MSc Mathematics

Master thesis

Estimating operational capacity of Dutch intermediate care using queueing models

by

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Preface

This thesis is part of the master's Mathematics educated at VU University Amsterdam. A seven-month internship at the Nederlandse Zorgautoriteit (NZa) is the basis of this thesis. During this internship, analyses are done at the NZa and the thesis is written, while I am part of the managing board Regulering and the unit Zorgbrede Regulering en Vernieuwing (ZRV).

The goal of the thesis is graduation on one hand and on the other hand giving insight to NZa within the subject of interest. The subject of interest is the capacity of ELV and GR within the Netherlands. The goal of the research done for this thesis is threefold, first to find the operational capacity of ELV and GR, second to determine if there is a shortage in capacity, and last to determine a capacity such that only 2% is blocked.

During the period in which I have written this thesis, I learned a lot about the organization of Dutch healthcare. This would not have been possible without the help of my two supervisors from the host organization, NZa: Elisabeth Wever and Matthew Pentecost, and their connections within NZa. Therefore, I would like to thank both Elisabeth and Matthew, as well as for their active involvement and their willingness to think along. On top of that, I would like to thank the whole unit ZRV, who all gave me a warm welcome to the team.

For mathematical and modeling advice I could count on my supervisor from the VU university: René Bekker. I would like to thank René for his enthusiastic supervision with numerous good ideas.

Next to my supervisors, I had contact with others inside and outside NZa. Several have thought along and provided me with information to successfully create this thesis. I would like to thank everybody from NZa that thought along and got me in touch with the people from the field. I would like to thank all external relations that provided me with interesting and essential information that I asked for.

Next to everyone who has helped me with content-related problems and questions, I would like to thank my family, my boyfriend, and my friends for supporting me throughout the period in which I was writing my thesis. They supported me mentally but also functioned as sparring partners.

Abstract

What is the operational capacity of intermediate care in the Netherlands and is there a capacity shortage? How many beds are needed to have a maximum blocking percentage of 2%? These questions are interesting for this thesis. Conclusions can be made by the use of historical declaration data and queueing models. The queueing models are used in two ways: by the use of analytical calculations and simulations.

In the 31 healthcare offices in the Netherlands, healthcare is autonomously organized, which causes a need for multiple models. Both the $M_t|M|\infty$ and $M_t|M|s|s$ queues are used. The operational capacity could be found by neither model. By the use of the models compared to the actual data, it can be stated that there is a capacity shortage for some types of intermediate care in selected regions. With the use of the two models, there is a range given for the needed capacity for a maximum of 2% blockage.

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Contents

1. Introduction

In the Netherlands, the healthcare system is well organized, but there are enough opportunities left to improve efficiency. One thing that could be optimized is the flow into some types of intermediate care. Patients that enter intermediate care can originate from the hospital or their home situation. The path a patient follows before ending up in an intermediate care facility is unique per patient. Similarly, it is unique how the patient enters the healthcare system, it could be with a scheduled surgery or with an acute demand for care. At the end of the healthcare system, intermediate care is found, which is mainly used to provide care for vulnerable elderly that are temporarily not able to live at home by themselves.

There are multiple types of intermediate care. A patient can receive extra help at home from district nursing or needs to go to some sort of nursing home. Patients that are not yet able to go home by themselves, but are in no need of staying in the hospital longer can block hospital outflow. If the intermediate care residences have no space, the patient cannot leave the hospital. The capacity problem can cause patients to stay in the hospital unnecessarily, which will be called 'wrong bed'.

A 'wrong bed' will give three main problems, the first is that the costs for a hospital bed are a lot higher than a bed in intermediate care. The second problem is that new patients are blocked, or also take a 'wrong bed' directly upon arrival. On top of that, older patients usually deteriorate while staying in the hospital, therefore it is not preferable to have patients unnecessarily in the hospital. Eventually, when there are a lot of 'wrong beds' occupied, this can cause that ED's are full, and cannot take any new emergency patients. These problems of a 'wrong bed' are not all caused by the intermediate care we consider, but they could partly be decreased by improving the efficiency of the intermediate care facilities.

During this research, which takes place at the Nederlandse Zorgautoriteit (NZa) (see section 2.1), the focus will be on patients that need to go to short-term residential care (in Dutch: Eerstelijnsverblijf), or geriatric rehabilitation (GR). From now on short-term residential care is referred to as ELV, by the Dutch abbreviation.

At the ELV there are three types of care: high and low complex care and palliative care. All three types of ELV are mostly meant for people that temporarily cannot live safely at home [15]. ELV is a type of inpatient care, so patients have a bed at a residence for medical care which is the responsibility of a general practitioner, a specialist elderly care, or a doctor for mentally disabled people. So, it is not medical-specialistic care. Usually, ELV is meant for short-term medical care [39].

GR is targeted at vulnerable elderly and is the responsibility of a specialist elderly care. The goal of this type of care is to help these patients to be able to return to their homes and to be able to participate as well as possible in society [38]. ELV and GR are more elaborately explained in Section 2.2.

Within some institutions that offer ELV or GR residences, it is unknown how much operational capacity they have. However, some institutions do know how much their operational capacity is. The operational capacity does not only depend on the capacity of physical beds, but also on the number of nurses specialized in either ELV or GR. The capacity of physical beds exceeds the number of operational beds. Some institutions can also have fluctuations in capacity, since an available bed for ELV can also be used as a GR bed, as some other type of intermediate care, or even as a WLZ bed. WLZ is long-term residential care (in Dutch: Wet Langdurige zorg), usually located at nursery homes. Besides, there are multiple types of care within ELV residences, all of which need a different amount of care, thus a different number of nurses. Altogether, the institutions do not have a clear insight into and overview of the available capacity.

That the operational capacity of intermediate care institutions is not generally known makes the outflow of hospital patients harder, which can have devastating results in the hospital. Hospital employees are manually searching for a bed in an intermediate care institution, by calling several institutions until they have found an available bed or give up. In which giving up means that the hospital employee stops searching for that day and the patient stays a day on a 'wrong bed'. Luckily, there are some regions within the Netherlands where this goes in a better-organized way. These regions have closer connections between hospital and intermediate care residences, such that it is more transparent how many beds are free and distributing patients is organized via a central point. When capacity would be regionally known per institution, better insight is generated, which increases the efficiency of allocating patients to available intermediate care beds.

The NZa focuses on the availability and affordability of healthcare in the Netherlands, so they want to prevent the devastating results that can be caused by a blockage in the outflow of patients. This leads to the objective of this research: estimating the operational capacity and the required operational capacity of ELV and GR for selected regions in the Netherlands. The objective will be reached with the use of available data over the years 2016 to 2020. This data includes stays of patients in some hospital departments and in ELV and GR intermediate care residences. A more elaborate explanation of what is included in the data can be found in Chapter 4.

With the use of queueing models, based on the available data, the situation will be simulated and calculations will be done. Multiple queueing models can be used to approximate the real situation best. With the use of analytical calculations some approximations can be done on the number of the operational capacity and the required capacity, with the use of simulations these results can be verified. Dependent on the type of model the number of beds can also follow from the simulation or the number of calculated beds is only verified with the use of simulation. An infinite server model can be used to simulate the number of needed beds, while the number of beds should be given as input for a blocking model. What type of model fits best to the data differs per region in the Netherlands, caused by the autonomy of the regions regarding healthcare organization. The results that follow from the models are compared to the data to find the best fit model. Per type of care and per region it should be decided which model is the best fit and what is the corresponding approximation of the number of beds.

The corresponding research questions can be formed as:

- *What is the operational capacity of ELV and GR in selected regions in the Netherlands?*
- *Is there a shortage of ELV or GR capacity within the Netherlands?*
- *How many beds are needed within the studied regions to produce a maximum blocking probability of 2%?*

To answer these questions this thesis is built up as follows. There is an informative chapter (Chapter 2) on the Dutch healthcare system including information about the host organization: Nederlandse Zorgautoriteit. Next, Chapter 3 about the theoretical concepts explains some distributions and some queueing models. After these two theoretical chapters, the data is analyzed, which is elaborately discussed in Chapter 4. During the data analysis, the distributions for the arrivals and the length of stay are found, but there is also looked into the occupancy, the relation between the arrivals and occupancy, the routes of patients, and the analysis on a regional level. Based on the data analysis and the theoretical concepts, an explanation is given on the used models in Chapter 5. Thereafter, the results are shown in Chapter 6, which is used to draw conclusions in Chapter 7. To end the thesis, Chapter 8 gives a discussion of the limitations of this research and recommendations for future research.

2. Dutch health system

In this chapter, the Dutch healthcare system is explained. First, the host organization NZa is discussed, with their role in Dutch healthcare (Section 2.1). Next, Section 2.2 is about the healthcare system and is divided into three subsections: the patient flows (Subsection 2.2.1), the considered healtcare departments (Subsection 2.2.2), and healthcare offices (Subsection 2.2.3).

2.1. Nederlandse Zorgautoriteit

The Dutch Healthcare Authority (in Dutch: Nederlandse Zorgautoriteit, shortly NZa) is an independent agency of the Dutch ministry of Health, welfare, and sport. NZa is established on October 1, 2006. The tasks of NZa are making rules for healthcare institutions and healthcare insurance companies and monitoring the abidance of these rules. [37]

To describe the NZa and her core values, the website of the NZa (www.nza.nl) is consulted. The goal of NZa is that all inhabitants of the Netherlands receive the care they need, now and in the future. The NZa focuses on good and affordable healthcare in the Netherlands. The ambition of the NZa is to take care that everything within healthcare delivers added value for the patient and contributes to the quality of life. To do all this, the NZa has 4 areas of focus for the upcoming years:

Appropriate care. The focus should be on the patient, the type of care needs to be necessary, it has to add value to the quality of life and it needs to connect to the needs and circumstances of the patient. The funding will be better connected to the demand for care and less targeted to production. [21]

Good management & professional business operations. Within this core value, the focus is on the accessibility and affordability of care. Healthcare money needs to be invested in appropriate care. The processes need to be well organized and it should be possible to trust the healthcare provider to deliver appropriate care and to declare correctly. NZa offers guidelines for healthcare providers, insurance companies, healthcare offices, and CAK[1]. Harmful behavior will be stopped as soon as possible and sanctions will follow whenever needed and useful. The management is good when a healthcare provider offers appropriate care, healthcare money is not wasted and a healthcare provider is open and transparent to those with interest. [21]

Data-driven management & execution. Data is used by NZa as the basis for

[1]CAK is a Dutch public law independent management element that is commissioned by the Dutch ministry of public health, well-being and sports (VWS). CAK has legal and administrative tasks in the field of healthcare and well-being.

decision-making and interpreting developments. By data-driven supervision, NZa enlarges her anticipating ability, and assessments and explorations based on data help NZa to regulate healthcare. Hence, NZa gives insight and overview. So, NZa can judge (sensitive) issues objectively and gives advice. [21]
Agile organization. NZa enlarges her impact by signalizing societal developments early, adapting rules, and monitoring. So, NZa needs to be proactive about what is needed and NZa needs to collaborate with healthcare professionals and healthcare insurances. Inside NZa there is a need for curiosity and decisiveness to see what happens outside and translate these occurrences to work NZa needs to do. [21]

2.2. Dutch healthcare system

2.2.1. Patient flows

Before the patient goes to the correct intermediate care department, the patient first needs to enter the care system, this can happen via several routes. A patient can have an emergency and enters the hospital for acute care, or a patient can have an appointment and enters the hospital for elective care. It can also happen that a patient does not need to go to the hospital, but is referred by a general practitioner to an intermediate care institution.

There are multiple routes a patient can take before ending up in ELV or GR. NZa created Figure 2.1 for acute care, in which it is made visible that acute care patients can enter intermediate care via three routes. Patients leave the (emergency department) ED and immediately enter intermediate care, or patients leave a hospital ward and then go to intermediate care, the last route is via the general practitioner or a specialist elderly care. Note that a patient who enters intermediate care does not necessarily originate from the hospital. The percentages of outflow for 2019-2020 are also given in Figure 2.1. From the ED 0.04% of the patients goes directly to GR and 0.16% goes directly to ELV. 33.4% of the ED patients goes to hospital wards. Of the ED patients that go to the hospital wards, 5.95% goes to GR afterward and 1.43% goes to ELV. [22]

The percentages given above are all for patients that started their route at the ED. This is only a part of all patients that go to ELV or GR. Even if patients do not need acute care, they follow about the same routes as in Figure 2.1. Thus, a patient can enter ELV or GR via a general practitioner or specialist elderly care while still being at home, or the patient leaves the hospital and then needs some extra care before going home. These routes are the only two possible to enter ELV or GR. The patients can also go from ELV to GR or the other way around, although this is not preferable.

2.2.2. Healthcare departments

Within the Dutch healthcare system, there are multiple types of care.

The two departments of intermediate care residences that will be considered are ELV and GR. In Chapter 1 there was already some explanation about these two. Now, these

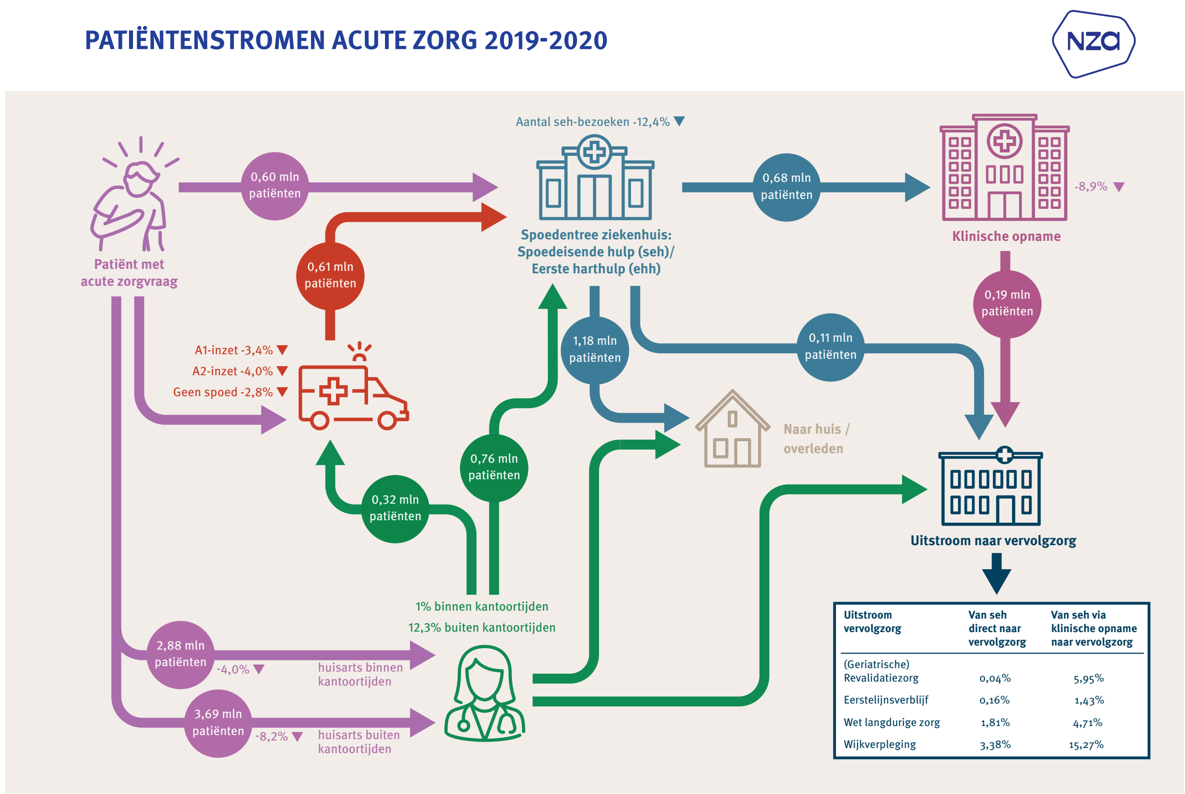


Figure 2.1.: Patient flows through the healthcare system in the period 2019-2020, made by NZa [19].

types of care are explained in more detail. In Appendix C definitions for some types of hospital care are given.

ELV is in Verenso's assessment tool [34] described as an intermediate care residence for patients that need a temporary residence for a period they are not able to live at home by themselves. ELV is a medical urgent short-term stay of minimally 24 hours and maximally 18 weeks. This type of intermediate care is meant for vulnerable people that can temporarily not live responsibly in their own living environment. These patients are in no need of a hospital stay, or another care residence with medical specialist treatment. The goal of an ELV stay is to analyze the condition(s) and/or the corresponding restrictions, and reduce these such that the patient can go back to his/her original living environment. For a patient to qualify for ELV he/she needs to satisfy one of the following situations:

- The health situation of the patient temporarily changes or threatens to change, such that the medical care at home is insufficient and serious health loss impends.
- The patient has finished treatment in an institution for healthcare or medical specialist care. The patient still has existing restrictions which need medical and

nursing care.

- An incident happens with the current caregiver (a person in the patient's network who helps with the daily activities), such that the needed care is no longer possible to give responsibly at home.

- District nursing at home cannot appropriately be organized.

For all of these situations, the other options need to be investigated and there are no options for the patient to stay at home and receive extra care. The patient can be of any age, but most of the time patients are classified as elderly (70+).

Important is that a patient at ELV can only stay temporarily. After the stay, the patient needs to go home or needs to be transferred to another type of care.

Within ELV there are three types of residence.

Low complex care is for patients that have a single condition that could be a threat to the patient's health. These patients need nursing and care close by, observation, signaling, and intervention. The patient receives support with daily tasks. The medical care is performed by the general practitioner. [34]

High complex care is for patients that have multiple conditions that can affect each other. Often there is a case of poly-pharmacy and its possible effects. The patients need close care and nursing. The treatment of a patient needs to take place in a live and treatment environment that is specific to the patient's conditions. The nurse supports daily tasks and the specialist handles both general and specialist care. [34]

Palliative care could be part of care within ELV. This type of care focuses on the improvement of the quality of life for patients that face problems with a life-threatening disease. Palliative care prevents and softens suffering by early recognition and excellent diagnostics, and by the use of pain relief. Palliative care mainly focuses on coaching in loss of functionalities and loss of quality of life. [34]

These three types of ELV take place at different locations. There are two types of nursing homes in the Netherlands. ELV patients with a low complex indication go to a nursing home in which the patient is taken care of but does not undergo any specific treatments (in Dutch: verzorgingshuis). ELV high complex patients can go to a nursing home in which treatment is provided (in Dutch: verpleeghuis) and the patients are being nursed as well. The beds that are used for patients with an ELV high complex indication can also be used for patients with a GR indication. It also happens that some institutions use a bed for both ELV low complex and ELV high complex. The division between low and high complex patients in the types of nursing homes is not strict. The beds for palliative care are usually specially reserved for these palliative patients, often in a hospice. [34]

GR is the other type of intermediate care we are considering during this thesis. Verenso did define GR in a triage tool [35] as follows: "integrated multidisciplinary care which targets expected recovery of functioning and participating of frail elderly after an acute condition or functional deterioration". This includes a patient on which medical specialist diagnostic/intervention has taken place, by which usually the patient was hospitalized and this treatment has finished. Subsequently, there is a need for medic rehabilitation treatment which requires an integral and multidisciplinary approach. Next to the condition for which the patient needs rehabilitation, the patient also struggles with other problems such as vulnerability or comorbidity, which together make it harder for the patient to learn and train. The rehabilitation follows a treatment plan that directly joins a medical intervention by a medical specialist. The treatment focuses on the return to the home environment or a nursing home.

Before a patient is admitted to a GR residence there is a screening. In the screening it is measured how vulnerable the patient is, usually a minimum age of 70 years is taken, however, younger patients can also qualify for GR. Instead of the calendar age, the biological age is taken into account for the screening of a patient. If a patient is qualified as vulnerable, there are two follow-up questions asked:

- Is there one or multiple conditions that could result in (permanent) restrictions?
- Does the patient need extra help with reducing the restrictions compared to the basic mono-disciplinary treatment and/or the basic nursing care?

Only if the patient answers both questions with ‘yes’, the triage continues. The next step is creating a patient profile. Within this patient profile, the diagnosis, the side diagnosis (multi-morbidity), the pre-morbidity, the actual functioning, and the medical stability together form insight into the functional prognosis of the patient. Based on the patient profile there is chosen between medical specialist rehabilitation care and geriatric rehabilitation care. This choice depends on the demand for care that follows from the functional prognosis, the medical stability, the needed intensity of the treatment, and later on the motivation of the patient. [35]

2.2.3. Healthcare offices

The Netherlands is divided into 31 regions for healthcare, Figure A.1 in Appendix A visualizes these regions. In these regions, healthcare is organized by healthcare offices. In all of these regions, there is at least one hospital, and multiple ELV and GR institutions. However, during 2018 the hospital MC Zuiderzee in the healthcare office region Flevoland went bankrupt, which caused this region to no longer have any hospitals. The healthcare offices are made for long-term residential care, and therefore are not especially used for ELV and GR. On the other hand, there are no preset regions on which ELV and GR are organized, so it is decided to use these healthcare office regions as well for ELV or GR within this research. Within NZa these healthcare offices are usually used for ELV and GR analyses.

3. Preliminaries

In this chapter mathematical concepts are explained and divided into two sections. The first section (3.1) explains distribution functions and the second section (3.2) explains queueing models. Firstly, there is no time dependency assumed, later there is time dependency assumed for the arrivals and thereafter time dependency for the length of stay is considered.

3.1. Distribution functions

During this thesis arrival processes and length of stay distributions are analyzed for patients in different sorts of intermediate care in the Dutch healthcare system. Some of these distributions are frequently used in queueing theory, while others are used less often.

Poisson distribution
For the arrival distribution, a common distribution is Poisson. A Poisson distribution is discrete, which means that there is an integer number of patients in the system, on top of that, the number of patients needs to be positive. The Poisson distribution results in the number of arrivals per time unit. During this research, the time unit is one day. The Poisson distribution takes one input, the arrival rate, which is the mean number of arrivals per time unit, in this case per day. The density function of the Poisson distribution, with arrival rate λ, is given by:

$$\mathbb{P}(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}.$$

For a Poisson arrival process, the interarrival times, the time between two arrivals, follows an exponential distribution. This is only possible because the events that happen (patients that arrive) are independent of one another, which can also be seen as the memory-less property. There is no planning involved if an arrival process is Poisson, it can be seen as random arrivals during a fixed period. A Poisson process is the only natural arrival process. There are no other known arrivals processes, however, the number of arrivals can also follow from a different distribution. For these other distributions, there are no known arrival processes where the interarrival times can be calculated. [24, 14]

Despite that the Poisson distribution is the only known distribution that induces an arrival process, there will be some other distributions fitted with the data during the data analysis, just for comparison.

Lognormal distribution
A fitted arrival distribution is the positive and continuous lognormal distribution. If random variable X is lognormally distributed, then its logarithm $Y = log(X)$, is normally distributed with mean $\mu_Y = \ln\left(\frac{\mu_X^2}{\sqrt{\mu_X^2+\sigma_X^2}}\right)$ and variance $\sigma_Y^2 = \ln\left(1+\frac{\sigma_X^2}{\mu_X^2}\right)$, in which μ_X and σ_X^2 are the mean and variance of the lognormal distribution. This distribution has not a clear distribution for the interarrival times, it is more the number of patients arriving at a certain period. By continuity, the number of arriving patients can theoretically be non-integer. The density function of the lognormal distribution is given by

$$f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right).$$

This distribution sometimes also occurs to be a length of stay distribution. [14]

Normal distribution
The normal distribution is also fitted as a possible arrival distribution, however, since the normal distribution can also have negative outputs, this is not representative. To have a complete image of the types of distributions, the normal distribution is considered anyway. With this distribution, there is a mean and variance parameter to determine the place and height of the peak of the distribution. If the mean and variance are equal and there are enough iterations, the normal distribution does approximate the Poisson distribution. The density function of the normal distribution is given by

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}. \text{ [24, 14]}$$

Logistic distribution
The last arrival distribution to discuss is the logistic distribution. The logistic distribution has a quite similar shape to the normal distribution. The difference between these two distributions is in the tails, the tails of the logistic distribution are heavier than the tails of the normal distribution. [36]

Besides the arrival distribution, we consider some distributions for the length of stay. The length of stay distribution is also called the service distribution, therefore it is notated by the use of random variable S.

Exponential distribution
For the length of stay distribution, a commonly used distribution is the exponential. The exponential distribution has a memory-less property, this means that for a patient that is present, the remaining time can be calculated as if the patient has just arrived. So with an exponential distribution with parameter μ the length of stay can be calculated at any time during the service of the patient. The density function of the exponential

distribution is given by

$$f_S(x) = \lambda e^{-\lambda x},$$

while the cumulative distribution function is given by

$$F_S(x) = \mathbb{P}(S \leq x) = 1 - e^{-\lambda x}.$$

To show the memory-less property, the cumulative distribution is used:

$$\begin{aligned}
\mathbb{P}(S > s + t | S > s) &= \frac{\mathbb{P}(S > s + t \cap S > s)}{\mathbb{P}(S > s)} \\
&= \frac{\mathbb{P}(S > s + t)}{\mathbb{P}(S > s)} \\
&= \frac{e^{-\lambda(t+s)}}{e^{-\lambda s}} \\
&= e^{-\lambda t} \\
&= \mathbb{P}(S > t).
\end{aligned}$$

Due to the memory-less property, the probability that a patient can leave is constant. That is, the leaving probability is equal for a patient at the start of his/her treatment compared to a patient that is already two months into treatment. In healthcare, this is not the case for every patient. For example, there exist some policies which cause that patients are only allowed for 18 weeks, as with ELV. [24, 14]

Weibull distribution

A length of stay distribution that is related to the exponential distribution is the Weibull distribution. The Weibull distribution only takes positive values and it requires two parameters, the shape (k) and the scale (λ) parameter. The scale parameter determines the spreading and height of the distribution. When λ is increased while k stays the same, the distribution will move to the right and the height will decrease. For a decrease of λ the distribution will come closer to 0 and the height will increase. The shape parameter affects the shape of the distribution. For $k < 1$ there is a larger probability that patients leave early. The probability that a patient leaves the system decreases the longer the patient is present, thus most departures are due to patients that stayed a short period. For $k = 1$ the Weibull distribution can be reduced to the exponential distribution, thus the probability of arrival is constant. For $k > 1$ the probability of leaving increases. The longer a patient is present, the higher the probability the patient can leave. The cumulative distribution function of the Weibull is as follows [1]:

$$\mathbb{P}(S \leq x) = \begin{cases} 1 - e^{-(x/\lambda)^k}, & x \geq 0, \\ 0, & x < 0. \end{cases}.$$

Gamma distribution

The last distribution to discuss is the gamma distribution as a possible length of stay distribution. The exponential distribution is a special case of the gamma distribution. The gamma distribution requires two parameters, one of which is the shape parameter (k) and the other can either be the scale parameter (θ) or the rate parameter ($\beta = \frac{1}{\theta}$). The density function of the gamma distribution is given by

$$f_S(x) = \frac{\beta^k}{\Gamma(k)} x^{k-1} e^{-\beta x},$$

where $\Gamma(k)$ is the gamma function, in which $\Gamma(1) = 1$. Therefore, if $k = 1$ the density function of the gamma distribution equals the density function of the exponential distribution. If you sum n independent exponential(λ) random variables, then the result is a random variable with a gamma(n, λ) distribution. [24]

3.2. Queueing models

Queueing models are convenient to model the paths of Figure 2.1. The main focus of this research is on the edges that enter the node intermediate care ("Uitstroom naar vervolgzorg").

It is arguable if in the Dutch healthcare system waiting lines are used. Patients will be admitted if they need care, however, on the other hand, if there is no space at the correct department a patient will be sent to a 'wrong bed', or home-care is temporarily intensified. There are several ways to handle these 'wrong beds', with the use of different models.

The models will be given in Kendall's notation [13]. Kendall's notation is of the form $A|B|c|d$, in which A represents the arrival process, B the length of stay distribution, c the number of beds, and d the total number of places in the queueing system. The number of places in the system (d) is the number of available beds (c) and the number of patients that can wait in line for a bed ($d - c$).

1. Consider a $G|G|s$ queueing model, which has a queue. Patients in the queue will not receive any care, but get full care whenever they are admitted.

2. Consider a $G|G|s|s$ queueing model, which has no queue. All patients that are not admitted are neglected and are not admitted at a later moment. In practice, these patients will go elsewhere or not receive the needed care.

3. Consider a $G|G|\infty$ model, which has no queue, but an infinite amount of beds. All patients are admitted and receive all the care they need, it could be seen as the demand for care. It is tracked how many patients are present. It can be computed how many patients are exceeding the capacity and are placed in the wrong bed.

4. Consider a model in which there is a queue, but patients in the waiting line can be 'impatient', which means they leave the queue. This can be seen as a $G|G|s + G$ model, in which patients that leave impatiently are noted by the $+G$ at the end

of the queue notation. Patients that enter the correct department will receive full service as if they were not taken care of while waiting in the queue. Patients that leave the queue were treated during their waiting period.

During this research, we will focus on two of these models: the model in which an infinite amount of patients can be admitted and the model in which patients are blocked. The infinite server model models the demand for care. If there are enough beds such that all patients that need care can be admitted, this model is the right model. However, if there is not a fixed capacity, but a capacity that varies over time, there is no known number of beds s, which makes the infinite server model to be a good choice. Unfortunately, for many regions the demand for care is higher than the number of beds available, therefore the infinite server model would not be a good choice within those regions. When there is a fixed capacity, a more realistic model is the blocking model. Within the blocking model, the number of beds is fixed, and the capacity cannot be exceeded. It is more likely that patients that cannot be admitted go somewhere else, therefore the blocking model is a good approximation for this situation.

For these two cases, we consider general arrival processes and general length of stay distributions. If the arrivals process is a Poisson process, then we have explicit formulas to calculate the number of occupied beds. With a Poisson arrival process, the blocking model ($M|G|s|s$) is called an Erlang B system. The third model ($G|G|\infty$) is called an infinite server queue regardless of the arrival process, however, if there is a Poisson arrival process there is a lot known about this type of model.

3.2.1. Stationary models

The models described above have a homogeneous arrival process, which makes these models called stationary models. For an infinite server queueing system with Poisson arrivals, it is possible to calculate the expected number of occupied beds. For a homogeneous Poisson process, Little's law can be used to calculate the expected number of occupied beds: $L = \lambda W$, in which λ represents the effective arrival rate and W is the expected length of stay [30]. The expected number of occupied beds also is seen as the offered load. Based on the offered load, the load per bed (a) can be calculated by $a = L/s$ in which s is the number of beds.

There not always is an infinite server model. As mentioned before, the blocking model will also be considered: $G|G|s|s$, there is no queue within this system. As soon as all beds are occupied, new patients are rejected and leave the system. The specific model we are interested in is the $M|G|s|s$ queue, thus with a Poisson arrival process. If the arrivals are homogeneous, the stationary distribution is given by:

$$\pi(i) = \frac{(\lambda \mathbb{E}S)^i/i!}{\sum_{j=0}^{s}(\lambda \mathbb{E}S)^j/j!} \tag{3.1}$$

This means, that the probability that there are i beds occupied, or similarly i patients being treated, is given by $\pi(i)$ for $0 \leq i \leq s$.

With the use of Formula 3.1, the blocking probability can be given. A patient will only be blocked if all beds are occupied, thus the probability that a patient is blocked equals the probability that s beds are occupied: $\pi(s)$. The blocking probability is defined as:

$$B(s, \rho) = \pi(s) = \frac{\rho^s/s!}{\sum_{j=0}^{s} \rho^j/j!}, \tag{3.2}$$

where ρ is the offered load: $\rho = \lambda \mathbb{E} S$.

3.2.2. Time-dependent arrivals

By adding time dependency to the arrivals of the above models, the arrival rate is split for arrivals at different moments in time. Time dependency influences the number of occupied servers since there is a difference in the number of arrivals during time [12]. In Dutch healthcare there is a clear weekly pattern, several healthcare providers are closed during weekends, and hardly any elective care will be scheduled during the weekends. Overall, this pattern will result in a lower arrival rate for weekends compared to the arrival rate for weekdays. A model with time-dependent arrivals has no stationary distribution, so it is called a non-stationary model.

For a time-dependent Poisson process, it is not possible to use Little's law, since there is no general arrival rate, but it differs in time. Thus the expectation of occupied beds can be calculated at a certain moment in time. Since the arrival rate depends on the time, the expected number of occupied beds does also depend on the time. The number of occupied beds is the expected number of patients that have arrived before t and that are still present at time t. The expected number of occupied beds in a time-dependent infinite server model is also referred to as the modified offered load. Important is that the modified offered load is calculated for an infinite server model, so $M_t|G|\infty$. This can be written as the following formula:

$$m(t) = \int_{-\infty}^{t} \lambda(x) \mathbb{P}(S > t - x) dx, \tag{3.3}$$

where $\lambda(x)$ is the arrival rate at time x and $\mathbb{P}(S > t - x)$ is the probability that a patient that entered at time x is still present at time t, so the probability that length of stay is longer than $t - x$. [12] By a change of variables $y = t - x$, formula 3.3 can be rewritten as:

$$m(t) = \int_{0}^{\infty} \lambda(t - y) \mathbb{P}(S > y) dy. \text{ [5]} \tag{3.4}$$

By the periodicity of the weekly pattern, it can be stated that $m(t) = m(t - 7)$ (see [6]). That is, in a stationary situation the expected number of occupied beds should be the same every Monday. Similarly, for every day of the week this equation holds. Assume that t is a moment during the week, where a week can be defined as the interval $[0, 7)$. In the week $[0, 1)$ represents Monday, $[1, 2)$ represents Tuesday, $[2, 3)$ represents Wednesday, $[3, 4)$ represents Thursday, $[4, 5)$ represents Friday, $[5, 6)$ represents Saturday, and $[6, 7)$ represents Sunday. By assumption, the arrival rate is the same over a single day. Denote the arrival rate of day d by $\lambda_d = \lambda([d, d+1))$, thus the arrival rate of Monday is given by λ_0.

Lemma 3.1. *For a time-dependent Poisson arrival process with exponential lengths of stay and periodicity of period $T = 7$, the modified offered load can be written as:*

$$m(t) = \frac{1 - e^{-\mu}}{1 - e^{-7\mu}} \frac{1}{\mu} \sum_{i=0}^{6} \lambda_{t-1-i} e^{-i\mu}. \tag{3.5}$$

Proof. Start with the modified offered load formula 3.4.

$$\begin{aligned}
m(t) &= \int_0^\infty \lambda(t-y)\mathbb{P}(S > y)dy \\
&= \int_0^\infty \lambda(t-y)e^{-y\mu}dy \\
&= \sum_{i=0}^{6} \int_i^{i+1} \lambda_{t-i-1}e^{-y\mu}dy + \int_7^\infty \lambda(t-y)e^{-y\mu}dy \\
&= \sum_{i=0}^{6} \lambda_{t-i-1} \left[-\frac{1}{\mu}e^{-y\mu}\right]_i^{i+1} + \int_0^\infty \lambda(t-(y+7))e^{-(y+7)\mu}dy \\
&= \sum_{i=0}^{6} \frac{\lambda_{t-i-1}}{\mu}(e^{-i\mu} - e^{-(i+1)\mu}) + e^{-7\mu}\int_0^\infty \lambda((t-7)-y)e^{-y\mu}dy \\
&= (1 - e^{-\mu})\frac{1}{\mu}\sum_{i=0}^{6} \lambda_{t-1-i}e^{-i\mu} + e^{-7\mu}m(t-7)
\end{aligned}$$

With the use of $m(t) = m(t-7)$, Formula 3.5 is found. □

The input of the modified offered load is the day on which the offered load should be calculated. If the day of interest for calculating the offered load is a Monday, then the input for the formula should be $t = 0$ for the beginning of Monday and $t = 1$ for the end of Monday and at the same time for the offered load at the beginning of Tuesday. Define the offered load of Monday as the offered load at the end of the day, thus $m(1)$ gives the offered load for Monday.

Formula 3.5 can only be used for models for which the length of stay distribution is exponential. If the length of stay distribution is not exponential, the integrals given in equations 3.3 and 3.4 need to be computed as a whole. Also when computing the complete integrals it is possible to simplify the integral by the weekly periodicity.

Assume that the arrival rate is the same for the weekdays, and the arrival rate is the same for the weekdays, this is $\lambda_0 = \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$ and $\lambda_5 = \lambda_6$. Define the arrival rate for weekdays as λ_{week} and the arrival rate for weekend days as λ_{weekend}. Then the arrival rate is summarized as:

$$\lambda_t = \begin{cases} \lambda_{\text{week}}, & \text{for } t \in [0, 5) \\ \lambda_{\text{weekend}}, & \text{for } t \in [5, 7), \end{cases} \tag{3.6}$$

in which $[0,5)$ represents the weekdays and $[5,7)$ represents the weekend days. During this research, it is assumed that the arrival rates are distributed as given in 3.6.

Every week can essentially be brought back to the interval $[0,7)$, since every week the same arrival pattern occurs. This can be written as $[0,7) \equiv [7i, 7(i+1)) \pmod 7$ with $i \geq 0$. Weekdays are defined by $[0,5) \equiv [7i, 7i+5) \pmod 7$ with $i \geq 0$, and weekend days by $[5,7) \equiv [7i+5, 7(i+1)) \pmod 7$ with $i \geq 0$.

Lemma 3.2. *For a time-dependent Poisson arrival process with general lengths of stay, and arrival rates as in 3.6, the modified offered load can be written as:*

$$m(t) = \sum_{i=0}^{\lfloor \frac{t}{7} \rfloor} \left(\int_{7i}^{\min(7i+5,t)} \lambda_{week}(1 - F(t-x))dx + \int_{\min(7i+5,t)}^{\min(7(i+1),t)} \lambda_{weekend}(1 - F(t-x))dx \right). \tag{3.7}$$

Proof. Formula 3.3 gives the modified offered load for systems started in the distant past, by assuming that the system has just started there is a small change in the formula: $m(t) = \int_0^t \lambda(x)\mathbb{P}(S > t-x)dx$, so it now starts from 0. This formula starts from zero, which makes it more intuitive when the system has started. In fact with the integral starting from 0, it is said that we start with an empty system, which first needs a warming-up period after which the number of customers converges to a periodic steady state. The formula for the modified offered load with a general length of stay can then be derived as follows:

$$\begin{aligned}
m(t) &= \int_0^t \lambda(x)\mathbb{P}(S > t-x)dx \\
&= \int_0^t \lambda(x)(1 - F(t-x))dx \\
&= \int_0^5 \lambda_{\text{week}}(1 - F(t-x))dx + \int_5^7 \lambda_{\text{weekend}}(1 - F(t-x))dx \\
&\quad + \int_7^{12} \lambda_{\text{week}}(1 - F(t-x))dx + \int_{12}^{14} \lambda_{\text{weekend}}(1 - F(t-x))dx + \ldots \\
&= \sum_{i=0}^{\lfloor \frac{t}{7} \rfloor} \left(\int_{7i}^{\min(7i+5,t)} \lambda_{\text{week}}(1 - F(t-x))dx + \int_{\min(7i+5,t)}^{\min(7(i+1),t)} \lambda_{\text{weekend}}(1 - F(t-x))dx \right).
\end{aligned}$$

□

Equation 3.7 is a summation over the weeks. For the last week, the boundaries of the integrals need to be considered closely, since it is not sure that the last week is completely taken into account. For example, if $t = 10$, the first week is completely considered, however, the second week only has the first 3 days that are relevant. To only consider these three days, the integral boundaries are limited by t, such that we have for the calculation in week two (with $i = 1$):

$$\int_7^{10} \lambda_{\text{week}}(1 - F(t-x))dx + \int_{10}^{10} \lambda_{\text{weekend}}(1 - F(t-x))dx.$$

Here the last integral is equal to zero, since the upper boundary is the same as the lower boundary, so for this week the modified offered load only is calculated up to and including Wednesday.

Independently of the length of stay distribution, the modified offered load can be calculated with the use of Formula 3.7. Whenever a model has a length of stay distribution that is exponential, then there are two types of calculating the modified offered load, both Formula 3.5 and Formula 3.7. However, there is a slight difference between the two ways of calculating the modified offered load. Formula 3.5 assumes the system to be in a stationary state already. While Formula 3.7 starts calculating from the start of the system. To calculate the modified offered load with the use of 3.7 correctly, the input parameter t should be large enough to be past the warming-up period of the system. For example, after 100 weeks the warming-up period should be over, the modified load for Monday can then be calculated with $m(100 \cdot 7 + 1) = m(701)$.

Let $N_s(t)$ denote the number of patients in the system at time t in a system with s beds. Then the number of patients in the $M_t|G|\infty$ model at time t is given by $N_\infty(t)$. By Eick, Massey, and Whitt [11], a basic $M_t|G|\infty$ result is given in Theorem 3.3.

Theorem 3.3. *For each t, $N_\infty(t)$ has a Poisson distribution with mean $m(t)$, where $m(t)$ is given by Formula 3.3.*

What follows from Theorem 3.3, is that $N_\infty(t) \sim Pois(m(t))$, thus

$$\mathbb{P}(N_\infty(t) = k) = \frac{m(t)^k}{k!} e^{-m(t)}, \tag{3.8}$$

which is among others also described by R. Bekker and A.M. de Bruin [5].

3.2.3. Time-dependent length of stay

Up till now, the assumption is made that the length of stay distribution is independent of the day of arrival. However, there often are fewer nurses available during the weekend days, which can cause only the ‘easy’ patients to be admitted, and the harder patients with more complex problems will not be admitted. If this split is made during the weekend, it is likely to assume that patients that enter during the weekend have a shorter length of stay. On the other hand, it could happen that patients that are admitted during the weekend do not immediately start their treatment, which causes these patients to have a longer length of stay than patients arriving during weekdays. Now we can see the patients that enter during weekdays and the patients that enter during the weekends as two different types of patients.

