frac{6.63 \times 10^{-34}}{4\pi \times 10^{-14}} \geq 0.5 \times 10^{-20} \text{ kg. m/s} \rightarrow (5)$$

By considering minimum uncertainty in the momentum of the electron

i.e., $(\Delta P_x)_{min} \geq 0.5 \times 10^{-20}$ kg.m/s = p $\rightarrow (6)$

Consider eqn (3)

$E^2 = p^2c^2 + m_o^2c^4 = c^2(p^2 + m_o^2c^2)$

Where $m_o = 9.11 \times 10^{-31}$ kg

If the electron exists in the nucleus its energy must be

$E^2 \geq (3 \times 10^8)^2[(0.5 \times 10^{-20})^2 + (9.11 \times 10^{-31})^2(3 \times 10^8)^2]$

i.e. $E^2 \geq (3 \times 10^8)^2[0.25 \times 10^{-40} + 7.4629 \times 10^{-44}]$

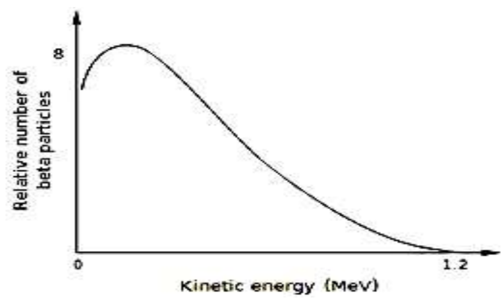
Neglecting the second term as it is smaller by more than the 3 orders of the magnitude compared to first term.

Taking square roots on both sides and simplifying

$$E \geq 1.5 \times 10^{-12}\ J \geq \frac{1.5 \times 10^{-12}}{1.6 \times 10^{-19}}\ eV \geq 9.4\ MeV$$

If an electron exists in the nucleus its energy must be greater than or equal to 9.4Mev. It is experimentally measured that the beta particles ejected from the nucleus during beta decay have energies of about 3 to 4 MeV. This shows that electrons cannot exist in the nucleus.

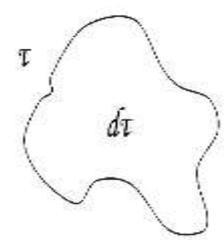
[**Beta decay:** In beta decay process, from the nucleus of an atom, when neutrons are converting into protons in releasing an electron (beta particle) and an antineutrino. When proton is converted into a neutron in releasing a positron (beta particle) and a neutrino. In both the processes energy sharing is statistical in nature. When beta particles carry maximum energy neutrino's carries minimum energy and vice-versa. In all other processes energy sharing is in between maximum and minimum energies. The maximum energy carried by the beta particle is called as the end point energy (E_{max}).



Wave Function:

A physical situation in quantum mechanics is represented by a function called wave function. It is denoted by 'ψ'. It accounts for the wave like properties of particles. Wave function is obtained by solving Schrodinger equation.

<u>Physical significance of wave function:</u>



<u>Probability density:</u> If ψ is the wave function associated with a particle, then $|\psi|^2$ is the probability of finding a particle in unit volume. If 'τ' is the volume in which the particle is present but where it is exactly present is not known. Then the probability of finding a particle in certain elemental volume dτ is given by $|\psi|^2 d\tau$. Thus $|\psi|^2$ is called probability density. The probability of finding an event is real and positive quantity. In the case of complex wave functions, the probability density is $|\psi|^2 = \psi^* \psi$ where ψ^* is Complex conjugate of ψ.

<u>Normalization:</u>

The probability of finding a particle having wave function 'ψ' in a volume 'dτ' is '$|\psi|^2 d\tau$'. If it is certain that the particle is present in finite volume 'τ', then

$$\int_0^{\tau} |\psi|^2 d\tau = 1$$

If we are not certain that the particle is present in finite volume, then

$$\int_{-\infty}^{\infty} |\psi|^2 d\tau = 1$$

In some cases $\int |\psi|^2 d\tau \neq 1$ and involves constant.

