MODULE-3
RANDOM VARIABLESB& PROBABILITY DISTRIBUTIONS

Random experiment:

An activity , that yield some results called the random experiment. The random variable means a real number, i.e. X associated with the outcomes of a random experiment.

Definition : Let S be a sample space associated with a random experiment with a real value function defined and taking its values is called a Random variable.

The random variables are two types, they are

1. Discrete random variables
2. Continuous random variables.

Discrete random variables: A Discrete random variables is a variable which can only take a countable number of values. For example if a coin is tossed three times, the number of heads can be obtained 0 ,1, 2 or 3. The probabilities of each of these probabilities can be tabulated as shown.

S={HHH,HHT,HTT,HTH,THH,TTH,THT,TTT}

x	**0**	**1**	**2**	**3**
$P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Continuous random variables:

A Continuous random variables is a random variable where the data can take infinitely many values. for example , a random variable measuring the time taken for something to be done is continuous since there are an infinite number of possible times that can be taken. Ex. temperature of the climate , age of a person ,etc.

Probability Mass Function :

Probability mass function is the probability distribution of a discrete random variable, and provides the possible values and their associated probabilities.

1. $P(x_i) \geq 0$

2. $\sum_{i=1}^{n} P(X = x_i) = 1$

3. $0 \leq P(x) \leq 1$

4. Mean $\mu = \sum_{i=1}^{n} x_i P(x_i)$

5. Variance $\sigma^2 = \sum_{i=1}^{n} x_i^2 P(x_i) - \mu^2$

Probability density function :

Probability density function is the probability distribution of a continuous random variable, and provides the possible values and their associated probabilities infinitely.

1. $P(x_i) \geq 0$ or $f(x) \geq 0$

2. $\int_{-\infty}^{\infty} f(x)dx = 1$

3. Mean $\mu = \int_{-\infty}^{\infty} xf(x)dx$

4. Variance $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) - \mu^2$

5. $P(a \leq x \leq b) = P(a < x \leq b) = P(a \leq x < b) = P(a < x < b) = \int_a^b f(x)dx$

PROBLEMS

1). **Show that the following probabilities one satisfying the properties of DRV , hence find its mean and variance .**

x	10	20	30	40
$P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Sol.

Let X be the random variable for the random values

$x_1 = 10, x_2 = 20, x_3 = 30, x_4 = 40$

And given $P(X = x_1) = P(x_1) = p_1 = \frac{1}{8}$,

$P(X = x_2) = P(x_2) = p_2 = \frac{3}{8}$,

$P(X = x_3) = P(x_3) = p_3 = \frac{3}{8}$ and $P(X = x_4) = P(x_4) = p_4 = \frac{1}{8}$

Let $\sum_{i=1}^{4} P(X = x_i) = P(X = x_1) + P(X = x_2) + P(X = x_3) + P(X = x_4)$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$

$= \frac{8}{8}$

$= 1$

Hence the given probabilities can satisfy the DRV property.

$$\mu = \sum_{i=1}^{4} x_i P(x_i)$$

$$\text{Mean } = 10 \times \frac{1}{8} + 20 \times \frac{3}{8} + 30 \times \frac{3}{8} + 40 \times \frac{1}{8}$$

$$= \frac{200}{8}$$

$$= 25$$

$$\sigma^2 = \sum_{i=1}^{4} x_i^2 P(x_i) - \mu^2$$

$$= 10^2 \times \frac{1}{8} + 20^2 \times \frac{3}{8} + 30^2 \times \frac{3}{8} + 40^2 \times \frac{1}{8} - 25^2$$

$$= 700 - 625$$

$$= 75$$

$$\text{SD} = \sqrt{Variance} = \sqrt{75} = 8.66$$

2) **Find the value of k, such that the following distribution represents discrete probability distribution, hence find Mean, SD, $P(x \le 1), P(x > 1)$ and $P(-1 < x \le 2)$.**

x	-3	-2	-1	0	1	2	3
$P(x)$	k	$2k$	$3k$	$4k$	$3k$	$2k$	k

Sol.

Let X be the random variable for the random values $x_1 = -3, x_2 = -2, x_3 = -1, x_4 = 0, x_5 = 1, x_6 = 2, x_7 = 3$ and given the respective probabilities are

$$P(X = x_1) = P(-3) = k$$
$$P(X = x_2) = P(-2) = 2k$$
$$P(X = x_3) = P(-1) = 3k$$
$$P(X = x_4) = P(0) = 4k$$
$$P(X = x_5) = P(1) = 3k$$
$$P(X = x_6) = P(2) = 2k$$
$$P(X = x_7) = P(3) = k$$

We know that

$$\sum_{i=1}^{7} P(X = x_i) = 1$$

$$\Rightarrow k + 2k + 3k + 4k + 3k + 2k + k = 1$$

$$\Rightarrow 16k = 1$$

$$\Rightarrow k = \frac{1}{16}$$

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
-3	K	-3k	9	9k
-2	2k	-4k	4	8k
-1	3k	-3k	1	3k
0	4k	0	0	0
1	3k	3k	1	3k
2	2k	4k	4	8k
3	K	3k	9	9k
$\sum$		0	-	40k

$$\mu = \sum_{i=1}^{n} x_i P(x_i)$$

$$= 0$$

Variance $\sigma^2 = \sum_{i=1}^{7} x_i^{\,2} P(x_i) - \mu^2 = 40k - 0 = 40k = 40 \times \frac{1}{16} = 2.5$

$$S.D. = \sqrt{2.5} = 1.5811$$

$$P(x \le 1) = P(-3) + P(-2) + P(-1) + P(0) + P(1)$$

$$\Rightarrow P(x \le 1) = k + 2k + 3k + 4k + 3k$$

$$\Rightarrow P(x \le 1) = 13k = \frac{13}{16} = 0.8125$$

$$P(x > 1) = P(2) + P(3) = 2k + k = 3k = \frac{3}{16} = 0.1875$$

$$P(-1 < x \le 2) = P(0) + P(1) + P(2) = 4k + 3k + 2k = 9k = \frac{9}{16} = 0.5625$$

3). **Find the value of k, such that the following distribution represents discrete probability distribution, hence find Mean, SD, $P(x \ge 5)$ and $P(3 < x \le 6)$.**

x	0	1	2	3	4	5	6
$P(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Sol.

Let X be the random variable for the random values

$x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 3, x_5 = 4, x_6 = 5, x_7 = 6$ and given the respective probabilities are

$$P(X = x_1) = P(0) = k$$
$$P(X = x_2) = P(1) = 3k$$
$$P(X = x_3) = P(2) = 5k$$
$$P(X = x_4) = P(3) = 7k$$
$$P(X = x_5) = P(4) = 9k$$
$$P(X = x_6) = P(5) = 11k$$
$$P(X = x_7) = P(6) = 13k$$

We know that

$$\sum_{i=1}^{7} P(X = x_i) = 1$$
$$\Rightarrow k + 3k + 5k + 7k + 9k + 11k + 13k = 1$$
$$\Rightarrow 49k = 1$$
$$\Rightarrow k = \frac{1}{49}$$

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	k	0	0	0
1	3k	3k	1	3k
2	5k	10k	4	20k
3	7k	21k	9	63k
4	9k	36k	16	144k
5	11k	55k	25	275k
6	13k	78k	36	468k
$\sum$		203k	-	973k

$$\mu = \sum_{i=1}^{n} x_i P(x_i)$$
$$= 203k$$
$$= \frac{203}{49}$$
$$= 4.1428$$

$$\sigma^2 = \sum_{i=1}^{7} x_i^2 P(x_i) - \mu^2 = 973k - 4.1428^2$$

Variance $= \frac{973}{49} - 17.1628$

$= 19.8571 - 17.1628$

$= 2.6943$

$S.D. = \sqrt{2.6943} = 1.6414$

$P(x \geq 5) = P(5) + P(6)$

$\Rightarrow P(x \geq 5) = 11k + 13k$

$\Rightarrow P(x \geq 5) = 24k = \frac{24}{49} = 0.4898$

$P(3 < x \leq 6) = P(4) + P(5) + P(6) = 9k + 11k + 13k$

$= 33k$

$= \frac{33}{49}$

$= 0.6734$

4).**A random variable X has a probability function for various values of x**

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find i) k **ii)** $P(x<6)$ **iii)** $P(x \geq 6), P(3 < x \leq 6)$ **, Also find the probability distribution and distribution function of** x**.**

Sol.