Since we assume Poisson arrivals for both groups, the summation of arrivals of these two types is still Poisson distributed [8]. The patients that arrive during the week have a positive arrival rate from Monday through Friday, while the arrival rate is equal to zero for the weekend days. The patients that arrive during the weekend days have a positive arrival rate for Saturday and Sunday and the rest of the week, the arrival rate equals zero. With this information and the assumption of exponential lengths of stay the modified offered load per group can be calculated with the use of Formula 3.5.

For weekday arrivals we have $\lambda_0 = \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_{\text{week}}$, and $\lambda_5 = \lambda_6 = 0$ for the arrival rates. There is a length of stay rate given by μ_{week}. This leads to the formula:

$$m_{\text{week}}(t) = \frac{1 - e^{-\mu_{\text{week}}}}{1 - e^{-7\mu_{\text{week}}}} \frac{1}{\mu_{\text{week}}} \sum_{i=0}^{6} \lambda_{t-1-i} e^{-i\mu_{\text{week}}}. \tag{3.9}$$

A similar situation occurs for the weekend arrivals, $\lambda_0 = \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$ and $\lambda_5 = \lambda_6 = \lambda_{\text{weekend}}$ are the arrival rates, and the length of stay rate is given by μ_{weekend}. We find about the same formula as for the arrivals through weekdays, however, the rates are changed.

$$m_{\text{weekend}}(t) = \frac{1 - e^{-\mu_{\text{weekend}}}}{1 - e^{-7\mu_{\text{weekend}}}} \frac{1}{\mu_{\text{weekend}}} \sum_{i=0}^{6} \lambda_{t-1-i} e^{-i\mu_{\text{weekend}}}. \tag{3.10}$$

Note that the arrival rates are defined differently for the group of patients that arrives during the weekend. The total modified offered load does not depend on the type of patient, it is the total number of expected occupied beds. Thus it is the number of patients that have arrived during weekdays and the number of patients that have arrived during weekend days. Together these two groups give the number of occupied beds. So with the use of the Formulas 3.9 and 3.10, we can find the modified offered load as:

$$m(t) = m_{\text{week}}(t) + m_{\text{weekend}}(t). \tag{3.11}$$

To apply Formula 3.9 and Formula 3.10 for calculating $m(t)$, exponential service times are needed. If there is a different distribution underlying the service times, the modified offered load can still be defined as given in Formula 3.11, however, $m_{\text{week}}(t)$ and $m_{\text{weekend}}(t)$ are calculated with the use of Formula 3.7 and the arrival rates as described above.

Let $N_\infty^{\text{week}}(t)$ be the number of present patients at time t that have arrived during weekdays. Similarly, let $N_\infty^{\text{weekend}}(t)$ be the number of present patients at time t that have arrived during weekend days. With the use of Theorem 3.3, we know for both groups that the number of present patients is Poisson distributed. So, $N_\infty^{\text{week}}(t) \sim Pois(m_{\text{week}}(t))$, and $N_\infty^{\text{weekend}}(t) \sim Pois(m_{\text{weekend}}(t))$. The total number of present patients is given by the sum of these two groups: $N_\infty(t) = N_\infty^{\text{week}}(t) + N_\infty^{\text{weekend}}(t)$. The sum of two independent Poisson distributions is Poisson [8], therefore $N_\infty(t) \sim Pois(m(t))$, where $m(t)$ as in Formula 3.11.

4. Data analysis

For queueing models, the focus is on both arrivals and the length of stay of patients. Within this chapter, we will first look into the available data, and then we are going to analyze these data. The first element to analyze is the arrivals in Section 4.2. Thereafter, the lengths of stay will be analyzed in Section 4.3. To be able to find a model that fits the data there also is a need for some more analyzes: the occupancy (Section 4.4), the relation between the arrivals and the occupancy (Section 4.5), the routes patients follow (Section 4.6), and the regional effects (Section 4.7). During this chapter analyzes are done for intermediate care facilities, these analyzes are also done for hospital care, however, this could be found in Appendix C.

4.1. Data

Healthcare data is collected by Vektis. Vektis has a unique database with all declaration data within the Dutch healthcare [33]. The NZa receives this data from Vektis and made the data that is relevant for this research available. The data that is available concerns the departments ELV, GR, VD, VB, and IC, which are explained in section 2.2, for the years 2016 to 2020.

The data was given in different data frame styles. The data of ELV is given in a data frame looking as follows:

Table 4.1.: Type of data frame ELV data.

Pseudo _BSN_ID	Institution	Start _Date	Length _of _Stay	End_ Date	VV_ code	Performance _Description
$xxx1$	$yyy1$	2016-01-01	14	2016-04-14	3	basic
$xxx2$	$yyy2$	2018-05-12	27	2018-06-07	6	intensive
$xxx2$	$yyy2$	2018-06-07	5	2018-06-11	10	palliative
$xxx3$	$yyy3$	2017-12-28	4	2017-12-31	3	basic
$xxx3$	$yyy3$	2018-01-01	5	2018-01-05	3	basic

This is not real data but an example to see how the data is delivered. The pseudo BSN ID in the data denotes the patient and is a pseudonym for the BSN of the patient. BSN is the Dutch abbreviation for citizen service number, a unique number that identifies a unique person. Due to privacy reasons a pseudo BSN ID is used, from which it is not possible to trace the real BSN of the patient. A patient ID helps with the routing of a patient, such that it is possible to follow the patient within the care system. Later in the analysis, this gives insight into the number of patients that go to ELV via a 'wrong bed' and faced a full ELV when they were ready to enter ELV.

To see where a patient has been treated, a code for the institution is given. With such a code the connection to the location of the institution can be made. There is a connecting chart available that connects the institution code to the name and address information of that institution. This information is useful for the analysis per region.

Start and end dates are the date of arrival and the date of departure, both included in the length of stay during this period. These dates are used to calculate the arrivals on a certain day, and to calculate the number of occupied beds per date. The length of stay is the number of days the patient spends at the healthcare institution.

The last two columns are connected and represent the VV_code and its description. There are three types of performances with their corresponding code: low complex (basic) care (3), high complex (intensive) care (6), and palliative care (10). Performance indicates the type of ELV care given to the patient.

In Table 4.1 there are five rows of example data given. Patient $xxx1$ has entered ELV once in institution $yyy1$, and stayed for 14 days, starting on January 1, 2016. Patient $xxx2$ stays for two consecutive stays, one of which is for high complex care and the other for palliative care, both stays take place in institution $yyy2$. In total patient $xxx2$ has stayed from May 12, 2018 until June 11, 2018, which is a total stay of 31 days. Note that this is not the same as the sum of the two lengths of stay, this is caused by the inclusion of both start and end dates in the length of stay. Patient $xxx2$ goes after 27 days from high complex care to palliative care, while staying at the same ELV institution. Patient $xxx3$ also stays two consecutive stays, but now both of them have the same type of care: low complex care. These stays are in the same institution, so we could see this as one stay. Due to administrative reasons, this is denoted as two stays. Later, it is explained how these two consecutive stays of the same type and in the same institution can be merged.

For GR and the three types of hospital stays, the data is given in the same types of data frames. The data for GR, VD, VB, and IC is given in a data frame looking as in Table 4.2.

Similarly as for the data of ELV, the patient is given by a pseudo BSN number and the institution is given by a code. Differently, as for ELV there only is given a start date. In the data, the start date means a declared day, since every declared day is seen as a starting day. A stay of a patient can thus be calculated by a period of consecutive starting dates.

Two overlapping ELV stays, or directly consecutive ELV stays could be combined into one stay. This is only done for the same patient, the same sort of care, and the same institution, so if the Pseudo_BSN_ID, Institution, VV_code, and Performance_Description are the same for the overlapping periods.

If two or more rows in GR, VD, VB, or IC data are consecutive dates for the same patient, and institution these could be combined into one stay. For Table 4.2 this would mean that patient $xxx1$ has one stay with starting date 2016-09-01 and departure date

Table 4.2.: Type of data frame GR, VD, VB, and IC data.

Pseudo _BSN_ID	Institution	Start_Date
$xxx1$	$yyy1$	2016-09-01
$xxx1$	$yyy1$	2016-09-02
$xxx1$	$yyy1$	2016-09-03
$xxx2$	$yyy2$	2019-02-14
$xxx2$	$yyy2$	2019-02-15
$xxx2$	$yyy2$	2019-02-16
$xxx2$	$yyy2$	2019-02-17
$xxx2$	$yyy2$	2019-02-18
$xxx3$	$yyy1$	2018-01-01

2016-09-03, which is a stay of 3 days.

The data frame for ELV will still look the same as before combining rows, it has the same columns, and the only difference is the number of rows. So Table 4.1 is still the data frame in which the data is notated.

For all other data, the data frame will have two new columns after combining rows: Length_of_Stay and End_Date. After combining the rows of Table 4.2, the resulting data frames are of the types given in Table 4.3.

Table 4.3.: Type of data frame GR, VD, VB, and IC data after combining consecutive rows.

Pseudo _BSN_ID	Institution	Start_Date	Length_of_Stay	End_Date
$xxx1$	$yyy1$	2016-09-01	3	2016-09-03
$xxx2$	$yyy2$	2019-02-14	5	2019-02-18
$xxx3$	$yyy1$	2018-01-01	1	2018-01-01

Due to the nature of the three types of ELV, these need to be split into two different categories. Patients that go to an ELV palliative type of care are not supposed to go home afterward, these patients stay until they pass away. The patients that go to ELV low complex care or ELV high complex care are temporarily at the ELV department until they can go home. There are administrative matters that could influence the length of stay for ELV low complex and ELV high complex, which are not present for ELV palliative care. Therefore, the two groups of ELV we are considering are ELV low/high complex and ELV palliative.

4.2. Arrivals

The interesting questions regarding arrivals are: What is the distribution of the arrivals?; Is there a pattern in the arrivals?; Is all data representative for the purpose of this research?

During the pre-analysis it was noted that there is an extremely high peak of arrivals on 2016-01-01, this is the starting date of the data sets. All patients that were already present in the hospital or intermediate care facility are in the data set seen as arriving patients that day. Since this is no good representation of reality, the data of 2016-01-01 is not taken into consideration for the arrivals.

During public holidays (see Appendix B, Table B.1) in the Netherlands there are dips in the number of arrivals. As we are interested in regular days, the dates of these public holidays are removed from the data, such that all remaining days are 'normal' days during the week or the weekend. This results in an arrival analysis for 'normal' days during the week and normal weekend days.

To have an idea of the distribution of the number of arrivals, a visualization of the intermediate care departments is given in Figure 4.1. For this figure, the public holidays are not removed, but just taken into account, to reduce the effect of these days. In appendix D.5, weekly arrivals for all departments are plotted, but for periods of one year.

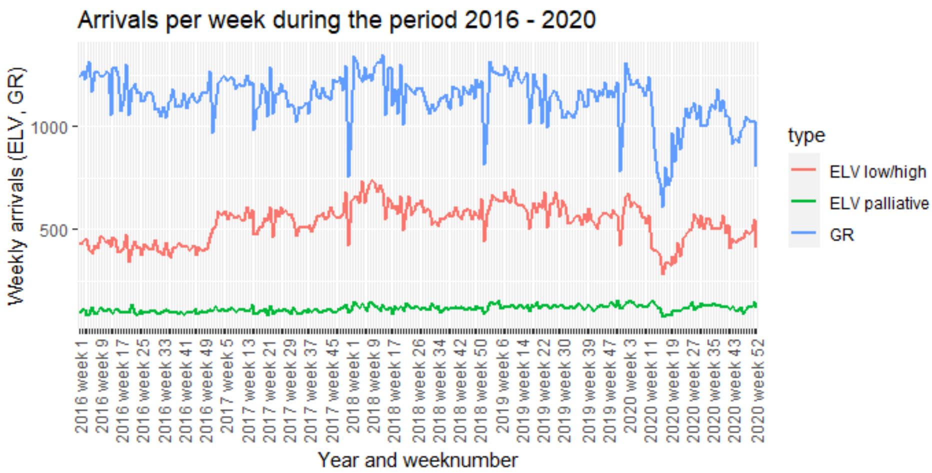
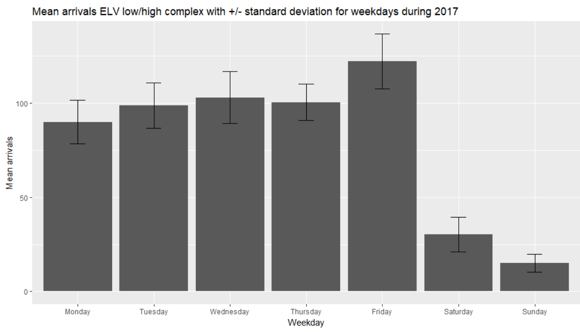


Figure 4.1.: Weekly arrivals in all departments over the period 2016 until 2020.

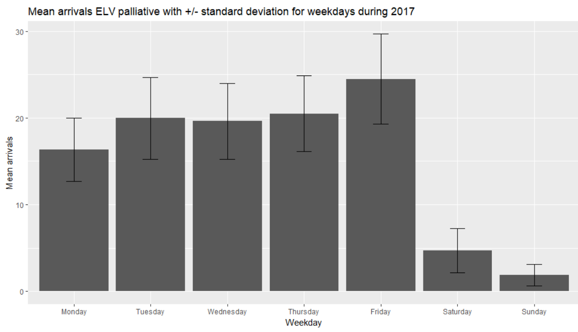
In Figure 4.1 the arrivals per week show erratic behavior, for some departments it is more erratic than for others. The arrivals are the least erratic for ELV palliative, possibly because there are very few arrivals compared to the other two departments. Especially around the first week of the years 2017-2020, there are fewer arrivals for GR, for which some are heavier dips than others. On top of that, around the first weeks in 2017 and 2018, there are increases for ELV low/high complex, which means that more ELV low/high complex beds became available at the start of these years. This increase is most visible in the transition from 2016 to 2017. For ELV palliative, the only period in which there are clear changes in the pattern is in 2020.

From Figure 4.1, there is a pattern visible that relates to the COVID-19 pandemic and its corresponding lockdowns. For the arrivals in 2020, some fluctuations differ from the pattern seen in the years 2016 until 2019. Due to this difference from now on the research will mainly focus on the years 2016 until 2019, and data from 2020 will not be analyzed further.

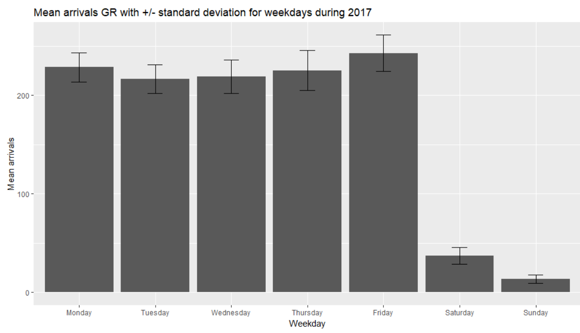
In Figure 4.2, a visualization is made on the mean number of arrivals per day during 2017 excluding the public holidays. For this year there were clear patterns visible, these patterns were also present for other years, but less clear. An error bar is added with one times the standard deviation above and below the mean value. These error bars indicate how much the number of arrivals on a certain day varies.



(a) ELV low/high complex



(b) ELV palliative



(c) GR

Figure 4.2.: Arrivals in intermediate care institutions.

Figure 4.2 is over the year 2017, in Appendices D.1 - D.3 these figures are available for the other years as well. On top of that, there is a figure with these mean arrivals per weekday for the period 2016-2020. Since the mean value differs per year, the standard deviation becomes smaller when considering only one year. Note from the figures in the Appendix that the error bars for 2020 are in general larger than for the other years.

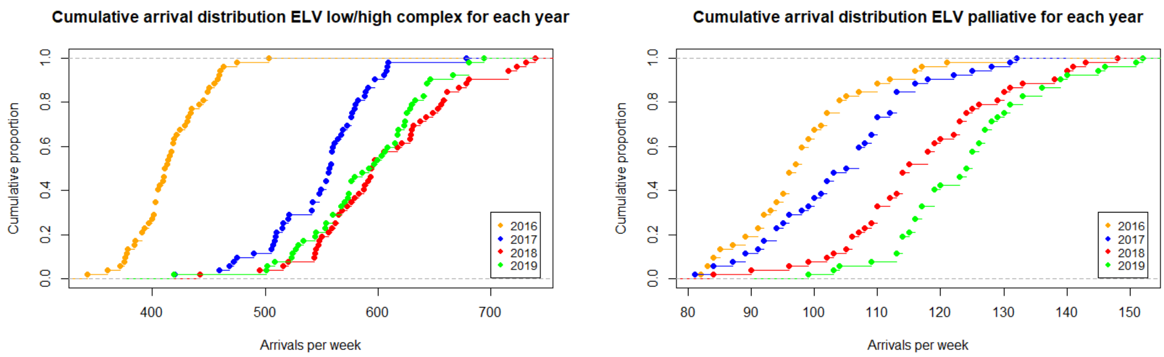
From the above figures, it follows that there is a weekly pattern that differs per department. What can be seen is that there are fewer arrivals during the weekend days.

For ELV low/high complex and ELV palliative (Figures 4.2a and 4.2b) the weekly pattern increases during the week from Monday to Friday, while the number of arrivals during the weekend becomes smaller again, but there are more arrivals on Saturday than on Sunday.

For GR (Figure 4.2c) the weekly pattern differs slightly from ELV. The GR weekly pattern is concave for the period Monday to Friday. Again the arrivals during the weekend are less than the rest of the week and Saturday has more than Sunday.

From the years 2016 until 2019 the data analysis is continued. A cumulative plot for the weekly arrivals is made in such a way that it adds up to 100%. By this plot, there is a better view of the difference per year. The dots in the figures represent the weeks, but not in chronological order. The horizontal axis represents the number of arrivals, so if two consecutive dots have x-value 400 and 415, that signifies that there were no weeks in which there were 410 arrivals.

In Figure 4.3 these cumulative figures can be found for the departments ELV low/high complex and ELV palliative, for the other departments, Appendix D.4 can be consulted.



(a) ELV low/high complex

(b) ELV palliative

Figure 4.3.: Cumulative weekly arrivals for all departments from 2016 until 2019.

Over the years there have been some changes in the distribution. As concluded from Figure 4.1 there have been increases in the number of arrivals for ELV low/high complex. In Figure 4.3 it is better visible how much these arrivals have increased. For ELV low/high complex the biggest increase in the number of arrivals is for the transition from 2016 to 2017, a smaller increase happens for the transition from 2017 to 2018. After 2018 there is no clear increase or decrease in the number of arrivals, so it seems to be stabilizing. The law changed for ELV from 2016 to 2017, which could cause an increase of arrivals in 2017. The mean number of weekly arrivals is given in Table 4.4.

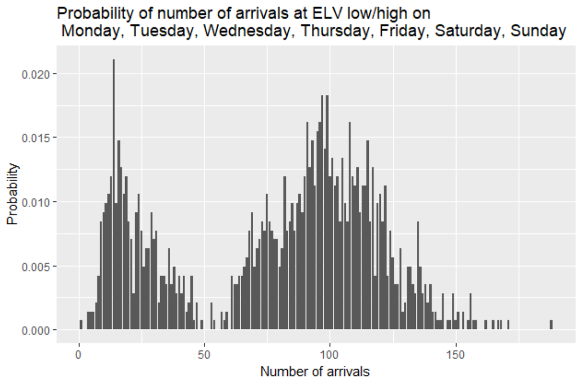
For ELV palliative the mean number of weekly arrivals increases from 2016 until 2019. Whereas for ELV low/high complex the mean number of weekly arrivals increases from 2016 until 2018 and decreases a little in 2019. However, the decrease from 2018 to 2019 is significantly smaller than the increases over the previous years.

For GR there are no big decreases or increases over the years. The largest difference is between 2017 and 2018, which is an increase of 1.3%. In Appendix D.4 the cumulative plot also visualizes this numeric result.

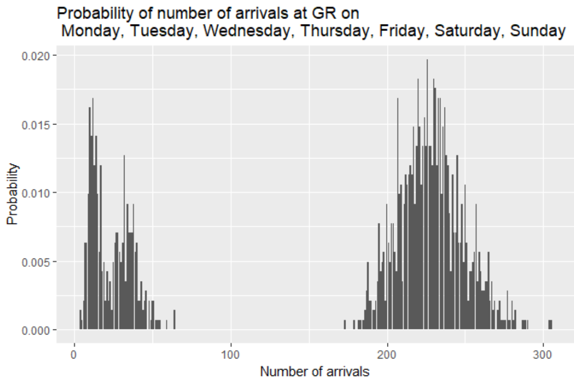
Table 4.4.: Mean values of the weekly arrivals for all intermediate care departments per year.

	ELV low/high	ELV palliative	GR
2016	415.8	98.3	1,165.4
2017	550.2	104.6	1,157.5
2018	605.9	117.4	1,174.8
2019	590.4	124.3	1,159.8

Next, we consider the distribution of the number of arrivals. To figure that out, the empirical distribution of the data is plotted for the complete period and for all days of the week together.



(a) ELV low/high complex



(b) GR

Figure 4.4.: Empirical arrival distributions.

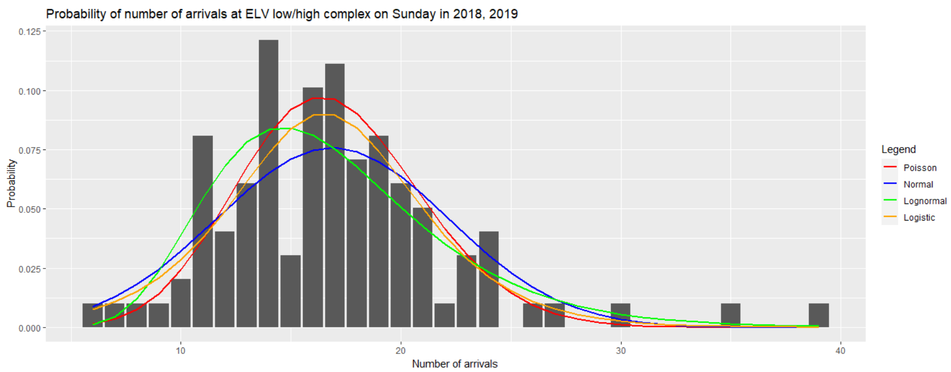
Especially for GR, there seem to be two separate distributions, one of which has more arrivals than the other. The same sort of separation is seen for ELV low/high complex, however, the separation is not as large as for GR. The distribution graphs for the other departments are given in Appendix D.6. Following from Figure 4.2 a good guess for splitting the arrivals is by weekdays and weekend days.

To find the best fit of the arrival distribution, the following distributions are fitted: Poisson distribution, normal distribution, lognormal distribution, and logistic distribution. By the literature (see e.g. [9, 7, 10][1]), the most likely distribution is the Poisson distribution since the arrivals are discrete and the Poisson distribution is the only discrete distribution. However, the data is not always a perfect fit, therefore the other distributions are added for comparison.

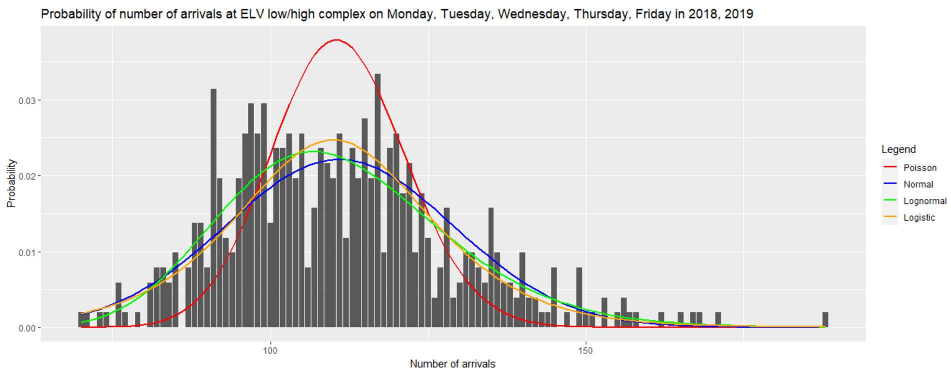
With the use of the function *fitdist* of R's package *fitdistrplus* the parameters for the best maximum likelihood estimator fit are calculated per distribution. Some of the corresponding graphs are included in Figures 4.5 - 4.7, such that it can be numerically

[1]All of these three articles refer to the Stabilization of inpatient bed occupancy through control of admissions. by Young JP. (1965), which I could not access at the time of writing this thesis.

determined which distribution would fit best. Not all best fits are good fits, but it is the best possible fit out of the previously mentioned distributions.
First, we look at the distributions for ELV low/high complex (Figure 4.5).



(a) The data and fitted distributions for the ELV low/high complex arrivals during Sundays in 2018 and 2019 are plotted with the following parameters: Pois(16.879), N(16.879, 5.258), lnorm(2.7780, 0.307), logistic(16.528, 2.767).



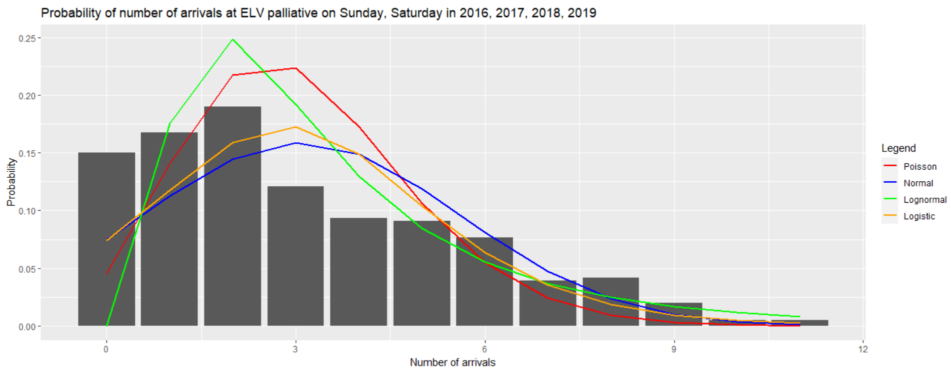
(b) The data and fitted distributions for the ELV low/high complex arrivals during the weekdays in 2018 and 2019 are plotted with the following parameters: Pois(110.923), N(110.923, 18.009), lnorm(4.696, 0.159), logistic(109.900, 10.115).

Figure 4.5.: Empirical arrival distribution ELV low/high complex with fitted distributions.

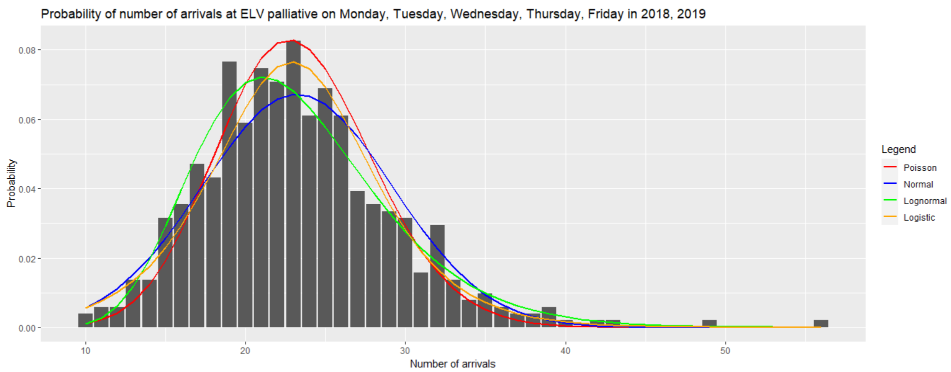
From Figure 4.5a it follows that the four distributions all approximate the ELV low/high complex data for Sundays in 2018 and 2019, however, the distribution that seems to have the best fit is the Poisson distribution. It is a bit hard to conclude Poisson to be the best since there appears a dip around 15 arrivals, which does not appear in one of the distributions.

For the weekdays, Figure 4.5b shows the arrivals during 2018 and 2019, here the Poisson distribution seems to be the worst. The lognormal distribution seems to be the best fit, but the normal and logistic distributions seem to fit equally well as the lognormal.

Next, we analyze the ELV palliative arrivals, with the use of the same distributions. In Figure 4.6 both the weekend days and the weekdays are plotted with the four dis-



(a) The data and fitted distributions for the ELV palliative arrivals during weekend days in 2016 - 2019 are plotted with the following parameters: Pois(3.089), N(3.089, 2.509), lnorm(1.068, 0.693), logistic(2.856, 1.445).

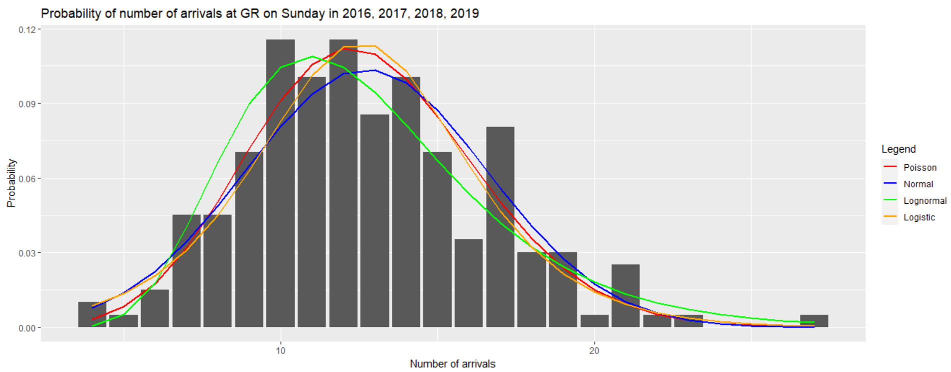


(b) The data and fitted distributions for the ELV palliative arrivals during the weekdays in 2018 and 2019 are plotted with the following parameters: Pois(23.230), N(23.230, 5.939), lnorm(3.113, 0.254), logistic(22.905, 3.266).

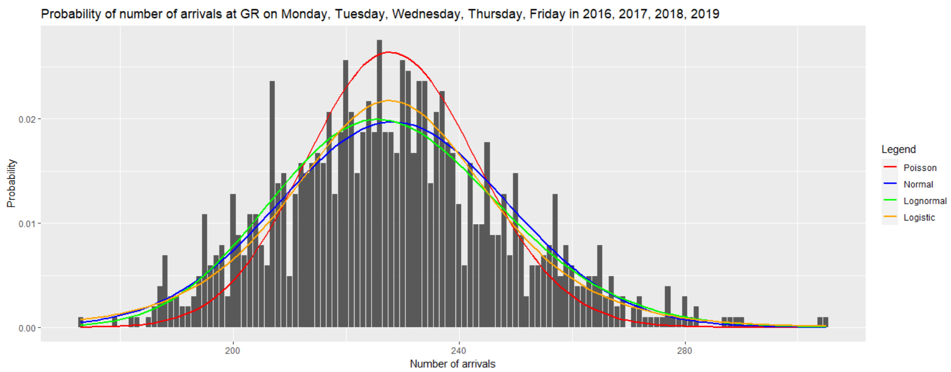
Figure 4.6.: Empirical arrival distribution ELV palliative with fitted distributions.

tributions that are fitted on the data. Since the lognormal distribution cannot take zero as a value, this distribution ignores all moments that there were no arrivals. This distribution does not fit very well. However, all other distributions also start too low on the probability of 0 arrivals. This causes an overestimation of 3 arrivals per day for all distributions. The arrivals during the weekdays are fitted better than for weekend days. The distribution that is the most likely is the Poisson distribution since it is quite a good fit and the only distribution that induces an arrival process.

For GR the same distributions are fitted, this results in Figure 4.7. The four distributions that are fitted for the GR arrivals on Sundays (see Figure 4.7a) are quite close to each other. So, the Sunday arrivals at GR can be simulated by multiple distributions relatively well. However, there is one distribution that seems to fit the best, and that is the Poisson distribution. The choice of only plotting the Sundays follows from the



(a) The data and fitted distributions for the GR arrivals during Sundays in 2016 - 2019 are plotted with the following parameters: Pois(12.724), N(12.724, 3.849), lnorm(2.495, 0.318), logistic(12.548, 2.183).



(b) The data and fitted distributions for the GR arrivals during the weekdays in 2016 - 2019 are plotted with the following parameters: Pois(228.301), N(228.301, 20.245), lnorm(5.427, 0.088), logistic(227.719, 11.492).

Figure 4.7.: Empirical arrival distribution GR with fitted distributions.

empirical distribution of both weekend days. In this empirical distribution, it became clear that the weekend days were separated from each other by the two peaks that were present.

In Figure 4.7b the arrivals during the weekdays are plotted with the fitted distributions. The fitted graphs of the normal distribution and the lognormal distribution are quite similar. However, the Poisson distribution has the best fit for the GR arrivals during weekdays.

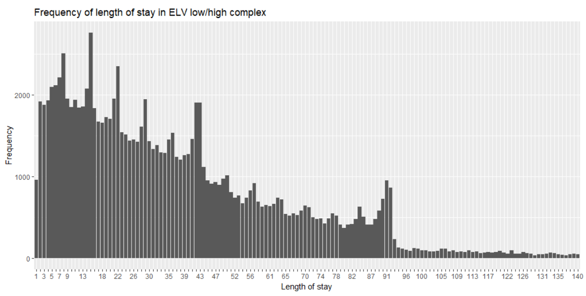
In Appendices D.7 - D.9 there are extra figures with the fitted distributions. Varying with the years and the days taken into account. The days and years that are not considered in this section can be found in one of these appendices.

4.3. Lengths of stay

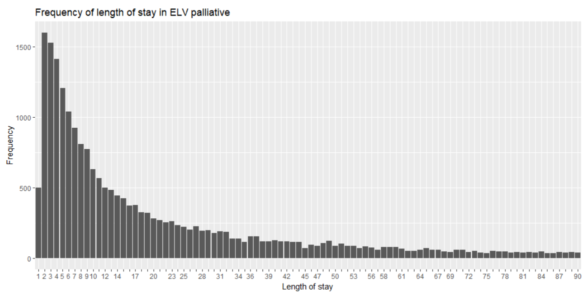
Depending on the department and the type of patient, the patient can stay either a short period of a few days or a longer period, for example, 3 months or even a year.

By definition, a day can only be declared as a nursing day if the patient has stayed over the night. The exact definition stated by the NZa is the following: “A nursing day is a registered calendar day which is part of the period of nursing (which contains minimally 1 overnight stay). The nursing period starts on the day of intake up and until the day of release, for which the intake (provided that the intake happened before 20.00 h) and the day of release are both marked as a calendar day to be registered.” [17] In this definition by overnight stay is meant a patient that arrives before 00.00 h and leaves after 07.00 h the subsequent calendar day. There are exceptions to this definition. The condition of an overnight stay does not apply to a definitive takeover of another institution on the day of intake, or for a death of the patient on the day of intake [17].

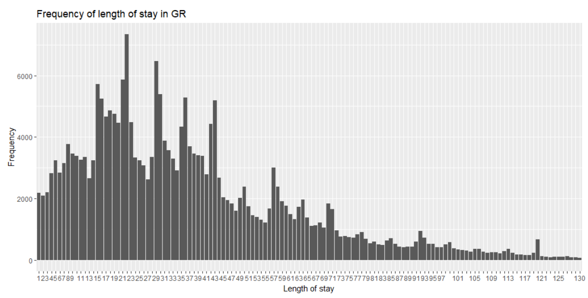
Now that the definition of a nursing day is clear, the lengths of stay (LoS) per department can be analyzed. As noted before there are a lot of arrivals during the first day in the data: 01-01-2016, these patients have stayed already for an unknown period at the bed they are occupying, therefore the data with this arrival date are not considered. The frequency of a certain LoS is visualized in Figure 4.8. To have a better insight into the dynamics, the long right tail is removed from the figures. The cutting point is decided by the looks of the original figures and around 1% is cut off, apart from ELV palliative where just below 7% of the data is cut off.



(a) ELV low/high complex



(b) ELV palliative



(c) GR

Figure 4.8.: Length of stay per department with the corresponding frequency.

Besides the given bar plots for the length of stay, a boxplot can give a different insight into the distribution. In Figure 4.9 these boxplots are given for all intermediate care departments. Included in the boxplots is the mean value for the length of stay for each department.

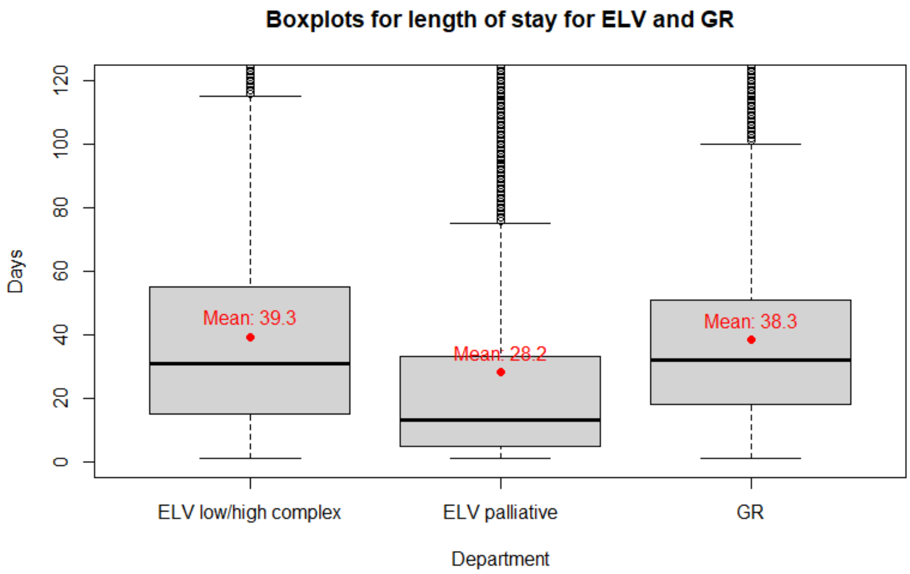


Figure 4.9.: Length of stay boxplot per intermediate care department with the corresponding mean value.

Within Figure 4.9 not all data is available, the y-axis is cut off in such a way that the first three quantiles are completely visible, but the outliers above are not visible. The length of stay ranges for each department to another value. Extra information on the LoS is given in Table 4.5.

Table 4.5.: Maximum, mean, standard deviation, and coefficient of variation of the LoS per department.

	ELV low/high	ELV palliative	GR
Maximum	1012	703	463
Mean	39.3	28.2	38.3
Standard deviation	42.8	34.2	28.9
Coefficient of variation[2]	0.87	1.52	0.75

For ELV low/high complex the mean value and the standard deviation, are closest to each other. If the standard deviation and mean are close to each other, that means that

[2]The coefficient of variation is given by $\frac{\text{standard deviation}}{\text{mean}}$

the coefficient of variation is close to 1.

Just as for the arrival distribution, now we are going to fit the best distribution for the length of stay. This is done with the use of *fitdist*, which uses the maximum likelihood estimator. Distributions that are potential candidates for the simulation of the length of stay are exponential distribution, lognormal distribution, Weibull distribution, and gamma distribution. Within this fitting of distributions, there is no distinction made in the arrival moment, just the length of stay is considered. This results in Figures 4.10, 4.11, and 4.12.

Since the length of stay distribution mainly has a high probability for small stays we zoom in on that part of the graph to see the best fitted distribution.

For every intermediate care department, the best fit distribution can be determined. However, it will not be a perfect fit, since data has some unpredictability in LoS by some events that cause longer or shorter stays and possible declaration mistakes. For every department, it is reasoned which distribution could simulate the lengths of stays the best per department.

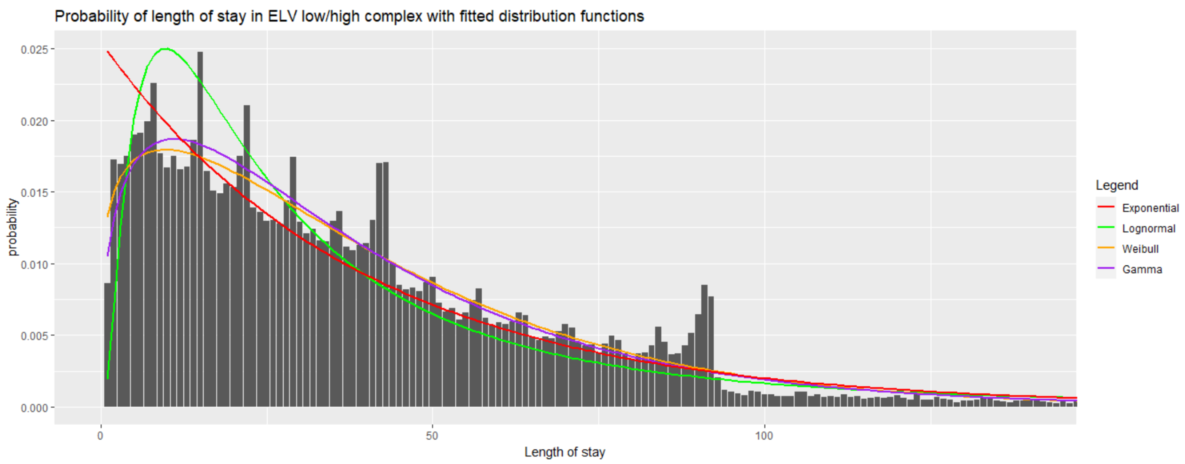


Figure 4.10.: The data and fitted distributions for the LoS of ELV low/high complex with the following parameters: Exp(0.025), lnorm(3.269, 1.000), Weibull(1.101, 38.801), gamma(1.205, 41.866).