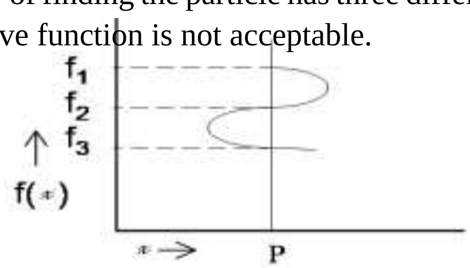
The process of integrating the square of the wave function within a suitable limits and equating it to unity the value of the constant involved in the wave function is estimated. The constant value is substituted in the wave function. This process is called as normalization. The wave function with constant value included is called as the normalized wave function and the value of constant is called normalization factor.

Properties of the wave function:

A system or state of the particle is defined by its energy, momentum, position etc. If the wave function 'ψ' of the system is known, the system can be defined. The wave function 'ψ' of the system changes with its state. To find 'ψ' Schrodinger equation has to be solved. As it is a second order differential equation, there are several solutions. All the solutions may not be correct. We have to select those wave functions which are suitable to the system. The acceptable wave function has to possess the following properties:

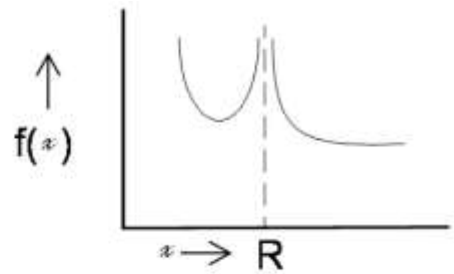
1) 'ψ' is single valued everywhere:

Consider the function f(x) which varies with position as represented in the graph. The function f(x) has three values f_1, f_2 and f_3 at x = p. Since $f_1 \neq f_2 \neq f_3$ it is to state that if f(x) were to be the wave function. The probability of finding the particle has three different values at the same location which is not true. Thus the wave function is not acceptable.



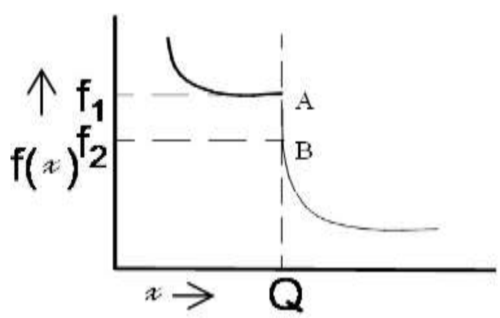
2) 'ψ' is finite everywhere:

Consider the function f(x) which varies with position as represented in the graph. The function f(x) is not finite at x =R but f(x)=∞. Thus it indicates large probability of finding the particle at a location. It violates uncertainty principle. Thus the wave function is not acceptable.



3) <u>'ψ' and its first derivatives with respect to its variables are continuous everywhere</u>:

Consider the function f(x) which varies with position as represented in the graph. The function f(x) is truncated at x =Q between the points A & B, the state of the system is not defined. To obtain the wave function associated with the system, we have to solve Schrodinger wave equation. Since it is a second order differential wave equation, the wave function and its first derivative must be continuous at x=Q. As it is a discontinuous wave function, the wave function is not acceptable.



4) For bound states 'ψ' must vanish at potential boundary and outside. If 'ψ*' is a complex function, then ψ* ψ must also vanish at potential boundary and outside.

The wave function which satisfies the above 4 properties are called *Eigen functions*.

<u>Time independent Schrodinger wave equation</u>

Consider a particle of mass 'm' moving with velocity 'v'. The de-Broglie wavelength 'λ' is

$$\lambda = \frac{h}{mv} = \frac{h}{P} \rightarrow (1)$$

Where 'mv' is the momentum of the particle.