Let X be the random variable for the random values $x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 3, x_5 = 4, x_6 = 5, x_7 = 6, x_8 = 7$ and given the respective probabilities are

$P(X = x_1) = P(0) = 0$

$P(X = x_2) = P(1) = k$

$P(X = x_3) = P(2) = 2k$

$P(X = x_4) = P(3) = 2k$

$P(X = x_5) = P(4) = 3k$

$P(X = x_6) = P(5) = k^2$

$P(X = x_7) = P(6) = 2k^2$

$P(X = x_8) = P(7) = 7k^2 + k$

We know that

$$\sum_{i=1}^{7} P(X = x_i) = 1$$
$$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$
$$\Rightarrow 10k^2 + 9k = 1$$
$$\Rightarrow 10k^2 + 9k - 1 = 0$$
$$\Rightarrow (10k - 1)(k + 1) = 0$$
$$\Rightarrow 10k - 1 = 0, k + 1 = 0$$
$$\Rightarrow k = \frac{1}{10}, k \neq -1$$

x	0	1	2	3	4	5	6	7
$P(x)$	O	0.1	0.2	0.2	0.3	0.01	0.02	0.17
CP	0	0.1	0.3	0.5	0.8	0.81	0.83	1

i)

$$P(x < 6) = 1 - P(x \geq 6)$$
$$\Rightarrow P(x < 6) = 1 - \{P(6) + P(7)\}$$
$$\Rightarrow P(x < 6) = 1 - (0.02 + 0.17)$$
$$\Rightarrow P(x < 6) = 0.81$$

Ii)

$$P(x \geq 6) = P(6) + P(7) = 0.02 + 0.17 = 0.19$$
$$P(3 < x \leq 6) = P(4) + P(5)(+P(6) = 0.3 + 0.01 + 0.02 = 0.33$$

5) A random variable has the following probability function for the various values of X=x

x	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	$2k$	0.3	k

Find i) Value of k

ii) $P(x < 1)$

ii) $P(x \geq -1)$

Sol.

Let X be the random variable for the random values

$x_1 = -2, x_2 = 1, x_3 = 0, x_4 = 1, x_5 = 2, x_6 = 3$ and given the respective probabilities are

$$P(X = x_1) = P(-2) = 0.1$$
$$P(X = x_2) = P(-1) = k$$
$$P(X = x_3) = P(0) = 0.2$$
$$P(X = x_4) = P(1) = 2k$$
$$P(X = x_5) = P(2) = 0.3$$
$$P(X = x_6) = P(3) = k$$

$$\sum_{i=1}^{6} P(X = x_i) = 1$$
$$\Rightarrow 0.1 + k + 0.2 + 2k + 0.3 + k = 1$$
$$\Rightarrow 4k + 0.6 = 1$$
$$\Rightarrow 4k = 0.4$$
$$\Rightarrow k = \frac{0.4}{4} = 0.1$$

$$P(x < 1) = P(-2) + P(-1) + P(0)$$
$$\Rightarrow P(x < 1) = 0.1 + k + 0.2$$
$$\Rightarrow P(x < 1) = k + 0.3 = 0.1 + 0.3 = 0.4$$
$$P(x \geq -1) = P(-1) + P(0) + P(1) + P(2)$$
$$\Rightarrow P(x \geq -1) = k + 0.2 + 2k + 0.3 + k$$
$$\Rightarrow P(x \geq -1) = 4k + 0.5$$
$$\Rightarrow P(x \geq -1) = 4(0.1) + 0.5$$
$$\Rightarrow P(x \geq -1) = 0.4 + 0.5 = 0.9$$

6)

A random variable has the following probability function for the various values of X=x

x	**0**	**1**	**2**	**3**	**4**	**5**
$P(x)$	k	$5k$	$10k$	$10k$	$5k$	k

Find i) Value of k

ii) $P(x \leq 1)$

ii) $P(0 \leq x < 3)$

Sol. Let X be the random variable for the random values

$x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 3, x_5 = 4, x_6 = 5$ and given the respective probabilities are

$P(X=x_1)=P(0)=k$
$P(X=x_2)=P(1)=5k$
$P(X=x_3)=P(2)=10k$
$P(X=x_4)=P(3)=10k$
$P(X=x_5)=P(4)=5k$
$P(X=x_6)=P(5)=k$

$$\sum_{i=1}^{6} P(X=x_i)=1$$

$$\Rightarrow k+5k+10k+10k+5k+k=1$$
$$\Rightarrow 32k=1$$
$$\Rightarrow k=\frac{1}{32}$$

$P(x \le 1)=P(0)+P(1)$

$\Rightarrow P(x \le 1)=k+5k$

$\Rightarrow P(x \le 1)=6k=\frac{6}{32}=0.1875$

$P(0 \le x < 3)=P(0)+P(1)+P(2)$

$\Rightarrow P(0 \le x < 3)=k+5k+10k$

$\Rightarrow P(0 \le x < 3)=16k$

$\Rightarrow P(0 \le x < 3)=\frac{16}{32}=0.5$

7). **Show that the function** $f(x)=\begin{cases} e^{-x} & ,x \ge o \\ 0 & ,x<0 \end{cases}$ **is pdf , hence find** $P(1.5<x<2.5)$**.**

Sol.

Given probability function

$$f(x)=\begin{cases} e^{-x} & ,x \ge o \\ 0 & ,x<0 \end{cases}$$

Let $\int_{-\infty}^{\infty} f(x)dx=\int_{-\infty}^{0} f(x)dx+\int_{0}^{\infty} f(x)dx$

$$= \int_{-\infty}^{0} 0dx + \int_{0}^{\infty} e^{-x} dx$$

$$== 0 + \left[\frac{e^{-x}}{-1}\right]_0^{\infty}$$

$$= -\left[e^{-\infty} - e^{0}\right]$$

=-(0-1)

=1

Hence the given probability function is pdf.

$$P(1.5 < x < 2.5) = \int_{1.5}^{2.5} f(x)dx$$

$$\Rightarrow P(1.5 < x < 2.5) = \int_{1.5}^{2.5} e^{-x} dx$$

$$\Rightarrow P(1.5 < x < 2.5) = -\left[e^{-x}\right]_{1.5}^{2.5}$$

$$\Rightarrow P(1.5 < x < 2.5) = -\left[e^{-2.5} - e^{-1.5}\right]$$

$$P(1.5 < x < 2.5) = \frac{1}{e^{1.5}} - \frac{1}{e^{2.5}}$$

6).**A random variable X has a probability density function** $f(x) = \begin{cases} kx^2 & ,0 \le x \le 3 \\ 0 & ,Otherwise \end{cases}$, **Evaluate**

i) k **ii)** $P(x \le 1)$ **iii)** $P(x > 1)$ **iv)** $P(1 \le x \le 2)$ **v)** $P(x \le 2)$ **vi)** $P(x \ge 2)$

Sol.