For ELV low/high complex the best distribution fit seems to be the Weibull distribution. There are some peaks in the data, but apart from these peaks the orange curve (Figure 4.10), representing the Weibull distribution with parameters $k = 1.101$ and $\lambda = 38.801$, does approximate the distribution quite well. For this distribution with parameter $k > 1$ it means that the probability a patient leaves becomes larger when the patient is present longer.

For ELV palliative none of the distributions does approximate the length of stay distribution very well. All the distributions have a too small probability of patients with a short LOS. This could potentially be caused by the short bar for a stay of length 1. The distribution that comes closest to the actual data distribution is the Weibull distribution or the lognormal distribution. In Figure 4.11 the distributions are visualized.

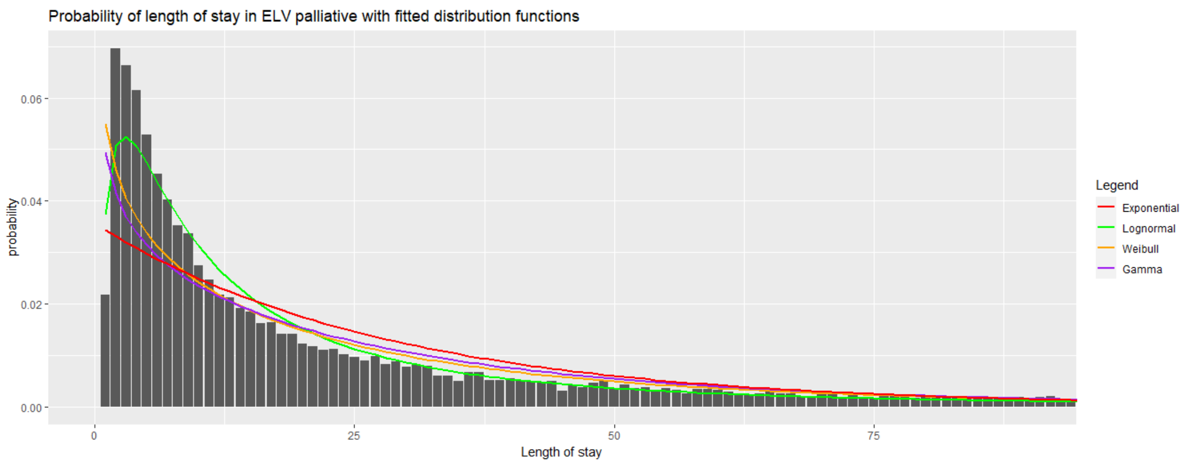


Figure 4.11.: The data and fitted distributions for the LoS of ELV palliative with the following parameters: Exp(0.036), lnorm(2.581, 1.244), Weibull(0.819, 24.788), gamma(0.787, 0.028).

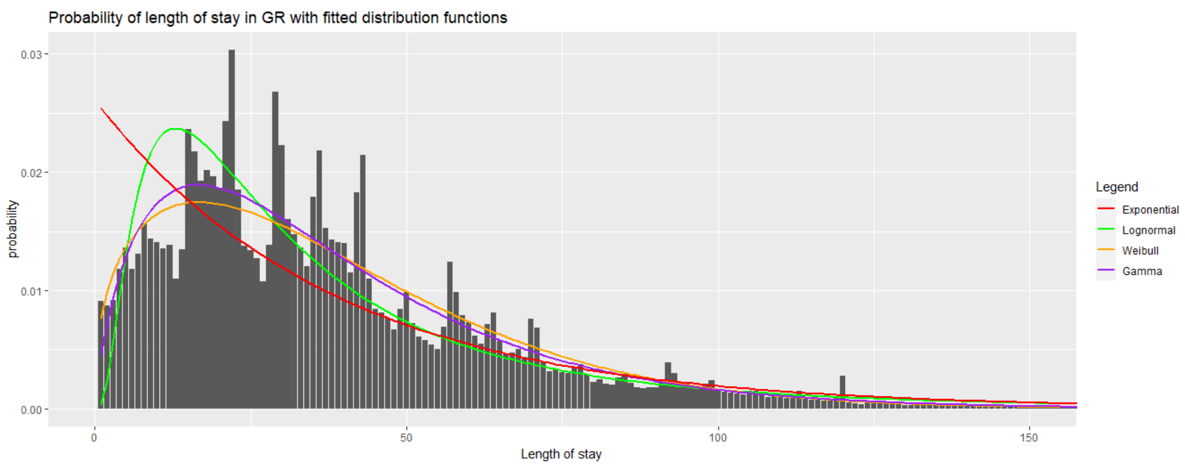


Figure 4.12.: The data and fitted distributions for the LoS of GR with the following parameters: Exp(0.026), lnorm(3.338, 0.877), Weibull(1.389, 42.071), gamma(1.771, 0.046).

In Figure 4.12 the distribution of the length of stay at a GR bed is visualized with the fitted distributions. The exponential distribution only decreases, therefore this is not a good fit. The other three distributions first increase and after a peak start decreasing, just like the actual distribution. The distribution that seems to be the best fit, is the gamma distribution, since the peak is about the height of the non-outlier peak of the data, and the behavior of the tail does approximate the data quite well.

So far, no distinction was made between patients that arrive during different moments in time. If the day of arrival influences the length of stay that would indicate that the patient groups are different during the days. A first analysis is done on the mean length of stay for the different days of the week. This results in Table 4.6.

Table 4.6.: Mean length of stay per intermediate care department and arrival day.

	ELV low/high	ELV palliative	GR
Monday	39.769	31.350	37.661
Tuesday	40.174	28.216	38.726
Wednesday	40.284	28.611	38.832
Thursday	39.979	28.158	38.995
Friday	39.489	27.170	38.844
Saturday	33.510	22.408	33.868
Sunday	29.651	19.226	24.572

For ELV low/high complex, ELV palliative, and GR there are shorter lengths of stay for patients that have arrived during the weekend days (Saturday and Sunday). The patients that arrived on Sundays had even shorter lengths of stay than patients that arrived on Saturdays. This difference does suggest that the easier patients are admitted during the weekends over the more complex patients, which causes a shorter stay for the patients arriving during the weekend.

The same type of fitted distribution figures as in Figures 4.10 - 4.12 could be made for the length of stay while making a distinction between the arrival days. However, these figures will not be made on a national basis. This will be done later, for the regional analysis in Section 4.7.

4.4. Occupied beds

With the use of the arrival moment and the length of stay, the occupancy can be determined. If the occupancy is determined nationally, the effect of fully occupied regions can cancel out against quiet regions. It is therefore useful to consider a few regions that are representative, separately.

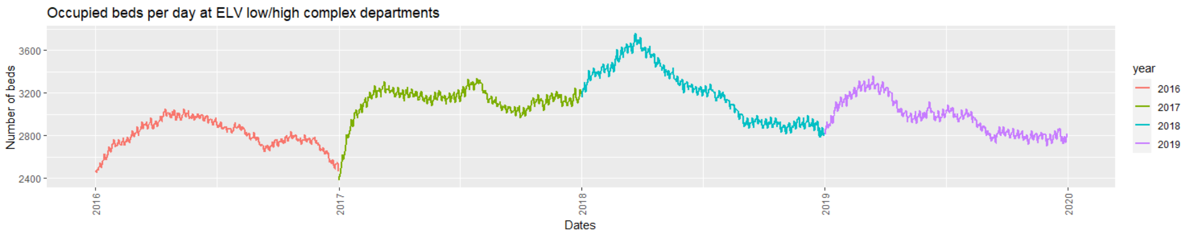
To get a representation of the number of beds that are occupied per day, new data sets are created for ELV low/high complex, ELV palliative, and GR. In these new data sets for every date, it is calculated how many beds are nationally occupied. It is not relevant which patient occupies the bed.

The occupancy for ELV low/high complex over time and the trends are given in Figure 4.13. The mean occupancy over the years is given in Table 4.7

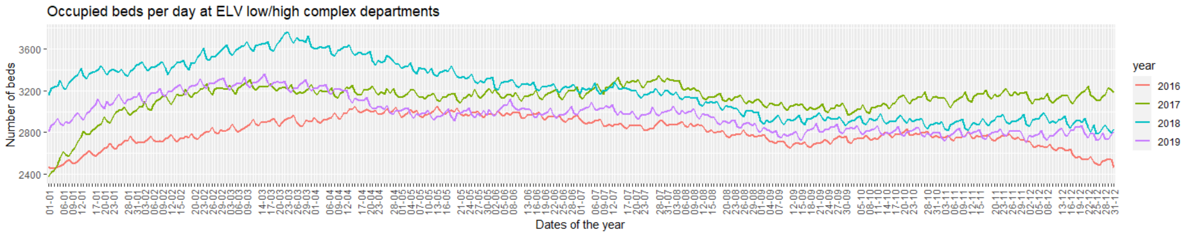
Table 4.7.: Mean occupancy of ELV low/high complex over the years.

2016	2017	2018	2019
2807	3124	3207	2975

In the first month of 2016, January, fewer patients occupy a bed, especially at the beginning of the month. The same is seen for the end of the last month, December, where the number of occupied beds also decreases toward the end. The number of present patients is about the same for the beginning of January 2016 and the end of December 2016, respectively 2466.6 and 2515.3 for the first and last five days of the year. Consequently, the first month of 2017 also has fewer occupied beds. In the first period, however, the number of occupied beds does increase significantly. For 2018 there is no dip in the number of occupied beds at the beginning of the year. During the first period in 2018, the number of occupied beds increased, after the peak it decreases. By the peak at the beginning of the year, the mean number of occupied beds has become higher than in 2017. The decreasing trend during 2018 does not continue during 2019. Since the start of 2019, there is a strong growth in the number of occupied beds. After a few months, a decrease occurs, which goes on for the rest of the year.



(a) Occupancy ELV low/high complex over the period 2016 - 2019.



(b) Occupancy ELV low/high complex for all years 2016 - 2019.

Figure 4.13.: Occupancy at ELV low/high complex departments.

Remark. *Note that the vertical axes in Figure 4.13 start from 2400, such that the underlying patterns are well visible.*

In Figure 4.13b it is visualized how the occupancy of ELV changes over the year. For all years there is an increase in the number of occupied beds in the first two and a half months of the year. Contrarily, the end of the years is not changing in the same manner. There is a clear decrease for 2016, while 2018 and 2019 stay about the same, and in 2017 there even seems to be an increase in the number of beds that are occupied.

On top of these yearly patterns, there also are some weekly patterns visible. The small peaks that are for all years visible represent the weeks. This is visualized more clearly in Figure 4.14, in which the mean number of occupied beds is visualized over the whole period 2016-2019.

Remark. *The scale of the vertical axis in Figure 4.14 is such that the difference between the several days is clearly visible, so there is not twice as much occupancy during Friday*

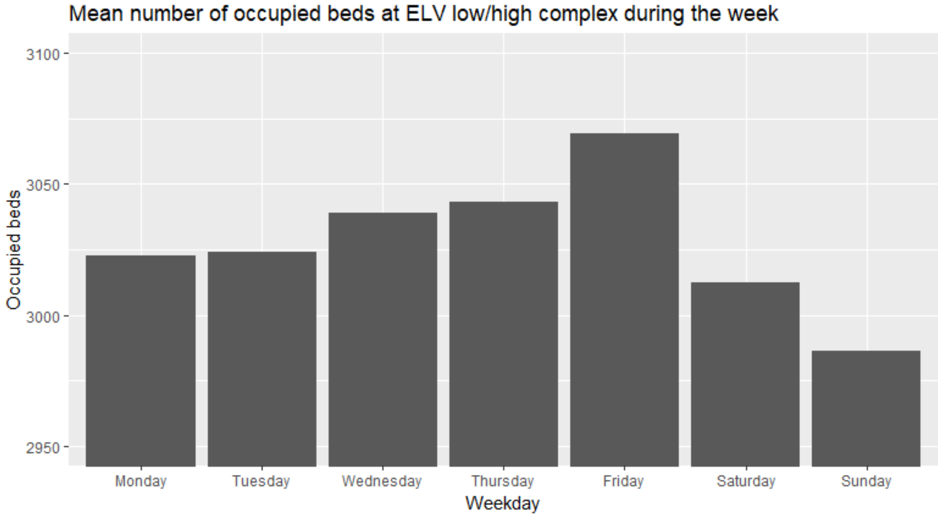


Figure 4.14.: Mean number of occupied ELV low/high complex beds per day of the week for the period 2016-2019.

compared to the weekend days.

In Figure 4.14 it is visualized that the number of occupied beds increases over the weekdays, while it decreases during the weekend. This pattern was also seen slightly in Figure 4.2a for the arrivals of patients at the ELV low/high complex departments.

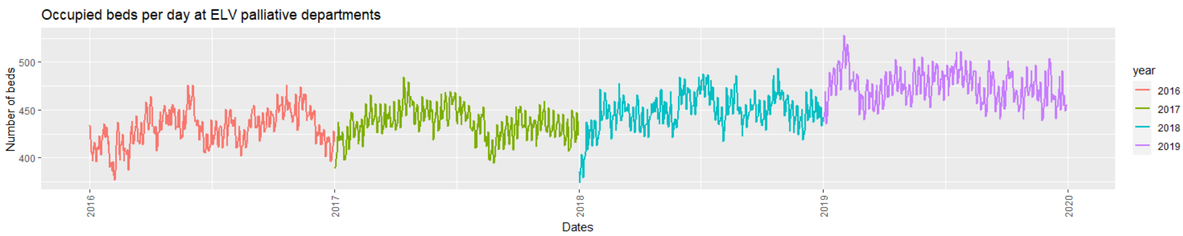
Similarly, the ELV palliative occupancy can be analyzed. The mean number of occupied beds is given in Table 4.8. The variety of occupancy is visualized in Figure 4.15.

Table 4.8.: Mean value of occupied beds for ELV palliative per year.

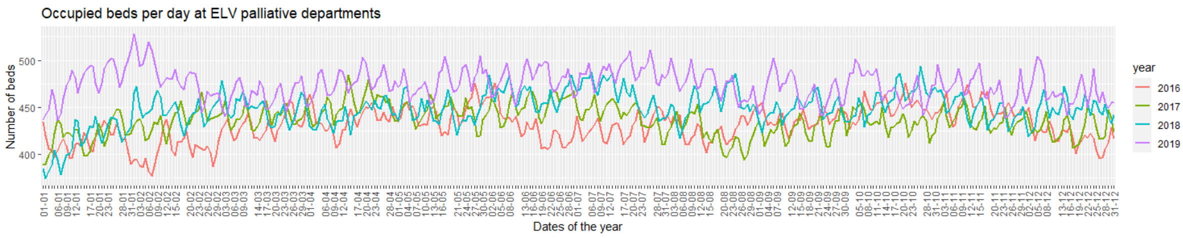
2016	2017	2018	2019
431	436	448	474

Remark. *Note that the vertical axes in Figure 4.15 start from 375, such that the underlying patterns are well visible.*

In Figure 4.15 it is visualized that the differences during the years are way smaller for ELV palliative than for ELV low/high complex. There is some fluctuation within the years, but in 4.15a it becomes visible that the occupancy increases over the years, which corresponds with the mean values of occupation per year that increases in Table 4.8. There are a few visible dips over the period 2016-2019: in the first quarter of 2016, during the end of 2016, the beginning of 2017, and the start of 2018. The start of 2018 looks quite weird since a large group of patients disappears overnight. This could probably be caused by an administrative mistake. There only is one peak that is significantly



(a) Occupancy ELV palliative over the period 2016 - 2019.



(b) Occupancy ELV palliative for all years 2016 - 2019.

Figure 4.15.: Occupancy at ELV palliative departments.

higher than the other peaks, this occurs in the first two months of 2019.

There seems to be a weekly pattern again for ELV palliative. To be sure about this pattern, the mean number of occupied beds per day of the week is given in Figure 4.16. This mean is calculated based on the data from 2016 up to and including 2019, shortly it is calculated over the complete period.

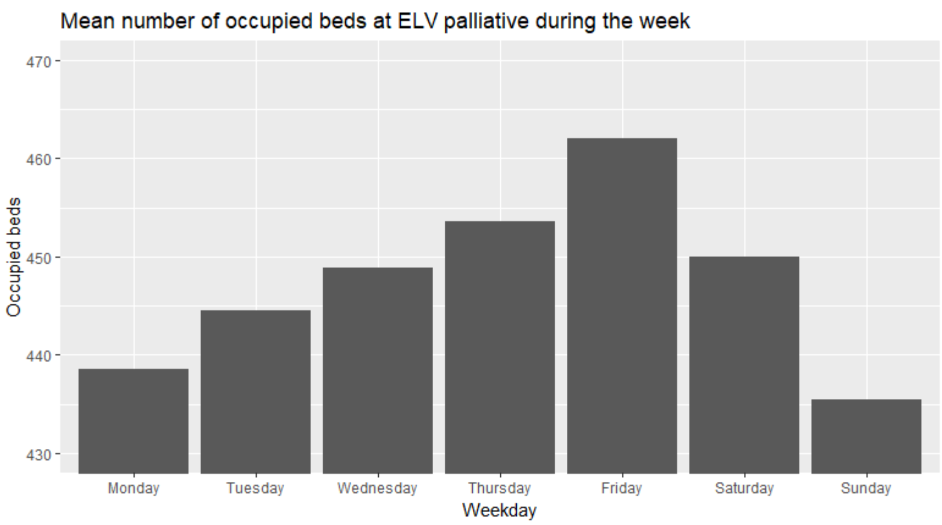


Figure 4.16.: Mean number of occupied ELV palliative beds per day of the week for the period 2016-2019.

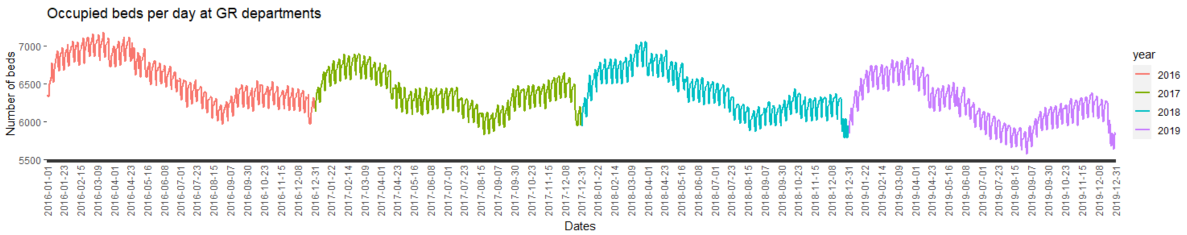
Remark. *The scale of the vertical axis in Figure 4.16 is such that the difference between the several days is visible.*

The weekly pattern for ELV palliative is similar to ELV low/high.

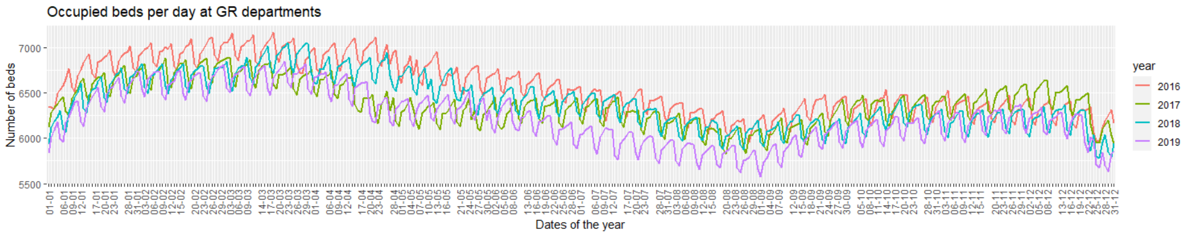
For GR the mean value of occupied beds in 2016, 2017, 2018, and 2019 are given in Table 4.9. There is more operational capacity in GR available compared to the operational capacity in ELV. This is also visualized in Figure 4.17, where the yearly pattern and the periodic pattern of the number of occupied beds can be seen.

Table 4.9.: Mean value of occupied beds for GR per year.

2016	2017	2018	2019
6562	6384	6399	6228



(a) Occupancy GR over the period 2016 - 2019.



(b) Occupancy GR for all years 2016 - 2019.

Figure 4.17.: Occupancy at GR departments.

Remark. *Note that the vertical axes in Figure 4.17 start from 5500, such that the underlying patterns are well visible.*

In Figure 4.17b it can be seen that the patterns over the years are approximately the same. In the first period of the year there is an increase in the number of occupied beds, later, around half of March there begins a decrease until the end of August. After the period of decrease, there starts a small increase where-after the number of occupied beds stabilizes for a few months. Then, during the last week of the year, the number of occupied beds does decrease again.

In Figure 4.17a the differences per year can be seen more clearly. The peak in the number of occupied beds is highest in 2016, for both 2017 and 2018 the peaks are about the same height and the peak is lowest in 2019.

For Figure 4.17, in which a period of one year and a period of four years is visualized, it is hard to see the weekly pattern, although there are peaks visible, which seem to be a weekly pattern. To have a better insight into this weekly pattern, Figure 4.18 is created. In Figure 4.18 the mean of the number of occupied beds is given for each day of the week. The mean is calculated over the whole period 2016-2019.

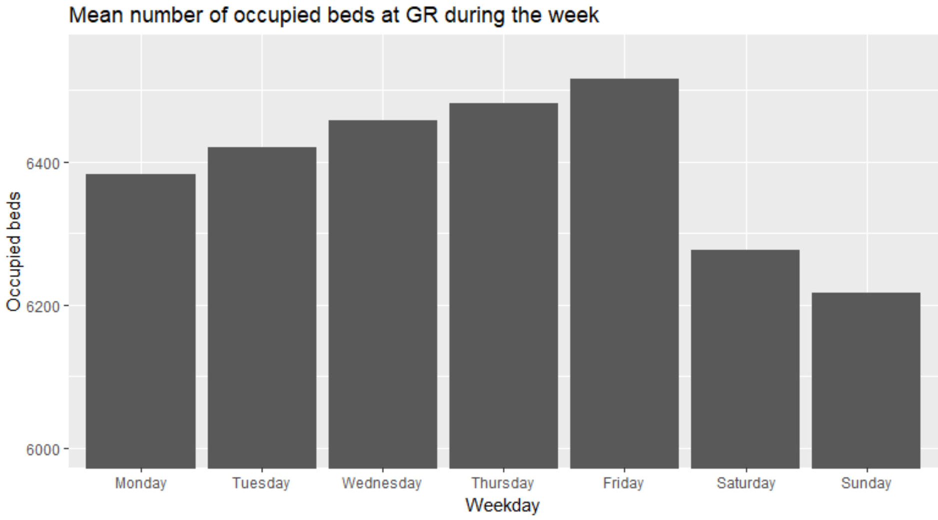


Figure 4.18.: Mean number of occupied GR beds per day of the week for the period 2016-2019.

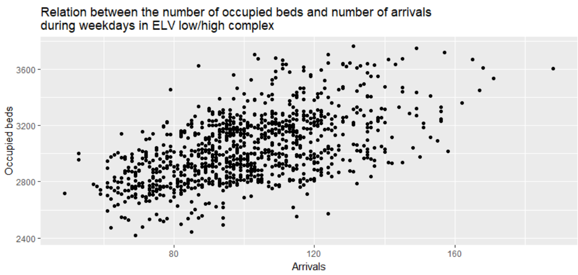
Remark. *The scale of the vertical axis in Figure 4.18 is such that the difference between the several days is visible.*

A similar pattern as for ELV low/high complex and ELV palliative is found in Figure 4.18. During the weekdays the number of occupied beds is increasing, while the number of beds decreases during the weekend. In contrast with the pattern found in ELV, the pattern during the week, for the mean number of occupied beds, does differ from the pattern of mean arrivals during the week. The mean number of arrivals is significantly smaller for the weekend days than the mean arrivals during weekdays, so by the fact that significantly fewer patients enter the GR during weekend days, there also are fewer patients that occupy a bed at the beginning of the week. This could be a declaration why the patterns are not the same.

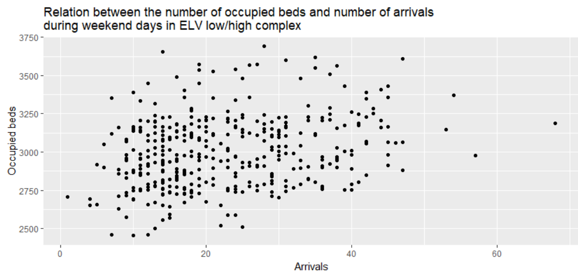
4.5. Arrivals versus Occupancy

The hypothesis is that there are less arrivals when there are a lot of beds occupied. This will result in a relation between the number of arrivals and the number of occupied beds. This could be the reason why there are less arrivals at some moments in time. On top of that, it could indicate if there happens a lot of blocking when many beds are occupied.

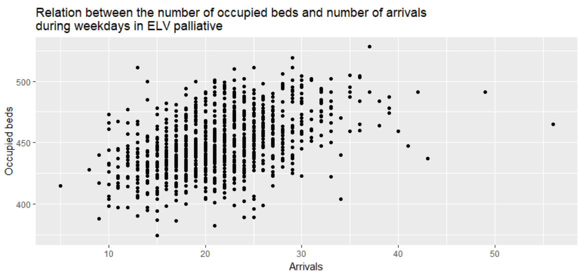
To see if the number of arrivals and the number of occupied beds are related a scatter plot is made for the three types of intermediate care. In this scatter plot the arrivals and occupancy for the same day are plotted, however, if the arrivals are plotted against the occupancy one day earlier about the same figures are made. The scatter plot for this relation is for all these three types of care given in Figure 4.19. Before creating these scatter plots, the dates of public holidays were removed, to eliminate the influence of these special days. There also is made a distinction between weekdays and weekend days as done before.



(a) ELV low/high complex during weekdays.

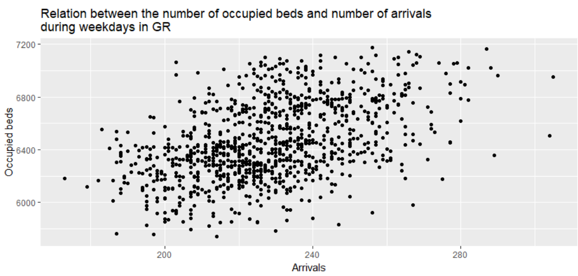


(b) ELV low/high during weekend days.

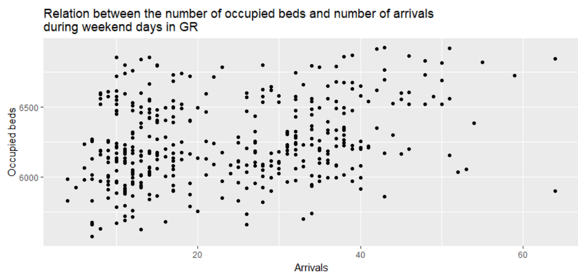


(c) ELV palliative during weekdays.

(d) ELV palliative during weekend days.



(e) GR during weekdays.



(f) GR during weekend days.

Figure 4.19.: Occupancy versus arrivals, weekend and weekdays separated.

Remark. *In the scatter plots of Figure 4.19 the arrivals are plotted against the occupancy for the same day, however, about the same figures result when the arrivals are plotted against the occupancy of one day earlier.*

The relation that the occupancy and arrivals have during weekdays for ELV low/high complex can be seen in Figure 4.19a. There is some positive relation between the number of arrivals and the number of occupied beds. Thus for the periods in which there are already many beds occupied, still lots of new patients arrive. However, the relation is stronger for less arrivals: the more beds are occupied, the wider the spread is within the figure. In Figure 4.19c the same sort of pattern for ELV palliative can be seen, however, it is less dense. Similarly for GR, there is the same sort of pattern (Figure 4.19e). So for all different departments, there is a small positive correlation visible for the weekdays. The correlation is strongest for ELV low/high complex.

In Figure 4.19b only the arrivals during the weekend days for ELV low/high complex are visualized. In Section 4.2, we have already seen that during the weekend days there are less arrivals, than for the other moments during the week. All data points that are visualized in the figure represent a weekend day with a relatively low number of arrivals. There is no clear relation between the occupancy and arrivals visible in the figure since the cloud of points does not seem to have any direction. This is the same for the correlation between the occupancy and arrivals of ELV palliative and GR.

The hypothesis that there might be less arrivals when there are a lot of beds occupied does not hold. So, we are not able to conclude that there is a connection between the occupancy and the arrivals. The only correlation that could be found was even slightly positive, and therefore the hypothesis certainly does not hold.

4.6. Patient routes

What is discussed in the section about arrivals so far, has only to do with the arrivals directly into the intermediate care facilities ELV or GR. To be able to analyze the arrivals properly it is useful to also look into the origin of the patient. So, the route of the patient will be analyzed.

In table 4.10 the number of patients is given that go from one department to ELV or GR. In this table, the total number of patients that go to ELV or GR is given but also the number of patients from which it is unknown where they come from. For the patients with unknown origin, it is clear that these did not go to VD, VB, IC, GR, ELV low/high complex, or ELV palliative. A patient cannot be referred to ELV or GR by himself, this can be done by a general practitioner (GP) or a specialist elderly care. Three options are possible: a patient had an intake at another department beforehand, a patient is referred by an ED, or a patient is referred by a GP. Unfortunately, there is no data available on patients that visit the ED or GP. On the other hand, there is data available on patients that visit other departments beforehand.

In Table 4.10 the left column indicates from where the patient leaves, the second column indicates the number of patients that go to ELV low/high complex, the third column gives how many patients go to ELV palliative, and the right column gives the number of patients that go to GR. These numbers are only the patients that are declared the same day or the next day in ELV or GR compared to the last day in the previous department.

For all three types of care, there is a part of the patients of which the origin is unknown. For ELV low/high complex from 54% of the patients, the origin is unknown, for ELV

Table 4.10.: Number of patients that follow the route from all departments to ELV or GR.

	ELV low/high	ELV palliative	GR
VD	37,052	9,058	161,658
VB	8,431	1,656	50,082
GR	3,555	1,571	3,753
ELV low/high	2,872	1,377	2,508
ELV palliative	85	509	10
IC	40	48	211
Unknown	60,623	8,945	24,510
Total	112,658	23,164	242,732

palliative that is 39%, and for GR that is 10%.

Remarkable is that the number of patients that went to a wrong bed directly before entering ELV or GR is relatively high. However, in the previous section (4.5) no relation between the occupancy and the number of arrivals was found.
It is also possible to see if a patient has been in the system for a longer route, so if the patient has been in three departments in consecutive periods. When analyzing the route of patients that went to three departments it was noticed that for some routes there were very few patients, for example, 3 or 4, but there even are routes none of the patients has taken. The routes with more than 1% of the patients that went through 3 departments and end up in ELV low/high complex are given in Table 4.11, the patients that end up in ELV palliative are given in Table 4.12, and the patients that end up in GR are given in Table 4.13. The routes in which a patient went to the same department consecutively are removed.

Table 4.11.: Number of patients that follow the route from and via other departments to ELV low/high complex with more than 1% of the total patients that go through 3 departments.

From	Via	To	Number of patients
VD	VB	ELV low/high complex	7,876
ELV low/high	VD	ELV low/high complex	3,607
VD	GR	ELV low/high complex	2,350
IC	VD	ELV low/high complex	2,199
VB	GR	ELV low/high complex	716
GR	VD	ELV low/high complex	347
Other			247
Total			17,342

The column From is the department from which the patient route has started, then the patient went to the department that is given in the column Via, after which the

Table 4.12.: Number of patients that follow the route from and via other departments to ELV palliative with more than 1% of the total patients that go through 3 departments.

From	Via	To	Number of patients
VD	VB	ELV palliative	1,565
VD	GR	ELV palliative	1,081
VD	ELV low/high	ELV palliative	440
IC	VD	ELV palliative	410
VB	GR	ELV palliative	320
GR	VD	ELV palliative	301
ELV low/high	VD	ELV palliative	232
VB	ELV low/high	ELV palliative	131
ELV palliative	VD	ELV palliative	71
Other			108
Total			4,659

Table 4.13.: Number of patients that follow the route from and via other departments to GR with more than 1% of the total patients that go through 3 departments.

From	Via	To	Number of patients
VD	VB	GR	47,454
IC	VD	GR	16,159
GR	VD	GR	12,314
ELV low/high	VD	GR	1,884
VD	ELV low/high	GR	1,034
Other			1,283
Total			80,128

patient ends in the department given in column To.
Notice that for both ELV and GR the most important routes almost all visit VD. For ELV low/high complex there only is one route in which VD does not appear, for ELV palliative two routes do not visit VD, and all routes to GR visit VD. In Table 4.10 it is visible that most patients that enter ELV or GR had a visit at VD directly before, and a smaller amount of patients had a visit at VB directly before. In Tables 4.11, 4.12, and 4.13 we see that the most popular route of length three starts in VD, then goes to VB and then goes to the intermediate care of need. Of the patients that go to ELV low/high complex after visiting a wrong bed, 93% originates from a VD, for ELV palliative and GR this percentage is 95%. So, mainly patients that attempt to go to some type of intermediate care from the hospital ward end up in a wrong bed beforehand. There are very few patients that do not originate from the hospital ward and end up in a wrong bed.

Apart from the percentages of patients following a certain route, it also is interesting how the arrivals at ELV low/high complex, ELV palliative, and GR are distributed when the origin of the patient is known. To calculate these distributions, the arrivals are partitioned into weekdays and weekend days this is done because Section 4.2 showed that this is a good partition. There also were some differences spotted per year, therefore the distribution is calculated for 2018 and 2019 for ELV low/high complex and ELV palliative, but for all years for GR.

Both ELV and GR care are sort of unplannable, that is, it is not known if and when a patient will need these types of care. For a patient that needs ELV low/high complex care, there is a temporary problem such that the patient cannot live at home (with extra home care), this patient does not have a particular path beforehand, such that it is not plannable when the patient will need ELV low/high complex. Similarly, for ELV palliative the patients only go there after there is concluded that the life expectancy is shorter than 3 months. For a patient that needs GR, it happens in most cases that the patient originates from the hospital, within the hospital, it is likely that the patient has undergone surgery. Since surgeries are planned, this will affect the GR arrivals. There are, also patients that came through a different route, which is less predictable. The less effect is passed by previously planned care, the more unplannable ELV and GR are, which causes independence between arrivals at these types of care. Given this independency of arrivals, an expected distribution would be a Poisson distribution for arrivals. For the Poisson distribution, the mean and variance have the same value. A measure of variation can therefore be given by the variance-to-mean ratio (VMR) $\frac{\text{Variance}}{\text{Mean}}$. If the VMR is equal to one, this indicates a completely random distribution, and thus a distribution that can be modeled by a Poisson process. [28]

The VMR is for each department to ELV low/high complex calculated and for each day of the week per year, so for example for all Mondays in 2016 the VMR of the arrivals at ELV low/high complex for patients that originate from VD is calculated, this has a value of 1.32. Per department the mean of the VMR over all days and years is calculated for both the arrivals at ELV and GR, to get an insight into the overall VMR for that department. Results can be seen in Table 4.14.

From Table 4.14 we find that almost all routes to ELV have a VMR that is close to 1, however, only the route from the hospital ward to ELV low/high complex has a VMR that is slightly higher with the value 1.22. Both the routes from the hospital ward via wrong beds to GR have a VMR that is 1.30, which is clearly higher than 1. For the other three routes to GR, the VMR is close to 1. Most routes to ELV and GR likely follow a Poisson arrival process. Thus if the origin of a patient is known, the arrival distribution often follows a Poisson distribution, when the origin of the patient was not taken into account this was not always possible as followed from Section 4.2.

4.7. Region analysis

As stated in Section 2.2, the Netherlands is divided into 31 regions. In table 4.15 there is an overview of the number of institutions that offer one of the five types of healthcare. Notice that the only institutions taken into consideration are the ones that have treated

Table 4.14.: Mean VMR for arrivals from different departments at ELV and GR.

From	To	VMR
ELV palliative	ELV low/high complex	1.01
VD	ELV low/high complex	1.22
VB	ELV low/high complex	1.08
IC	ELV low/high complex	1.01
GR	ELV low/high complex	1.05
ELV low/high complex	ELV palliative	1.10
VD	ELV palliative	1.02
VB	ELV palliative	0.99
IC	ELV palliative	0.98
GR	ELV palliative	0.99
VD	GR	1.30
VB	GR	1.30
IC	GR	1.04
ELV low/high complex	GR	1.08
ELV palliative	GR	1.00

10 unique patients or more over a period of one year. All other institutions are more likely to treat a patient more by coincidence than regularly.

Due to multiple causes, the number of ELV and GR institutions differ over the years. The most obvious cause is closure of institutions and new institutions, however, that will not happen very often, therefore there should be other causes why this happens. Due to irregularities and inconsistencies, organizations some years declare on the organizational level and other years on the location level. Besides, there was a policy change for ELV from 2016 to 2017 which could cause changes in the number of ELV institutions, since some could have received a new AGB-code (which is used to indicate unique care institutions). Simultaneously, there were changes in the way of contracting healthcare insurance.

To have an idea of the number of institutions that offer ELV, Table 4.15 gives the number of institutions that treated 10 or more unique patients in 2019. Note that this is the number of institutions that offer ELV low/high complex, ELV palliative, or both. This is the most recent available data that is not affected by COVID-19. To give an insight into the number of institutions over the other years a column is added with the mean number of institutions over the years 2017, 2018, and 2019. 2016 is left out of this mean because there were changes over the year caused by the change in policy.

In Table 4.15 the column GR 2019 gives the number of institutions that treated 10 or more unique patients in 2019. GR mean gives the mean number of institutions that offer GR over the period 2016 up to 2019. The changes are smaller for GR compared to ELV, therefore the means are closer to the number of institutions in 2019 for GR.

Apart from Flevoland, every region has a hospital. There used to be a hospital in

Flevoland, however, it went bankrupt in 2018. Some other hospitals went bankrupt within other regions, however, these regions had other hospitals that kept existing. Moreover, there were some merges of hospitals and new hospitals have arisen.

Table 4.15.: Number of institutions in the healthcare office region that offer the five types of healthcare.

Healthcare office	VD	IC	VB	ELV 2019	ELV mean	GR 2019	GR mean
'T Gooi	2	2	1*	8	8.33	4	4.00
Amstelland en de Meerlanden	2	2	2*	8	7.00	1	1.00
Amsterdam	6*	5*	5*	12	15.67	4	4.25
Apeldoorn/ Zutphen e.o.	1	1	1	15	15.67	2	2.00
Arnhem	4	4	4	25	25.00	9	9.00
Delft Westland Oostland	2*	2*	1	12	12.00	7	7.75
Drenthe	3*	2*	1*	11	12.33	4	5.00
Flevoland	1*	1*	1*	22	19.67	2	2.00
Friesland	4	4	3*	30	26.67	5	5.75
Groningen	3	3	3*	15	16.67	7	7.00
Haaglanden	4*	3	2*	20	19.67	5	5.75
Kennemerland	2*	1	1	18	14.33	5	5.50
Midden-Brabant	2*	1	1	9	8.67	3	3.00
Midden-Holland	1	1	1	9	7.67	2	1.25
Midden-IJssel	1	1	1	7	6.67	2	2.25
Nijmegen	3	3	3*	6	5.33	5	5.00
Noord-Holland Noord	2	2	2	17	15.33	3	3.00
Noord-Limburg	4	3	2	24	20.67	3	4.00
Noordoost Brabant	3	3	3*	8	9.33	6	6.00
Rotterdam	6*	6*	6*	10	14.67	6	6.75
Twente	2	2	2	8	7.00	7	7.00
Utrecht	5*	4*	3*	27	24.33	8	7.75
Waardenland	2	2	1*	12	11.33	5	5.75
West-Brabant	2	2	2*	18	18.33	5	6.00
Zaanstreek/ Waterland	2*	1*	1	9	11.67	2	2.00
Zeeland	2*	2	2*	10	10.00	5	4.50
Zuid-Holland Noord	2	2	2	19	15.67	3	3.00
Zuid-Hollandse Eilanden	2	1	1	14	13.67	2	2.00
Zuid-Limburg	2	2	1	5	5.33	5	5.00
Zuidoost-Brabant	4	4	4	15	16.33	6	6.00
Zwolle	3	3	3	17	16.00	6	6.00

* Within these regions there are differences in the number of hospitals and the types of care provided. The given number is the number of hospitals that were present over the longest amount of time.

Remark. *There is no physical department for patients on a wrong bed, the number of institutions with VB given in Table 4.15 are the hospitals that have declared patients with a wrong bed indication.*

From Table 4.15 it is clear that there are some differences between the regions. Some regions have multiple hospitals and others have more ELV institutions. Note that the number of ELV institutions is the number of locations where ELV (low/high complex and/or palliative) is offered. Apart from 10 unique patients, there are no restrictions on the locations that offer ELV. Therefore, one region could have 27 locations in which ELV is offered and another region has only 6 locations in which ELV is offered, but both regions do have about the same number of ELV patients. There is no information available on the number of beds these locations have, so it could be that there are ten available beds, but it could also be that there are thirty available beds.

Within the regions in the Netherlands intermediate care is organized differently. In some of the regions, it is better organized than in other regions. To have an overview of the regions which have organized it better, and the regions which have organized it worse, see Table 4.16, in which the percentage is given of patients that occupy a wrong bed before entering ELV or GR.

Remark. *The 'wrong bed' indication should only be used for patients that go to long-term care (WLZ), however, it also is used for patients that go to short-term intermediate care. Due to a financial incentive, the administration of VB is not done very accurately. Therefore the information given in Table 4.16 should be seen as a lower bound.*

In Appendix E.1 the information from Table 4.16 is visualized in two bar plots.

