ODREĐENI INTEGRALI

1. $\displaystyle\int_1^2 x^3 dx = \frac{x^4}{4}\Big|_1^2 = \frac{1}{4}\left(2^4 - 1^4\right) = \frac{15}{4}$

2. $\displaystyle\int_1^4 \sqrt{x}\,dx = \int_1^4 x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_1^4 = \frac{2}{3}\left(\underbrace{\sqrt{4^3}}_{8} - \sqrt{1}\right) = \frac{2}{3}\cdot 7 = \frac{14}{3}$

3. $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \sin x\,dx = -\cos x\Big|_{\frac{\pi}{2}}^{\pi} = -\left(\underbrace{\cos\pi}_{-1} - \underbrace{\cos\frac{\pi}{2}}_{0}\right) = 1$

4. $\displaystyle\int_e^{e^2} \frac{\ln^3 x}{x} dx = \left|\begin{array}{l} \ln x = t \\ \frac{1}{x} dx = dt \\ x = e \Rightarrow t = \ln e = 1 \\ x = e^2 \Rightarrow t = \ln e^2 = 2 \end{array}\right| = \int_1^2 t^3 dt = \frac{t^4}{4}\Big|_1^2 = \frac{1}{4}\left(2^4 - 1\right) = \frac{15}{4}$

5. $\displaystyle\int_1^2 e^{2x} dx = \left|\begin{array}{l} 2x = t \\ 2dx = dt \Rightarrow dx = \frac{dt}{2} \\ x = 1 \Rightarrow t = 2 \\ x = 2 \Rightarrow t = 4 \end{array}\right| = \frac{1}{2}\int_2^4 e^t dt = \frac{1}{2} e^t\Big|_2^4 = \frac{1}{2}\left(e^4 - e^2\right)$

6. $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \underbrace{\sin^3 x}_{\sin^2 x\cdot\sin x}\,dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (1 - \cos^2 x)\cdot\sin x\,dx = \left|\begin{array}{l} \cos x = t \\ -\sin x\,dx = dt \\ x = \frac{\pi}{3} \Rightarrow t = \cos\frac{\pi}{3} = \frac{1}{2} \\ x = \frac{\pi}{4} \Rightarrow t = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \end{array}\right|$

$\displaystyle = -\int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}} (1 - t^2)\,dt = -\left(t - \frac{t^3}{3}\right)\Big|_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}} = \left(\frac{t^3}{3} - t\right)\Big|_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}} = \frac{2\sqrt{2}}{24} - \frac{\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right)$

7. $\displaystyle\int_1^2 \frac{x}{x^2+2} dx = \left|\begin{array}{l} x^2 + 2 = t \\ 2x\,dx = dt \\ x\,dx = \frac{dt}{2} \\ x = 1 \Rightarrow t = 3 \\ x = 2 \Rightarrow t = 6 \end{array}\right| = \frac{1}{2}\int_3^6 \frac{dt}{t} = \frac{1}{2}\ln|t|\Big|_3^6 = \frac{1}{2}\left(\ln 6 - \ln 3\right) =$

$\displaystyle = \frac{1}{2}\ln\frac{6}{3} = \frac{1}{2}\ln 2$

8. $\int_e^{e^2} x \ln x\,dx = \left| \begin{array}{l} u = \ln x \Rightarrow du = \frac{1}{x}dx \\ dv = x\,dx \Rightarrow v = \int x\,dx = \frac{x^2}{2} \end{array} \right| = \ln x \cdot \frac{x^2}{2} \Big|_e^{e^2} - \frac{1}{2}\int_e^{e^2} x^2 \frac{1}{x}\,dx =$

$= \underbrace{\ln e^2}_{\underbrace{2\ln e}_{1} = 2} \cdot \frac{e^4}{2} - \underbrace{\ln e}_{1} \cdot \frac{e^2}{2} - \frac{1}{2}\,\frac{x^2}{2}\Big|_e^{e^2} = e^4 - \frac{e^2}{2} - \frac{1}{4}\left(e^4 - e^2\right)$

9. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x \cdot \cos x\,dx = \left| \begin{array}{l} u = x \Rightarrow du = dx \\ dv = \cos x\,dx \Rightarrow v = \int \cos x\,dx = \sin x \end{array} \right| = x \cdot \sin x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x\,dx =$

$= \frac{\pi}{2} \cdot \underbrace{\sin\frac{\pi}{2}}_{1} - \frac{\pi}{4}\underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}} + \cos x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{\pi}{2} - \frac{\pi\sqrt{2}}{8} + \cancel{\cos\frac{\pi}{2}}^{\,0} - \underbrace{\cos\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}$