Given probability density function

$$f(x) = \begin{cases} kx^2 & ,0 \le x \le 3 \\ 0 & ,Otherwise \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{3} f(x)dx + \int_{3}^{\infty} f(x)dx = 1$$

We know that $\Rightarrow 0 + \int_{0}^{3} kx^2 dx + 0 = 1$

$$\Rightarrow k\left[\frac{x^3}{3}\right]_0^3 = 1$$

$$\Rightarrow 9k = 1$$

$$\Rightarrow k = \frac{1}{9}$$

$$P(x<1) = \int_{-\infty}^{1} f(x)dx$$

$$\Rightarrow P(x<1) = \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx$$

$$\Rightarrow P(x<1) = 0 + \int_{0}^{1} kx^2 dx$$

$$\Rightarrow P(x<1) = k\left[\frac{x^3}{3}\right]_0^1$$

$$\Rightarrow P(x<1) = \frac{k}{3} = \frac{1}{27}$$

$$P(x>1) = \int_{1}^{\infty} f(x)dx$$

$$\Rightarrow P(x>1) = \int_{1}^{3} f(x)dx + \int_{3}^{\infty} f(x)dx$$

$$\Rightarrow P(x>1) = 0 + \int_{1}^{3} kx^2 dx$$

$$\Rightarrow P(x<1) = k\left[\frac{x^3}{3}\right]_1^3$$

$$\Rightarrow P(x<1) = \frac{26k}{3} = \frac{26}{27}$$

$$P(1 \le x \le 2) = \int_{1}^{2} f(x)dx$$

$$\Rightarrow P(1 \le x \le 2) = \int_{1}^{2} kx^2 dx$$

$$\Rightarrow P(1 \le x \le 2) = k\left[\frac{x^3}{3}\right]_1^2$$

$$\Rightarrow P(1 \le x \le 2) = \frac{7k}{3} = \frac{7}{27}$$

$$P(x \le 2) = \int_{-\infty}^{2} f(x)dx$$

$$\Rightarrow P(x \le 2) = \int_{-\infty}^{0} f(x)dx + \int_{0}^{2} f(x)dx$$

$$\Rightarrow P(x \le 2) = 0 + \int_{0}^{1} kx^2 dx$$

$$\Rightarrow P(x \le 2) = k\left[\frac{x^3}{3}\right]_0^2$$

$$\Rightarrow P(x \le 2) = \frac{8k}{3} = \frac{8}{27}$$

$$P(x \ge 2) = \int_{2}^{\infty} f(x)dx$$

$$\Rightarrow P(x \ge 2) = \int_{2}^{3} f(x)dx + \int_{3}^{\infty} f(x)dx$$

$$\Rightarrow Px \ge 2) = 0 + \int_{2}^{3} kx^2 dx$$

$$\Rightarrow P(x \ge 2) = k\left[\frac{x^3}{3}\right]_2^3$$

$$\Rightarrow P(x \ge 2) = \frac{19k}{3} = \frac{19}{27}$$

Binomial distribution: Let X be discrete random variable , p be the probability of success and let q be the probability of failure , then the probability mass function of the Binomial distribution can be defined as (n, the number of trials is finite)

$$P(X=x)=b(n,p,x)=\begin{cases} n_{c_x}\, p^x q^{n-x} & ,x\geq 0 \\ 0 & ,Otherwise \end{cases}$$

Where n , p are called the parameters ..

Note : 1. $P(X=x)=b(n,p,x)\geq 0$

2. $p+q=1$

3. $\sum_{x=0}^{n} n_{c_x}\, p^x q^{n-x}=1$

4. The mean of B.D. $\mu=np$, Variance $\sigma^2=npq$ and S.D. is $\sigma=\sqrt{npq}$

PROBLEMS:

1. Let X be a binomially distributed random variable based on 6 repetitions of an experiment. If p=0.3, evaluate the following probabilities i) P(X≤ 3) ii) P(X > 4)

Sol.

Given P=0.3 and n = 6 Hence q=1-p = 0.7 and $P(X=x)=$ b (6,0.3.x) = $6_{c_x}(0.3)^x(0.7)^{6-x}$

i) P(X≤ 3)=P(0)+P(1)+P(2)+P(3)

$$=6_{c_0}(0.3)^0(0.7)^{6-0}+6_{c_1}(0.3)^1(0.7)^{6-1}+6_{c_2}(0.3)^2(0.7)^{6-2}+6_{c_3}(0.3)^3(0.7)^{6-3}$$

ii) P(X > 4)=P(5)+P(6)

$$=6_{c_5}(0.3)^5(0.7)^{6-5}+6_{c_6}(0.3)^6(0.7)^{6-6}$$

2. The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected at random, find the probability that

I) Exactly two pens will be defective

II) At most two pens will be defective

III) None will be defective

Sol.

Let the probability that a pen manufactured is defective = p.

Then p=0.1 q=1-p = 0.9 and n =12

Hence b(n, p, x) = $12_{c_x}(0.1)^x(0.9)^{12-x}$ = P(x) say

i) Probability that exactly two pens will be defective = P(X=2)= $12_{c_2}(0.1)^2(0.9)^{12-2}$=0.2301

ii) Probability that at most 2 pens will be defective =P(X≤2) =P(0)+P(1)+P(2)

$$=12_{c_0}(0.1)^0(0.9)^{12-0}+12_{c_1}(0.1)^1(0.9)^{12-1}+12_{c_2}(0.1)^2(0.9)^{12-2}$$

iii) Probability that none of the pens will be defective =P(X=0)

=P(0)=$12_{c_0}(0.1)^0(0.9)^{12-0}$

=0.2824295

iv) Probability that at least 2 pens will be defective= $P(x \ge 2) = 1 - \{P(0) + P(1)\}$

$=1-\{12_{c_0}(0.1)^0(0.9)^{12-0} + 12_{c_1}(0.1)^1(0.9)^{12-1}\}$

=0.3411

3) **The number of telephonic lines busy at an instant line is a binomial veriate with a probability 0.1 . If 10 lines are chosen at random, what is the probability that (i) No line is busy (ii) All lines are busy (iii) At least one line is busy (iv) At most two lines are busy.**
Sol.

Let the probability that a telephonic line is busy $p = 0.1 \Rightarrow q = 1 - p = 1 - 0.1 = 0.9$

And the total number of lines chosen n=10

Let X be the binomial variate , $P(X = x) = 10_{C_x}(0.1)^x(0.9)^{10-x}$

i) The probability of getting no line is busy is $P(x=0) = 10_{C_0}(0.1)^0(0.9)^{10-0} = 0.3486$

ii) The probability of getting all the lines are busy is $P(x=10) = 10_{C_{10}}(0.1)^{10}(0.9)^{10-10} = (0.1)^{10}$

iii) The probability of getting at least one line is busy

$$P(x \ge 1) = 1 - P(x < 1)$$
$$\Rightarrow P(x \ge 1) = 1 - P(0)$$
$$\Rightarrow P(x \ge 1) = 1 - 0.3486$$
$$\Rightarrow P(x \ge 1) = 0.6514$$

iv) The probability of getting at most two lines are busy

$$P(x \le 2) = P(0) + P(1) + P(2)$$
$$\Rightarrow P(x \le 2) = 10_{C_0}(0.1)^0(0.9)^{10-0} + 10_{C_1}(0.1)^1(0.9)^{10-1}10_{C_2}(0.1)^2(0.9)^{10-2}$$
$$\Rightarrow P(x \le 2) = 0.3486 + 0.3874 + 0.1937$$
$$\Rightarrow P(x \le 2) = 0.9219$$

4) **When a coin is tossed 4 times , find the probability of getting i) Exactly One head ii) At most three heads iii) At least two heads.**

Sol.