As said before, there are differences between the regions, but as can be seen in Table 4.16 and Figure E.1 there also are differences in the percentages of wrong beds before admission to ELV and GR. This can be caused by the treatment before. A patient that goes to GR comes more often from a hospital bed (87.3%) than patients that go to ELV low/high complex (40.4%) or ELV palliative (46.5%). That means that the majority of patients that go to ELV is redirected by the general practitioner or the emergency care unit without staying in the hospital. Since these patients are not in the hospital beforehand, they have to stay at home for some period until they can be admitted. These patients will receive extra care via district nursing, or family and/or friends step up for a short period. For patients that are waiting at home, there is no administration as for wrong beds, so the percentage of wrong beds does not necessarily match the percentage of patients waiting for ELV care.

Also for GR patients, the percentage of waiting patients is likely not the same as the percentage of wrong beds. However, since more patients are already in the hospital, there will be more patients registered as wrong bed patients, since these patients are not sent home for the waiting time.

There also is a possibility in the differences that can be caused by the consistency of the healthcare organizations within a region. For some regions, there are no patients

Table 4.16.: Percentage of patients that went to a wrong bed right before entering the intermediate care facility per region.

Healthcare office	Wrong beds before ELV low/high	Wrong beds before ELV palliative	Wrong beds before GR
'T Gooi	0.3	1.3	0.0
Amstelland en de Meerlanden	1.6	0.6	0.0
Amsterdam	12.9	28.2	15.9
Apeldoorn/Zutphen e.o.	6.8	21.5	15.5
Arnhem	12.4	21.8	3.8
Delft Westland Oostland	5.8	6.8	2.9
Drenthe	1.6	9.2	0.7
Flevoland	0.2	1.0	0.4
Friesland	4.0	23.3	4.4
Groningen	2.9	17.9	3.0
Haaglanden	4.8	8.8	7.1
Kennemerland	4.6	9.7	2.5
Midden-Brabant	7.3	26.5	11.0
Midden-Holland	1.0	24.6	5.2
Midden-IJssel	0.2	22.8	0.0
Nijmegen	1.4	14.4	2.4
Noord Holland Noord	17.3	42.8	5.0
Noord-Limburg	9.5	26.9	1.9
Noordoost Brabant	2.7	13.1	2.1
Rotterdam	10.2	22.5	8.3
Twente	16.4	46.4	17.1
Utrecht	1.5	12.3	4.5
Waardenland	2.5	3.9	2.6
West-Brabant	7.3	18.8	5.5
Zaanstreek/Waterland	4.6	6.7	6.8
Zeeland	1.4	2.9	1.4
Zuid-Holland Noord	1.3	4.7	1.0
Zuid-Hollandse eilanden	1.5	0.9	0.4
Zuid-Limburg	2.8	10.6	4.7
Zuidoost-Brabant	10.9	41.1	17.4
Zwolle	1.4	6.8	1.5

that go to a wrong bed before entering an ELV palliative institution. The first option is that this region has enough beds for ELV palliative available such that every patient that needs such a bed can be admitted directly. The second option is that the hospital which treats the patient does not register the patient as a patient on a wrong bed, while the patient actually is waiting for ELV palliative care. For these patients, the hospital could probably find a reason for which the patient needs to be treated, such that the patient does not have to lay in a wrong bed. There could be a financial incentive underlying this decision.

A region that has a high percentage of wrong beds can signify a very consistent hospital that correctly declares all wrong beds. Or it could be a region with problems in capacity for intermediate care. Similarly, a region with low percentages of wrong beds, it could be the case that there is not so much consistency in declaring wrong beds, such that there are small percentages. Or there actually are short waiting times or no waiting times at all. From this data, it is not possible to find out what would be the case. Anyhow, a high percentage of wrong beds indicates an issue regarding the throughput of patients from the hospital to a type of intermediate care. If these issues are solely caused by a capacity shortage or other factors underlie the issue is unclear. To clarify the percentages of wrong beds more interviews are needed with experts from the field.

Since the 31 regions are different from one another, there needs to be taken a choice in the regions that are taken into account for the analysis during this thesis. To have a bit of a representation of the Netherlands, it is decided to analyze and model two regions. The first region will be Zuid-Limburg, and the second region will be Amsterdam. These two regions are quite different from one another. Zuid-Limburg is a region that is very separated from the rest of the Netherlands, there only is a small border of about 10 kilometers with Noord-Limburg (in Figure A.1 it is the border with the region called Noord-en Midden-Limburg). Since there only is a small border, Zuid-Limburg can be seen as a self-contained region. Zuid-Limburg contains the city Maastricht, some other smaller cities, and some small villages. Amsterdam is quite another region, the healthcare region Amsterdam, only contains the cities Amsterdam and Diemen. The healthcare region is surrounded by multiple other regions (Zaanstreek/Waterland, Kennemerland, 'T Gooi, and Amstelland en de Meerlanden). Since the border between Amsterdam and other healthcare regions is quite large, there are more possibilities for interchanging patients between regions.

4.7.1. Region Zuid-Limburg

The first region to analyze is the region Zuid-Limburg. To have a bit of an idea about the region Zuid-Limburg and the types of patients we will potentially analyze, we will first give some population information about Maastricht, and then for the whole province Limburg. This information is not available for the area of the healthcare office region. The information about Maastricht [4] and about Limburg [2] are presented by https://allecijfers.nl/.

Maastricht counts 121,151 inhabitants, of which 47.8% is male and 52.2% is female. 24% of these inhabitants are in the age class 25-45, 24% is in the age class 45-65, and 22% is in the age class 65+. The educational level of the inhabitants is high for 35.6%, medium for 38.8%, and low for 25.6%. The population density within this city is 2,147 inhabitants per km^2. [4]

The numbers are slightly different over the whole of Limburg, which can be caused by the fact that there are not many large cities and a lot of countryside. The complete province Limburg counts 1,118,302 inhabitants, of which 49.8% is male and 50.2% is female. In the age class 25-45, 22% of the inhabitants can be found, that is 29% for the age class 45-65, while 25% of the inhabitants falls in the age class of 65+. The population density is lower for the province than for Maastricht only with 520 inhabitants per km^2. There is 26% of the inhabitants with a high educational level, 43% with a medium educational level, and 31% with a low educational level. [2]
The actual numbers will lay somewhere between the numbers given.

For the healthcare office regions Zuid-Limburg, is seen as one region, however, there seem to be three regions within this region: Heuvelland, Oostelijk Zuid-Limburg, and Westelijke Mijnstreek. Even though Zuid-Limburg is divided into 3 regions, during this research the assumption is made that Zuid-Limburg acts as one region. Firstly, the data were analyzed only for Zuid-Limburg. Based on the data, parameters are found to simulate queueing models.

ELV low/high complex Zuid-Limburg

From 2016 to 2017 there have been some changes in the system for ELV registration. It went from one type of law to another, which also causes possible mistakes and irregularities in the declaration data. As will be explained later, there was a centralization of dividing patients over the region during 2017. For these reasons, the analysis is done for 2018 and 2019.

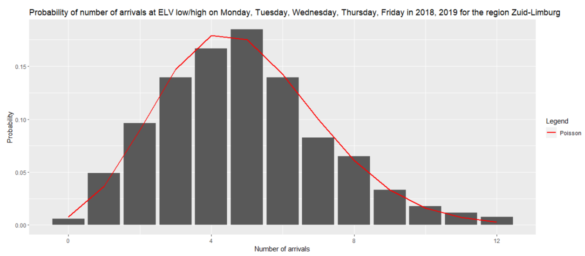
To know what type of model would fit best, multiple persons from the field are interviewed. The first to interview is Frank Amory, who was the project leader for setting up a regional ELV-point in Maastricht Heuvelland, from which all low complex ELV and high complex ELV patients were divided over that part of the region Zuid-Limburg. This ELV-point is founded in October 2017, afterwards, Frank Amory stayed involved with the ELV-point. Currently, the ELV-point has real-time insight into the available beds of ELV within the region. To reach this insight, multiple institutions that offer ELV within the region have agreed on a cooperation covenant. Before this was realized, the ELV-point was only a central point to sign-up patients, after which the ELV-point had to call every institution in the region to find an available bed for the patient. The cooperation of institutions and a regional point of dividing patients have caused all ELV low complex and ELV high complex patients can enter currently ELV as they need to within 1 day. Before October 2020 the ELV-point was not yet as efficient as it is nowadays, the time it took to handle administrative matters and find a bed before the patient could enter was longer than 1 day. Within the period 2018 and 2019 the handling time before admission had an average time of 5.4 days for ELV low/high complex according to Frank Amory. This is a large difference from which the efficiency of the ELV-point comes forward.

In Zuid-Limburg there is no fixed capacity only for ELV low/high complex. The largest ELV institution within the region Zuid-Limburg is Envida which offers both low and high complex ELV care, next to that they offer WLZ care (long-term care).

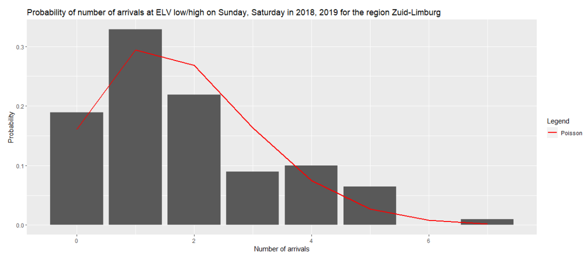
The operational beds at Envida are never reserved for any type of care. Whenever a bed becomes free, there are people within the organization that decide which patient can enter. If a patient is waiting for ELV that patient will likely be admitted, if no ELV patient is waiting and there is a WLZ patient actively waiting, then that patient is being admitted. The second largest organization that offers ELV in Zuid-Limburg is Sevagram. Sevagram does have a separate department, especially for patients with a short-term stay. At this department, there are 124 beds available for patients that receive ELV low/high complex care, GR care, or patients for whom it is not certain that they can go home afterward, but are not entitled to long-term care as well [27]. All these beds are located at the same location, so it is not divided over the organization locations.

What follows from this information already is that even in one region different institutions can already have a different organization of capacity. However, for both the interviewed organizations it can be concluded that there is not a strict capacity for ELV. Even though there is not a fixed amount of capacity for ELV, there is a maximum capacity for ELV.

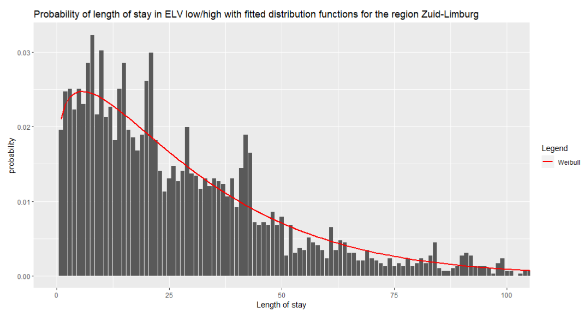
The arrival distributions and the length of stay distribution for ELV low/high complex are given in Figure 4.20, together with the best fitted Poisson distribution for the arrivals and the best fitted Weibull distribution for the length of stay.



(a) Distribution of arrivals during weekdays, with fitted Pois(4.886) distribution.



(b) Distribution of arrivals during weekend days, with fitted Pois(1.826) distribution.



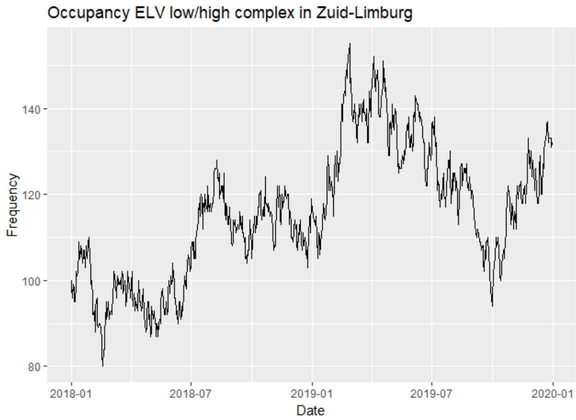
(c) Distribution of length of stay, with fitted Weibull(1.162, 31.044) distribution.

Figure 4.20.: Distributions needed to create a simulating model for ELV low and high complex.

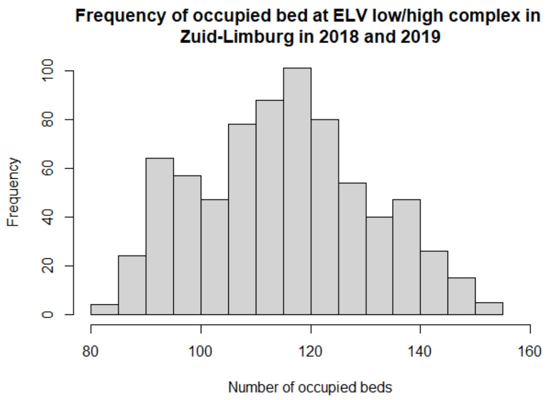
In Figure 4.20 the fitted distributions are given for the arrivals of ELV low/high complex patients during weekdays and during weekend days and the fitted distribution is given for the length of stay, this is all based on the data of 2018 and 2019 for the region Zuid-Limburg. For all fitted distributions the parameters are given, the arrival rate during weekdays is 4.886, and the arrival rate during weekend days is 1.826, these arrival rates are the mean number of arrivals per day. The distribution with the best fit for the length of stay is the Weibull distribution with shape parameter 1.162 and scale parameter 31.044. The mean length of stay can be calculated with the use of the gamma function $31.044 \cdot \Gamma(1 + \frac{1}{1.162}) \approx 29.456$.

The number of occupied ELV low/high complex beds in Zuid-Limburg changes over time, there are more beds in 2019 compared to 2018. This can cause some irregularities in the data, however, we assume that the parameters fitted on the data of both years are a good fit, so we keep working with these parameters.

The difference between 2018 and 2019 can also be seen in Figure 4.21, where there seem to be two peaks. The left peak in Figure 4.21b is caused by the data from 2018 and the right peak is caused by the data from 2019. This visualizes clearly that there is more occupancy within 2019, which also is visualized in Figure 4.21a.



(a) Dynamics of the occupancy.



(b) Frequency of number of occupied beds.

Figure 4.21.: Occupancy of ELV low/high complex in Zuid-Limburg.

Since the arrival rate differs for weekdays and weekend days, the mean number of occupied beds is calculated per day of the week. In Table 4.17 the mean occupied beds per day of the week is given.

Table 4.17.: The mean number of occupied beds per day of the week for ELV low complex and ELV high complex in Zuid-Limburg.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
115.905	115.552	115.471	115.644	117.288	115.202	114.135

The patients for ELV low/high complex can always enter the care of need (however, this could take about 5 days on administrative matters), therefore it is likely to assume that there is a model with infinite beds.

During the length of stay analysis, it was found that the arrival date influenced the length of stay. The mean length of stay is also calculated for the ELV within Zuid-Limburg and given in Table 4.18.

Table 4.18.: Mean length of stay for ELV low/high complex per arrival day.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
33.324	32.754	34.549	32.543	33.365	25.470	20.450

The largest difference occurs during the weekend, over the week the mean length of stay does not fluctuate too much, and as soon as it becomes weekend the mean length of stay drops. For patients that arrive on Saturdays the mean length of stay is higher than for patients that arrive on Sundays. Although Saturday and Sunday still have different mean lengths of stay we take these days together and go and estimate the parameters for a fitted distribution. On the other hand, the weekdays are taken together to find the parameters for the fitted distribution.

When we use a week-weekend split for the ELV low/high complex arrivals in Zuid-Limburg, the parameters for the Weibull distribution will be as follows:

Table 4.19.: Fitted parameters for the Weibull distribution for the time-dependent length of stay for ELV low/high complex in Zuid-Limburg.

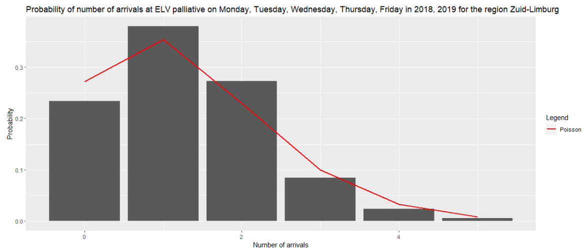
	Shape	Scale
Week	1.182	32.330
Weekend	1.097	22.952

ELV palliative Zuid-Limburg

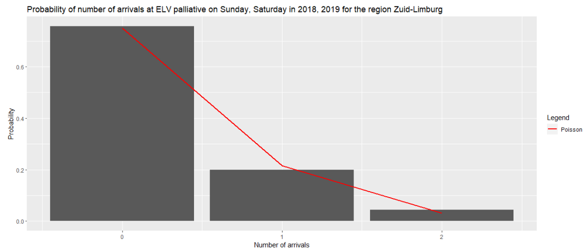
For ELV palliative, things are slightly different compared to ELV low/high complex, there are specified hospices for ELV palliative, which have a fixed capacity. However, patients with an ELV palliative indication are not the only patients that qualify for a bed in a hospice. Some patients already have a WLZ indication that go to a hospice, this is declared as WLZ, and on top of that can patients that receive district nursing qualify for a bed in a hospice, while they are not indicated with an ELV palliative. [23, 40] This makes it likely that the best model to use has a fixed number of beds where patients have to wait in a queue if every bed is occupied, however, the patients that are indicated with ELV palliative will not occupy all hospice capacity. Therefore the needed capacity can be approximated by a model with infinite capacity (infinite beds).

Before a model can be made, the distributions need to be determined.

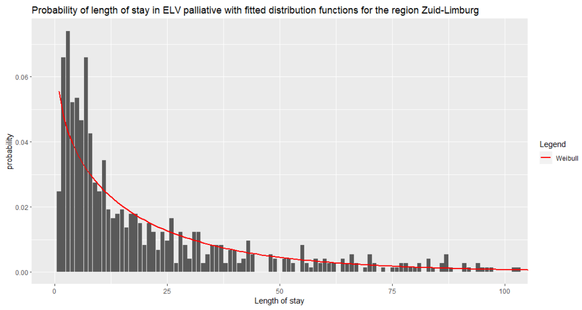
Figure 4.22 gives the fitted distributions for the arrivals at ELV palliative departments, often hospices, during weekdays, and during weekend days, besides the fitted distribution for the length of stay at ELV palliative care beds is given. These fitted distributions are based on the declaration data for 2018 and 2019. The arrival rate for weekdays is



(a) Distribution of arrivals during weekdays, with fitted Pois(1.303) distribution.



(b) Distribution of arrivals during weekend days, with fitted Pois(0.289) distribution.



(c) Distribution of length of stay, with fitted Weibull(0.869, 21.863) distribution.

Figure 4.22.: Distributions needed to create a simulating model for ELV palliative.

fitted at 1.303, and for weekend days it is fitted at 0.289, which are the mean numbers of arrivals per day at ELV palliative. The fitted shape and scale parameters for the Weibull distribution are $k = 0.869$ and $\lambda = 21.863$. The mean length of stay is then: $21.863 \cdot \Gamma(1 + \frac{1}{0.860}) \approx 23.463$ days.

Over 2018 and 2019 the number of occupied ELV palliative beds does not change significantly over time, during the period 2018-2019 the overall line of occupied beds stayed about the same. In Figure 4.23 the occupancy over time and the frequency of the number of occupied beds are given.

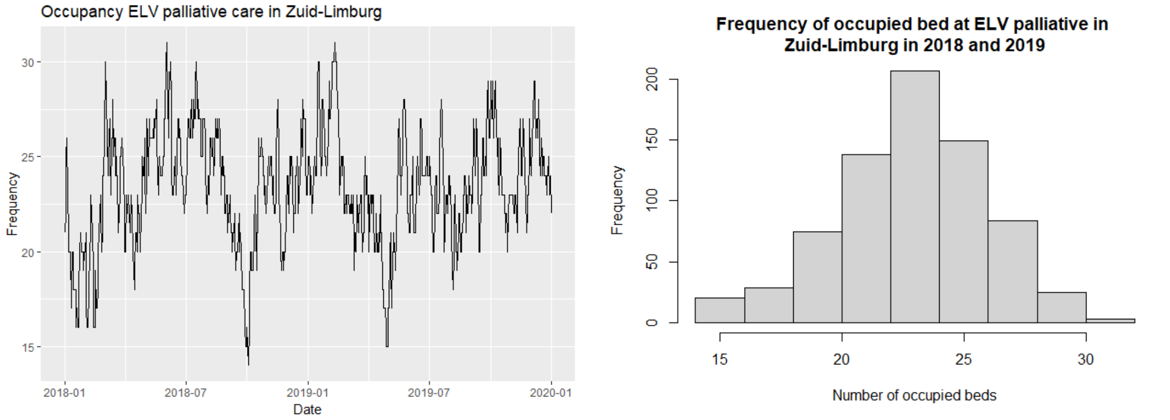
The mean number of occupied beds per day of the week is given in Table 4.20.

Table 4.20.: The mean number of occupied beds per day of the week for ELV palliative in Zuid-Limburg.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
22.905	23.295	23.490	23.808	24.087	23.413	22.577

During the length of stay analysis, it was found that the arrival date influenced the length of stay. In Table 4.6 the national mean lengths of stay are given based on the day of arrival. This is also calculated for the ELV within Zuid-Limburg. in Table 4.21

For ELV palliative there is not a clear split between the weekend and the weekdays. Remarkably, the arrivals for Monday to Wednesday have longer lengths of stay than the rest of the week, while the patients that arrive on Sunday have a shorter length of stay



(a) Dynamics of the occupancy. (b) Frequency of number of occupied beds.

Figure 4.23.: Occupancy of ELV palliative in Zuid-Limburg.

Table 4.21.: Mean length of stay for ELV palliative per arrival day.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
29.914	29.495	27.237	23.429	22.751	24.742	18.185

than the remaining days. A better split would be into three types of days: Monday to Wednesday, Thursday to Saturday, and Sunday. Unfortunately, the data does not have enough arrivals on Sundays to calculate reliable parameters for Sunday. Therefore, Sunday is taken with the days of the end of the week. For the two groups remaining the parameters for the fitted distributions are calculated.

When we use a Monday-to-Wednesday and Thursday-to-Sunday split for the ELV palliative arrivals in Zuid-Limburg, the parameters for the Weibull distribution will be as follows:

Table 4.22.: Fitted parameters for the Weibull distribution for the time-dependent length of stay for ELV palliative in Zuid-Limburg.

	Shape	Scale
Monday - Wednesday	0.870	23.917
Thursday - Sunday	0.875	19.649

GR Zuid-Limburg

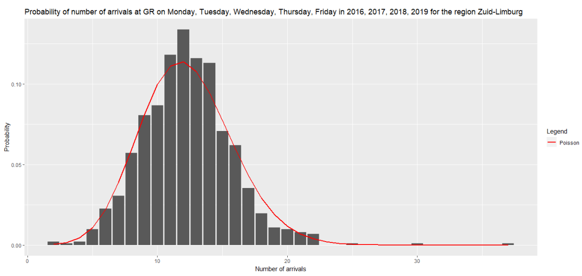
Unlike ELV, there were no severe changes for GR organization over the period 2016-2019, therefore, the data from 2016 up to and including 2019 can be used to fit a model.

The rules to receive a GR indication are stricter than the rules to receive an ELV indication. There usually are typical treatments connected to a previous surgery or there is a typical plan of revalidation for a patient that has followed a certain route. There are only 6 types of institutions that offer GR within Zuid-Limburg.

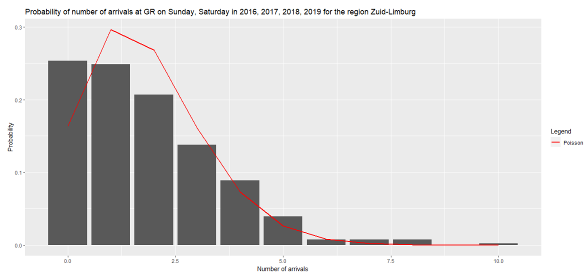
For one of the institutions, Cicero Zorggroep, the expected end date is determined by the intake, such that there is a goal to work to [29]. At this institution there are special departments for revalidation care, however, next to GR there also is care for patients that are recovering, but not in need of medical care in the hospital.

As explained in the part about ELV, Sevagram has a department for GR and other types of short-term care. Within the region Zuid-Limburg, Sevagram is the largest provider of GR. The second largest provider is Stichting Zuyderland zorg, which is part of the hospital Zuyderland Medisch Centrum. In this institution, there are three departments each with 30 beds for GR. [41] Again, there are differences between the different institutions offering GR.

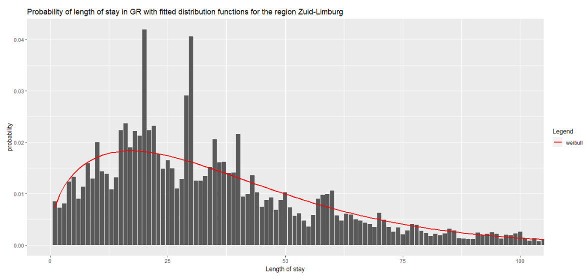
The arrival distribution of GR is split into weekdays and weekend days, just as done for ELV. These split arrivals are fitted with a Poisson distribution, this is shown in Figure 4.24, just as the length of stay with a fitted Weibull distribution.



(a) Distribution of arrivals during weekdays, with fitted Pois(12.284) distribution.



(b) Distribution of arrivals during weekend days, with fitted Pois(1.808) distribution.

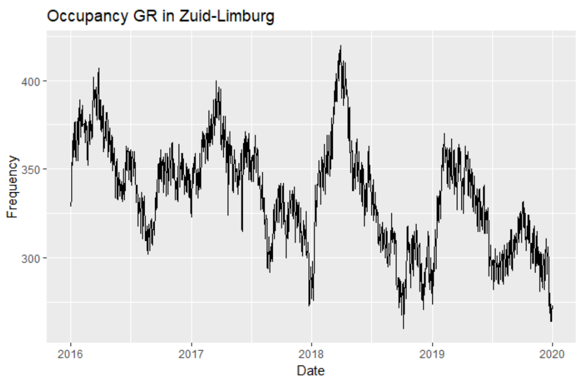


(c) Distribution of length of stay, with fitted Weibull(1.422, 40.222) distribution.

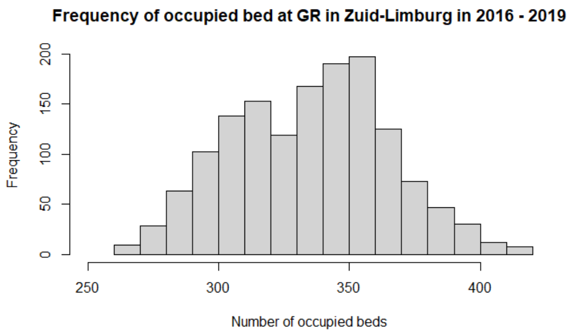
Figure 4.24.: Distributions needed to create a simulating model for GR.

The fitted distributions to both the arrivals and the length of stay of GR are visualized in Figure 4.24. The arrivals are fitted with a Poisson distribution, for the weekdays the corresponding arrival rate parameter is 12.284, and for the weekend days the corresponding arrival rate parameter is 1.808. These are the mean number of arrivals during weekdays and weekend days respectively. The fitted shape and scale parameters for the Weibull distribution are $k = 1.422$ and $\lambda = 40.222$. The mean length of stay is then: $40.222 \cdot \Gamma(1 + \frac{1}{1.422}) \approx 36.574$ days.

The occupancy of GR does not vary a lot over the years in Zuid-Limburg, the same pattern occurs over the years. At the beginning of the year the occupancy goes up quite fast, which peaks around the beginning of quarter three, after the peak, the occupancy starts to decrease. For some years there is again an increase for the last period of the year and then a stable amount of capacity, while for other years this does not happen. To have a better insight into the dynamics of the occupancy, itis given in Figure 4.25 over the complete period, together with the frequency plot.



(a) Dynamics of the occupancy.



(b) Frequency of number of occupied beds.

Figure 4.25.: Occupancy of GR in Zuid-Limburg.

The mean number of occupied days of the week is given in Table 4.23.

Table 4.23.: The mean number of occupied beds per day of the week for GR in Zuid-Limburg.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
334.818	337.220	339.663	340.894	343.426	327.990	325.330

Just as for the ELV, there is a difference in the length of stay based on the time of arrival. This results in the mean lengths of stay per day of arrival as given in Table 4.24

Table 4.24.: The mean length of stay per day of arrival for GR in Zuid-Limburg.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
34.051	37.104	37.549	37.722	37.175	32.923	30.190

From these numbers, we could conclude that patients that arrive from Tuesday to Friday have a similar length of stay. The arrivals during weekend days have a smaller length of stay, Monday arrivals also have a shorter length of stay than the arrivals on the rest of the weekdays, but it is higher than for the weekend arrivals. When splitting the length of stay data into the different arrival days, it is not possible to fit a distribution for the weekend days (see Figure 4.26).

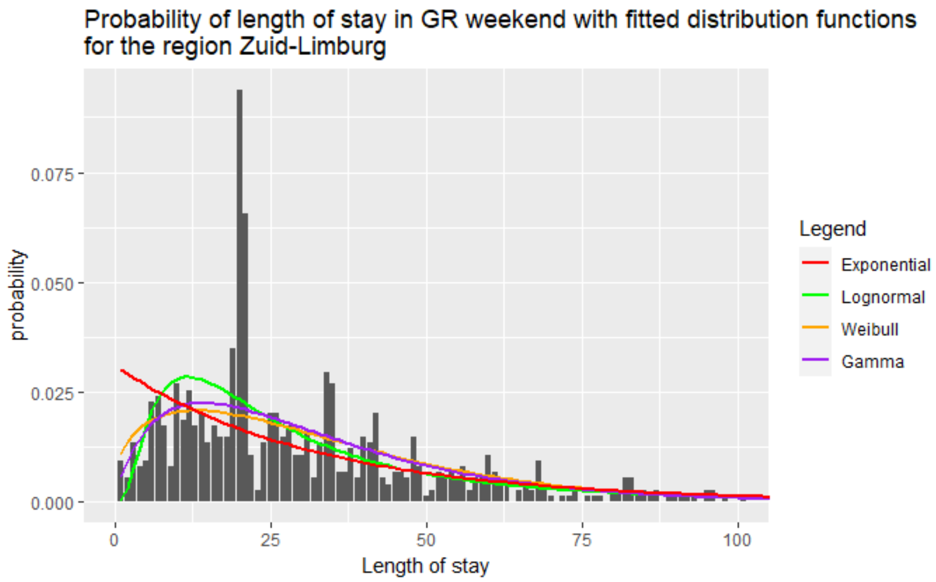


Figure 4.26.: Length of stay data with fitted distributions for patients that arrive during the weekend.

None of the distributions is a good fit for the actual data, this is caused by the peak around 21 days. Most likely this peak is caused by a fixed revalidation procedure for a certain type of patient. The data for Monday does not have enough data points to fit a good distribution, therefore the natural split is taken: week vs weekend. If we would fit the Weibull distribution, just as for the length of stays without time dependency, then we would find the parameters given in Table 4.25.

Table 4.25.: Fitted parameters for the Weibull distribution for the time-dependent length of stay for GRZ in Zuid-Limburg.

	Shape	Scale
Week	1.427	40.517
Weekend	1.352	35.296

Note that these values are not a good representation of the data, but these give the best approximation with the use of a Weibull distribution.

4.7.2. Region Amsterdam

The second region to analyze is the healthcare region Amsterdam, containing the city Amsterdam and the village Diemen. As said, this region is different from Zuid-Limburg for geographical reasons, there is a lot more border with other healthcare office regions. On top of that, the population differs, and thus the patients in intermediate care differ. https://allecijfers.nl/ provides information about the inhabitants of the municipality Amsterdam [3]. There are 882,633 inhabitants in Amsterdam, and the ratio male to female is 49.6:50.4. These inhabitants live with 5,277 per km^2. The educational level of the inhabitants of Amsterdam is divided as follows: 48% high education level, 29.3% medium education level, and 22.7% low education level. Then we are interested in the age of the inhabitants of Amsterdam, 36% of the inhabitants are in the age class 25-45, 23% is in the age class 45-65, and 13% is in the age class 65+.

ELV low/high complex Amsterdam

By the changes in law 2016 is not a representative year to consider, however from 2017 on the data is representative, therefore the years 2017, 2018, and 2019 are considered.

Through some interviews, information is gathered about the organization of ELV in the region Amsterdam. Unlike Zuid-Limburg, Amsterdam has specified departments only for ELV patients. So, there are no mixed departments for patients with ELV and some other healthcare indications. With the use of this organization, there is no chance of varying the number of available beds. Five organizations within Amsterdam offer ELV, and all of these have a fixed capacity for ELV patients. From the start of ELV, some institutions began with more beds than they currently have. Shortly, institutions have cut the capacity for ELV patients over time. For example, Cordaan had 100 ELV (ELV low/high complex and ELV palliative summed) beds at the start of ELV, while it has nowadays about 60 beds left. This is most likely also the case for the period of three years that we are considering during this analysis. This should be kept in mind while estimating the capacity.

There are two routes possible to be taken to enter ELV in Amsterdam.

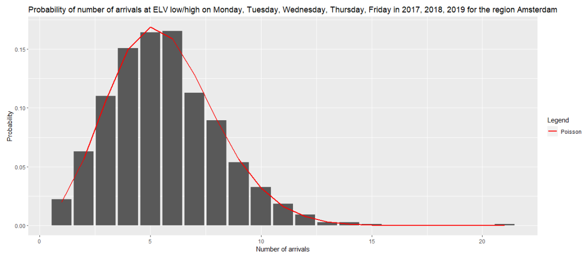
1. A patient goes to the general practitioner or ED and is signed up via the signup portal, which searches for an ELV bed for the patient within Point.

2. A patient stayed at the hospital and the transfer nurse of that hospital searches for an ELV bed for the patient within Point.

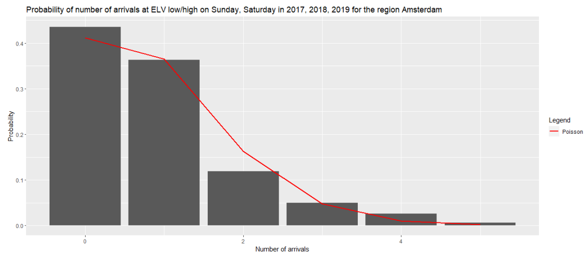
These two routes are now discussed. Within the region Amsterdam there is a central ELV sign-up portal, from which patients are divided over the ELV institutions in Amsterdam. General practitioners and ED doctors only have to call this central sign-up portal, before 2017 they needed to call all institutions within the region to find an available bed. If the sign-up portal cannot find an available bed, the general practitioner or ED doctor can decide to place the patient on the waiting list. Whenever a bed becomes available, the sign-up portal notifies the doctor of the first patient on the waiting list. However, some patients receive care in the hospital before entering ELV, these patients are not signed up at the sign-up portal first, but are signed up via a transfer nurse from the hospital. Both the transfer nurse and the employees of the sign-up portal are checking for an available bed in the system Point. In Point, all ELV providers need to keep track of their beds and fill in the number of available beds and the date from which these beds will be available.

Point is an online and centralized system for all ELV beds in Amsterdam. However, since this is not real-time updated, there is some delay and therefore the possibility of multiple sign-ups for one bed. Despite this drawback of the system, we can say that patients are distributed over Amsterdam via one central system. Therefore it is decided to see the region Amsterdam as one large ELV system, instead of multiple smaller systems.

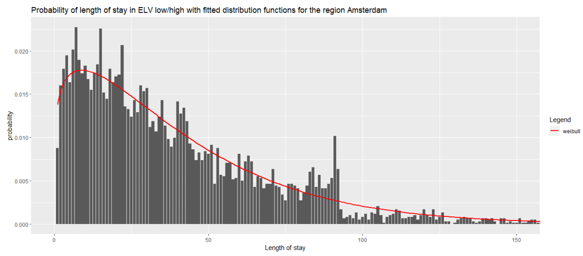
The arrivals for the ELV low/high complex are Poisson distributed, in Figure 4.27 the fitted distributions are seen for both weekdays and weekend days. On top of that, the distribution of the length of stay for patients at ELV low/high complex is given.



(a) Distribution of arrivals during weekdays, with fitted Pois(5.642) distribution.



(b) Distribution of arrivals during weekend days, with fitted Pois(0.888) distribution.



(c) Distribution of length of stay, with fitted Weibull(1.182, 42.732) distribution.

Figure 4.27.: Distributions needed to create a simulating model for ELV low/high complex.

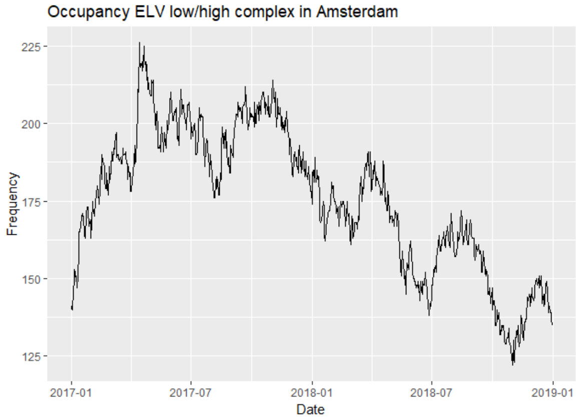
For the arrivals of the ELV low/high complex patients, a Poisson distribution is fitted both for the weekdays and for the weekend days. For the length of stay, a Weibull distribution is fitted. In Figure 4.27 the data and fitted distributions are plotted, given are the parameters for these fitted distributions. For the weekly arrivals, the arrival rate equals 5.642, while the arrival rate for weekend days equals 0.888. That are simultaneously the mean arrivals per weekday and per weekend day, respectively. The parameters for the length of stay are $k = 1.182$ as the shape parameter and $\lambda = 42.732$ as the scale parameter. With these parameters, we find the mean length of stay as $42.732 \cdot \Gamma(1 + \frac{1}{1.182}) \approx 40.361$.

As mentioned before, the number of ELV low/high complex decreased, which is also visible in Figure 4.28a. The decrease over time results in a widely spread frequency for the occupancy, this is visualized in Figure 4.28b.

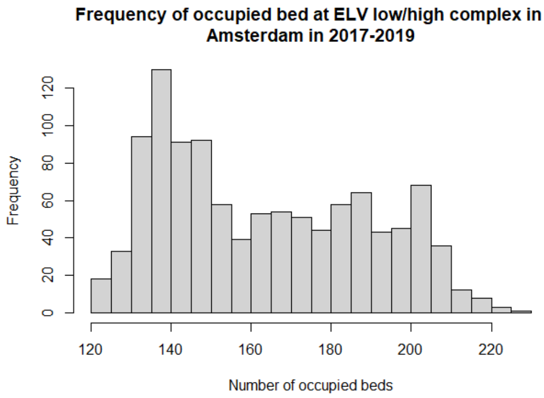
The number of occupied beds per day of the week is given in Table 4.26.

Just as for the ELV in Zuid-Limburg, the lengths of stay are dependent on the day of arrival. The mean lengths of stay are given per day of arrival in Table 4.27.

The ELV low/high complex does have a clear difference between the patients that arrive during the weekdays and the arrivals during the weekend days. For the best fit of



(a) Dynamics of the occupancy.



(b) Frequency of number of occupied beds.

Figure 4.28.: Occupancy of ELV low/high complex in Amsterdam.

Table 4.26.: The mean number of occupied beds per day of the week for ELV low complex and ELV high complex in Amsterdam.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
159.522	159.187	159.870	160.188	161.732	158.627	157.555

Table 4.27.: Mean length of stay of ELV low/high complex based on arrival day.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
138.718	41.256	41.236	41.088	41.444	33.724	29.637

the Weibull distribution, we have found the following parameters.

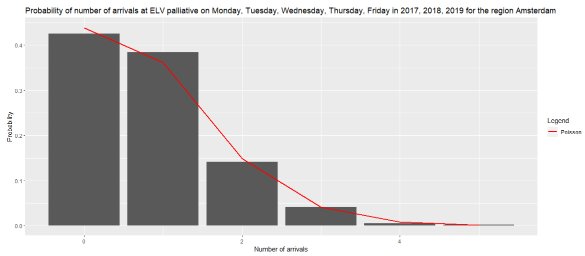
Table 4.28.: Fitted parameters for the Weibull distribution for the time-dependent length of stay for ELV low/high complex in Amsterdam.

	Shape	Scale
Week	1.186	41.814
Weekend	1.076	31.708

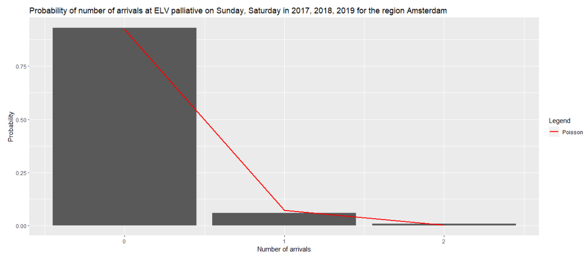
ELV palliative Amsterdam

Of the five organizations that offer ELV in Amsterdam, only three offer ELV palliative. However, over the years it happened a few times that the other two organizations treated an ELV palliative patient. This only happened incidentally, and therefore we can use that only three organizations are offering ELV palliative. The organization which has treated the most ELV palliative patients is Amstelring, however from current numbers Cordaan has 1 bed more than Amstelring. Amsta does offer ELV palliative but only has treated 35 patients over the three years. The latest information gives that Amsta has 4 ELV palliative beds, Amstelring has 11 ELV palliative beds and Cordaan has 12 ELV palliative beds. However, this could have changed over the years, just as for ELV low/high complex beds.

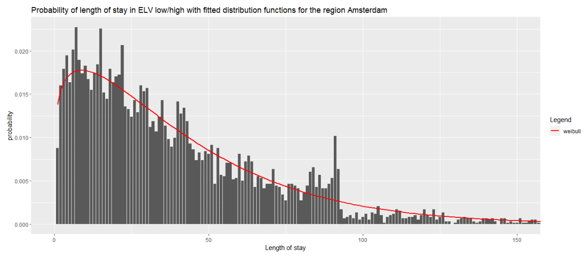
Again, the data is fitted with a Poisson distribution for the arrivals and with a Weibull distribution for the length of stay. In Figure 4.29 the data is plotted together with the fitted distributions.