The wave eqn is

$$\psi = Ae^{i(kx - \omega t)} \rightarrow (2)$$

Where ‘A’ is a constant and ‘ω’ is the angular frequency of the wave.
Differentiating equation (2) with respect to ‘t’ twice

$$\frac{d^2\psi}{dt^2} = -A\omega^2 e^{i(kx-\omega t)} = -\omega^2\psi \rightarrow (3)$$

The equation of a travelling wave is

$$\frac{d^2y}{dx^2} = \frac{1}{\text{v}^2}\frac{d^2y}{dt^2}$$

Where ‘y’ is the displacement and ‘v’ is the velocity.

Similarly for the de-Broglie wave associated with the particle

$$\frac{d^2\psi}{dx^2} = \frac{1}{\text{v}^2}\frac{d^2\psi}{dt^2} \rightarrow (4)$$

where ‘ψ’ is the displacement at time ‘t’.

From eqns (3) & (4)

$$\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{\text{v}^2}\psi$$

But ω = 2πυ and v = υ λ where ‘υ’ is the frequency and ‘λ’ is the wavelength.

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi \quad \text{or} \quad \frac{1}{\lambda^2} = -\frac{1}{4\pi^2\psi}\frac{d^2\psi}{dx^2} \rightarrow (5)$$

$$K.E = \frac{1}{2}m\text{v}^2 = \frac{m^2\text{v}^2}{2m} = \frac{P^2}{2m} \rightarrow (6)$$

$$\Rightarrow \frac{h^2}{2m\lambda^2} \rightarrow (7)$$

Using eqn (5)

$$K.E = \frac{h^2}{2m}\left(\frac{1}{\lambda^2}\right) = \frac{h^2}{2m}\left(-\frac{1}{4\pi^2\psi}\frac{d^2\psi}{dx^2}\right) = -\frac{h^2}{8\pi^2 m\psi}\frac{d^2\psi}{dx^2} \rightarrow (8)$$

Total Energy E = K.E + P.E

$$E = -\frac{h^2}{8\pi^2 m\psi}\frac{d^2\psi}{dx^2} + V$$

$$E - V = -\frac{h^2}{8\pi^2 m\psi}\frac{d^2\psi}{dx^2}$$

$$\frac{d^2\psi}{dx^2} = -\frac{8\pi^2 m}{h^2}(E - V)\psi$$

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0}$$

This is the time independent Schrodinger wave equation.

What are Eigen functions and Eigen values?

Eigen functions: Eigen functions are those wave functions of quantum mechanics which possess the properties that they are single valued & finite everywhere, and also their first derivatives with respect to their variables are continuous everywhere.

Eigen values: Eigen values are the possible values of a physical quantity like momentum, energy, etc., of a system that are observed when eigen functions are operated by quantum mechanical operators.

i.e., If an operator $\hat{A}$ operating on the well – behaved function Ψ produces back the wave function multiplied by a constant λ i.e.,

$$\hat{A}\Psi = \lambda\Psi$$

then the function Ψ is called as the **eigen function** and the value λ is called the **eigen value** of the operator $\hat{A}$ associated with eigenfunction Ψ.

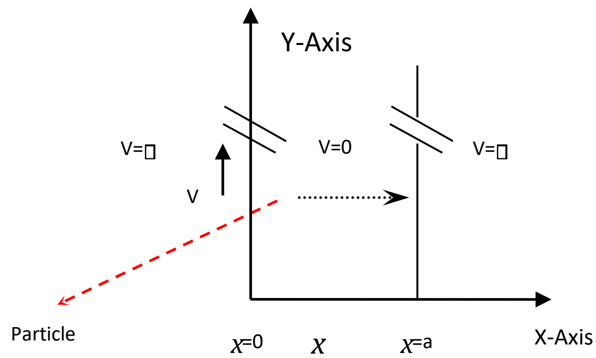
Example:

$$\frac{d}{dx}e^{ax} = ae^{ax}$$

Here, $\frac{d}{dx}$ is the operator, e^{ax} is the eigen function and a is the eigen value.