The number of times tossed a coin n=4

Let x be the binomial variate getting head $p = 0.5$

The probability of getting tail $q = 0.5$

$$\therefore P(X = x) = 4_{Cx}(0.5)^x(0.5)^{4-x}$$

i) The probability of getting exactly one head $P(x=1) = 4_{C1}(0.5)^1(0.5)^{4-1} = 0.25$

ii) The probability of getting at most three heads $P(x \le 3) = 1 - P(x=4)$

$$= 1 - 4C_4(0.5)^4(0.5)^{4-4}$$

=0.9375

iii) The probability of getting at least two heads

$$P(x \ge 2) = 1 - P(x=0) - P(x=1)$$

$$\Rightarrow P(x \ge 2) = 1 - 4_{C0}(0.5)^0(0.5)^{4-0} - 4_{C1}(0.5)^1(0.5)^{4-1}$$

$$\Rightarrow P(x \ge 2) = 1 - 0.0625 - 0.25 = 0.6875$$

5) **The probability of germination of a seed in a packet of seeds is found to be 0.7. If 10 seeds are taken for experimenting on germination in a laboratory, find the probability that (i) 8 seeds germinate**
(ii) at least 8 seeds germinate
(iii) at most 8 seeds germinate.

Sol.

Let X be the binomial variate of the seed germination.

Given the number of seeds packets taken for the experiment in the laboratory n=10

The probability of germination of a seed in a packet of seeds is p =0.7 $q = 1 - 0.7 = 0.3$

The probability mass function of Binomial distribution is $\therefore P(X = x) = 10_{Cx}(0.7)^x(0.3)^{10-x}$

i) The probability of 8 seeds germinate $= P(x=8)$

$$\Rightarrow P(x=8) = 10_{C8}(0.7)^8(0.3)^{10-8} = 90 \times 0.05764 \times 0.09 = 0.4670$$

ii) The probability of at least 8 seeds germinate= $P(x \ge 8) = P(8) + P(9) + P(10)$

$$\Rightarrow P(x \ge 8) = 10_{C8}(0.7)^8(0.3)^{10-8} + 10_{C9}(0.7)^9(0.3)^{10-9} + 10_{C10}(0.7)^{10}(0.3)^{10-10}$$

$$\Rightarrow P(x \ge 8) = 0.4670 + 0.12106 + 0.02824 = 0.6163$$

iii) The probability of at most 8 seeds germinate= $P(x \le 8) = 1 - P(x > 8)$

$$\Rightarrow P(x \le 8) = 1 - P(9) - P(10)$$

$$\Rightarrow P(x \le 8) = 1 - 0.12106 - 0.02824 = 0.8507$$

Poisson Distribution:

Let X be the discrete random variable , for any real value λ (Parameter), such that the probability mass function of poison distribution can be defined as,

$$P(X=x)=\begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!} & ,x=0,1,2,3.....n \\ 0 & Otherwise \end{cases}$$

Note: 1. $P(X=x)=P(x)\geq 0$

2. $\sum_{x=0}^{n} P(x)=\sum_{x=0}^{n}\frac{e^{-\lambda}\lambda^{x}}{x!}=1$

3. Mean $\mu=np=\lambda$

4. Variance $\sigma^{2}=\lambda$, S.D.= $\sqrt{\lambda}$

PROBLEMS:

1. **The number of accidents in a year to a taxi drivers in a city follows a poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of drivers with i) No accident in a year ii) More than three accidents in a year.**

Sol. Let X be the poison variate follows accident in the year of the poison distribution.

The probability mass function of the poison distribution is $P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!}$

Given the mean of poison distribution is $\mu=\lambda=3$

$\therefore P(X=x)=\frac{e^{-3}3^{x}}{x!}$ --------(1)

i) No accident in the year out of 1000 taxi drivers $=1000\times P(x=0)$

$=1000\times\frac{e^{-3}3^{0}}{0!}$

$=1000\times 0.05$

$=50$

$\therefore$ 50 members of the drivers out of 1000 drivers having no accident in the year.

ii) More than three accidents in the year out of 1000 drivers $=1000\times P(x>3)$

$$= 1000 \times [1 - P(x \le 3)]$$
$$= 1000 \times [1 - P(0) - P(1) - P(2]) - P(3)]$$
$$= 1000 \times \left[1 - \frac{e^{-3}3^0}{0!} - \frac{e^{-3}3^1}{1!} - \frac{e^{-3}3^2}{2!} - \frac{e^{-3}3^3}{3!}\right]$$
$$= 1000 \times [1 - 0.05 - 0.15 - 0.225 - 0.225]$$
$$= 1000 \times 0.5$$
$$= 500$$

$\therefore$ 500 members done accident of more than three accidents in the year out of 1000 drivers

2.In a certain factory turning out razor blades there is a small probability of $1/500$ for any blade to be defective .The blades are supplied in a packets of 10. use poisson distribution to calculate approximate number of packets containing i) No defective ii) Two defective iii) Three defective in consigment of 10000 packets.

Sol.

Let X be the poison variate follows the blades to be defective of the poison distribution.

The probability mass function of the poison distribution is $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given the $n = 10, p = 1/500 = 0.002$

$$\Rightarrow \mu = np = 10 \times 0.002 = 0.02 = \lambda$$

$$\therefore P(X = x) = \frac{e^{-0.02}(0.02)^x}{x!} \text{ --------(1)}$$

i) No blades are defective out of 10000 packets $= 10000 \times P(x = 0)$

$$= 10000 \times \frac{e^{-0.02}(0.02)^0}{0!}$$
$$= 10000 \times 0.9802$$
$$= 9802$$

$\therefore$ 9802 blades are no defective out of 10000 packets

ii) Two defective blades out of 10000 packets $= 10000 \times P(x = 2)$

$$= 10000 \times \frac{e^{-0.02}(0.02)^2}{2!}$$
$$= 10000 \times 0.0002$$
$$= 2$$

$\therefore$ 2 blades are defective out of 10000 packets of only two defective.

iii) Three defective blades out of 10000 packets $= 10000 \times P(x = 3)$

$$= 10000 \times \frac{e^{-0.02}(0.02)^3}{3!}$$
$$= 10000 \times 0.0000$$
$$= 0$$

$\therefore$ No blades are defective out of 10000 packets of only three defective.

3. If the probability of bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals more than two will get a bad reaction.

Sol.

Let X be the poison variate follows the bad reaction of the injection.

WKT

The probability mass function of the poison distribution is $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given the $n = 2000, p = 0.001$
$$\Rightarrow \mu = np = 2000 \times 0.001 = 2 = \lambda$$

$$\therefore P(X = x) = \frac{e^{-2}(2)^x}{x!} \text{--------(1)}$$

The probability that of more than two individuals will get bad reaction $= P(x > 2)$
$$= 1 - P(x \le 2)$$
$$= 1 - \left[P(0) + P(1) + P(2)\right]$$
$$= 1 - \frac{e^{-2}(2)^0}{0!} - \frac{e^{-2}(2)^1}{1!} - \frac{e^{-2}(2)^2}{2!}$$
$$= 1 - \frac{5}{e^2}$$
$$= 0.32$$

4) The Probability that a news reader commits no mistakes in reading the news is $\frac{1}{e^3}$. Find a probability on a particular news broadcast he commits only i) Two mistakes ii) More than three mistakes iii) At most three mistakes.

Sol.