(a) Distribution of arrivals during weekdays, with fitted Pois(0.824) distribution.



(b) Distribution of arrivals during weekend days, with fitted Pois(0.079) distribution.



(c) Distribution of length of stay, with fitted Weibull(0.804, 27.503) distribution.

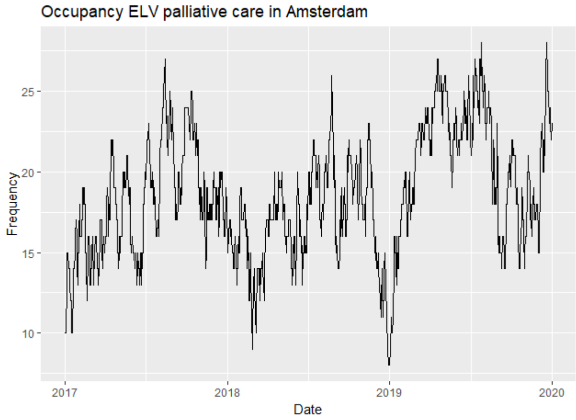
Figure 4.29.: Distributions needed to create a simulating model for ELV palliative.

What could be seen in Figure 4.29 is that the fitted distribution for the week arrivals is a Poisson(0.824) distribution, which means that the arrivals rate is equal to 0.824. During the weekend days, the arrival rate is lower, the fitted distribution is a Poisson(0.079) distribution. This means that on average there is fewer than one patient arriving per day for ELV palliative. During the weekend days most often there is not even an arrival, on average there is 1 arrival per 12.625 days, for the weekdays there is on average 1 arrival per 1.213 days. The length of stay distribution is fitted with a Weibull distribution, the shape parameter is $k = 0.804$, and the scale parameter is $\lambda = 27.503$. These parameters result in a mean length of stay of 31.039 days.

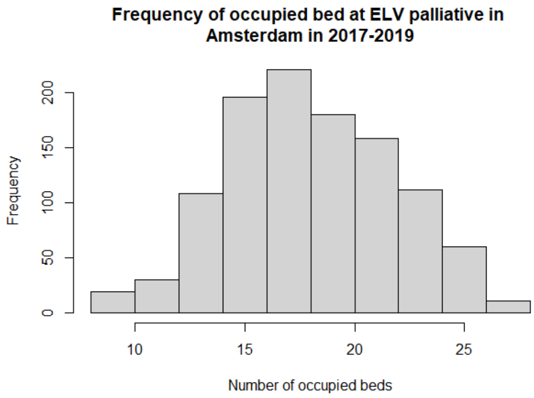
Compared to ELV palliative in Zuid-Limburg, there is more variation in Amsterdam. Especially in the first half of 2019, the occupancy is higher than the rest of the period. In Figure 4.30 the dynamics of the occupancy can be seen.

The number of occupied beds per day of the week is given in Table 4.29.

Just as for the ELV in Zuid-Limburg, the lengths of stay are dependent on the day of arrival. The mean lengths of stay are given per day of arrival in Table 4.30.



(a) Dynamics of the occupancy.



(b) Frequency of number of occupied beds.

Figure 4.30.: Occupancy of ELV palliative in Amsterdam.

Table 4.29.: The mean number of occupied beds per day of the week for ELV palliative in Amsterdam.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
18.144	18.282	18.394	18.712	18.928	18.378	17.813

Table 4.30.: Mean length of stay of ELV based on arrival day.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
43.873	26.052	32.257	31.766	27.286	27.348	10.083

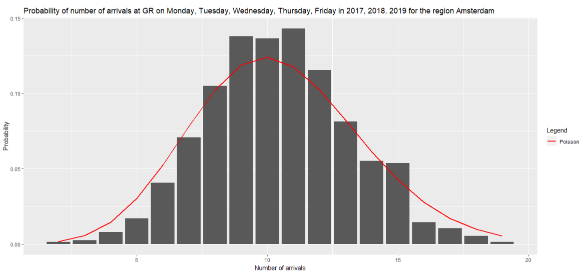
The ELV palliative in Amsterdam does not have a clear division during the weekdays, the patients that arrive on Mondays stay the longest with almost 44 days on average, while patients that arrive on Sundays have the shortest length of stay, with a little above 10 days on average. For the arrivals on all other days of the week, the lengths of stay are in between 10 and 44 days, but most lengths of stay are around 30 days on average. Therefore, we could try three groups: Monday, Tuesday to Saturday, and Sunday. However, this would likely result in bad approximations caused by too few data points. The ELV palliative in Amsterdam had over the years 2017, 2018, and 2019 only 7 arrivals during Sundays, therefore it is not possible to fit a distribution over this period. Due to this reason, there is no time dependency considered for the ELV palliative in Amsterdam.

GR Amsterdam

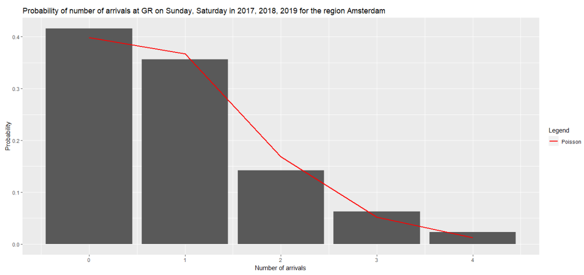
Similarly as for Zuid-Limburg, we can consider the period 2016 up to and including 2019. There is no clear trend in the number of occupied beds within the region Amsterdam. Over the complete period, the number of occupied beds stays roughly the same, excluding some outlier periods.

There are four organizations offering GR care within Amsterdam. Amstelring and Cordaan are the largest two (respectively 4045 and 4036 unique patients over the four years), while Amsta is smaller with 773 unique patients over the four years, Zorggroep Amsterdam-Oost is in between with 2347 unique patients over the four years. Cordaan has three locations in which GR care is offered. One of these locations is not in Amsterdam but is located in Nieuw Vennep, due to administrative matters, this is declared as if it is within Amsterdam. Therefore it is taken into account for this analysis. Together the three locations have 110 beds for GR, of which 24 beds are in Nieuw Vennep.

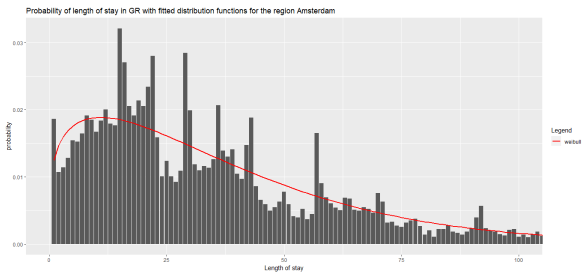
As done for the other situations (region and type of care), again a Poisson distribution is fitted for the arrivals during weekdays and the arrivals during weekend days, and a Weibull distribution is fitted for the length of stay.



(a) Distribution of arrivals during weekdays, with fitted Pois(10.437) distribution.



(b) Distribution of arrivals during weekend days, with fitted Pois(0.921) distribution.



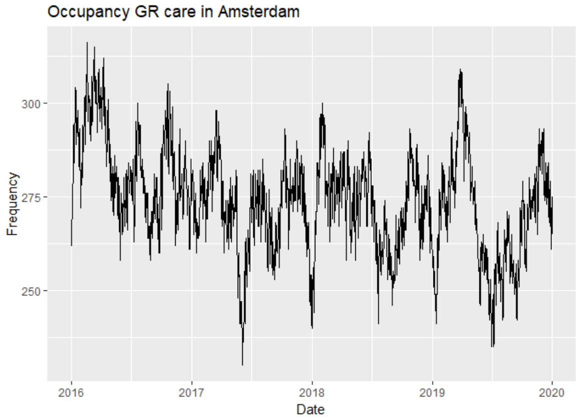
(c) Distribution of length of stay, with fitted Weibull(1.253, 39.292) distribution.

Figure 4.31.: Distributions needed to create a simulating model for GR.

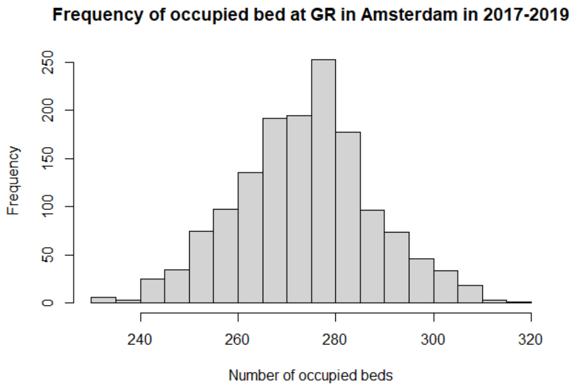
In Figure 4.31 the fitted distributions are plotted together with the data. The arrivals during weekdays are fitted with a Poisson distribution with an arrival rate of 10.437, which is the mean number of patients entering GR in Amsterdam per weekday. For the weekend days, the fitted Poisson distribution has an arrival rate of 0.921. The length of stay distribution is fitted with a Weibull distribution with shape parameter $k = 1.253$ and scale parameter $\lambda = 39.292$. The mean length of stay can be calculated by $39.292 \cdot \Gamma(1 + \frac{1}{1.253})$, which gives us an approximate mean length of stay of 36.577 days.

The number of occupied beds per day of the week is given in Table 4.31.

The time dependency in the length of stay is considered for the other intermediate care types and remaining regions, it lasts us to still do this for the GR in Amsterdam.



(a) Dynamics of the occupancy.



(b) Frequency of number of occupied beds.

Figure 4.32.: Occupancy of GR in Amsterdam.

Table 4.31.: The mean number of occupied beds per day of the week for GR in Amsterdam.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
274.488	274.842	276.135	277.221	278.694	269.689	267.392

To incorporate the time dependency, we have calculated the mean lengths of stay for the arrivals per day of the week. This results in Table 4.32.

Table 4.32.: The mean length of stay for arrivals per day of the week for GR in Amsterdam.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
35.874	37.065	37.021	36.742	37.048	28.993	26.655

This could be split into two groups, arrivals during the weekend and arrivals during the weekdays. Similarly as for the GR in Zuid-Limburg, the length of stay for arrivals during the weekend have not a good fit, however, the approximation is a lot better than for the weekend arrivals for GR in Zuid-Limburg. A side note is that the better-fitted distribution is the lognormal distribution instead of the Weibull distribution. Therefore the length of stay for GR in Amsterdam has to be calculated with two different types of distributions. For the weekdays the Weibull distribution is a better fit, so we find the following parameters.

Table 4.33.: Fitted parameters for the time-dependent length of stay of GR in Amsterdam.

Weibull	Shape	Scale
Week	1.255	39.572
Lognormal	**Logmean**	**Log Standard deviation**
Weekend	3.026	0.807

5. Mathematical queueing model

By the data analysis of Chapter 4 there is already some insight into the approach needed to find the operational capacity. Chapter 4 provides the parameters and distributions required in the queueing models that are discussed below.

With the use of the arrival distribution, and the length of stay distribution a queueing model can be made. To find the best queueing model that fits the data, multiple models are considered. These models are used in multiple manners. Firstly there will be done analytical calculations, how that is done can be found in Section 5.1. Next, there are simulations done with the use of the same models, these algorithms to do these simulations are explained in Section 5.2. Finally, to check the models with the data validation is done, this is discussed in Section 5.3.

Division into regions
Since ELV and GR cannot be seen as one big department in the Netherlands, a division is made such that the Netherlands is divided into 31 regions. These regions are defined as healthcare office regions, which are described in Section 2.2. Within such a healthcare region the care is organized for the complete region, that is ideally, there is a central administrating office that distributes patients over the multiple ELV and GR institutions. Unfortunately, this is not the case for every region within the Netherlands, causing some regions to be significantly better organized than others. However, since 1 April 2018, there is a national obligation that there is a covering network of regional coordination locations for ELV. This obligation caused every healthcare office region now has a regional coordination location for ELV. [25]

Assumption 5.1. *During this research, we assume that within a region ELV can be seen as two large departments: ELV low/high complex, and ELV palliative. GR can be seen as one large department over the region.*

This can be backed by the fact that patients prefer to enter an intermediate care institution close to their home and family and therefore the intermediate care facility would likely be in the same region. Additionally, some of the types of care are under the responsibility of the general practitioner, who is mostly close to the patient's home. A GP is not likely to travel across the country to help its patients, so the patient should stay close enough.

Base cases
The two base cases to model, Zuid-Limburg and Amsterdam, have two different types of organization. Where the ELV and GR institutions in Zuid-Limburg have a varying

amount of capacity, the ones in Amsterdam have a fixed amount of capacity. This causes differences in the way of modeling the queue.

In Sections 4.7.1 and 4.7.2 we have seen that the arrivals for ELV low/high complex care, ELV palliative, and GR follow approximately an inhomogeneous Poisson process for both Zuid-Limburg and Amsterdam. As stated before, we assume that arrivals occur according to a time-dependent Poisson process. Besides, we assume that the Poisson distribution gives the correct arrival process for all other regions within the Netherlands. That leads to the following assumption.

Assumption 5.2. *The arrivals occur according to a time-dependent Poisson process for ELV low/high complex, ELV palliative, and GR in all regions within the Netherlands.*

The length of stay is not the same for every type of care in the base cases. Moreover, the length of stay has not the best fit with the exponential distribution. Therefore, the length of stay follows a general distribution. On top of that, there seem to be differences in the length of stay based on the arrival moment for some situations. This is best seen in the ELV low/high complex length of stay, and the worst in the ELV palliative, for GR there are more influences from the policy, which makes it harder to split the data time-dependently.

There is a difference in the type of model for the number of beds. The two possible methods to consider are $M_t|G|\infty$, and $M_t|G|s|s$. As described in Chapter 2, the $M_t|G|s|s$ queue can be approximated with the use of an $M_t|G|\infty$ queue. Therefore, we will first model both base cases with the use of the $M_t|G|\infty$ queue.

First, assume a system with a varying amount of capacity. This capacity can be expanded based on the demand, therefore one could say that there is an infinite amount of capacity. Thus, here the $M_t|G|\infty$ model would be a good fit. Multiple calculations can be done on the $M_t|G|\infty$ model, as already explained in Chapter 2.

5.1. Analytical calculations

The used models are described above, these models will calculate analytically how many beds are needed to fulfill a requirement of maximally 2% blocking or exceeding probability.

To do the calculations, first the parameters are needed, for the three types of care in the two regions, these parameters are calculated and given in the data analysis. With the use of these parameters, the modified offered load is calculated for the infinite server models. To calculate the modified offered load Formulas 3.5, 3.7, and 3.11 are used. This results in a modified offered load for each day of the week. With the modified offered load per day of the week, the number of needed beds can be calculated per day of the week in two ways, dependent on the used model.

The $M_t|G|s|s$ queue has a time-dependent arrival process, therefore we are not able to use Formula 3.2 as the blocking probability; however, the modified offered load of the $M_t|G|s|s$ queue can be approximated by its infinite server counterpart, the $M_t|G|\infty$ queue. The only difference occurs if there are s beds occupied, for the $M_t|G|s|s$ queue patients are blocked and leave the system, while in the $M_t|G|\infty$ queue the patients are admitted, and being served. For all situations in which there are fewer than s patients present, the two queues are the same. With the use of the modified offered load from the infinite server queue, the blocking probability of the $M_t|G|s|s$ queue can be approximated. This method is called the Modified-Offered-Load (MOL) approximation. The MOL approximation is by W.A. Massey and W. Whitt [16] defined as:

$$B_t \approx B(s, m(t)) = \frac{m(t)^s/s!}{\sum_{j=0}^{s} m(t)^j/j!}. \tag{5.1}$$

By Formula 3.11 the modified offered load for the $M_t|G_t|\infty$ model can be calculated. Since the modified offered load is known, we can also use Theorem 3.3 to conclude that the number of present patients at time t, $N_\infty(t)$, is Poisson distributed with mean $m(t)$. Similarly, with the use of the approximated MOL for the $M_t|G_t|s|s$ the number of occupied beds $N_s(t)$ is Poisson distributed with its approximated MOL.

Remark. *In the blocking model there is made use of an approximation, which makes it uncertain that the result is correct. To check if the approximation gives a correct result, simulations are needed to compare results.*

1. Infinite server model: By Theorem 3.3 the number of present patients is distributed by $N_\infty(t) \sim \text{Pois}(m(t))$, so the minimum needed number of beds s to have a maximum of 2% exceeding that capacity is calculated by: $\mathbb{P}(N_\infty(t) \geq s) \leq 0.02$, with t fixed.
Note that this calculation is done per day of the week, so define N_∞^{day} as the number of present patients on the given day of the week, then $N_\infty^{\text{day}}(t) \sim \text{Pois}(m_{\text{day}}(t))$ and $\mathbb{P}(N_\infty^{\text{day}}(t) \geq s) \leq 0.02$.

2. Blocking model: By the use of the MOL approximation and Formula 5.1, the minimum number of needed beds can be calculated such that the blocking percentage is maximally 2%. To do so s is found such that $\frac{m(t)^s/s!}{\sum_{j=0}^{s} m(t)j/j!} \leq 0.02$. Again, this calculation is done separately per day of the week, since each day has its own value for $m(t)$, thus there will be a different value for s for each day of the week.

5.2. Simulation

To do simulations, the software of Rstudio [26] is used. There are multiple packages used: data.table, DescTools, dplyr, EnvStats, fitdistrplus, fst, ggplot2, graphics, lubridate, plyr, RODBC, stringr, and tidyr. All are freely available via Rstudio.

A simulation is done with a given start moment and end moment. The start and end moment should simulate over a period of 11 years, in which the first year can be seen as a warming-up period. So there is a period of 10 years which represents the real-world situation of demand for short-term intermediate care beds and length of stay. As explained in Section 5.1, the calculations for the $M_t|G|\infty$ model give the correct values, therefore the simulation and the calculations should give the same result. However, for the $M_t|G|s|s$ model the calculations are based on approximations, therefore the simulation can be used as an extra check to see if the approximations are correct.

Both the infinite server model and the blocking model will be simulated with the use of a Poisson arrival process. Both of these simulations need parameters for the arrival process and the length of stay distribution as inputs, as well as the period which will be simulated. In the simulation for the blocking model, the number of beds also is needed as an input.

To start the simulation a data frame is created with given parameters, a given start moment, and a given end moment. Since the arrivals follow an inhomogeneous Poisson process the interarrival times are exponentially distributed with the arrival rate as parameter. This is used in the simulation, from the starting moment a loop is made such that the next arrival is determined with the use of the interarrival time distribution. Fewer arrivals occur during the weekend compared to during the weekdays.

To take the weekdays-weekend days difference into account the Acceptance-Rejection method is used, in which a probability of rejection is implemented. The probability that an arrival during the weekend days is not rejected is $\frac{\text{arrival rate weekend}}{\text{arrival rate weekdays}}$, this is done such that it is possible to use the interarrival rate based on the weekdays while calculating for the weekend. If there is not an infinite capacity, but a strict capacity in the system, then the arrivals can also be blocked if the system is full. If this is the case, then there is a need for a more complicated model in which the length of stay and occupancy are directly tracked while creating the arrivals data frame.

After creating simulated arrivals in the data frame, the service times (length of stay) and the end of service moment are added to the data frame. This is done by a random general distribution. If the distribution is Weibull, the fitted parameters are used to pick a random Weibull value for the service time. The end of the stay is calculated by adding the service time to the arrival moment. Eventually, this will result in a long table with every patient over the complete period. In Algorithm 1 the pseudo-code is given to create a data frame with the arrivals and the service (length of stay) information.

Algorithm 1 Create data frame with arrivals and length of stay for $M_t|G|\infty$ model.

Require: Parameters, start_date, end_date, empty data frame

```
Set interarrival rate = arrival rate weekdays
Set date equal to start_date
while date ≤ end_date do
    Set interarrival time with random exponential with λ = interarrival rate
    Arrival time = date + interarrival time
    if Arrival time is a weekday then
        date = arrival time
        Service time drawn from random general distribution
        Calculate end of service
        Add row to data frame with arrival time, service time, and end of service
    else Arrival time is a weekend day
        Accepting probability = arrival rate weekend / arrival rate week
        Draw random uniform number to decide if the arrival is accepted
        if Arrival is accepted then
            date = arrival time
            Service time drawn from random general distribution
            Calculate end of service
            Add row to data frame with arrival time, service time, and end of service
        else
            date = arrival time
        end if
    end if
end while
Return data frame
```

For every arrival moment, it is calculated how many patients are present at that moment in time. This is added to the data frame with the arrival moments the service time and the end time of service. The pseudo-code for this process is given in Algorithm 2.

Algorithm 2 Add occupancy to data frame for $M_t|G|\infty$ model.

Require: data frame with arrivals, length of stay, and end of stay

```
for Each row in data frame do
    Check how many are present by arrival time in current row
    Add column with occupancy at arrival
end for
Return Updated data frame
```

For the $M|G|s|s$ model the occupancy need to be calculated directly during the creation of the arrival data frame, since an arrival only is accepted if the occupancy is not exceeded. In fact, the arrival is firstly simulated and then it is checked how many patients were already present by arrival, if this number equals the capacity, then the arrival is deleted again. It is tracked how many patients are blocked, so when the row is deleted which would exceed the capacity, then the blocked patients is raised by one. Therefore the simulation of an $M_t|G|s|s$ model is given in the pseudo-code of Algorithm 3.

Using the data frame with the arrival, length of stay, and occupancy information just created, a new data frame is created with the occupancy per date. The start date is one year later than the starting moment of the simulation. This is done because the warming-up period will be over after one year, and we are not interested in the warming-up period. The pseudo-code is given in Algorithm 4.

With the data frames created, calculations can be done based on the simulations. The mean number of occupied beds, the maximum number of occupied beds, and how many beds are needed such that only a chosen percentage of the patients would be blocked, in case of an $M_t|G|s|s$ model, or that the chosen percentage of the patients exceed that capacity, in case of the $M_t|M|\infty$ model. The blocking probability for the $M_t|G|s|s$ queue is calculated with the use of the modified offered load that results from the $M|G|\infty$ model. However, with the use of the $M_t|G|\infty$ there is no blocking probability calculated, by the number of present patients being Poisson distributed, it can be calculated what number of present patients only appears in a maximum chosen percentage of the time. So, it can be calculated how many beds are needed such that the capacity is exceeded in the chosen percentage of the time.

During the data analysis, parameters are found to simulate a queueing model and do perform calculations based on queueing models. For both ELV (low/high complex and palliative) and GR the first model to make is the $M_t|G|\infty$ model. The second model to use is the $M_t|G|s|s$. By the calculations that follow from the $M_t|G|\infty$ model and the results that follow from the simulation, an approximation for the operational capacity can be done for an infinite server model. Thereafter, the calculated modified offered load can be used for the $M_t|G|s|s$ queue, such that the number of operational beds (s) can be calculated for a fixed blocking percentage. For the $M_t|G|\infty$ model the capacity can be calculated since it is the capacity that only is exceeded a given percentage of the time. So for the $M_t|G|\infty$ model the tail of the Poisson distribution is calculated.

Next to the best fit distribution for the length of stay, the exponential distribution also is used. Resulting are $M_t|M|\infty$ and $M_t|M|s|s$ queues. The simulation and calculations are done similarly as for the better-fitted length of stay distributions. However, some formulas are easier to calculate with an exponential distribution, for example, the formula for the modified offered load $m(t)$.

5.3. Validation

To check the results that follow from the analytical model and the simulation model, these need to be compared to the data. To make this comparison insightful there are two types of plots to be made. The first type of plot will give the occupancy over the period

Algorithm 3 Create data frame with arrivals, service, and occupancy information for $M_t|G|s|s$ model.

```
Require: Parameters, start_date, end_date
Set interarrival rate = arrival rate weekdays
Set date equal to start_date
Set blocked patients to zero
while date ≤ end_date do
    Set interarrival time with random exponential with λ = interarrival rate
    Arrival time = date + interarrival time
    if Arrival time is a weekday then
        date = arrival time
        Service time drawn from random general distribution
        Calculate end of service
        Calculate number of present patients
        Add row with arrival, service, occupancy, and blocked patients
        if Present patients > s then
            Delete last row which exceeds the capacity
            Set blocked patients 1 higher than it was
        end if
    else Arrival time is a weekend day
        Accepting probability = arrival rate weekend / arrival rate week
        Draw random uniform number to decide if the arrival is accepted
        if Arrival is accepted then
            date = arrival time
            Service time drawn from random general distribution
            Calculate end of service
            Calculate number of present patients
            Add row with arrival, service, occupancy, and blocked patients
            if Present patients > s then
                Delete last row which exceeds the capacity
                Set blocked patients 1 higher than it was
            end if
        else
            date = arrival time
        end if
    end if
end while
Return data frame
```

together with a horizontal line that represents the calculated number of needed beds. This makes it insightful how often the number of calculated needed beds is exceeded and in what period. Possibly, it could happen that the trend has changed over the period such that all times the number of beds is exceeded happens in one period, or there could

Algorithm 4 Create data frame with the occupancy per day.

Require: Data frame with arrivals, service, and occupancy, start_date, end_date
Create a sequence with all dates from start_date + 1 year to end_date
Set up a data frame with all dates and 0 occupied beds per date
for Each date in the data frame **do**
    Select all new arrivals for the corresponding date
    if The corresponding date has 0 arrivals **then**
        Take the occupation of the previous row
    else The corresponding date has ≥ 1 arrivals
        Set occupancy to the maximum occupancy of the newly arrived patients
    end if
end for
Return data frame with occupied beds

be multiple moments in time that the number of beds is exceeded.

The second type of plot is a frequency plot, which gives the frequency per number of present patients. Over this data, a line is plotted with the calculated distribution of present patients. This distribution is different for the infinite server model compared to the blocking model.

For the infinite server model, the number of present patients is calculated with the use of the Poisson distribution, since the number of present patients is distributed with Pois$(m(t))$. That is, for every day of the week there is a Poisson distribution with the number of present patients with the modified offered load of that day as the rate. For every day of the week the distribution N_∞^{day} is translated to the frequency. For Monday it is first counted how many Mondays occur in the period, and then the probability is multiplied by that number to get the frequency. This is done for each day of the week, such that it results in 7 distributions, by adding these distributions the total distribution is found. This results in the following distribution:

$$\mathbb{P}(N_\infty(t) = i) = \sum_{\text{day}=1}^{7} n(\text{day}) \cdot \mathbb{P}(N_\infty^{\text{day}} = i).$$

In the above distribution $n(\text{day})$ is the number of times that a certain day appears in the period. For example, in 2019 there were 52 Mondays and 53 Tuesdays, thus $n(\text{Monday}) = 52$, and $n(\text{Tuesday}) = 53$ if the period is the year 2019.

For the blocking model, this goes almost the same, however, the number of present patients is not Poisson distributed. The probability of k patients present is given by $\frac{\mathbb{P}(N=k)}{\mathbb{P}(N\leq s)}$, where s is the number of beds. Similarly as for the infinite server model, this probability is calculated per day of the week and then multiplied by $n(\text{day})$ to result in the frequency. Eventually, the frequencies are summed to the overall frequency, which is given in the following equation.

$$\mathbb{P}(N(t) = i) = \sum_{\mathrm{day}=1}^{7} n(\mathrm{day}) \cdot \frac{\mathbb{P}(N^{\mathrm{day}} = i)}{\mathbb{P}(N^{\mathrm{day}} \leq s)}.$$

6. Results

As explained before, multiple models will be used. We are going to vary between infinite server models and blocking models, between exponential lengths of stay and Weibull length of stay, and between time dependency for the length of stay and no time dependency. So the models that will pass are: $M_t|M|\infty$, $M_t|M|s|s$, $M_t|G|\infty$, $M_t|G|s|s$, $M_t|M_t|\infty$, $M_t|M_t|s|s$, $M_t|G_t|\infty$, and $M_t|G_t|s|s$. Not for all region, type of care combinations all of the 8 models will be used. However, for the ELV low/high complex, they will all be discussed. Since the time dependency for the length of stay distributions was not found for ELV palliative, there are only 4 possible models left.

If the infinite server model and blocking model have the same arrival distribution and the same length of stay distribution, then the two models use the same modified offered load. For the infinite server model, the MOL is the exact value, while it is an approximation for the blocking model. Since the models are based on the data the results should fit the data. The simulation should give the exact same results for infinite server models, while for the blocking models a check with the simulation is needed to see if the modified offered load approximation gives a reliable result.

Within this chapter we will first look extensively into the region Zuid-Limburg in Section 6.1, then we will look into the region Amsterdam in a more concise manner in Section 6.2. In both regions about the same structure is found. Firstly, the analytical calculations (Subsections 6.1.1, 6.2.1) are done, where-after simulations are done (Subsections 6.1.2, 6.2.2). With the results of the simulation, the analytical calculations are checked, and then the analytical calculations are compared to the data in the validation (Subsections 6.1.3, 6.2.3). After the more elaborate results of these two regions, Section 6.3 gives results for the remaining regions and for a national level.

6.1. Region Zuid-Limburg

6.1.1. Analytical queueing model

ELV low/high complex

The ELV low/high complex care within Zuid-Limburg will be approximated with the use of multiple models. For all of the models, there is a time-dependent Poisson process with arrival rate 4.886 during weekdays and 1.826 during weekend days. Since there are different types of length of stay distributions, there are different parameters needed for the length of stay for every model. For a homogeneous exponential length of stay, the service rate is 0.034. If time dependency is added two service rates are needed, the service rate for weekdays is 0.033, while the service rate for weekend days is 0.045. For the homogeneous Weibull distribution the length of stay parameters are 1.162 (shape) and 31.044 (scale). When adding time dependency to the Weibull distribution we find 1.182 (shape) and 32.330 (scale) for the weekdays and for the weekend days 1.097 (shape) and 22.952 (scale). These models will have different modified offered loads. Table 6.1 gives the modified offered load per day of the week and for the four different length of stay distributions. This MOL is calculated with the different formulas given in Chapter 3 for $m(t)$.

Table 6.1.: Modified offered load per day of the week for all models for ELV low/high complex in Zuid-Limburg.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
$M_t\|M\|\infty$	116.553	117.456	118.328	119.172	119.987	117.766	115.619
$M_t\|M_t\|\infty$	116.573	117.469	118.343	119.194	120.024	117.811	115.654
$M_t\|G\|\infty$	116.860	117.758	118.638	119.499	120.339	118.124	115.951
$M_t\|G_t\|\infty$	116.820	117.708	118.590	119.461	120.322	118.118	115.931

The differences between the modified offered load per model do not vary too much. The largest difference appears between the $M_t|M|\infty$ and the $M_t|G|\infty$ models, of which the difference on Saturday is the largest.

With the use of the modified offered load, the calculations with the number of needed beds can start. In Chapter 5.1 it is explained how the 2% border is calculated for the different types of models. These calculations together with the modified offered load given in Table 6.1 result in the needed numbers of beds that are given in Table 6.2.

Table 6.2.: Number of needed beds per day of the week for all models for ELV low/high complex in Zuid-Limburg.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
$M_t\|M\|\infty$	139	140	141	142	143	141	138
$M_t\|M_t\|\infty$	139	140	141	142	143	141	138
$M_t\|G\|\infty$	140	141	142	142	143	141	139
$M_t\|G_t\|\infty$	140	141	142	142	143	141	139
$M_t\|M\|s\|s$	130	131	132	133	133	131	129
$M_t\|M_t\|s\|s$	130	131	132	133	133	131	129
$M_t\|G\|s\|s$	130	131	132	133	134	131	129
$M_t\|G_t\|s\|s$	130	131	132	133	134	131	129

What can be seen in Table 6.2 is that there are very small differences between the different types of length of stay distributions. There is a clear difference between the infinite server model and the blocking model. For the blocking model, there are fewer beds needed, which could be explained by the fact that the blocking model assumes that patients disappear from the system. In the infinite server model, patients that exceed the capacity stay in the system, which causes that the capacity will be exceeded for a longer period. Especially for the blocking models, the difference in the number of needed beds is very small, there only is one bed extra needed on Fridays with the use of the Weibull distribution compared to the exponential distribution for the length of stay. For the infinite server model there are a bit more differences visible, however, adding time dependency in the length of stay distribution does not give any difference in the needed number of beds. The maximum difference for the number of needed beds with infinite server models is one as well, that is, when using the Weibull distribution for modeling the length of stay, there is one bed extra needed for four days of the week. The day for which the most number of beds is needed is Friday, for both types of models, this is the case. This makes sense because the modified offered load is highest during Fridays for all types of models. Due to the weekly pattern, predictably, the expected number of present patients is highest on Fridays. Most patients arrive during the weekdays, and fewer patients arrive during the weekend, therefore at the end of the period of more arrivals most patients will be present.

A choice can be made on the way to determine the number of needed beds. Since the number of needed beds is calculated per day of the week, there is not yet a capacity such that 2% of the overall patients are blocked, but 2% of the patients per day of the week. By taking the mean number of needed beds, this will cause a higher blocking percentage for Friday. It is not desirable that during Fridays the blocking percentage is higher than 2%, therefore it is decided to take the maximum number of needed beds as the actual number of needed beds. So that is, for the use of infinite server models there are 143 beds needed to fulfill the maximum of 2% exceeding the capacity, while there are 133 beds needed when using a blocking model with exponential lengths of stay, and when the lengths of stay are Weibull distributed, then the number of needed beds is 134.

As explained, Friday is the busiest day, and therefore the number of needed beds is based on the occupancy on Friday. From now on the number of needed beds is only calculated for the Fridays, based on the modified offered load on Friday.

By a different percentage of the exceeding probability and the blocking probability, a different number of beds is needed. To see the difference we compare an exceeding probability of 1%, 5%, and 10% next to the 2% which is already used, we also consider the blocking probabilities of 1%, 5%, and 10%. In Table 6.3 the number of needed beds is given for the infinite server model $M_t|M|\infty$ and the Erlang blocking model $M_t|M|s|s$.

Table 6.3.: Number of needed ELV low/high complex beds for several exceeding and blocking probabilities.

	$M_t\|M\|\infty$	$M_t\|M\|s\|s$
1%	146	138
2%	143	133
5%	138	125
10%	134	115

In Table 6.1 it is found that the mean number of present patients on Friday is about

120, from Table 6.3 it is found that with a blocking probability of 10% even fewer beds are needed. This can be explained by the 10% of blocked patients that leave the system. Obviously, it is undesirable that there are fewer beds than the mean occupancy.

To go from a blocking percentage of 10% to 5%, there are 10 beds needed, if the exceeding percentage decreases from 10% to 5%, there are only 4 beds extra needed. So, when decreasing the exceeding probability, the number of extra needed beds is fewer than the same reduction for the blocking probability. This makes sense, since with the blocking model the patients leave the system, thus the occupancy will be bigger when these patients are admitted.

This type of care in Zuid-Limburg is explained elaborately, however, it followed that only the number of needed beds for Friday is used. For the remaining types of care in Zuid-Limburg, it is reduced to the results only based on Fridays. This will also be done for all types of care within Amsterdam.

ELV palliative

For ELV palliative it is found that the length of stay dependency on the arrival moment is not divided into weekdays and weekend days. Since the distribution is not organic, it is chosen to omit time dependency for the length of stay, therefore there are fewer models to consider. The maximum modified offered load during the week of the infinite server models with exponential lengths of stay is given in Table 6.4.

Table 6.4.: Maximum modified offered load for all models for GR in Zuid-Limburg.

$M_t\|M\|\infty$	$M_t\|G\|\infty$
24.748	24.456

For ELV palliative care there are fewer beds needed than for ELV low/high complex since the average number of patients is lower. In Table 6.5 the number of needed beds is given per type of model and per blocking and exceeding percentage.

Table 6.5.: Number of needed ELV palliative beds in Zuid-Limburg for several exceeding and blocking probabilities.

	$M_t\|M\|\infty$	$M_t\|G\|\infty$	$M_t\|M\|s\|s$	$M_t\|G\|s\|s$
1%	37	37	36	35
2%	35	35	34	33
5%	33	33	30	30
10%	31	31	27	27

Due to the smaller numbers, by adding 2 or 3 beds the blocking and exceeding percentages can be significantly decreased.

GR

For the GR departments, there are treated more patients, which will result in a higher modified offered load. As explained in the data analysis, there is not a good fit for the length of stay when there is a split between weekdays and weekend days. However, we still consider these to see the impact. Therefore we have calculated the modified offered load for the four infinite server models.

Table 6.6.: Maximum modified offered load for all models for GR in Zuid-Limburg.

$M_t\|M\|\infty$	$M_t\|M_t\|\infty$	$M_t\|G\|\infty$	$M_t\|G_t\|\infty$
346.163	346.182	347.301	347.309

Almost the same pattern occurs for the modified offered load for GR as for ELV low/high complex, however, the inhomogeneous Weibull distribution has a slightly higher MOL for GR, while it is slightly lower for ELV low/high complex compared to the MOL for the $M_t|G|\infty$ model.

The number of needed beds for GR in Zuid-Limburg is analytically calculated as well. The results are summarized in Table 6.7.

Table 6.7.: Number of needed GR beds in Zuid-Limburg for several exceeding and blocking probabilities.

	$M_t\|M\|\infty$ $M_t\|M_t\|\infty$	$M_t\|G\|\infty$ $M_t\|G_t\|\infty$	$M_t\|M\|s\|s$ $M_t\|M_t\|s\|s$	$M_t\|G\|s\|s$ $M_t\|G_t\|s\|s$
1%	390	391	371	372
2%	385	386	361	362
5%	377	378	342	343
10%	370	371	320	321

In table 6.7 both for the time-dependent and for the time-independent lengths of stay, the number of needed beds is given. Note that the time dependency of the length of stay has no influence, and thus the number of needed beds is exactly the same for the lengths of stay dependent and independent of time. Between exponential lengths of stay and Weibull lengths of stay, there is a difference of 1 bed.

6.1.2. Simulated queueing model

Apart from the analytical calculations done above, the number of needed beds can be approximated with the use of simulation. The simulations will be done for all of the models discussed above.

The simulation is done for 10 years, such that the results will be as representative as possible. However, since the simulation is not deterministic there could be differences per simulation, therefore the simulation is performed 100 times and the number of needed beds is the average of the numbers that resulted from these 100 simulations. The calculations in the above section give sometimes a different number of needed beds per model. We are interested in the number of needed beds that follow from the simulations. Therefore this number is given in the tables below. As explained, the calculations done for the infinite server models should give the same values as the simulations, since there are no approximations done for the calculations.

For the simulation of the infinite server models, after every patient is admitted it is calculated for which number of beds only 1%, 2%, 5%, and 10% exceeds that capacity.

For simulating a blocking model the number of available beds is firstly needed, therefore the number of needed beds resulting from the analytical calculations (Tables 6.3, 6.5, and 6.7) are taken into account, note that this is based on an approximation. The percentage of patients that are blocked is subsequently calculated. This comparison is done to check if the number of needed beds that follow from the approximations is similar to the results from the simulation.

ELV low/high complex

The results from the infinite server models for ELV low/high complex are visualized in Table 6.8.

Table 6.8.: Number of needed ELV low/high complex beds Zuid-Limburg for several exceeding probabilities on average for 100 simulations of 10 years.

	$M_t\vert M\vert\infty$	$M_t\vert M_t\vert\infty$	$M_t\vert G\vert\infty$	$M_t\vert G_t\vert\infty$
1%	144.3	144.5	145.3	145.3
2%	141.6	141.5	142.2	142.2
5%	137.1	137.0	137.5	137.4
10%	132.9	132.8	133.4	133.3

In Table 6.2 we have seen that the analytical calculations for the infinite server models all resulted in 143 beds. From Table 6.8 in the row of 2% we see that the mean needed number of beds is between 141.5 and 142.2, to have an integer number of beds, this is rounded up to 142 and 143. The difference in the number of beds for the analytical calculations and the simulations therefore is small. That a slightly smaller value results from the simulation can be explained by the consideration of the complete week instead of just Friday.

The mean number of arrivals per year is calculated by 100 simulations over 10 years, so there are 1,000 years simulated to find out how many arrivals there are on a yearly basis. There are on average 1467 arrivals per year for ELV low/high complex in Zuid-Limburg. This means that by a blocking percentage of 10%, 147 patients are blocked, while there are 73 patients blocked by a blocking percentage of 5%, 29 by a 2% blocking percentage, and 15 by a 1% blocking percentage. By roughly adding 3 beds, there will be 14 patients per year that are being helped with that.

The results of simulating with the use of the blocking model for ELV low/high complex is given in Table 6.9.

Table 6.9.: Blocking probabilities on average for 100 simulations of 10 years for the calculated needed number of ELV low/high complex beds in Zuid-Limburg.

	$M_t\|M\|s\|s$	$M_t\|M_t\|s\|s$	$M_t\|G\|s\|s$	$M_t\|G_t\|s\|s$
Blocking probability	1.7%	1.8%	1.6%	1.5%

In Table 6.9 it is seen that the percentages are nicely below 2%, this means that the number of needed beds that followed from the analytical calculations fulfill the maximum of 2% blocking probability.

ELV palliative

For ELV palliative the same calculations are done, however, for ELV palliative, there only are 2 models to consider. The results are given in Table 6.10.

Table 6.10.: Number of needed ELV palliative beds in Zuid-Limburg for several exceeding probabilities on average for 100 simulations of 10 years.

	$M_t\|M\|\infty$	$M_t\|G\|\infty$
1%	37.1	36.5
2%	35.6	35.1
5%	33.4	32.9
10%	31.4	31.0

For ELV palliative, the blocking probabilities that follow from the simulations are given in Table 6.11.