Application of Schrodinger wave equation:

Energy Eigen values of a particle in one dimensional, infinite potential well (potential well of infinite depth) or of a particle in a box



Consider a particle of a mass 'm' free to move in one dimension along positive x -direction between x =0 to x =a. The potential energy outside this region is infinite and within the region is zero. The particle is in bound state. Such a configuration of potential in space is called infinite potential well. It is also called particle in a box. The Schrödinger equation outside the well is,

$$E \;\square\square\square\;\square\; 0 \rightarrow (1) \qquad \because V = \infty$$

For outside, the equation holds good if ψ = 0 & |ψ|² = 0. That is particle cannot be found outside

The Schrodinger's equation inside the well is:

$$\frac{d^2\square}{\;^2} \square \frac{8\square^2 m}{h^2} E\square \;\square 0 \rightarrow (2) \qquad \because V = 0$$

$d_2\square$ $8\square_2 m$

$dx_2\square$ h_2

the well and also at the walls

dx

$8\square_2 mE\square k_2 \rightarrow (3)$

Let h_2

$dd x_2 \square_2 \square\ _2$

$0\ k\square\square$

The solution of this equation is:

ψ = C cos k x + D sin k x → (4)

at x = 0 → ψ = 0

0 = C cos 0 + D sin 0

∴ C = 0

Also x = a → ψ = 0

0 = C cos ka + D sin ka

But C = 0

∴ D sin ka = 0 ⟶ (5)

D□0 (because the wave concept vanishes)

i.e. ka = nπ where n = 0, 1, 2, 3, 4… (Quantum number)

$$k = \frac{n\square}{a} \rightarrow (6)$$

sub eqn (5) and (6) in (4) $\frac{n\square}{a} x$ → (7)

$\psi_n = D\sin$

This gives permitted wave functions.

The Energy Eigen value given by

Substitute equation (6) in (3)

$8\pi^2 mE / h^2 = k^2 = n^2\pi^2 / a^2$

For n = 0 $E = \frac{}{8ma^2} =$ is not acceptable inside the well because ψ_n = 0. It means that the electron is not present inside the well which is not true. Thus the lowest energy value for n = 1 is called zero point energy value or ground state energy.

i.e. $E_{zero\text{-}point} = \frac{h^2}{8ma^2}$

The states for which n >1 are called exited states.

To find out the value of D, normalization of the wave function is to be done.

i.e. $\int_0^a |\psi_n|^2 dx = 1 \rightarrow (8)$

using the values of ψ_n from eqn (7)

$\int_0^a D^2 \sin^2 \frac{n\pi}{a} x \, dx = 1$

$$D^2 \int_0^a \frac{[1 - \cos(2n\pi/a)x]}{2}\, dx = 1 \qquad \because \sin^2\theta = \frac{[1 - \cos 2\theta]}{2}$$

$$\frac{D^2}{2}\left[\int_0^a dx - \int_0^a \cos\frac{2n\pi}{a} x\, dx\right] = 1$$

$$\frac{D^2}{2}\left\{[x]_0^a - \frac{a}{2n\pi}\left[\sin\frac{2n\pi}{a} x\right]_0^a\right\} = 1$$

$$\frac{D^2}{2}[a - 0] = 1$$

$$\frac{D^2}{2} a = 1$$

$$D = \sqrt{\frac{2}{a}}$$

Substitute D in equation (7) the normalized wave functions of a particle in one dimensional infinite potential well is:

$$\boxed{\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x \rightarrow (9)}$$

<u>Wave functions, probability densities and energy levels for particle in an infinite potential well</u>:

Let us consider the most probable location of the particle in the well and its energies for first three cases.