Let X be the poison variate follows the news reader do mistakes of the poisson distribution.

WKT

The probability mass function of the poison distribution is $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given $P(X = 0) = \frac{1}{e^3}$

$$\Rightarrow \frac{e^{-\lambda}\lambda^0}{0!} = \frac{1}{e^3}$$

$$\Rightarrow \frac{1}{e^{\lambda}} = \frac{1}{e^{3}} \Rightarrow \lambda = 3$$

$$\therefore P(X = x) = \frac{e^{-3}3^{x}}{x!}$$

i) The probability that the news reader can do two mistakes $= P(x=2) = \frac{e^{-3}3^{2}}{2!} = 0.2240$

ii) The probability that the news reader can do mistakes of more than three $= P(x>3)$

$$\Rightarrow P(x>3) = 1 - P(x \le 3)$$

$$\Rightarrow P(x>3) = 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$$

$$\Rightarrow P(x>3) = 1 - e^{-3}\left[\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!}\right]$$

$$\Rightarrow P(x>3) = 1 - 0.05(1+3+4.5+4.5)$$

$$\Rightarrow P(x>3) = 1 - 0.65$$

$$\Rightarrow P(x>3) = 0.3500$$

The probability that the news reader can do at most three mistakes $= P(x \le 3)$

$$\Rightarrow P(x \le 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$\Rightarrow P(x \le 3) = e^{-3}\left[\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!}\right]$$

$$\Rightarrow P(x \le 3) = (0.05)(1+3+4.5+4.5)$$

$$\Rightarrow P(x \le 3) = 0.6500$$

5)

Suppose 300 misprints are randomly distributed throughout a book of 500 pages, find the probability that a given page contains (i) Exactly three misprints
(ii) Less than three misprints
and (iii) Four or more misprints.

Sol. Let X be the poisson variate of misprints througout a book of 500 pages

Given

Suppose 300 misprints are randomly distributed throughout a book of 500 pages

$$\therefore Mean = \lambda = \frac{300}{500} = 0.6$$

WKT

The probability mass function of the poison distribution is $P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

The probability of exactly three misprints= $P(X=3) = \frac{e^{-0.6}(0.6)^3}{3!} = 0.01975$

The probability of less than three misprints= $P(x<3) = P(0) + P(1) + P(2)$

$$\Rightarrow P(x<3)=\frac{e^{-0.6}(0.6)^0}{0!}+\frac{e^{-0.6}(0.6)^1}{1!}+\frac{e^{-0.6}(0.6)^2}{2!}$$
$$\Rightarrow P(x<3)=e^{-0.6}[1+0.6+0.18]$$
$$\Rightarrow P(x<3)=0.5488\times 1.78$$
$$\Rightarrow P(x<3)=0.9788$$

The probability of four and more misprints= $P(x\geq 4)=1-P(x<4)$

$$\Rightarrow P(x\geq 4)=1-P(0)-P(1)-P(2)-P(3)$$
$$\Rightarrow P(x\geq 4)=1-\frac{e^{-0.6}(0.6)^0}{0!}-\frac{e^{-0.6}(0.6)^1}{1!}-\frac{e^{-0.6}(0.6)^2}{2!}-\frac{e^{-0.6}(0.6)^3}{3!}$$
$$\Rightarrow P(x\geq 4)=1-e^{-0.6}(1+0.6+0.18+0.036)=1-0.5488\times 1.816=0.00338$$

Exponential distribution:

Let X be a continuous random variable , then the probability density function of an Exponential distribution is $f(x)=\begin{cases}\alpha e^{-\alpha x} & for x\geq 0\\ 0 & Otherwise\end{cases}$

Note:

1. $f(x)\geq 0$
2. $\int_{-\infty}^{\infty} f(x)dx=1$
3. Mean of Exponential distribution $\mu=\frac{1}{\alpha}$
4. Variance of Exponential distribution $\sigma^2=\frac{1}{\alpha^2}$
5. S.D. of Exponential distribution $\sigma=\frac{1}{\alpha}$

PROBLEMS:

1) If X is an exponential variate with mean 3 , then find $P(x>1)$ & $P(x<3)$.

Sol.

Given X be a continuous random variable of an Exponential distribution is

$$f(x)=\begin{cases}\alpha e^{-\alpha x} & for x\geq 0\\ 0 & Otherwise\end{cases}$$

And given the mean of Exponential distribution is 3

$$\Rightarrow \frac{1}{\alpha}=3\Rightarrow \alpha=\frac{1}{3}$$

$$\therefore f(x)=\begin{cases}\frac{1}{3}e^{-\frac{x}{3}} & for x\geq 0\\ 0 & Otherwise\end{cases}$$

i) $P(x>1)=\int_1^\infty f(x)dx=\frac{1}{3}\int_1^\infty e^{\frac{-x}{3}}dx=-\left[e^{\frac{-x}{3}}\right]_1^\infty=-\left[e^{-\infty}-e^{\frac{-1}{3}}\right]=-\left[0-e^{\frac{-1}{3}}\right]=e^{\frac{-1}{3}}$

ii) $P(x<3)=\int_{-\infty}^{3} f(x)dx=\int_{-\infty}^{0} f(x)d+\int_0^3 f(x)dx=0+\frac{1}{3}\int_0^3 e^{\frac{-x}{3}}dx=-\left[e^{\frac{-x}{3}}\right]_0^3=-\left[e^{-1}-e^{0}\right]=1-\frac{1}{e}$

2) If X is an exponential variate with mean 4 , then find $P(0<x<1), P(x>2)$ & $P(-\infty<x<10)$.

Sol.-

Given X be a continuous random variable of an Exponential distribution is

$$f(x)=\begin{cases}\alpha e^{-\alpha x} & for x\geq 0\\ 0 & Otherwise\end{cases}$$

And given the mean of Exponential distribution is 3

$$\Rightarrow \frac{1}{\alpha}=4\Rightarrow \alpha=\frac{1}{4}$$

$$\therefore f(x)=\begin{cases}\frac{1}{4}e^{-\frac{x}{4}} & for x\geq 0\\ 0 & Otherwise\end{cases}$$

$$P(0<x<1)=\int_0^1 fr(x)dx=\frac{1}{4}\int_0^1 e^{\frac{-x}{4}}dx=-\left[e^{\frac{-x}{4}}\right]_0^1=-\left[e^{\frac{-1}{4}}-e^{0}\right]=1-\frac{1}{e^{\frac{1}{4}}}$$

$$P(x>2)=\int_2^\infty fr(x)dx=\frac{1}{4}\int_2^\infty e^{\frac{-x}{4}}dx=-\left[e^{\frac{-x}{4}}\right]_2^\infty=-\left[e^{-\infty}-e^{\frac{-2}{4}}\right]=e^{\frac{-1}{2}}$$

$$P(-\infty<x<10)=\int_{-\infty}^{10} fr(x)dx=\int_{-\infty}^{0} fr(x)dx+\int_0^{10} fr(x)dx=0+\frac{1}{4}\int_0^{10} e^{\frac{-x}{4}}dx$$

$$\Rightarrow P(-\infty<x<10)=-\left[e^{\frac{-x}{4}}\right]_0^{10}=-\left[e^{\frac{-10}{4}}-e^{0}\right]=1-\frac{1}{e^{\frac{5}{2}}}$$

3) In a certain town the duration of shower has mean 5 minutes , what is the probability that shower will last for

i) 10 minutes and more

ii) Less than 10 minutes

iii) Between 10 and 12 minutes.

Sol.