Table 6.11.: Blocking probabilities on average for 100 simulations of 10 years for the calculated needed number of ELV palliative beds in Zuid-Limburg.

	$M_t\|M\|s\|s$	$M_t\|G\|s\|s$
Blocking probability	1.2%	1.6%

Both these percentages are below 2%, which is the required maximum blocking percentage.

GR

For GR the infinite server models result in the number of needed beds given in Tables 6.10 and 6.12.

For GR the blocking probabilities are given in Table 6.13.

For GR, the analytical calculations based on the approximated modified offered load for the needed number of beds do not exceed the blocking probability when using simulations.

Table 6.12.: Number of needed GR beds in Zuid-Limburg for several exceeding probabilities on average for 100 simulations of 10 years.

	$M_t\|M\|\infty$	$M_t\|M_t\|\infty$	$M_t\|G\|\infty$	$M_t\|G_t\|\infty$
1%	384.0	384.5	385.9	385.0
2%	379.0	379.4	380.6	379.8
5%	371.1	371.4	372.5	371.8
10%	364.1	364.4	365.3	364.7

Table 6.13.: Blocking probabilities on average for 100 simulations of 10 years for the calculated needed number of GR beds in Zuid-Limburg.

	$M_t\|M\|s\|s$	$M_t\|M_t\|s\|s$	$M_t\|G\|s\|s$	$M_t\|G_t\|s\|s$
Blocking probability	1.6%	1.5%	1.6%	1.6%

6.1.3. Validation

So far the needed number of beds for a maximum blocking percentage of 2% is calculated analytically and with the use of simulations based on the different models. To have an idea of how this looks compared to the actual declaration data, different lines are plotted over the occupancy corresponding to the results from the different types of models.

ELV low/high complex

Multiple models have passed, and these models did result in a different number of needed beds for the ELV low/high complex in Zuid-Limburg. To compare the different models to the real situation in 2018 and 2019, we have plotted the actual occupancy over this period and added lines for the different values for the needed beds. To summarize the analytical results, the number of needed beds is 133 for the $M_t|M|s|s$ model and the $M_t|M_t|s|s$ model, it is 134 for the $M_t|G|s|s$ model and the $M_t|G_t|s|s$, for all models with infinite capacity the number of needed beds is 143, thus that is for the $M_t|M|\infty$, $M_t|M_t|\infty$, $M_t|G|\infty$, and $M_t|G_t|\infty$ models. For the infinite server models, we also have found the needed number of beds based on the simulations, there we have found both 142 and 143 as needed capacities.

What can be seen in Figure 6.1 is that the data fluctuates over time, thus only in 2019 the calculated needed number of beds is exceeded. The simulated models have the same distribution over the whole period as the analytical calculations, while there are differences during the years and between the years visible in the real data.

To have a better insight into the distribution of the number of present patients a bar plot is made with the frequency of present patients. In comparison to the data, the analytically calculated frequency of present patients is plotted for multiple models. All infinite server models have a similar distribution, which means that the plotted distributions of the models are largely overlapping. For that reason, it is decided to only plot the $M_t|M|\infty$ model compared to the data. Note that from Figure 6.1 it follows

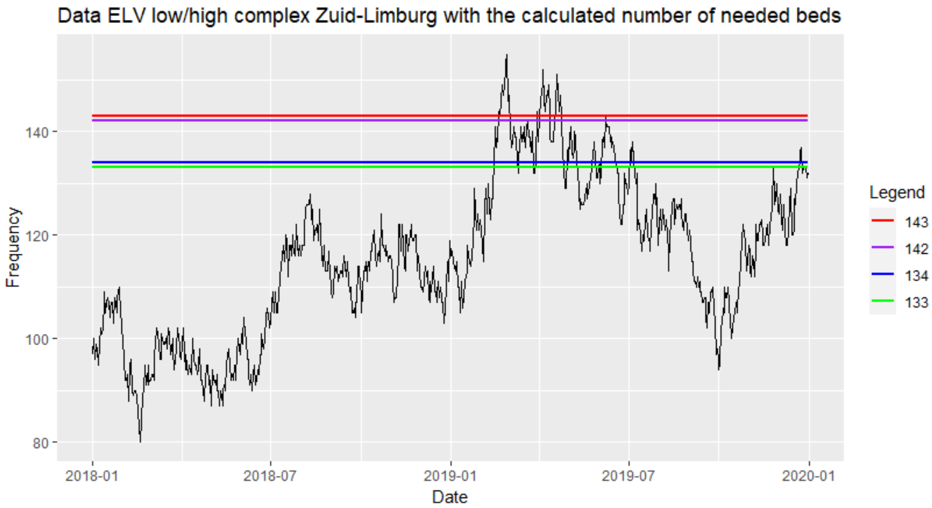


Figure 6.1.: Occupancy in the data compared to calculated number of needed beds for ELV low/high complex in Zuid-Limburg.

that the occupancy varies, the occupancy in 2019 is higher than in 2018, which could cause the model probably to be more centralized than the data.

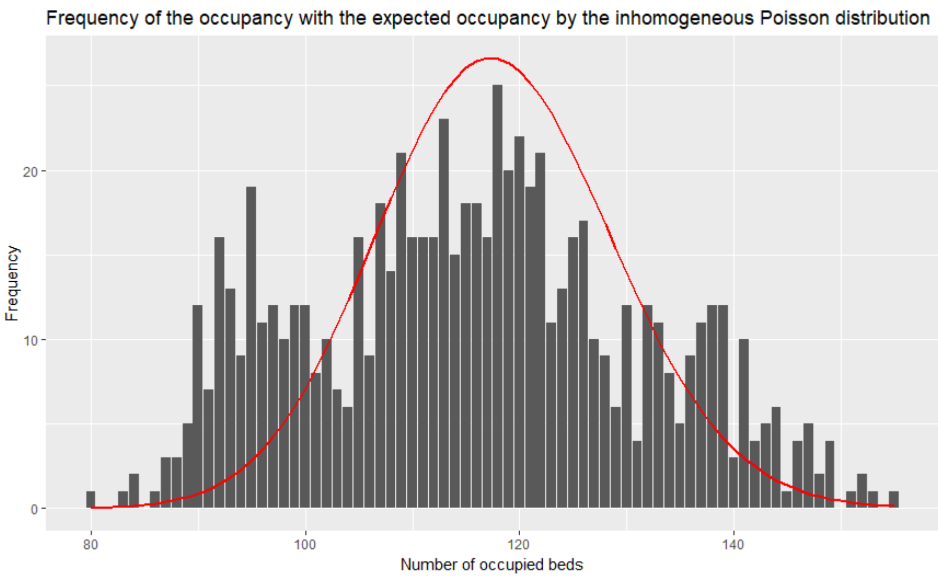


Figure 6.2.: The frequency of occupancy of the data of 2018 and 2019 compared to the results from the analytical $M_t|M|\infty$ model.

As expected, the data is more spread over the occupations, while the analytical approximation of the occupation is more centralized. It could be a better fit to only consider 2019. So the parameters for 2019 are calculated and the occupancy of 2019 is calculated to create Figure F.1b. There also is a figure for the occupancy only for 2018, this also is given in Appendix F. Note that the yearly figures in Appendix F show a better fit compared to the two-year period in Figure 6.2. However, the fit is still not perfect.

However, the same sort of figure can be created for the $M_t|M|s|s$ model, where s is the maximum number of present patients calculated by the model. In Figure 6.3 the red line represents the distribution. For the model, there are no possibilities that the given number of needed beds is exceeded since that is not the nature of the blocking model.

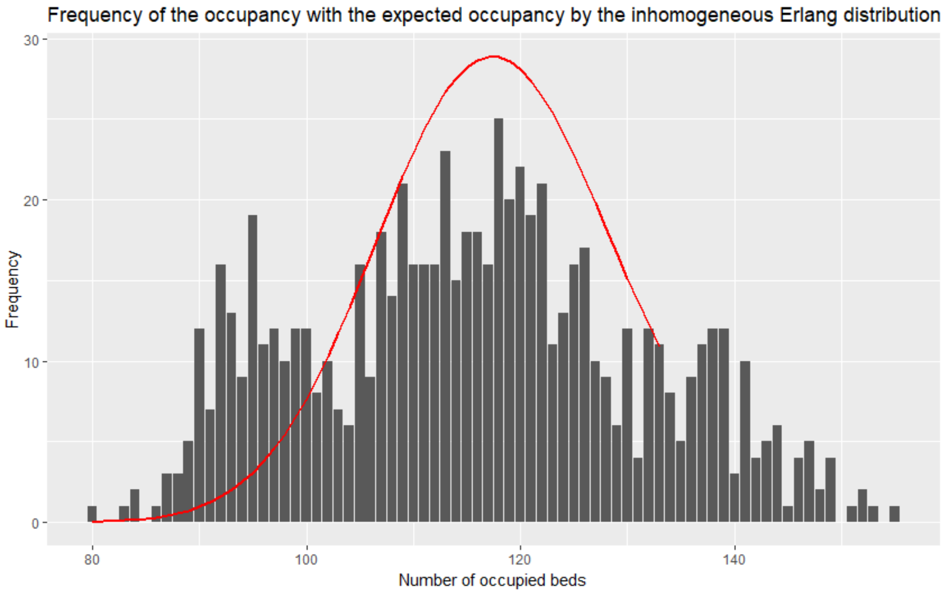
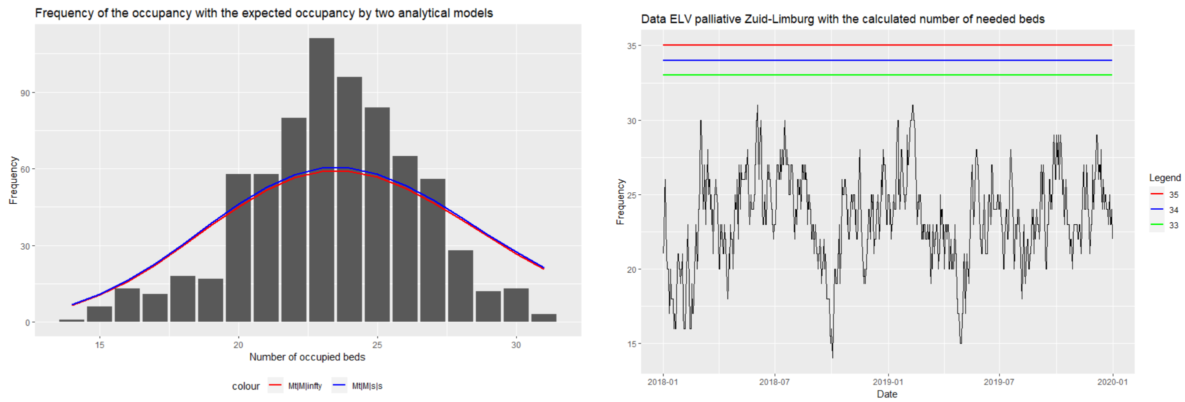


Figure 6.3.: The frequency of occupancy of the data of 2018 and 2019 compared to the results from the analytical $M_t|M|s|s$ model.

ELV palliative

Apart from the ELV low/high complex, the models should be compared to the data for ELV palliative as well. Therefore the occupancy over time is plotted together with the calculated needed beds and the occupancy bar plot is extended with the distribution of the analytical model.

In Figure 6.4a it is visualized that the peak of the data is higher than the peak of the analytical model, around the tails the analytical model has higher values than the data. This means that the data has less variability compared to the analytical models, induced by the inhomogeneous Poisson distribution. For the calculated number of needed beds, it is visualized that the data has not exceeded these boundaries over the period of interest. The data has not changed significantly over the period of interest, which is also visible in the frequency of Figure 6.4b. What also should be noted from Figure 6.4b is that all three boundaries of needed beds that follow from the analytical models are not exceeded for the period which is considered. The highest occupancy value found in the data is 31, which would be the 10% boundary for the infinite server model. This suggests that there was less capacity available than the calculated 33 and 34. By splitting these



(a) Frequency data vs. analytical. (b) Occupancy data vs. analytical.

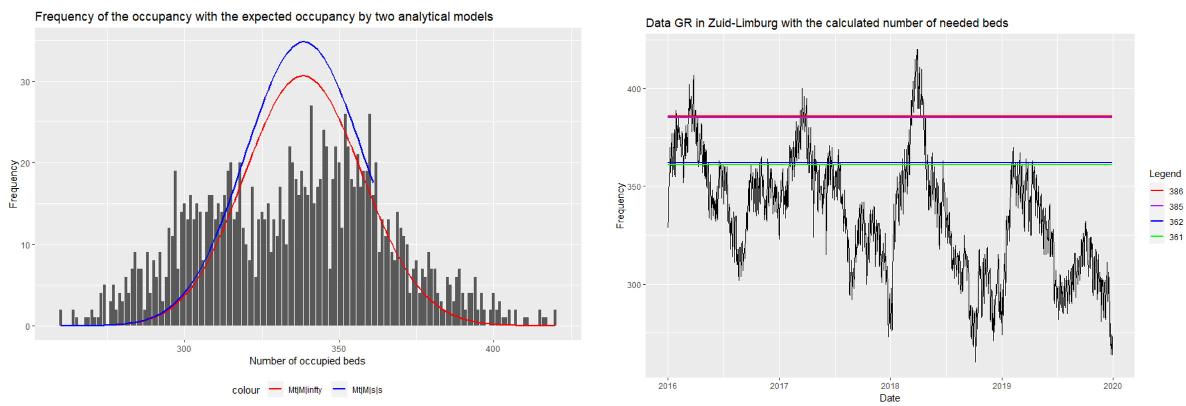
Figure 6.4.: Comparison of the ELV palliative data with the analytical results.

results per year it is visualized that there was a shortage in capacity in 2018. Figure F.2 in Appendix F shows the distribution of the data for 2018 and the fitted model. The steep jump downward indicates that the demand for ELV palliative in Zuid-Limburg was higher than the available number of beds. From this figure, the number of available beds seems to be 27, which is smaller than the calculated number of beds needed to fulfill a maximum of 2% blockage.

In Appendix F the frequency figures are also given for ELV palliative for periods of one year.

GR

The GR department is considered for the years 2016 up to 2019, which is one year longer than the period of ELV, as mentioned in Chapter 4.



(a) Frequency data vs. analytical. (b) Occupancy data vs. analytical.

Figure 6.5.: Comparison of the GR data with the analytical results.

Figure 6.5 shows on the left the frequency per number of present patients together with the analytically calculated distribution for the number of present patients. On the right, the occupancy of the data is visualized over time, where the horizontal lines give the calculated needed number of beds for the different models. In Table 6.7 it can be found from what model the created lines result. There is a clear distinction between the two types of models. The infinite server models result in higher lines than the blocking models.

What in Figure 6.6b can be seen is that the variation is strongest in 2018, which also is visible in the occupancy distribution in Figure F.3c. Similarly as for ELV low/high complex and ELV palliative, the frequency figures are also given for GR periods of one year in Appendix F.

6.2. Region Amsterdam

Due to the small differences between the multiple types of models for the region Zuid-Limburg, we only consider the homogeneous exponential length of stay distribution for Amsterdam. Since there is a fixed capacity for the ELV institutions, we only consider the blocking models. Therefore, the results for Amsterdam will be less extensive than the results for Zuid-Limburg.

6.2.1. Analytical queueing model

The modified offered load is used to approximate the mean number of present patients per day of the week. Similarly as for the region Zuid-Limburg, the number of needed beds is based on the modified offered load of Friday. Again, Friday has the highest modified offered load, and will thus give the highest number of needed beds.

The modified offered load for ELV low/high complex on Friday is 152.574, which says that there are on average 152.574 beds occupied. For ELV palliative and GR, the approximated modified offered loads are 20.609 and 282.799, respectively. Based on these approximated modified offered loads the approximations of the needed number of beds are given in Table 6.14.

Table 6.14.: Needed number beds in Amsterdam for the $M_t|M|s|s$ model for multiple blocking probabilities.

	1%	2%	5%	10%
ELV low/high complex	190	184	173	160
ELV palliative	31	29	26	23
GR	310	301	285	266

6.2.2. Simulated queueing model

With the use of simulations, the above-given numbers of beds are checked. The number of beds to have a maximum of 2% blockage is given as input for the $M_t|M|s|s$ simulation, thus $s = 167$ for ELV low/high complex, for example. By performing 100 simulations over 10 years, the blocking percentage is calculated 100 times, and the average over these percentages is given in Table 6.15.

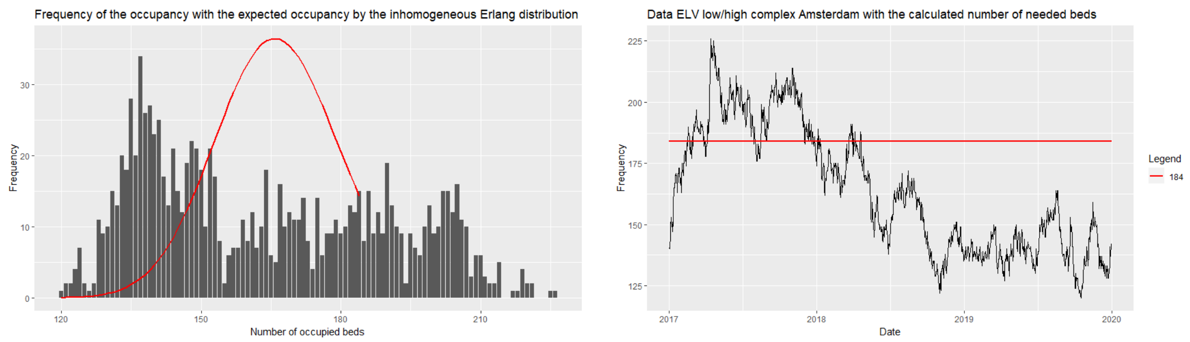
Table 6.15.: Blocking probabilities on average for 100 simulations of 10 years for the calculated needed number of in Amsterdam.

	ELV low/high complex	ELV palliative	GR
Blocking probability	1.5%	1.3%	1.5%

What can be seen in Table 6.15 is that the approximated number of needed beds results in blocking percentages below 2% when performing simulations.

6.2.3. Validation

Now we are going to compare the results above with the actual data. First, the ELV low/high complex data is compared to the needed number of beds for a 2% blocking probability.



(a) Frequency data vs. analytical.

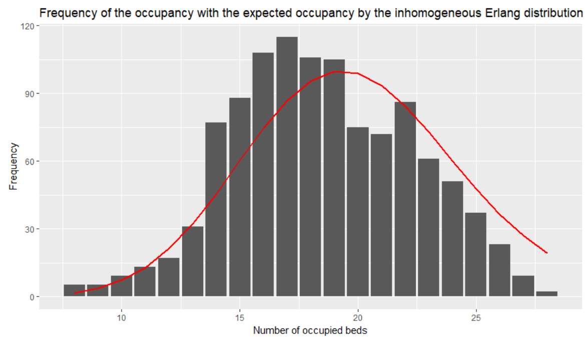
(b) Occupancy data vs. analytical.

Figure 6.6.: Comparison of the ELV low/high complex data of Amsterdam with the analytical results.

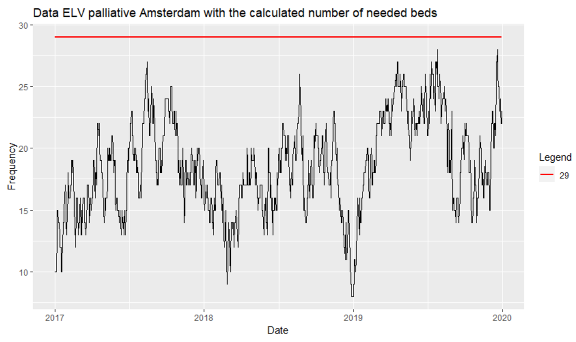
In Figure 6.6 it is visualized that the ELV low/high complex data over the years 2017-2019 is spread over a range from about 120 to 225, while the number of needed beds that is calculated with the model is 184. In Figure 6.6b it can be seen that the occupancy was higher in 2017 and at the beginning of 2018, after which it stabilizes before the start of 2019. So, in the first period, the calculated number of beds is exceeded relatively often, while it is never exceeded from halfway 2018 on.

Similarly, these figures can be made for ELV palliative and GR. These are both visualized in Figure 6.7

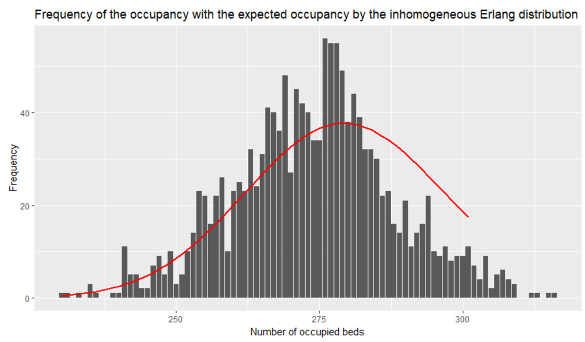
From Figure 6.7 it follows that both ELV palliative and GR seem to have a shortage in capacity. The model has higher probabilities of higher occupancy than found in the data. Besides, the calculated number of needed beds for ELV palliative is never exceeded. For



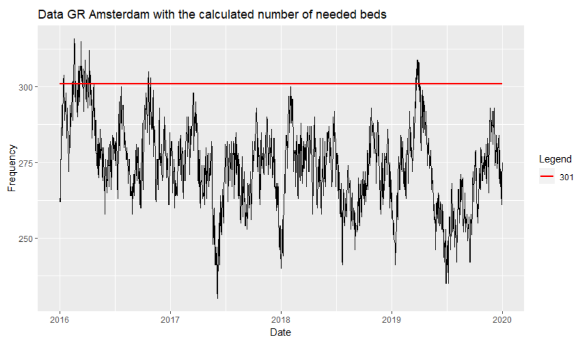
(a) ELV palliative



(b) ELV palliative



(c) GR



(d) GR

Figure 6.7.: Comparison of the data with the analytical results for ELV palliative and GR in Amsterdam.

GR the calculated number of needed beds is exceeded sometimes, however, this happens more incidentally compared to Zuid-Limburg.

The frequency together with the model figures are also made per year and given in Appendix F. For ELV low/high complex, the model becomes a better fit with the data on a yearly basis. Although, there still are some irregularities visible. The year 2017 gives the best fit with the model. For ELV palliative the yearly models are about as well fitted as the model with a 3-year-period. Lastly, the GR models for one year seem to be a worse fit compared to the model for a longer period.

6.3. Other regions

Since it is unknown how ELV and GR are organized within other regions it is decided to calculate 2 types of models. The infinite server model and the blocking model. This is done since these models will give a wide range in the number of needed beds. Where the blocking model will give a lower limit, the infinite server model will give an upper limit. The blocking and exceeding percentages are taken at 2%. It is debatable which years to take into account. On one hand the most recent year of the data could be used to do the calculations, on the other hand, a more complete image is given by the use of multiple years. Due to these two options, it is decided to calculate the number of needed beds twice. For ELV it is calculated once for the most recent year of the data, 2019, and once for a longer period 2017-2019. For GR it is decided to calculate it once for 2019 and once for the period 2016-2019. Both Zuid-Limburg and Amsterdam are also included in the tables below since the effect of the whole period vs 2019 alone has not been given in the results for these regions by themselves.

Table 6.16.: Number of needed ELV low/high complex beds for a 2% blocking probability for all remaining regions.

Healthcare office	$M_t\|M\|\infty$		$M_t\|M\|s\|s$	
Years	2017-2019	2019	2017-2019	2019
'T Gooi	139	135	130	126
Amstelland en de Meerlanden	54	67	51	63
Amsterdam	197	172	184	160
Apeldoorn/Zutphen e.o.	63	52	59	49
Arnhem	188	163	175	152
Delft Westland Oostland	153	140	142	131
Drenthe	171	171	159	160
Flevoland	39	35	37	34
Friesland	158	157	147	146
Groningen	124	115	115	107
Haaglanden	221	216	206	202
Kennemerland	114	107	106	100
Midden-Brabant	61	48	57	45
Midden-Holland	104	124	97	116
Midden-IJssel	61	49	57	46
Nijmegen	77	68	72	63
Noord Holland Noord	122	121	114	113
Noord-Limburg	51	53	48	50
Noordoost Brabant	100	111	93	103
Rotterdam	288	235	270	220
Twente	185	168	173	157
Utrecht	238	240	223	224
Waardenland	90	91	84	85
West-Brabant	131	111	123	104
Zaanstreek/Waterland	107	108	100	101
Zeeland	106	93	99	87
Zuid-Holland Noord	134	137	125	128
Zuid-Hollandse eilanden	72	80	67	74
Zuid-Limburg	128	155	119	145
Zuidoost-Brabant	78	78	73	73
Zwolle	105	100	98	93

Table 6.17.: Number of needed ELV palliative beds for a 2% blocking probability for all remaining regions.

Healthcare office	$M_t\|M\|\infty$		$M_t\|M\|s\|s$	
Years	2017-2019	2019	2017-2019	2019
'T Gooi	12	8	12	9
Amstelland en de Meerlanden	13	14	13	14
Amsterdam	30	32	29	30
Apeldoorn/Zutphen e.o.	15	16	15	16
Arnhem	37	40	35	37
Delft Westland Oostland	23	23	22	22
Drenthe	24	27	24	26
Flevoland	9	10	9	10
Friesland	15	16	15	16
Groningen	27	30	26	29
Haaglanden	31	32	29	31
Kennemerland	18	21	18	20
Midden-Brabant	34	38	33	36
Midden-Holland	14	14	14	14
Midden-IJssel	18	16	17	15
Nijmegen	21	21	20	21
Noord Holland Noord	9	9	10	10
Noord-Limburg	23	24	22	23
Noordoost Brabant	35	34	33	33
Rotterdam	60	58	56	54
Twente	21	21	21	20
Utrecht	16	16	16	16
Waardenland	21	22	21	21
West-Brabant	42	46	39	43
Zaanstreek/Waterland	16	17	16	17
Zeeland	21	23	21	22
Zuid-Holland Noord	7	7	8	7
Zuid-Hollandse eilanden	11	10	11	10
Zuid-Limburg	36	35	34	34
Zuidoost-Brabant	39	39	36	37
Zwolle	27	30	36	28

In Tables 6.16, 6.17, and 6.18 it can be seen that for some regions the number of needed beds is higher for the period of multiple years compared to 2019 separately. For other regions, it is vice versa, so there are more needed beds when only 2019 is considered.

If there are more beds needed for ELV low/high complex when only considering 2019, then it is not necessarily true that there are more beds needed for ELV palliative and GR when only considering 2019. For example, in the region Delft Westland Oostland

Table 6.18.: Number of needed GR beds for a 2% blocking probability for all remaining regions.

Healthcare office	$M_t\|M\|\infty$		$M_t\|M\|s\|s$	
Years	2017-2019	2019	2017-2019	2019
'T Gooi	247	231	231	215
Amstelland en de Meerlanden	68	68	64	63
Amsterdam	322	316	301	296
Apeldoorn/Zutphen e.o.	125	121	116	113
Arnhem	318	312	398	292
Delft Westland Oostland	334	341	312	320
Drenthe	282	293	264	274
Flevoland	57	55	53	52
Friesland	348	332	326	311
Groningen	379	376	355	352
Haaglanden	364	337	341	315
Kennemerland	304	397	284	278
Midden-Brabant	163	155	152	145
Midden-Holland	89	121	83	133
Midden-IJssel	109	108	102	100
Nijmegen	202	198	189	185
Noord Holland Noord	171	171	159	159
Noord-Limburg	163	149	152	139
Noordoost Brabant	226	219	211	204
Rotterdam	547	503	515	473
Twente	245	244	229	228
Utrecht	447	454	419	426
Waardenland	216	212	202	198
West-Brabant	318	304	297	285
Zaanstreek/Waterland	205	189	191	177
Zeeland	134	140	125	130
Zuid-Holland Noord	193	187	180	174
Zuid-Hollandse eilanden	58	55	54	52
Zuid-Limburg	385	368	361	345
Zuidoost-Brabant	283	272	265	255
Zwolle	270	263	253	246

the $M_t|M|\infty$ model finds 153 needed ELV low/high complex beds, while there are 140 needed when only considering 2019, thus there is a decrease in time. However, for ELV palliative, there is no difference, for the multiple years period, there are 23 beds needed just as for the period that only contains 2019. For GR the number of needed beds does even increase from 334 needed beds in the period 2016-2019, to 341 needed beds in the period 2019.

ELV low/high complex, ELV palliative, and GR are not dependent on each other, if there is a region that has the most ELV low/high complex beds, this will not necessarily mean that there also are most ELV palliative beds. To visualize this, three regions are selected, and only for 2019 in Table 6.19. To see the effect there only is one type of model needed, thus we only look into the results from the $M_t|M|\infty$ model.

Table 6.19.: Selection of three regions and the needed beds calculated with the $M_t|M|\infty$ model for 2019.

Healthcare region	ELV low/high complex	ELV palliative	GR
Noord Holland Noord	121	9	171
Amstelland en de meerlanden	67	14	68
Apeldoorn/Zutphen e.o.	52	16	121

From Table 6.19 it follows that Noord Holland Noord has the most ELV low/high complex beds, and Apeldoorn/Zutphen e.o. the least. This order is reversed for ELV palliative, where Apeldoorn/Zutphen e.o. has the most ELV palliative beds and Noord Holland Noord the least. For GR this order is even different since now Amstelland en de meerlanden has the least capacity for GR.

The results per region can be used to give an indication of the total number of needed beds within the Netherlands. The number of needed beds on a national level is calculated by summing up the regional results for both types of models based on the data of 2019. These summed-up values are given for the three types of intermediate care in Table 6.20.

Table 6.20.: Summed number of needed beds on a national level based on 2019.

Type of intermediate care	$M_t\|M\|\infty$	$M_t\|M\|s\|s$
ELV low/high complex	3700	3457
ELV palliative	749	721
GR	7391	6915

For all of the healthcare office regions given in the tables 6.16 - 6.18 two types of models are used. Both of these models are not certain to be the best fit within that region. Besides, the needed number of beds is rounded up to an integer number of beds. By summing these calculated numbers of needed beds the margin of error becomes larger. However, the values in Table 6.20 give a rough indication of the number of needed beds within the Netherlands.

7. Conclusion

The purpose of this thesis was to apply mathematical models to be able to do calculations and simulations that result in approximations of the capacity in intermediate care within the Netherlands. To achieve these results, the Erlang blocking model ($M_t|M|s|s$) and the infinite server model ($M_t|M|\infty$) are used to do these calculations and simulations for ELV and GR per region. Historical ELV declaration data over the years 2017-2019 and historical GR declaration data over the years 2016-2019 were used to select the best-fitting arrival processes and length of stay distributions. Based on these arrival processes and length of stay distributions, models are built. The results that follow from the models are used to answer the research questions.
Based on preliminary research the following questions are to be answered:

- *What is the operational capacity of ELV and GR in selected regions in the Netherlands?*
- *Is there a shortage of ELV or GR capacity within the Netherlands?*
- *How many beds are needed within the studied regions to produce a maximum blocking probability of 2%?*

Furthermore, the significance to the host organization will also be discussed.

Operational capacity
The first question about the operational capacity has no conclusive result. It follows that it is not possible to give the number of operational beds based on the available data. The observed occupancy does vary over the years and even within the years. The maximum found in these figures therefore cannot be taken as the operational capacity. Furthermore, the calculated number of beds ensures that there is a maximum of 2% blocking, however, this is not necessarily the number of beds that actually was available during the period of interest. The regions studied vary in their flexibility. For some regions there is a fixed capacity, for others, the capacity can change depending on the demand. These differences between regions make it even more difficult to find a generic method for determining capacity.

Shortage of capacity
To determine if there is a shortage in ELV and/or GR capacity, we examine the results on a regional level. A shortage in one region does not imply a shortage within all other regions. Shortage can be different per type of care, even within one region. With the use of the queueing models, a theoretical framework is created such that it becomes possible to see remarkable patterns. In the regions Zuid-Limburg and Amsterdam we can

conclude that there are some capacity shortages. For ELV palliative in Zuid-Limburg in 2018, we see that the capacity could not fulfill the demand, which indicates a shortage in capacity. Similarly, for ELV palliative in Amsterdam, we find that the calculated number of beds is higher than the highest occupancy in the data, which is an indication of capacity shortage. In Amsterdam we see that for GR the occupancy does exceed the calculated number of needed beds; however, this does not happen often and constantly, therefore it is likely that there is a capacity shortage here as well. For ELV low/high complex in both Zuid-Limburg and Amsterdam and GR in Zuid-Limburg, there seems to be no capacity shortage, however, there is no conclusive evidence that there is not. For all remaining regions, further research is needed to draw conclusions based on the calculated capacities.

Needed capacity
The number of beds required for a 2% blockage of patients differs per region. Similarly, the model needed to calculate this does also differ per region. For ELV low/high complex and GR in the region Zuid-Limburg the infinite server model is the best fit since there is no blocking involved. For this case, therefore a 2% exceeding probability is calculated instead of a blocking probability. For ELV palliative in Zuid-Limburg and all types of care in Amsterdam, the blocking model is used, so it is possible to find a 2% blocking probability. The numbers of needed beds with these 2% probabilities are given in Chapter 6. For these models, the needed numbers of beds in Zuid-Limburg are 143 for ELV low/high complex, 34 for ELV palliative, and 385 for GR. For Amsterdam, it becomes 184 for ELV low/high complex, 29 for ELV palliative, and 301 for GR. For all remaining regions, more research is needed, but the values in Tables 6.16 - 6.18 give a range in which the number of needed beds should be according to both the blocking model and the infinite server model. Summing these numbers up across regions gives us a range for each type of care on a national level (see Table 6.20).

Significance to the host organization
Prior to the research the host organization, the Nederlandse Zorgautoriteit (NZa), received signals that there are problems with the capacity for both ELV and GR institutions within the Netherlands. This turned out to be a difficult problem to study: not only did the NZa lack data on the operational capacity of the types of care, but it was not even clear if the providers themselves possessed this information. The research done for this thesis has provided greater insight into the capacity in Zuid-Limburg and Amsterdam, even though there is still haziness. Despite the uncertainty of the remaining regions, there is some insight given to the capacity with the use of the 2% exceeding or blocking probabilities. The calculated numbers of needed beds with the use of the 2% probability give the NZa a starting point to facilitate regional parties and insurance companies for negotiations about the needed number of beds.

Beyond a potential role in negotiations, the data analysis has also led to new insights for NZa, such as the regional differences, the use of ‘wrong beds’, and different groups of patients during the weekend compared to the weekdays.

8. Discussion

8.1. Restrictions

While performing the research explained within this thesis there were some difficulties to face.

Actual arrivals
The available data is declaration data, which means that it is the data of all patients that are admitted. Patients that are not admitted are not visible in the data. Since it is not known what happened before admission there is a lack of critical information. It is unknown if patients that could not be admitted had to wait, or are sent somewhere else. Furthermore, it would be interesting how long patients had to wait in case they had to wait before admission. If the arrival rate of the actual arrivals is known, there are two options possible. The first option is that the arrival rate is the same, thus there were no patients blocked. The second option is that the actual arrival rate is higher than the arrival rate from the declaration data, which means that there were problems with capacity, since some patients were not admitted.

Arrival times
The declaration data causes the information about the period of stay to be on a daily level. The arrival and departure times are not available. This makes it impossible to see if there are patients that arrive, right after another patient has left, or that two patients have an overlapping period in the day. In case an institution is fully occupied this could make a difference in the arrival process, since then patients are only admitted if other patients leave. Besides, it is interesting to see when these arrivals occur to investigate if there are problems with admission outside office hours.

Habits and policy effects
Secondly, when analyzing the length of stay of the patients in different types of intermediate care, it came forward that some lengths of stay appeared more often such as multiplies of 1 complete week. This is clearly visible in Figure 4.8a. This is not based on the patients' needs or policy, but on the habits of healthcare organizations. Although, there are some influences based on policy and funding. For ELV the institutions receive a budget for the exact number of days the patients was present, for GR this is divided into groups, which causes that it can be beneficial to keep a patient a little longer than needed such that the length of stay is covered by a higher budget.

Partly due to the effects of habits and policy the data is not distributed smoothly, which causes it to be harder to fit a distribution for the length of stay. The distribution that is found as the best fit takes the effects of habits and policy into account, however, the fitted distribution is smooth, so these effects are spread smoothly over the distribution. It is not desirable to take these effects very detailed into account. You rather want the underlying demand of needed care.

Fluctuations in occupancy
Over the years there is some fluctuation in the occupancy of the types of care, especially for ELV low/high complex. Since there are significant changes in occupancy over the years and even within the years, it is hard to find a good model that fits the data.

Healthcare offices
The chosen scale to split the data on a regional level is the healthcare office, this is the scale in which long-term care in the Netherlands is organized. Officially, this is not the division of ELV and GR. However there is no strict division in regions for ELV and GR, in practice, the healthcare offices are a good approximation of how short-term care is organized.

8.2. Future research

Due to the restricted amount of time available for finishing this research, it can still be expanded. In this section, some recommendations for future research are listed.

Other regions
An obvious expansion to this research is made by considering and investigating other regions. The models chosen now are not with certainty appropriate for all regions, therefore it would be necessary to find out how other regions than Zuid-Limburg and Amsterdam organize their intermediate care. Based on the information from other regions, the known models can be used or other models should be considered.

Seasonal/yearly pattern
In the data, we have seen that some effects can be related to the season or time of the year. When performing extra research to this extent, the conclusions might differ per time of the year.

Time-dependent arrivals
In this research the arrivals are split into two groups, weekdays and weekend days, it could be a more precise approximation when the arrival rate is determined per day of the week.

Departure dependency
The length of stay is influenced by the arrival day as seen in the data analysis. However, the length of stay also is dependent on the day of departure. The difference between the

mean length of stay for different departure days is smaller than seen for arrival days.

Actual arrival rate
As already explained as a restriction the actual arrival rate is unknown. When the number of needed beds in a blocking model is known it is possible to calculate back what the actual arrival rate was. This actual arrival rate is based on a capacity that follows from the arrival rate from the declaration data, therefore it is not yet optimal, but it could improve the results.

Statistical tests
During this research, the distributions are mainly fitted by visual aspects. To verify the hypothesis that the arrivals follow a Poisson process statistical tests could be used. These tests could also be used for the length of stay distribution. Statistical tests would add some extra credibility to the choices made.

Dynamical capacity
During this research, the calculated capacity is based on a period and resulted in a capacity for that complete period. It would also be possible to make this dynamic, which means that the capacity at a certain point is based on a period with a fixed length back in time, such as the moving average. This makes the needed number of beds or the needed capacity dynamic. This could be done with the use of queueing theory, but it also is possible to use fewer sophisticated methods.

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A. Healthcare office regions

Figure A.1.: Healthcare offices within the Netherlands with the largest healthcare insurance company per region. Made by NZa and CBS (Centraal Bureau voor de Statistiek) [20].

B. Dutch public holidays

Table B.1.: Public holidays in the Netherlands for 2016 to 2020.

	2016	2017	2018	2019	2020
New year's day	01-01	01-01	01-01	01-01	01-01
First easterday	03-27	04-16	04-01	04-21	04-12
Easter monday	03-28	04-17	04-02	04-22	04-13
Kingsday	04-27	04-27	04-27	04-27	04-27
Liberation day	05-05	05-05	05-05	05-05	05-05
Ascension day	05-05	05-25	05-10	05-30	05-21
Pentecost	05-16	06-04	05-20	06-09	05-31
Whit Monday	05-17	06-05	05-21	06-10	06-01
Christmas day	12-25	12-25	12-25	12-25	12-25
Boxing day	12-26	12-26	12-26	12-26	12-26

C. Data analysis hospital departments

Similarly as done for ELV and GR, there is a data analysis done for the departments in the hospital. So the hospital wards (VD), the wrong beds (VB), and the intensive care (IC) departments. Before the data analysis is executed, first the definitions of the types of hospital care are given.

Hospital ward (VD) (in Dutch: Verpleegafdeling) is the department with a hospital bed for research and treatment of the patient's condition. This definition is given by UMC Utrecht [14]. There are different types of hospital wards, for example Neurology and Oncology. For this research, all types of hospital wards are seen as the hospital ward, so we will not distinguish the different types of hospital wards. A patient only is hospitalized at VD if the patients stays for at least one night.

IC stands for Intensive Care, by UMC Utrecht [15], this is a department where more nurses and doctors take care of the patients and the patients are usually seriously diseased with one or multiple important body functions have dropped out or are threatened. There is a specially educated team of nurses that observes, treats and cares intensively. With the use of medical equipment the patient is monitored 24 hours a day.

The last type of hospital care used in this research is not a physical department, but it is a different type of administration of the patient on some type of hospital department. For the patients with this type of administration it can be a patient that should not be in the hospital anymore, but it is also possible that the patient does belong in the hospital. This is caused by the **wrong bed (VB)**, in which a patient occupies a bed in the hospital on the wrong place. A wrong bed is potentially caused by intermediate care residences with a shortage in capacity, so the patient cannot go to the intermediate care the patient needs. It is possible that a patient occupying a wrong bed worsens and needs to return to the hospital ward for extra treatment, then the patient does not occupy a wrong bed anymore, but a clinical bed. Besides, it is possible that a patient is waiting on the correct bed for such a long time that it is not even necessary anymore, and the patient goes home.

C.1. Arrivals

The number of arrivals per week are plotted for the arrivals of VD, VB, and IC.