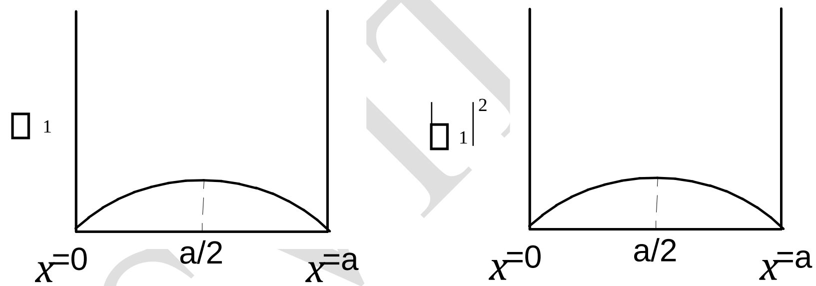
<u>Case I → n=1</u> It is the ground state and the particle is normally present in this state.

The Eigen function is

$$\psi_1 = \sqrt{\frac{2}{a}}\, Sin \frac{\pi x}{a} \quad \text{from eqn (7)}$$

$\psi_1 = 0$ for $x = 0$ and $x = a$

But ψ_1 is maximum when $x = a/2$.



The plots of ψ_1 versus x and $|\psi_1|^2$ verses x are shown in the above figure.

$|\psi_1|^2 = 0$ for $x = 0$ and $x = a$ and it is maximum for $x = a/2$. i.e. in ground state the particle cannot be found at the walls, but the probability of finding it is maximum in the middle.

The energy of the particle at the ground state is

$$E_1 = \frac{h^2}{8ma^2} = E_0$$

Case II → n=2

In the first excited state the Eigen function of this state is

$$\psi_2 = \sqrt{\frac{2}{a}} \, Sin \, \frac{2\pi x}{a}$$

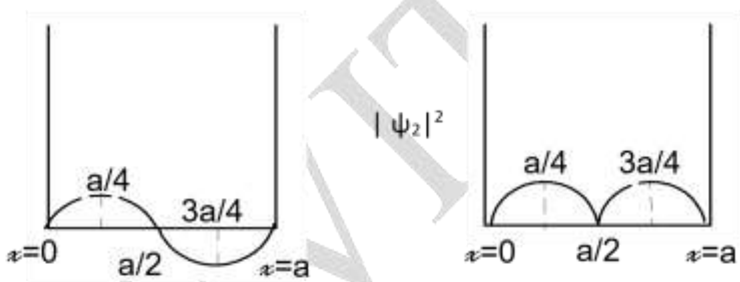
$\psi_2 = 0$ for the values $x = 0, a/2, a$.

Also ψ_2 is maximum for the values $x = a/4$ and $3a/4$.

These are represented in the graphs.

$|\psi_2|^2 = 0$ at $x = 0, a/2, a$, i.e. particle cannot be found either at the walls or at the centre.

$$|\psi_2|^2 = \text{maximum} \quad for \; x = \frac{a}{4}, \; x = \frac{3a}{4}$$



The energy of the particle in the first excited state is $E_2 = 4E_0$.

Case III → **n=3**

In the second excited state,

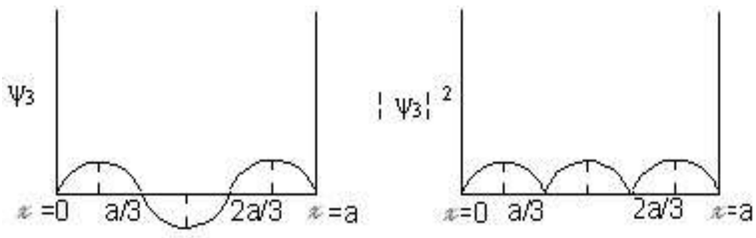
ψ_2

$$\psi_3 = \sqrt{\frac{2}{a}} \, Sin \, \frac{3\pi x}{a}$$

$\psi_3 = 0$, for $x = 0, a/3, 2a/3$ and a.

ψ_3 is maximum for $x = a/6, a/2, 5a/6$.

These are represented in the graphs.



$|\psi_3|^2 = 0$ for $x = 0$, $a/3$, $2a/3$ and a. $|\psi_3|^2 =$ maximum $for\ x = \frac{a}{6}, x = \frac{a}{2}, x = \frac{5a}{6}$

The energy of the particle in the second excited state is $E_3 = 9\,E_0$.

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