Given X be a continuous random variable of an Exponential distribution is

$$f(x)=\begin{cases}\alpha e^{-\alpha x} & for x\geq 0\\ 0 & Otherwise\end{cases}$$

And given the mean of Exponential distribution is 3

$$\Rightarrow \frac{1}{\alpha}=5\Rightarrow \alpha=\frac{1}{5}$$

$$\therefore f(x)=\begin{cases}\frac{1}{5}e^{-\frac{x}{5}} & for x\geq 0\\ 0 & Otherwise\end{cases}$$

i) The probability that the shower will last 10 minutes and more is

$$P(x\geq 10)=\int_{10}^{\infty} f(x)dx=\int_{10}^{\infty}\frac{1}{5}e^{-\frac{x}{5}}dx=\frac{1}{5}\int_{10}^{\infty}e^{-\frac{x}{5}}dx=-\left[e^{-\frac{x}{5}}\right]_{10}^{\infty}=-\left[0-e^{-2}\right]=\frac{1}{e^{2}}$$

The probability that the shower will last 10 minutes and more is

$$P(x<10)=\int_{-\infty}^{10} f(x)dx=\int_{-\infty}^{0} f(x)dx+\int_{0}^{10} f(x)dx$$

$$=0+\int_{0}^{10}\frac{1}{5}e^{-\frac{x}{5}}dx=\frac{1}{5}\int_{0}^{10}e^{-\frac{x}{5}}dx=-\left[e^{-\frac{x}{5}}\right]_{0}^{10}=-\left[e^{-2}-1\right]=1-\frac{1}{e^{2}}$$

The probability that the shower will last between 10 and 12 minutes is

$$P(10<x<12)=\int_{10}^{12} f(x)dx=\int_{10}^{12}\frac{1}{5}e^{-\frac{x}{5}}dx=\frac{1}{5}\int_{10}^{12}e^{-\frac{x}{5}}dx$$

$$\Rightarrow P(10<x<12)=-\left[e^{-\frac{x}{5}}\right]_{10}^{12}=-\left[e^{\frac{-12}{5}}-e^{-2}\right]=\frac{1}{e^{\frac{12}{5}}}-\frac{1}{e^{2}}$$

Normal Distribution:

Let X be continuous random variable , for any $\mu,\ \sigma^2$ the probability density function of the Normal distribution is defined as

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$, where $-\infty<x<\infty, -\infty<\mu<\infty$ and $\sigma^2>0$

Whereas $f(x)\geq 0$, $\int_{-\infty}^{\infty} f(x)dx=1$

and μ is called mean and σ is called standard deviation of the Normal distribution.

NOTE:

The graph of the probability density function $f(x)$ is always a bell shaped curve , symmetric about the line $x=\mu$ and it is called the normal probability curve and the line

$x = \mu$ divides the curve into two equal parts with the area of 0.5 sq units. and its total area of the normal probability curve is 1sq.units.

Standard Normal variate :

By the definition of Normal distribution ,

$$\text{we have } f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$P(a \le x \le b) = \int_a^b f(x)dx$$

$$\Rightarrow P(a \le x \le b) = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-\frac{z^2}{2}} dz = P(z_1 \le z \le z_2)$$

Where $z = \dfrac{x-\mu}{\sigma}$, $z_1 = \dfrac{a-\mu}{\sigma}$, $z_2 = \dfrac{b-\mu}{\sigma}$

and $F(z) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ is called the standard normal function

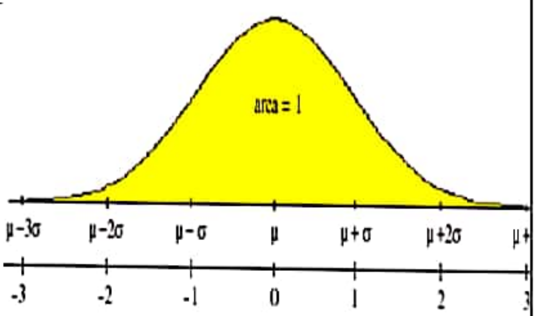
and $z = \dfrac{x-\mu}{\sigma}$ is called the standard normal variate.

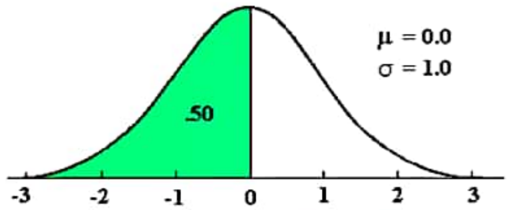
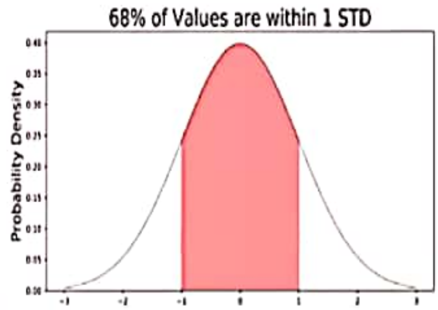
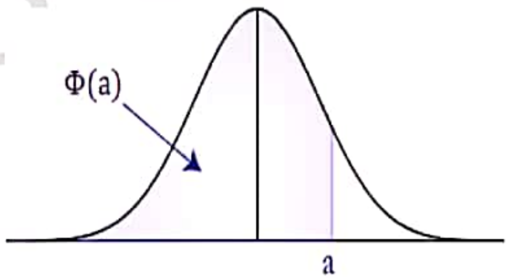
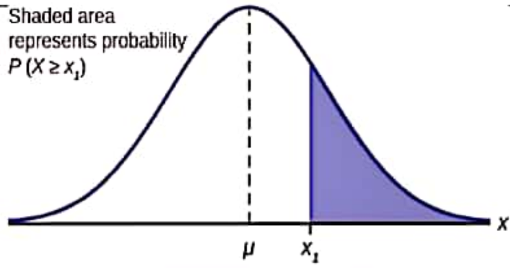
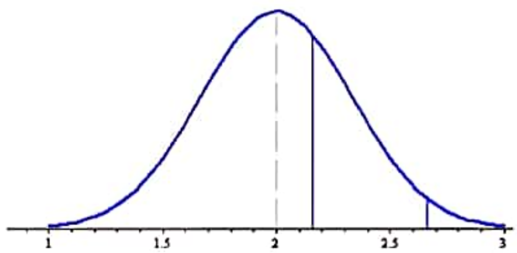
when $z_1 = 0$, $z_2 = z$, then the normal curve over 0 to z is defined as

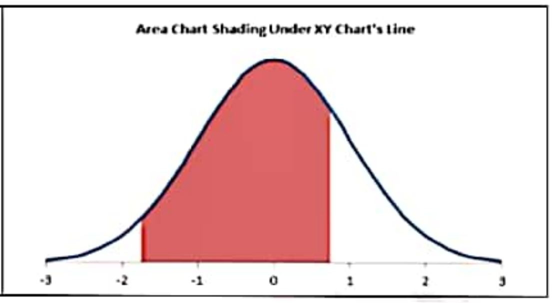
$$A(z) = \phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{z^2}{2}} dz$$

where these values will be taken from Area table of normal distribution.

Some important results:

Sl.No.	Probability Range	Result	Graph
1	$P(-\infty < z < \infty)$	1	

2	$P(-\infty < z < 0) = P(0 < z < \infty)$	0.5	
3	$P(-z_1 < z < z_1) = 2P(0 < z < z_1)$	$2A(z_1)$	
4	$P(-\infty < z < z_1) = 0.5 + P(0 < z < z_1)$	0.5+ $A(z_1)$	
5	$P(z_1 < z < \infty) = P(0 < z < \infty) - P(0 < z < z_1)$	0.5- $A(z_1)$	
6	$P(z_1 < z < z_2) = P(0 < z < z_2) - P(0 < z < z_1)$	$A(z_2)$ - $A(z_1)$	

7	$P(-z_1 < z < z_2) = P(0 < z < z_2) + P(0 < z < z_1)$	$A(z_2)$ + $A(z_1)$	

PROBLEMS:

1) **The marks of 1000 students in an examination follows normal distribution with mean 70 and standard deviation 5. Find the number students whose marks will be**
   i) Less than 65
   i) More than 75
   ii) Between 65 and 75. [A(1)=0.3413]

Sol.