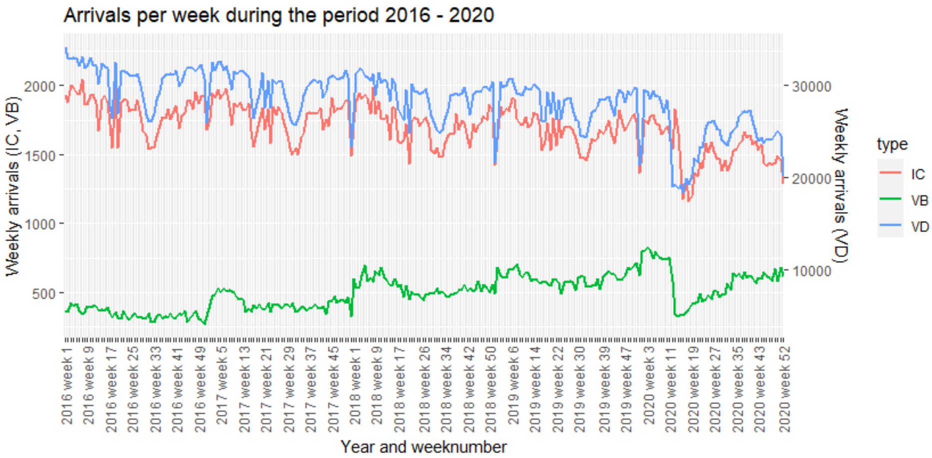


Figure C.1.: The weekly number of arrivals for hospital departments for the period 2016-2020.

For the years 2016 - 2019 there is a trend visible in which the arrivals for VD go through a dip during the summer period. Also, the IC arrivals follow this pattern, although it is fewer strong. Over the years, the number of VB arrivals increases, which means that more patients arrive at a 'wrong bed'.

In Figure C.1 there are more variations than explained, one of the causes is the differences during the week. To see how much variation there is over the week, the mean number of arrivals per day of the week is given in Figure C.2.

For VD (Figure C.2a) the weekly pattern decreases from Monday to Saturday, while it starts increasing from Saturday to Sunday. The number of arrivals during the weekend days is about half of the arrivals during weekdays.

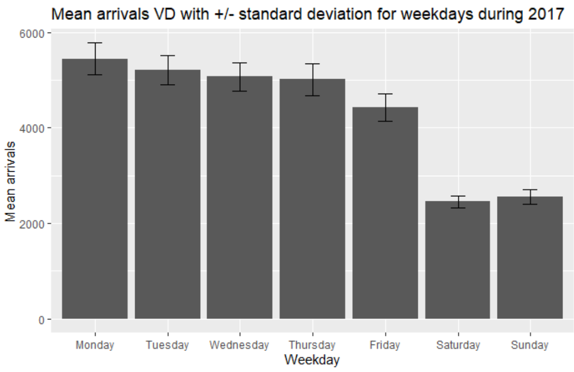
For IC (Figure C.2c) the weekly patterns do vary most per year, for most years there is a small wavy pattern during the week, while the weekend is more stable. Weekdays do have about twice as much arrivals as weekend days.

The only department for which the weekly pattern is significantly different is the VB (Figure C.2b). Instead of a smaller amount of arrivals during the weekend the smaller amount of arrivals is shifted one day, to Sunday and Monday. During the rest of the week there is a slight wavy pattern. Compared to the other departments there also are relatively bigger error bars for VB.

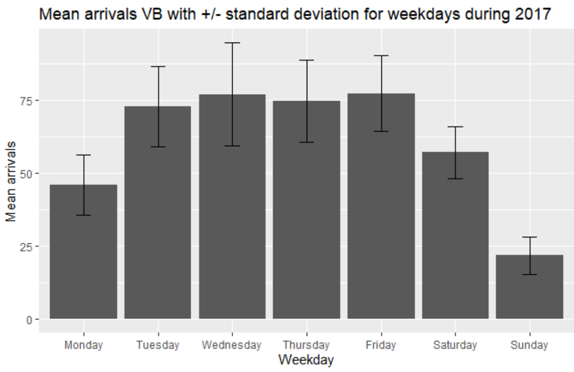
In Appendix D.4 the number of arrivals are given in a cumulative plot, for all types of hospital care. With the use of these cumulative plots, the differences between the several years becomes more clear.

For VB an increase happens every year transition. For VD we do see a decrease in the number of arrivals over the years, although, this is a fewer strong decrease. This decrease is not very clear in Figure C.1.

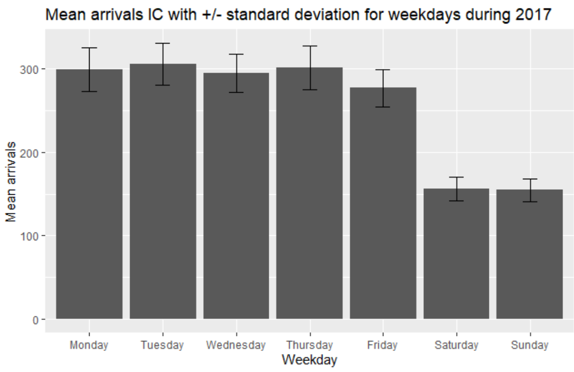
A better overview is given with the mean number of arrivals per year:



(a) VD



(b) VB



(c) IC

Figure C.2.: Arrivals in hospital departments.

Table C.1.: Mean values of the weekly arrivals for the hospital departments per year.

	VB	IC	VD
2016	343.4	1,817.3	30,804.9
2017	422.5	1,771.6	29,838.6
2018	537.3	1,714.2	28,773.7
2019	607.1	1,682.6	28,102.4

For VB, the wrong beds, the mean number of weekly arrivals does increase, with an increase of about 77% from 2016 to 2019.

For the IC department the mean weekly arrivals are decreasing. The decrease is about 7.5% over the period 2016 to 2019.

For VD the mean number of arrivals are also decreasing, just as for IC. The total decreasing percentage over the period 2016-2019 is 8.8%.

As for the intermediate care the arrivals can be split up into weekdays and weekend days, since that causes the separation in the arrival distribution. For VB this split could possible be better done by Sunday and Monday on one hand and Tuesday to Saturday on the other hand.

In Figure D.9b it is visible that there are very narrow bars for the frequency of arrivals of VD, this causes that some of the bars are not visible. The number of arrivals at the hospital ward is so high that the variation is higher, but the difference of 1 arrival is proportionally smaller. Therefore these arrivals are binned into bins of width 20. That results in Figure C.3, which has the same separation as for other departments.

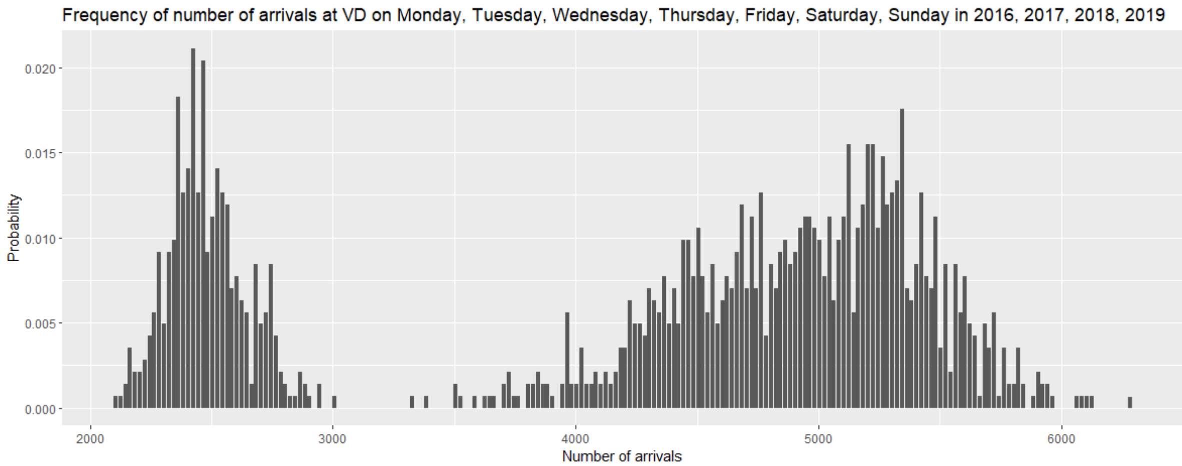
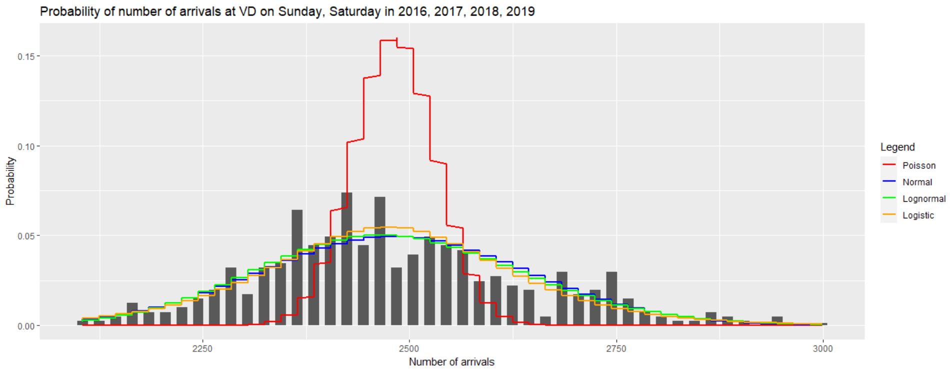
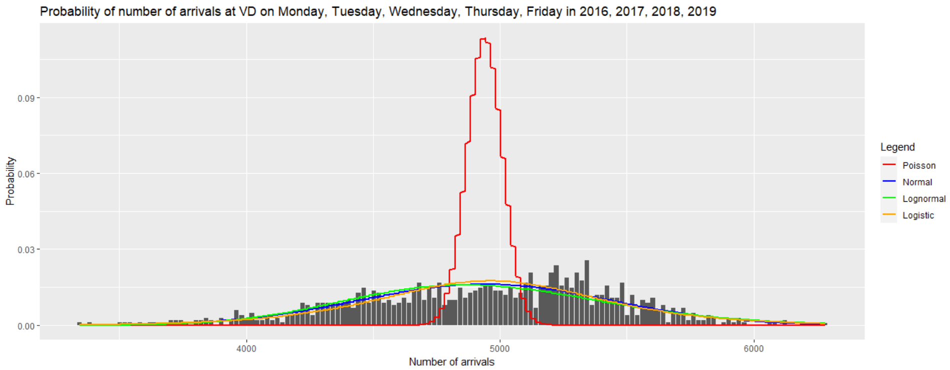


Figure C.3.: Empirical distribution of the binned arrivals.

Now the data is split, and for both weekdays and weekend days there are some distributions plotted. As mentioned before, the empirical distribution for VD is plotted in bins of width 20, therefore the fitted distributions also have flat parts of width 20. This gives a bit a weird graph, but the comparison with the empirical distribution can be made. These graphs are given in Figure C.4.



(a) The data and fitted distributions for the VD arrivals during weekend days in 2016 - 2019 are plotted with the following parameters: Pois(2481.754), N(2481 .754, 160.934), lnorm(7.815, 0.064), logistic(2473.865, 91.389).



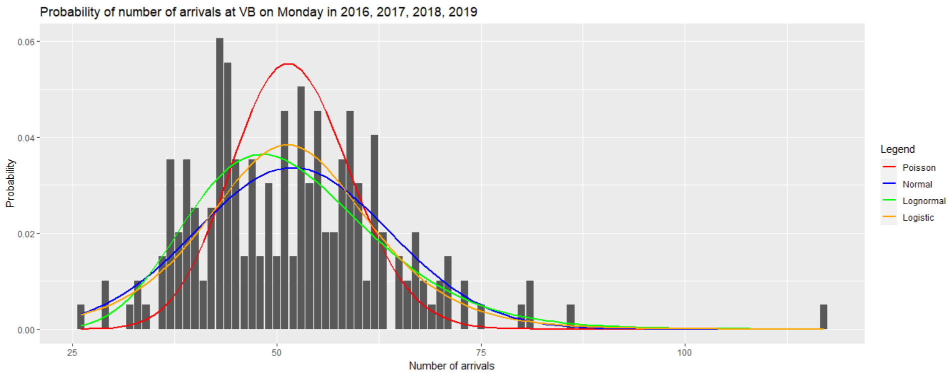
(b) The data and fitted distributions for the VD arrivals during the weekdays in 2016 - 2019 are plotted with the following parameters: Pois(4939.669), N(4939.669, 492.728), lnorm(8.500, 0.103), logistic(4960.259, 285.362).

Figure C.4.: Empirical arrival distribution VD with fitted distributions.

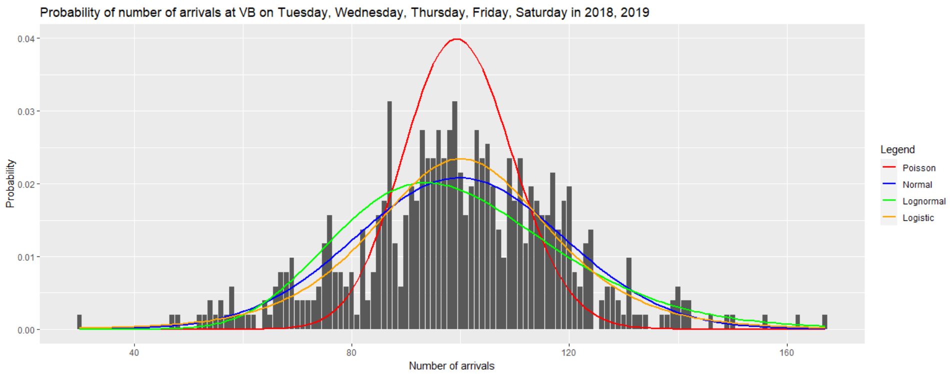
For both Figure C.4a as Figure C.4b the Poisson distribution is not a good fit. For weekend days the fitted normal distribution, lognormal distribution and the logistic distribution are close to each other. One of these three can simulate the VD arrivals of weekend days best. The peak of the lognormal distribution is most left of the three, that also appears to happen in the arrivals of the data, so the lognormal will be the best fit for simulating weekend arrivals at the hospital wards.

For the arrivals during weekdays again the normal, lognormal and logistic distribution graphs are very close. However, none of these seems to be the best fit for the VD arrivals during weekdays. The peak of arrivals is slightly moved to the right, just like the peak of the logistic distribution. With these reasoning the logistic distribution seems to fit best for the VD arrivals during weekdays.

The weekend effect, significantly fewer arrivals, has shoved one day for the wrong beds, this means that there are fewer arrivals during Sundays and Mondays, instead of the 'usual' Saturday and Sunday. In Figure C.5 the graphs of VB arrivals with the fitted distributions are given for Monday and for Tuesday to Saturday.



(a) The data and fitted distributions for the VB arrivals during Mondays in 2016 - 2019 are plotted with the following parameters: Pois(51.879), N(51.879, 11.842), lnorm(3.924, 0.222), logistic(51.271, 6.498).



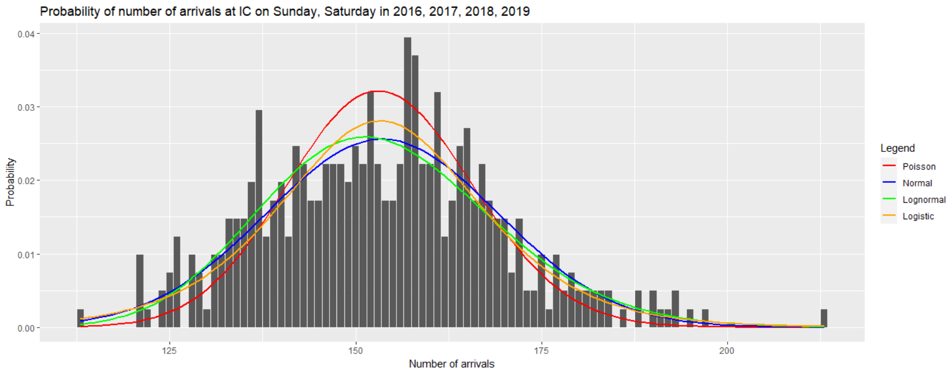
(b) The data and fitted distributions for the VB arrivals during Tuesday - Saturday in 2018 and 2019 are plotted with the following parameters: Pois(99.763), N(99.763, 19.153), lnorm(4.583, 0.207), logistic(100.043, 10.657).

Figure C.5.: Empirical arrival distribution VB with fitted distributions.

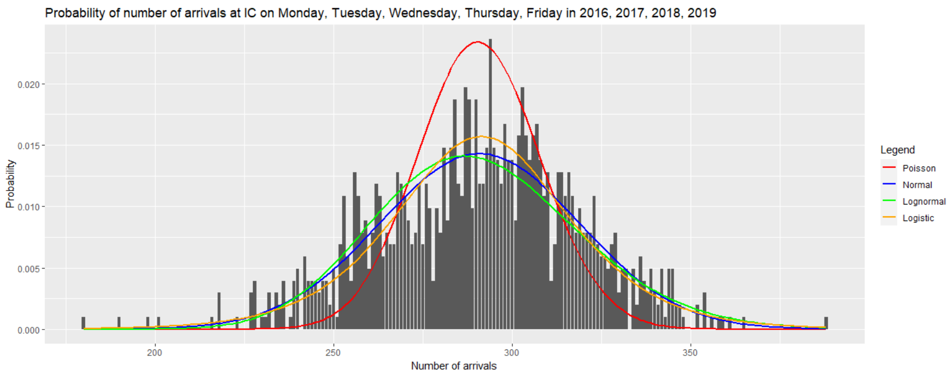
In the arrival figure of VB on the Monday and Sunday together there were two peaks available. Therefore these days were taken into account separately. In Figure C.5a the VB arrivals for the Monday are given with the fitted distributions. It is debatable which distribution is best in this case. By the low bars around 50 alternated with higher bars, there is not a good fit. By the most arrivals being somewhat to the left the lognormal distribution seems to be the best fit. However, by looking mostly at the higher probabilities of arrivals, the height of the Poisson distribution seems to be better.

In Figure C.5b the arrivals for Tuesday to Saturday for 2018 and 2019 are plotted with the fitted distributions. There are to be two peaks if all years were considered, this disappears when this is split into two periods: 2016, 2017 and 2018, 2019. The best fit seems to be the normal distribution. However, an optimal solution lies between the normal and the Poisson distribution. Not all days are considered in Figure C.5 not all days and years are considered, therefore in appendix D.10 some extra days are tried with different selection of years.

The arrivals and the fitted distributions for IC are given in Figure C.6.



(a) The data and fitted distributions for the IC arrivals during weekend days in 2016 - 2019 are plotted with the following parameters: Pois(153.532), N(153.532, 15.584), lnorm(5.029, 0.101), logistic(153.253, 8.899).



(b) The data and fitted distributions for the IC arrivals during the weekdays in 2016 - 2019 are plotted with the following parameters: Pois(290.879), N(290.879, 27.898), lnorm(5.668, 0.098), logistic(291.461, 15.925).

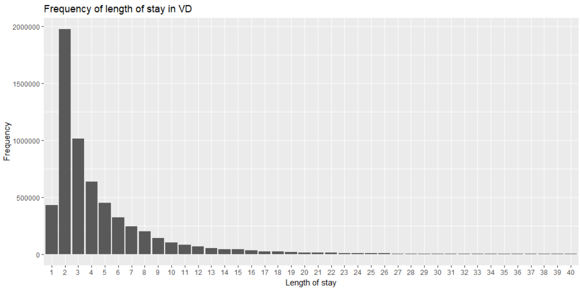
Figure C.6.: Empirical arrival distribution IC with fitted distributions.

Figure C.6a shows the IC arrivals during the weekends from 2016 to 2019 with the fitted distributions. The distribution that seems to fit best is the normal distribution.

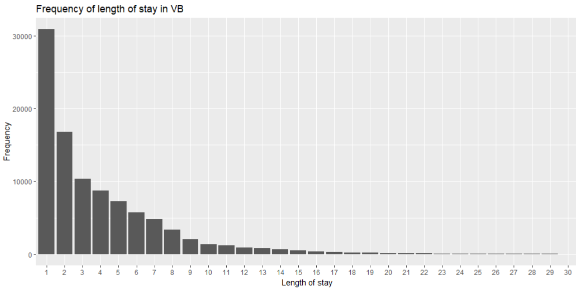
For the weekdays, in Figure C.6b, the fitted distributions for lognormal and normal are close to each other, but the normal distribution has its peak slightly more to the right. However, the arrival peak of the data is even more to the right, which appears in the logistic distribution as well, therefore the distribution that seems to fit best is the logistic distribution.

C.2. Length of stay

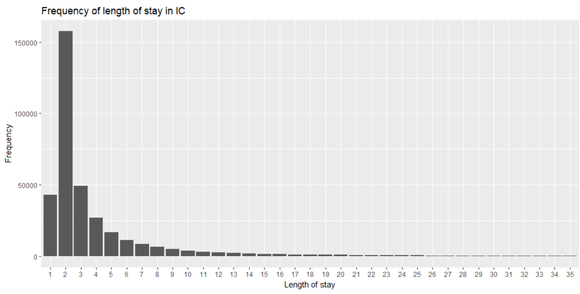
The frequency of a certain length of stay is visualized in Figure C.7. To have a better insight into the the dynamics, the long right tail is removed from the figures. The cutting point is decided by the looks of the original figures and around 1% is cut off.



(a) VD



(b) VB



(c) IC

Figure C.7.: Length of stay per department with the corresponding frequency.

Note that C.7a and C.7c both have a short bar for length of stay is one. By looking at the definition of a nursing day the shortness of this bar can be declared. A patient does only have one nursing day if the patient enters between 20.00 h and 00.00 h and only stays over one night, or the patient comes on an arbitrary moment during the day, but is transferred to a different institution that same day, or the patient comes on an arbitrary moment during the day, but dies that same day.

For the wrong bed department, VB, (see Figure C.7b) there is a high bar for length of stay equals one. A hospital aims for patients to be at the right department, and the patients that end up on a wrong bed to go to the right department as soon as possible.

The distribution of the length of stay becomes more clear when visualized in a boxplot. Figure C.8 gives these boxplots for the three types of hospital care.

The top of the boxplot is cut off, this means that the outliers are not visualized, therefore the maximum length of stay is given in Table C.2, together with some more information about the lengths of stay.

If the standard deviation to mean ratio is close to 1, then an exponential distribution is likely for the length of stay. For VD and VB the standard deviation to mean ratio is close to 1, however for IC this value is higher.

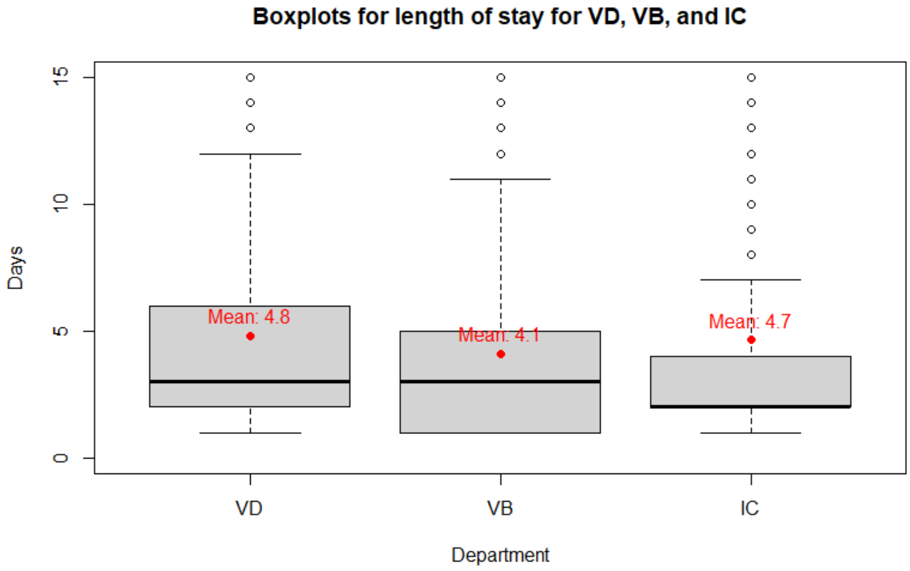


Figure C.8.: Length of stay boxplot per hospital care department with the corresponding mean value.

Table C.2.: Maximum, mean, standard deviation, and coefficient of variation of the LoS per department.

	VD	VB	IC
Maximum	367	158	601
Mean	4.8	4.1	4.7
Standard deviation	5.3	4.6	8.3
Coefficient of variation	1.10	1.11	1.77

We are interested in the distribution for the LOS, therefore we fit a few distributions on the data.

As described earlier there are fewer patients that stay 1 day at the hospital ward than patients that stay 2 days, after 2 days the probability of staying that many days starts decreasing again. This is visualized in Figure C.9 together with the fitted distributions. The best fit distribution is the lognormal distribution, since both the exponential and Weibull distribution are not increasing in the first place and the gamma distribution does only increase slightly.

The length of stay of VB seems to be best simulated by the exponential distribution (see Figure C.10). However the lognormal distribution starts higher, just like the data, the rest of the days is better approximated by the exponential distribution with rate about 0.244, which results in a mean of 4.1 days of stay. With a standard deviation to mean ratio of 1.11, this is an outcome that could be expected.

The last department to consider is the intensive care unit, the distribution is plotted

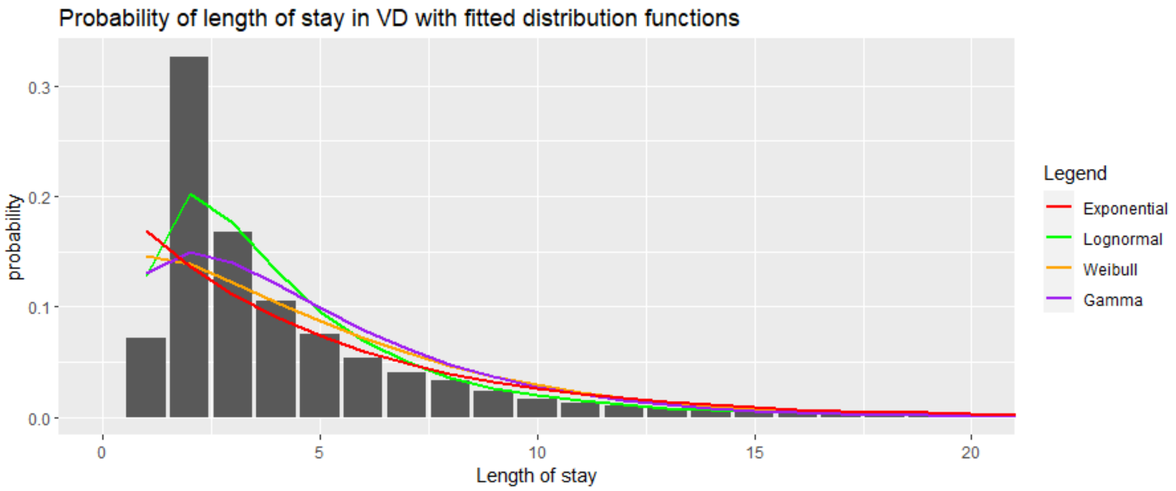


Figure C.9.: The data and fitted distributions for the LoS of VD with the following parameters: Exp(0.207), lnorm(1.253, 0.740), Weibull(1.186, 5.175), gamma(1.706, 0.354).

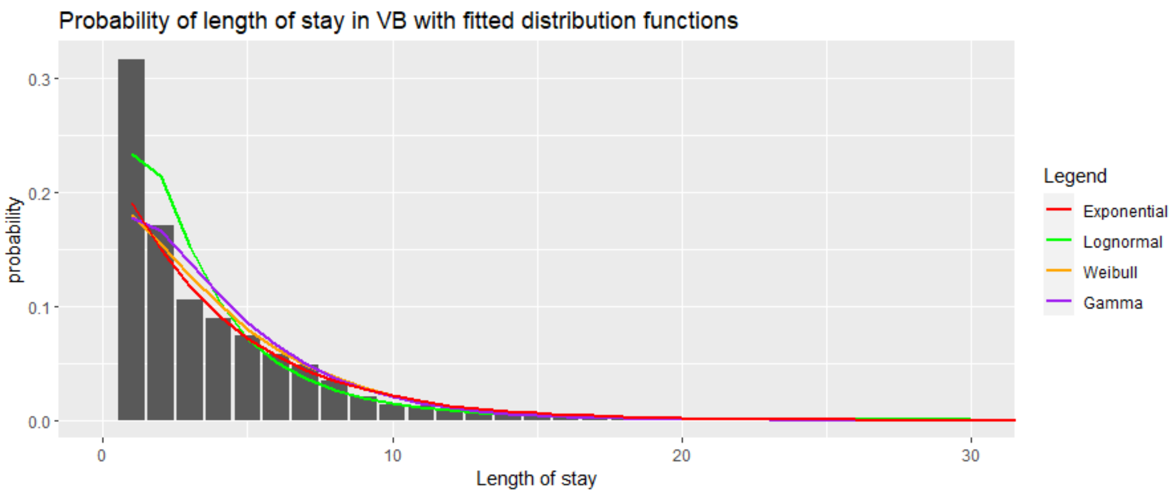


Figure C.10.: The data and fitted distributions for the LoS of VB with the following parameters: Exp(0.244), lnorm(1.010, 0.870), Weibull(1.110, 4.294), gamma(1.386, 0.338).

together with some fitted distributions in Figure C.11. Similar as for the hospital ward there is a peak at LoS of 2. The best fit seems to be the lognormal distribution.

The mean length of stay per day of the week is given in Table C.3 for the hospital departments.

For VD and IC patients that arrive during the weekend have a longer mean length of stay. For the hospital wards (VD) the patients that arrive on Friday also have a bit longer length of stay, but in fact the period from Monday to Saturday the mean length of stay is increasing, then it decreases a little from Saturday to Sunday.

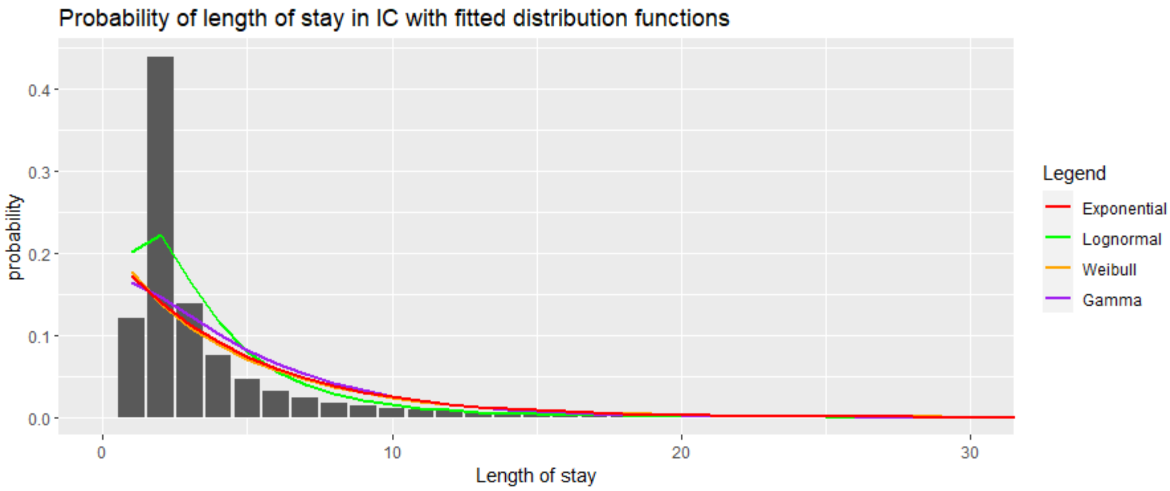


Figure C.11.: The data and fitted distributions for the LoS of IC with the following parameters: Exp(0.214), lnorm(1.078, 0.800), Weibull(0.967, 4.569), gamma(1.220, 0.261).

Table C.3.: Mean length of stay per hospital department and arrival day.

	VD	VB	IC
Monday	4.582	4.069	4.357
Tuesday	4.646	3.854	4.361
Wednesday	4.680	3.900	4.468
Thursday	4.755	4.267	4.475
Friday	5.044	4.606	4.639
Saturday	5.468	4.090	5.705
Sunday	5.111	3.352	5.581

For VB there is no clear pattern in the effect of arrival day, however the day with on average the longest length of stay is the Friday, which could be caused by the weekend effect, however this is not clear.

D. Arrival figures

D.1. ELV low/high complex

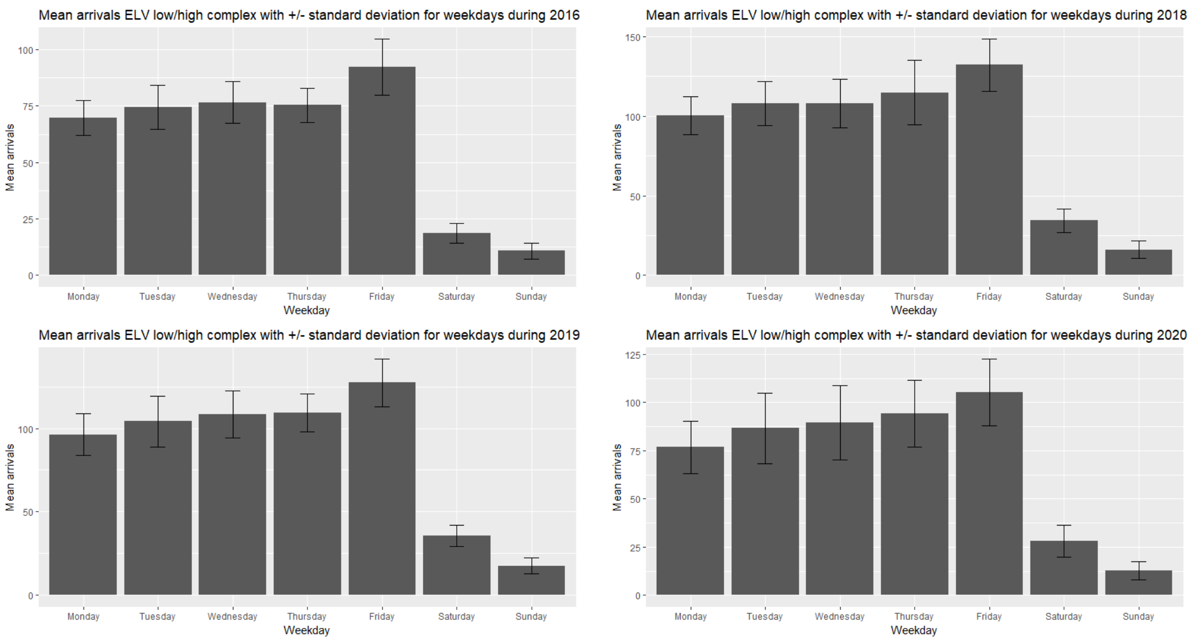


Figure D.1.: Arrivals mean and standard deviation at ELV low/high complex per year.

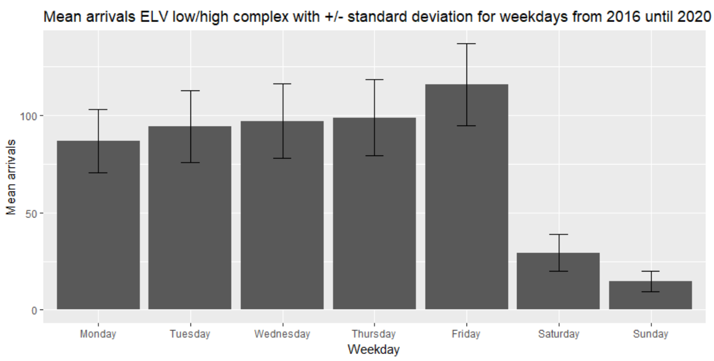


Figure D.2.: Arrivals mean and standard deviation at ELV low/high complex for 2016-2020.

D.2. ELV palliative

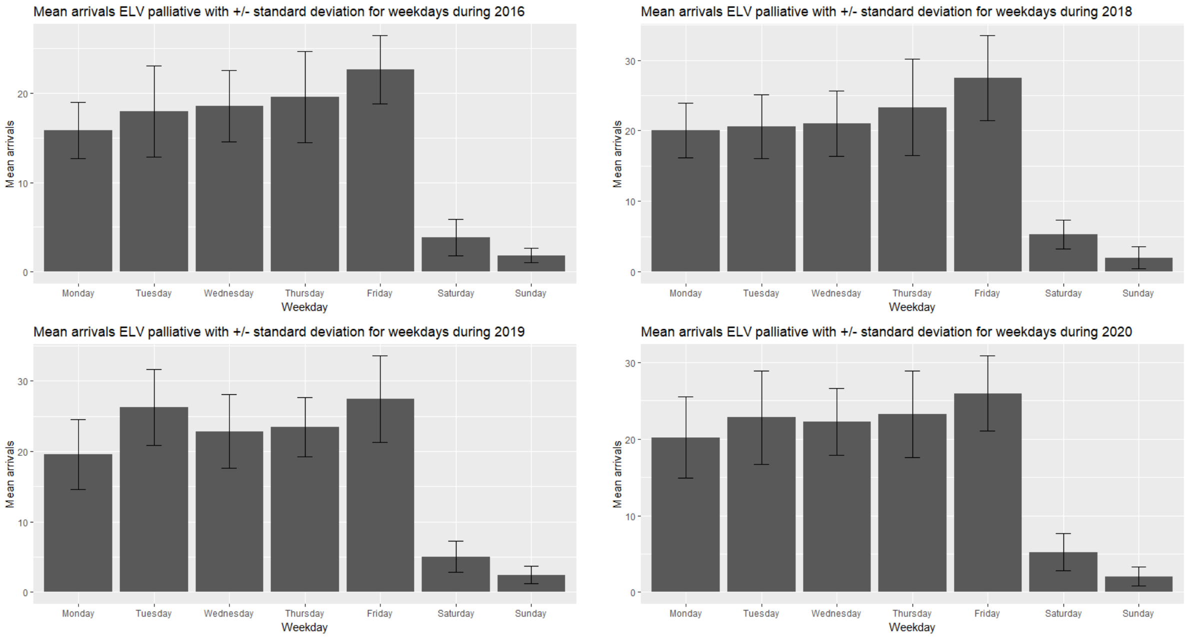


Figure D.3.: Arrivals mean and standard deviation at ELV palliative per year.

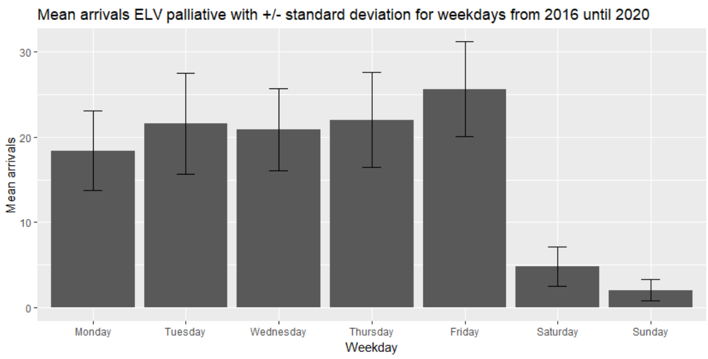


Figure D.4.: Arrivals mean and standard deviation at ELV palliative for 2016-2020.

D.3. GR

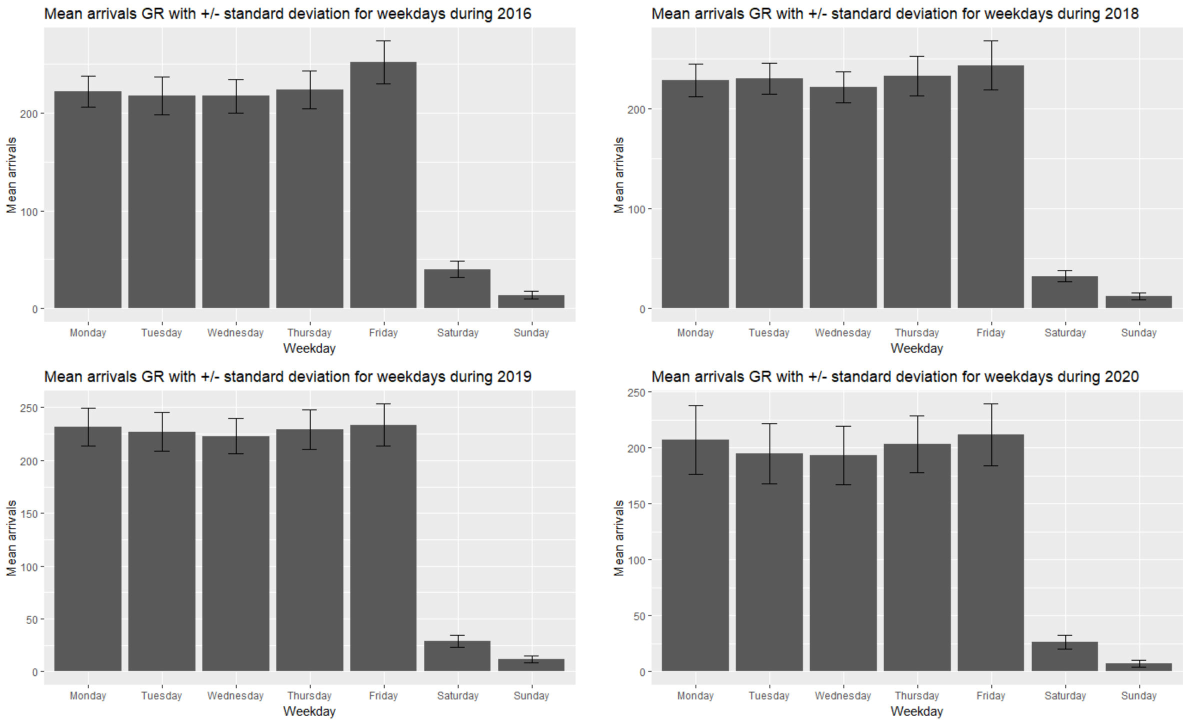


Figure D.5.: Arrivals mean and standard deviation at GR per year.