Let X be the continuous random variable
Given
Mean of the Normal distribution $\mu = 70$
Standard deviation of the Normal distribution $\sigma = 5$
$\therefore$ The standard normal variate $z = \dfrac{x-\mu}{\sigma} \Rightarrow z = \dfrac{x-70}{5}$

When x=65 then $z = \dfrac{65-70}{5} = -1$

When x=75 then $z = \dfrac{5-70}{5} = 1$

i) No.0f students scored less than 65 marks= $P(x<65) = P(z<-1) =$
$P(z>1) = 0.5 - A(1) = 0.5 - 0.3413 = 0.1587$

No.0f students scored less than 65 marks out of 1000 students=1000x0.1587=158.7=159

ii) No.0f students scored more than 75 marks= $P(x>75) = P(z>1) =$
$P(z>1) = 0.5 - A(1) = 0.5 - 0.3413 = 0.1587$

No.0f students scored more than 75 marks out of 1000 students=1000x0.1587=158.7=159

iii) No.0f students scored marks between 65 and 75 $= P(65 < x < 75) = P(-1 < z < 1)$

$$= 2P(0 < z < 1)$$
$$= 2A(1)$$
$$= 2 \times 0.3413$$
$$= 0.6826$$

No.0f students scored between65 and 75 marks out of 1000 students=1000x0.6826
=682.6=683

2) **200 students appeared in an examination, distribution of marks is assumed to be**

normal with mean 30 and standard deviation 6.25, how many students are expected to get marks .

i) Between 20 and 40

ii) Less than 35 [A(1.6)=0.4452 , A(0.8)=0.2881]

Sol.

Let X be the continuous random variable

Given

Mean of the Normal distribution $\mu = 30$

Standard deviation of the Normal distribution $\sigma = 6.25$

$\therefore$ The standard normal variate $z = \dfrac{x-\mu}{\sigma} \Rightarrow z = \dfrac{x-30}{6.25}$

When x=20 then $z = \dfrac{20-30}{6.25} = -1.6$

When x=40 then $z = \dfrac{40-30}{6.25} = 1.6$

When x=35 then $z = \dfrac{35-30}{6.25} = 0.8$

The probability that number of students expected to score between 20 and 40 marks:

$$P(20 < x < 40) = P(-1.6 < z < 1.6)$$
$$\Rightarrow P(20 < x < 40) = 2 \times P(0 < z < 1.6)$$
$$\Rightarrow P(20 < x < 40) = 2 \times A(1.6)$$
$$\Rightarrow P(20 < x < 40) = 2 \times 0.4452 = 0.8904 = 0.9$$

The number of students expected to score between 20 and 40 marks out of 200:

The probability that number of students expected to score less than 35:

$$= P(x < 35) = P(z < 0.8)$$
$$= A(0.8)$$
$$= 0.2881 = 0.3$$

The number of students expected to score less than 35 marks out of 200:

= 200x0.3=60

3) **The weekly wages of workers in a company are normally distributed with mean of Rs.700 and S.D. of Rs.50.Find the probability that the weekly wage of randomly chosen workers is i) Between Rs.650 and Rs.750 ii) More than Rs.750.**

Sol.

Let X be the continuous random variable

Given

Mean of the Normal distribution $\mu = 700$

Standard deviation of the Normal distribution $\sigma = 50$

$\therefore$ The standard normal variate $z = \frac{x-\mu}{\sigma} \Rightarrow z = \frac{x-700}{50}$

When x=650 then $z = \frac{650-700}{50} = -1$

When x=750 then $z = \frac{750-700}{50} = 1$

The probability of the weekly wages between Rs.659 and Rs.750 is:

$$P(650 < x < 750) = P(-1 < z < 1)$$
$$\Rightarrow P(650 < x < 750) = 2 \times P(0 < z < 1)$$
$$\Rightarrow P(650 < x < 750) = 2 \times A(1)$$
$$\Rightarrow P(650 < x < 750) = 2 \times 0.3413 = 0.6826$$

The probability of the weekly wages of more than Rs.750 is:

$$= P(x > 750) = P(z > 1)$$
$$= 0.5 - P(z < 1)$$
$$= 0.5 - 0.3413$$
$$= 0.17065$$

4) In a test on 2000 electric bulbs , it was found that the life of a particular make was normally distributed with an average life of 2040 hours and SD of 60 hours . Estimate the number of bulbs likely to burn for...

i) More than 2150 hours

ii) Less than 1950 hours

iii) Between 1920 and 2160 hours.

Sol.

Let X be the continuous random variable

Given

Mean of the Normal distribution $\mu = 2040$

Standard deviation of the Normal distribution $\sigma = 60$

$\therefore$ The standard normal variate $z = \frac{x-\mu}{\sigma} \Rightarrow z = \frac{x-2040}{60}$

When x=2150 then $z = \frac{2150-2040}{60} = 1.83$

When x=1950 then $z = \frac{1950-2040}{60} = -1.5$

When x=1920 then $z=\frac{1920-2040}{60}=-2$

When x=2160 then $z=\frac{2160-2040}{60}=2$

i) The probability that the number of bulbs likely to burn of more than 2150 hours:

$$P(x>2150)=P(z>1.83)=$$

$$P(z>1.83)=0.5-A(1.83)=0.5-0.4664=0.0336$$

The number of bulbs likely to burn of more than 2150 hours out of 2000 bulbs :

=2000x0.0336

=67.2=67

ii) The probability that the number of bulbs likely to burn of less than 1950 hours:

$$=P(x<1950)=P(z<-1.5)=$$

$$P(z>1.5)=0.5-A(1.5)=0.5-0.4332=0.0668$$

The number of bulbs likely to burn of less than 1950 hours out of 2000 bulbs :

=2000x0.0668

=133.6=137

iii) The probability that the number of bulbs likely to burn between 1920 and 2160 hours

$$=P(1920<x<2160)=P(-2<z<2)$$

$$=2P(0<z<2)$$

$$=2A(2)$$

$$=2\times 0.4772$$

$$=0.9544$$

The number of bulbs likely to burn between 1920 and 2160 hours out of 2000 bulbs :

=2000x0.9544

=1908.8=1909

5) In a normal distribution , 7% of items are under 35 and 89% of the items are under 63.Find the mean and standard deviation of the distribution.

Sol.

Let X be the continuous random variable

Given

Let μ and σ be the Mean and Standard deviation of the distribution

$\therefore$ The standard normal variate $z=\frac{x-\mu}{\sigma}$ --------------(1)

When x=35 the standard normal variate $z = \frac{35-\mu}{\sigma} = z_1 (Say)$

When x=63 the standard normal variate $z = \frac{63-\mu}{\sigma} = z_2 (Say)$

Given

$$P(x<35) = P(z<z_1) = 0.07$$
$$\Rightarrow P(z<z_1) = P(-\infty<z<0) - P(0<z<z_1) = 0.07$$
$$\Rightarrow 0.5 - A(z_1) = 0.07$$
$$\Rightarrow A(z_1) = 0.5 - 0.07$$

$$\Rightarrow A(z_1) = A(-1.47)$$
$$\Rightarrow z_1 = -1.47$$
$$\Rightarrow \frac{35-\mu}{\sigma} = -1.47$$
$$\Rightarrow \mu - 1.47\sigma = 35 ------(2)$$

And $P(x<63) = P(z<z_2) = 0.89$

$$\Rightarrow P(z<z_2) = P(-\infty<z<0) + P(0<z<z_2) = 0.89$$
$$\Rightarrow 0.5 + A(z_2) = 0.89$$
$$\Rightarrow A(z_2) = 0.89 - 0.5 = 0.39$$
$$\Rightarrow A(z_2) = A(1.23)$$
$$\Rightarrow z_2 = 1.23$$
$$\Rightarrow \frac{63-\mu}{\sigma} = 1.23$$
$$\Rightarrow \mu + 1.23\sigma = 63 --------(3)$$

Solving eq(2) and (3)
we get
$\mu = 50.2915$
$\sigma = 10.332$

6) **In a normal distribution , 31% of items are under 45 and 8% of the items are 0ver 64.Find the mean and standard deviation of the distribution.**

Sol.

Let X be the continuous random variable
Given
Let μ and σ be the Mean and Standard deviation of the distribution

$\therefore$ The standard normal variate $z = \dfrac{x-\mu}{\sigma}$ --------------(1)

When x=35 the standard normal variate $z = \dfrac{45-\mu}{\sigma} = z_1 (Say)$

When x=63 the standard normal variate $z = \dfrac{64-\mu}{\sigma} = z_2 (Say)$

Given

$$P(x<35) = P(z<z_1) = 0.31$$
$$\Rightarrow P(z<z_1) = P(-\infty<z<0) - P(0<z<z_1) = 0.31$$
$$\Rightarrow 0.5 - A(z_1) = 0.31$$
$$\Rightarrow A(z_1) = 0.5 - 0.31 = 0.19$$

$$\Rightarrow A(z_1) = A(0.5)$$
$$\Rightarrow z_1 = 0.5$$
$$\Rightarrow \frac{45-\mu}{\sigma} = 0.5$$
$$\Rightarrow \mu + 0.5\sigma = 45 ------(2)$$

And $P(x>64) = P(z>z_2) = 0.08$

$$\Rightarrow P(z>z_2) = 0.5 - P(0<z<z_2) = 0.08$$
$$\Rightarrow 0.5 - A(z_2) = 0.08$$
$$\Rightarrow A(z_2) = 0.08 - 0.5 = -0.42$$
$$\Rightarrow A(z_2) = A(1.4)$$
$$\Rightarrow z_2 = 1.4$$
$$\Rightarrow \frac{64-\mu}{\sigma} = 1.4$$
$$\Rightarrow \mu + 1..4\sigma = 64 ---------(3)$$

Solving eq(2) and (3)
we get

$$\mu = 50$$
$$\sigma = 10$$

7) In an examination 7% of the students scored less than 35% of the marks and 89% of the students scored less than 60% of the marks .Find the mean and standard deviation if marks are normally distributed.

Sol.

Let X be the continuous random variable
Given

Let μ and σ be the Mean and Standard deviation of the distribution

$\therefore$ The standard normal variate $z = \dfrac{x-\mu}{\sigma}$ --------------(1)

When x=35 the standard normal variate $z = \dfrac{35-\mu}{\sigma} = z_1 (Say)$

When x=63 the standard normal variate $z = \dfrac{60-\mu}{\sigma} = z_2 (Say)$

Given

$$P(x<35) = P(z<z_1) = 0.07$$
$$\Rightarrow P(z<z_1) = P(-\infty<z<0) - P(0<z<z_1) = 0.07$$
$$\Rightarrow 0.5 - A(z_1) = 0.07$$
$$\Rightarrow A(z_1) = 0.5 - 0.07$$

$$\Rightarrow A(z_1) = A(-1.47)$$
$$\Rightarrow z_1 = -1.47$$
$$\Rightarrow \frac{35-\mu}{\sigma} = -1.47$$
$$\Rightarrow \mu - 1.47\sigma = 35 -------(2)$$

And $P(x<60) = P(z<z_2) = 0.89$

$$\Rightarrow P(z<z_2) = P(-\infty<z<0) + P(0<z<z_2) = 0.89$$
$$\Rightarrow 0.5 + A(z_2) = 0.89$$
$$\Rightarrow A(z_2) = 0.89 - 0.5 = 0.39$$
$$\Rightarrow A(z_2) = A(1.23)$$
$$\Rightarrow z_2 = 1.23$$
$$\Rightarrow \frac{60-\mu}{\sigma} = 1.23$$
$$\Rightarrow \mu + 1.23\sigma = 60 ---------(3)$$

Solving eq(2) and (3)
we get

$\mu = 48.65$

$\sigma = 9.25$

8) The IQ of students in a certain college is assumed to be normally distributed with mean 100 and variance 25. If two students are selected at random , find the probability that
(i) both of them have IQ between 102 and 110
(ii) at least one of them have IQ between 102 and 110

(iii) at most one of them have IQ between 102 and 110.

Sol.

Let X be the continuous random variable

Given

Let μ and σ be the Mean and Standard deviation of the distribution

$\therefore$ The standard normal variate $z = \dfrac{x-\mu}{\sigma}$ --------------(1)

and given

$\mu = 100, \sigma^2 = 25 \Rightarrow \sigma = 5$

$\therefore z = \dfrac{x-100}{5}$ ------(2)

When x=102 , the standard normal variate $z = \dfrac{102-100}{5} = 0.4$

When x=110 , the standard normal variate $z = \dfrac{110-100}{5} = 2$

i) The probability of of both of them have IQ between 102 and 110 is

$$P(102 < x < 110) = P(0.4 < z < 2)$$

$$\Rightarrow P(0.4 < z < 2) = P(0 < z < 2) - P(0 < z < 0.4)$$

$$\Rightarrow P(0.4 < z < 2) = A(2) - A(0.4)$$

$$\Rightarrow P(0.4 < z < 2) = 0.4772 - 0.1554$$

$$\Rightarrow P(0.4 < z < 2) = 0.3218 = p \Rightarrow q = 1 - p = 1 - 0.3218 = 0.6782$$

$\therefore$ The probability that both the students have I.Q. between 102 and 110 is $(0.3218)^2 = 0.1036$

ii) The probability that at least one of two students have I.Q. between 102 and 110 is (By using Binomial distribution) $= P(x \geq 1) = 1 - P(x < 1)$

$$\Rightarrow P(x \geq 1) = 1 - P(x = 0)$$

$$\Rightarrow P(x \geq 1) = 1 - \left[2_{c0}\, p^0 q^{2-0}\right]$$

$$\Rightarrow P(x \geq 1) = 1 - \left[1 \times 1 \times (0.6782)^2\right]$$

$$\Rightarrow P(x \geq 1) = 1 - 0.4600$$

$$\Rightarrow P(x \geq 1) = 0.5400$$

iii) The probability that at most one of them have IQ between 102 and 110 is

$$P(x \leq 1) = 1 - P(x > 1)$$

$\Rightarrow P(x \le 1) = 1 - P(x = 2)$

$\Rightarrow P(x \le 1) = 1 - 2_{C2}\, p^2 q^{2-2}$

$\Rightarrow P(x \le 1) = 1 - \left[2_{C2} (0.3218)^2 (0.6782)^{2-2}\right]$

$\Rightarrow P(x \le 1) = 1 - [1 \times 0.1036 \times 1]$

$\Rightarrow P(x \le 1) = 1 - 0.1036$

$\Rightarrow P(x \le 1) = 0.8964$