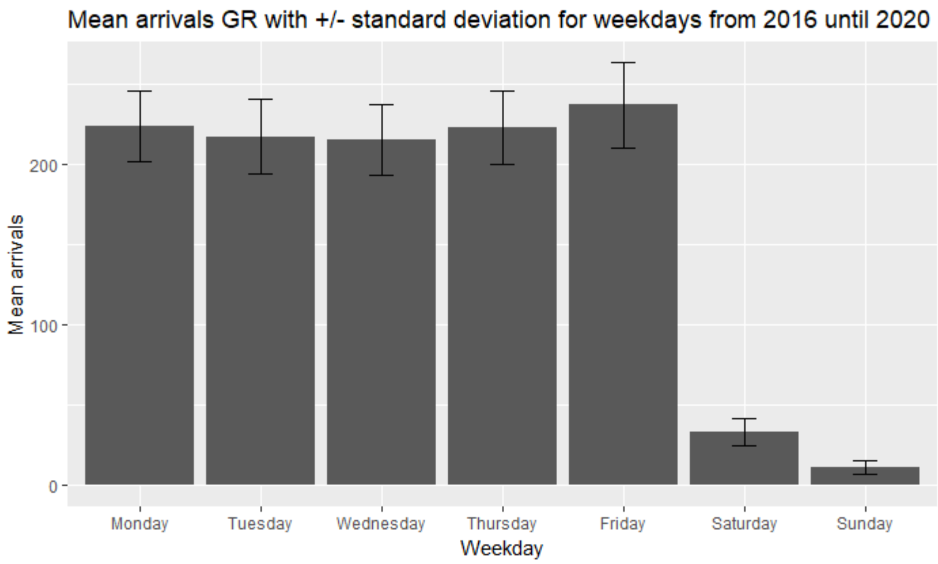
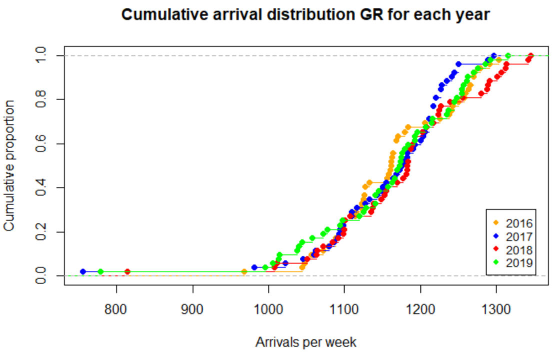
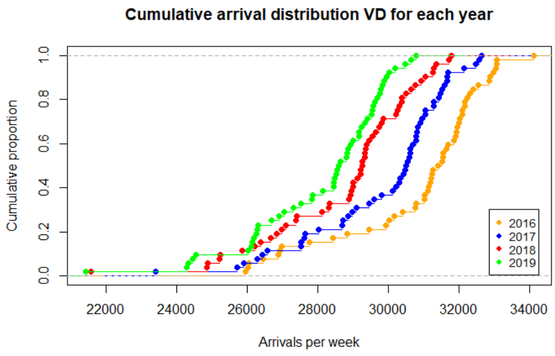


Figure D.6.: Arrivals mean and standard deviation at GR for 2016-2020.

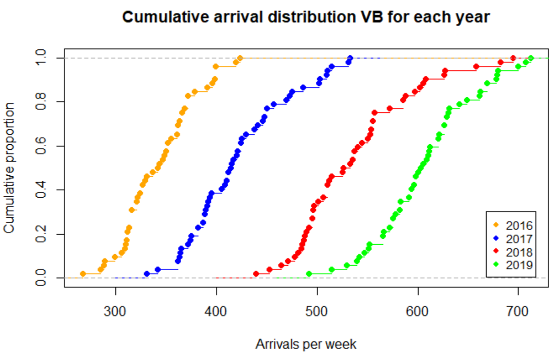
D.4. Cumulative arrivals



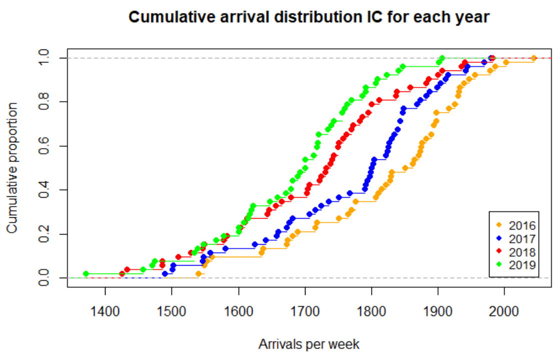
(a) GR



(b) VD



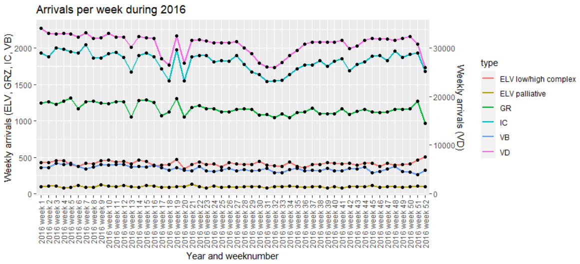
(c) VB



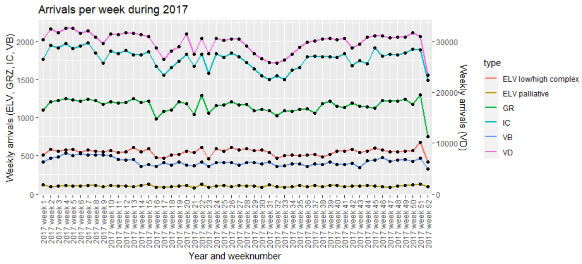
(d) IC

Figure D.7.: Cumulative arrival plots.

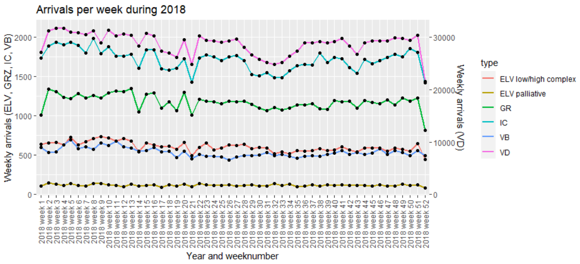
D.5. Weekly arrivals



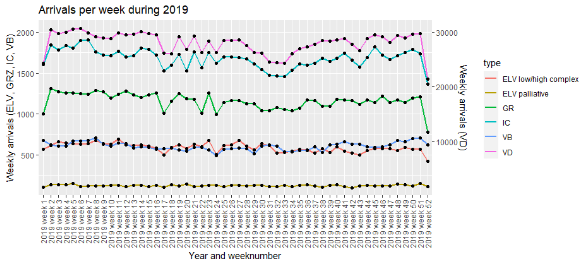
(a) 2016



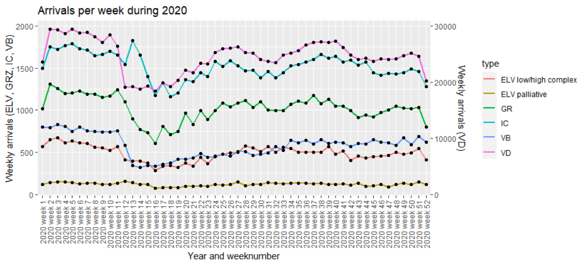
(b) 2017



(c) 2018



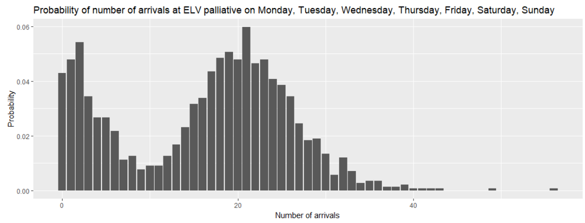
(d) 2019



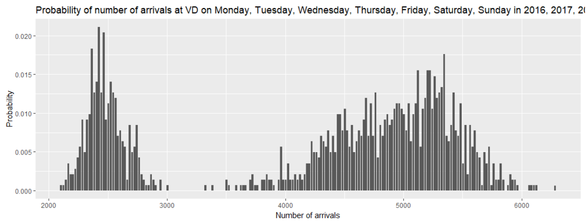
(e) 2020

Figure D.8.: Arrivals per week in 2016 until 2020 for all departments.

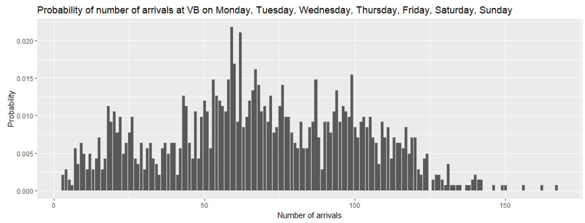
D.6. Empirical arrival distributions



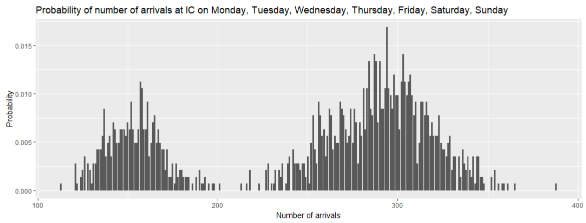
(a) ELV palliative



(b) VD



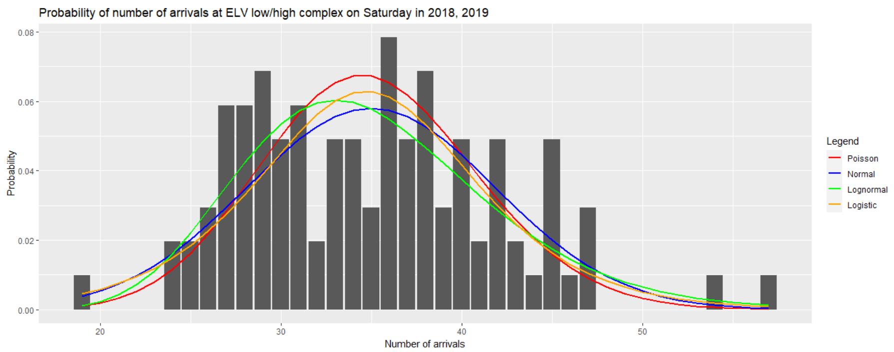
(c) VB



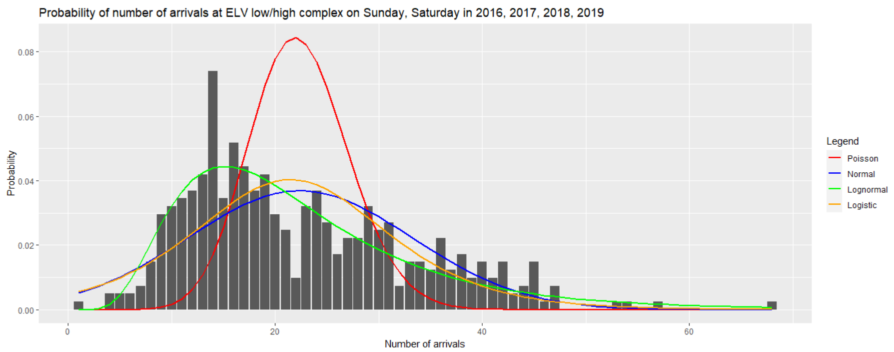
(d) IC

Figure D.9.: Empirical arrival distributions 2016-2019 all days of the week.

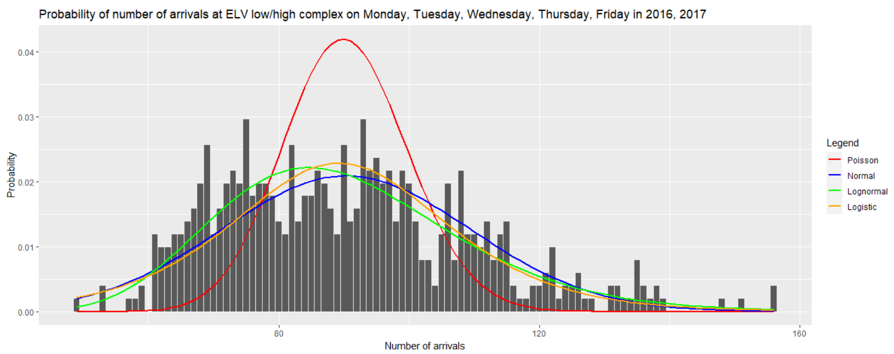
D.7. Fitted arrival distributions ELV low/high complex



(a) The data and fitted distributions for the ELV low/high complex arrivals during the saturdays in 2018 and 2019 are plotted with the following parameters: Pois(34.990), N(34.990, 6.876), lnorm(3.536, 0.196), logistic(34.709, 3.966).



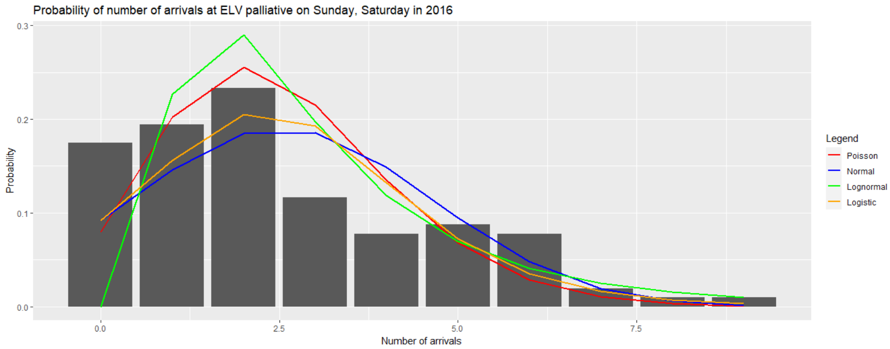
(b) The data and fitted distributions for the ELV low/high complex arrivals during the weekend days over all years are plotted with the following parameters: Pois(22.394), N(22.394, 10.799), lnorm(2.986, 0.518) logistic(21.442, 6.193).



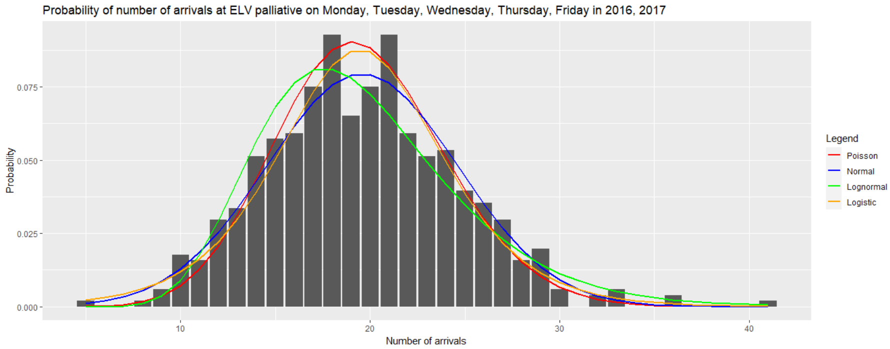
(c) The data and fitted distributions for the ELV low/high complex arrivals during the weekdays in 2016 and 2017 are plotted with the following parameters: Pois(90.347), N(90.347, 19.078), lnorm(4.482, 0.208), logistic(89.238, 10.924).

Figure D.10.: Arrival data ELV low/high complex with fitted distributions.

D.8. Fitted arrival distributions ELV palliative



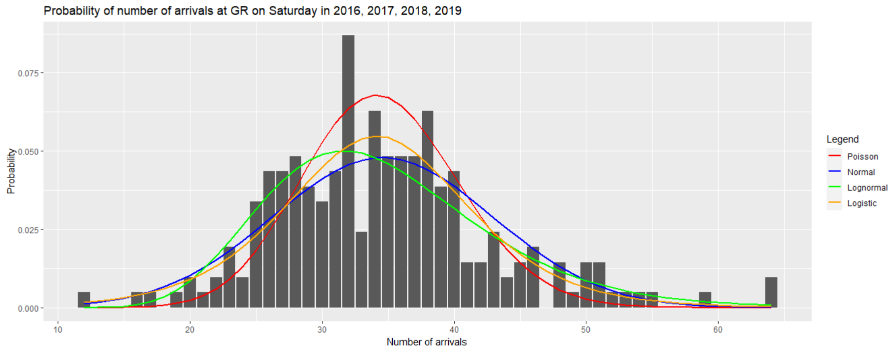
(a) The data and fitted distributions for the ELV palliative arrivals during the weekend days in 2016 are plotted with the following parameters: Pois(2.524), N(2.524, 2.094), lnorm(0.916, 0.648), logistic(2.317, 1.198).



(b) The data and fitted distributions for the ELV palliative arrivals during the weekdays in 2016 and 2017 are plotted with the following parameters: Pois(19.572), N(19.572, 5.019), lnorm(2.940, 0.269), logistic(19.446, 2.851).

Figure D.11.: Arrival data ELV palliative with fitted distributions.

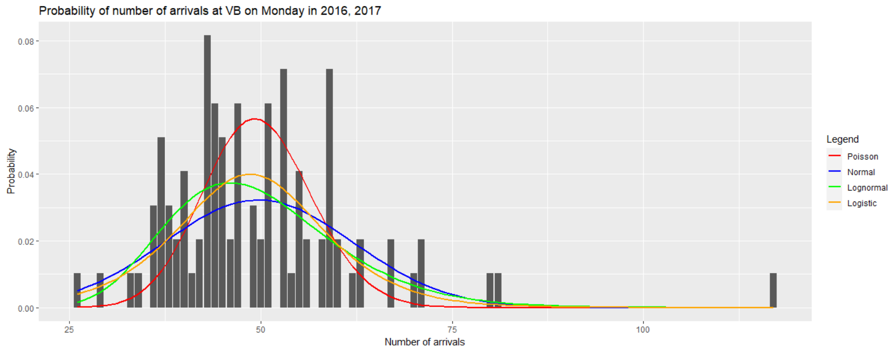
D.9. Fitted arrival distributions GR



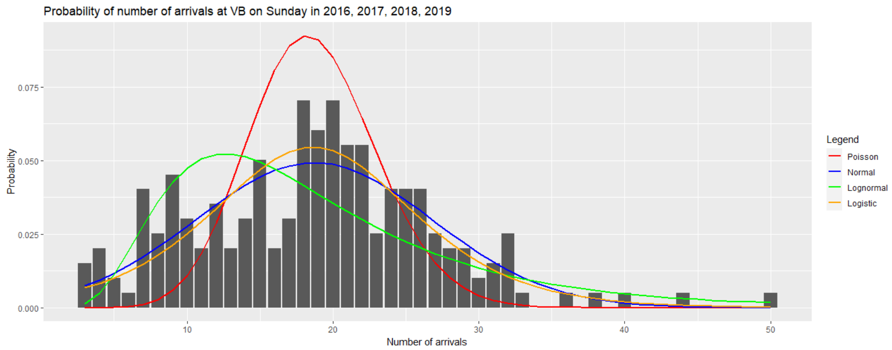
(a) The data and fitted distributions for the GR arrivals during saturdays in 2016 - 2019 are plotted with the following parameters: Pois(34.585), N(34.585, 8.327), lnorm(3.514, 0.244), logistic(34.130, 4.566).

Figure D.12.: Arrival data GR with fitted distributions.

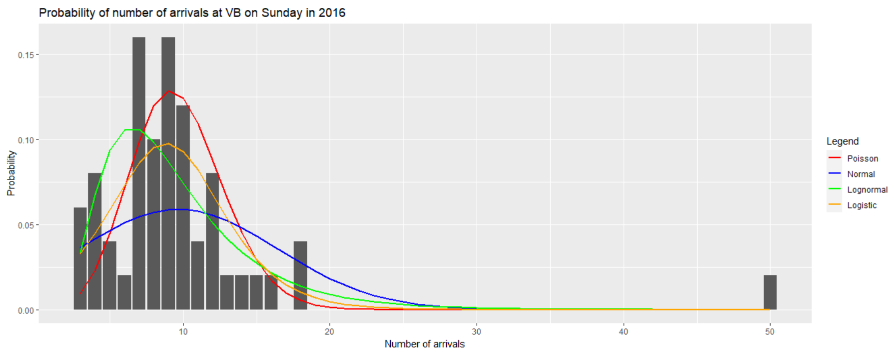
D.10. Fitted arrival distributions VB



(a) The data and fitted distributions for the VB arrivals during the Mondays in 2016 and 2017 are plotted with the following parameters: Pois(49.694), N(49.694, 12.345), lnorm(3.879, 0.226), logistic(48.586, 6.244).



(b) The fitted distributions for the VB arrivals during the Sundays in 2016 - 2019 are plotted with the following parameters: Pois(51.879), N(51.879, 11.842), lnorm(3.924, 0.222), logistic(18.617, 4.582).



(c) The data andfitted distributions for the VB arrivals during the Sundays in 2016 are plotted with the following parameters: Pois(53.469), N(53.469, 13.214), lnorm(3.954, 0.214), logistic(8.823, 2.553).

Figure D.13.: Arrival data VB with fitted distributions.

E. Region Analysis

E.1. Wrong beds

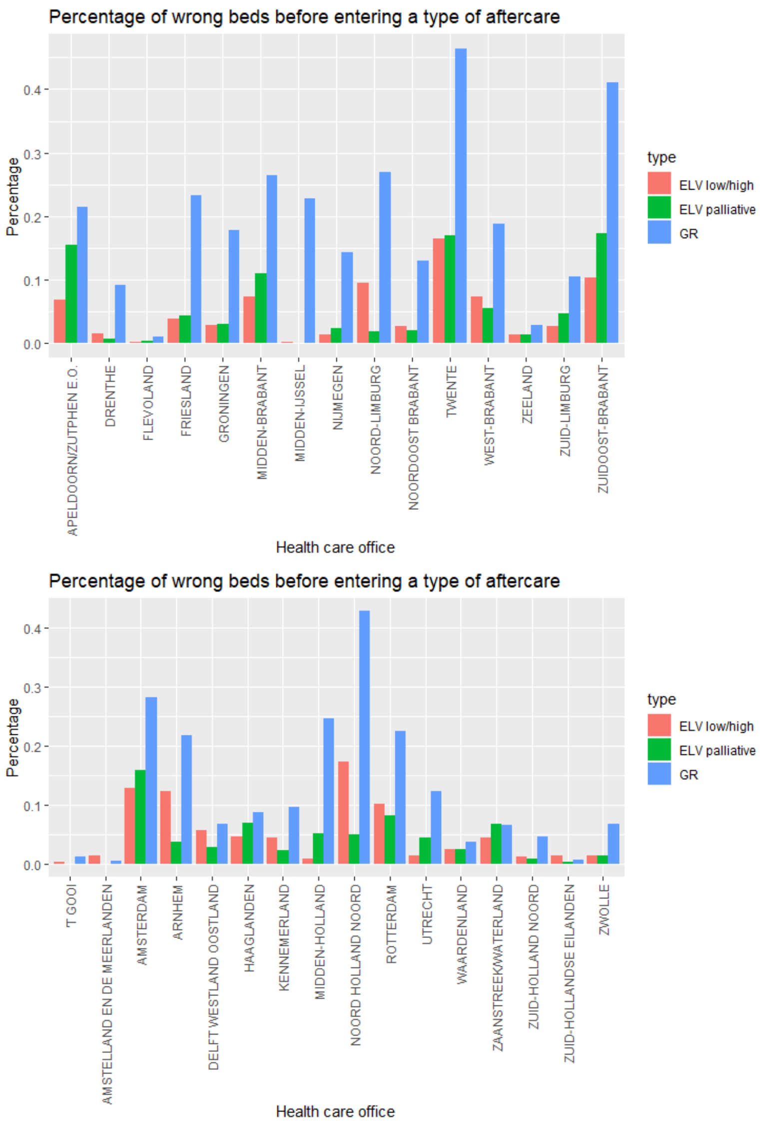


Figure E.1.: Percentages of wrong beds per region for all three types of intermediate care.

E.2. Zuid-Limburg

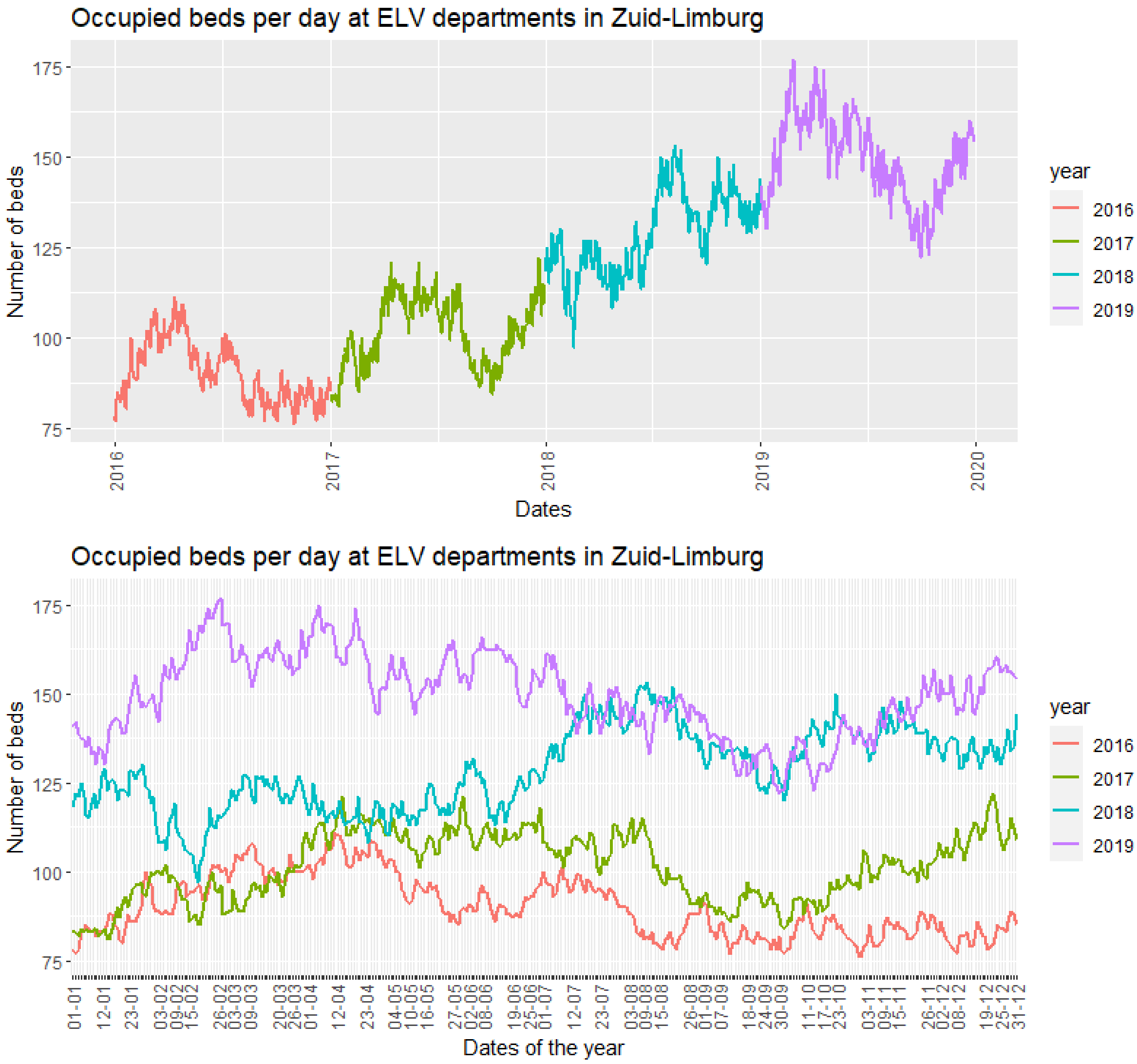
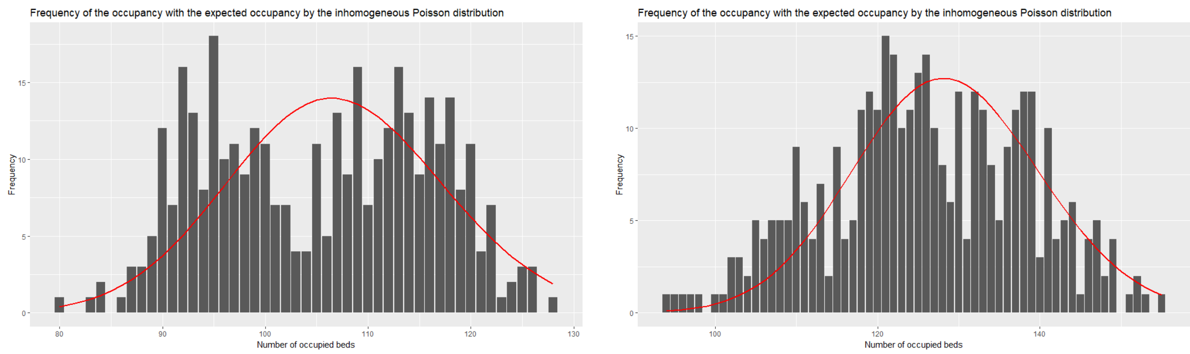


Figure E.2.: Number of occupied beds over the year.

F. Results

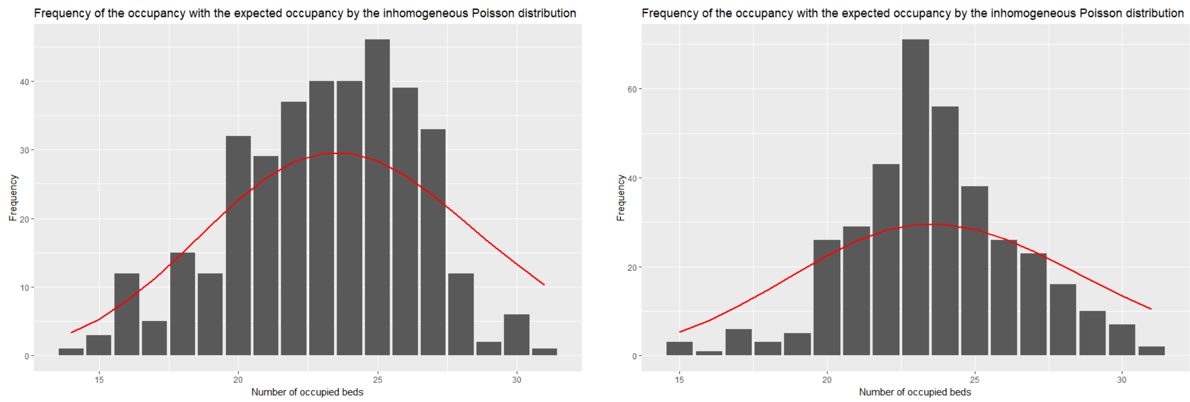
F.1. Data versus Analytical model

F.1.1. Zuid-Limburg



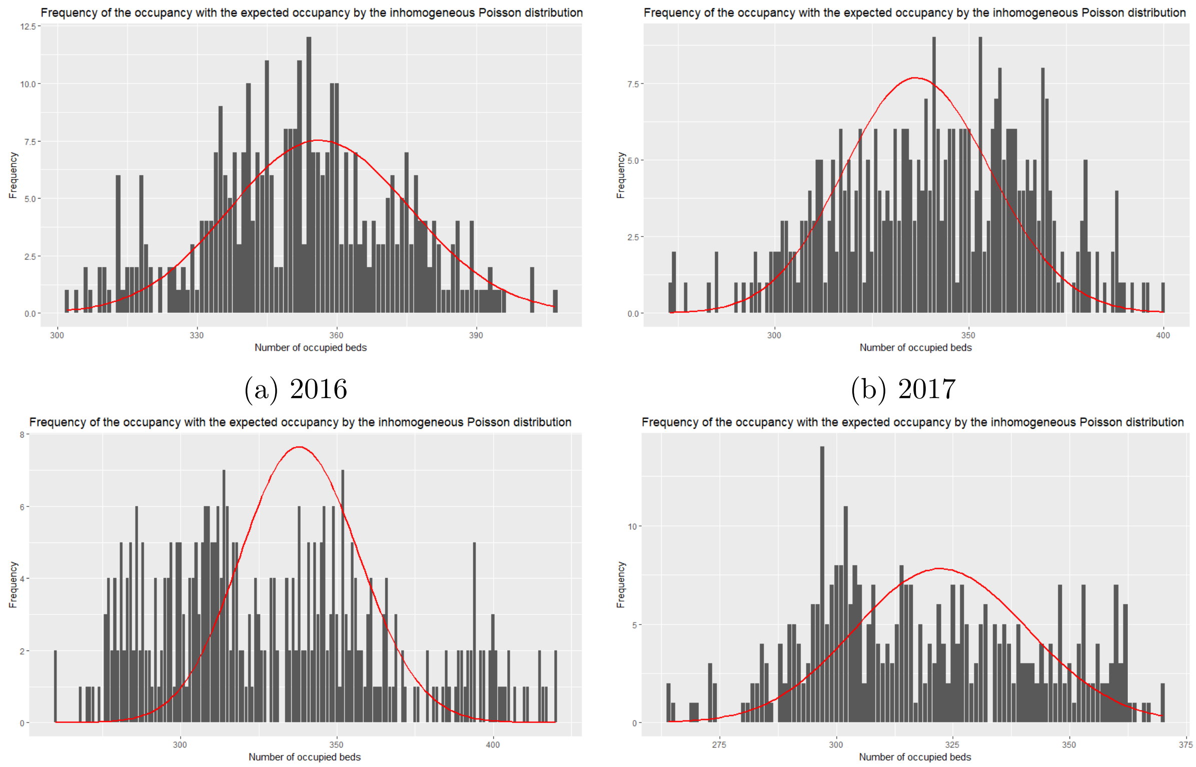
(a) 2018 (b) 2019

Figure F.1.: Analytical model based on the data compared to the data for ELV low/high complex during one year.



(a) 2018 (b) 2019

Figure F.2.: Analytical model based on the data compared to the data for ELV palliative during one year.



(a) 2016

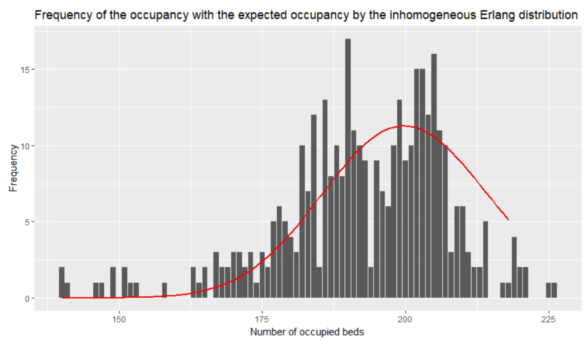
(b) 2017

(c) 2018

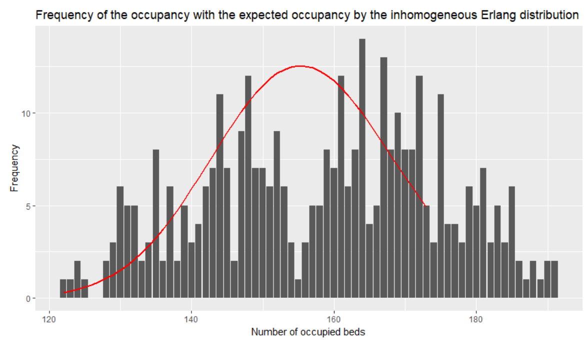
(d) 2019

Figure F.3.: Analytical model based on the data compared to the data for GR during one year.

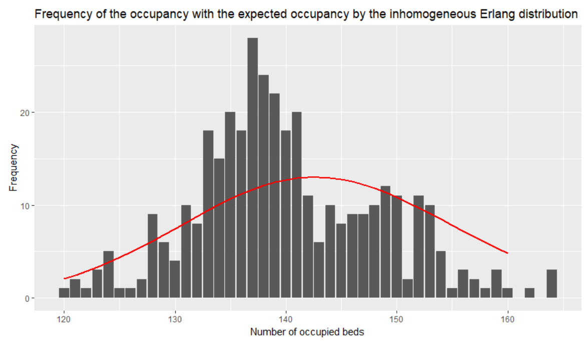
F.1.2. Amsterdam



(a) 2017

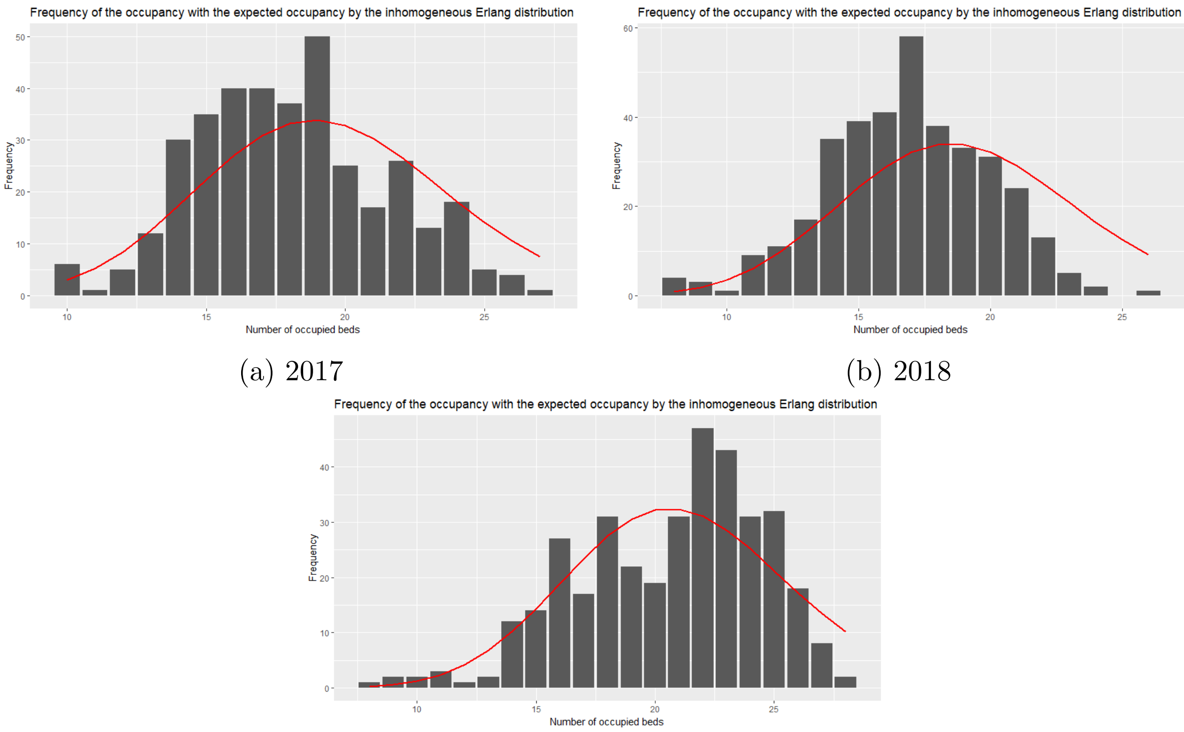


(b) 2018



(c) 2019

Figure F.4.: Analytical model based on the data compared to the data for ELV low/high during one year.

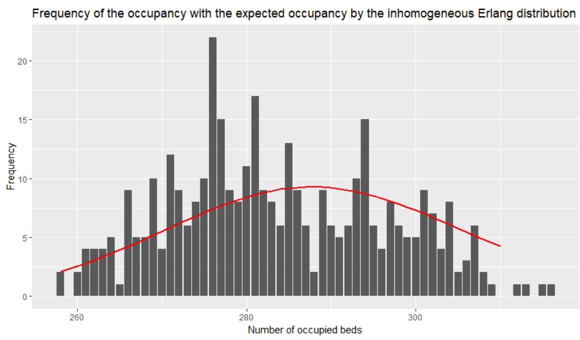


(a) 2017

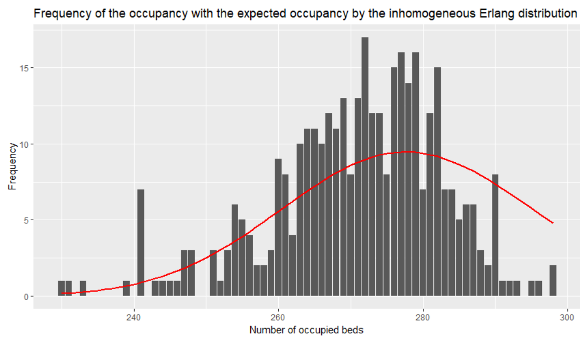
(b) 2018

(c) 2019

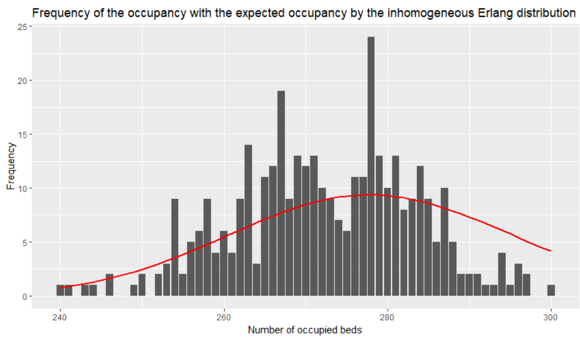
Figure F.5.: Analytical model based on the data compared to the data for ELV palliative during one year.



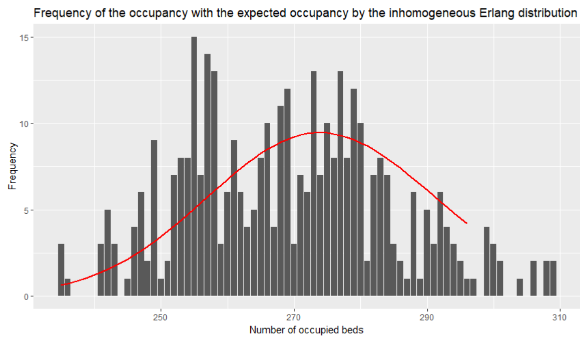
(a) 2016



(b) 2017



(c) 2018



(d) 2019

Figure F.6.: Analytical model based on the data compared to the data for GR during one year.