TABLIČNI INTEGRALI

1. a) $\int 3dx = 3\int dx = 3x + C$
(MOŽE i $3(x+C)$)

b) $\int x\,dx = \frac{x^2}{2} + C$

c) $\int x^5 dx = \frac{x^6}{6} x + C$

d) $\int \frac{1}{x^3} dx = \int x^{-3} dx = \frac{x^{-2}}{-2} + C = \frac{-1}{2x^2} + C$

e) $\int \frac{1}{x^8} dx = \int x^{-8} dx = \frac{x^{-7}}{-7} + C = \frac{-1}{7x^7} + C$

f) $\int \sqrt{x}\, dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2\sqrt{x^3}}{3} + C$

g) $\int \sqrt[4]{x^5}\, dx = \int x^{\frac{5}{4}} dx = \frac{x^{\frac{9}{4}}}{\frac{9}{4}} + C = \frac{4\sqrt[4]{x^9}}{9} + C$

h) $\int \frac{1}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x} + C$

i) $\int \frac{1}{\sqrt[4]{x^5}} dx = \int x^{-\frac{5}{4}} dx = \frac{x^{-\frac{1}{4}}}{\frac{1}{4}} + C = \frac{1}{4\sqrt[4]{x}} + C$

j) $\int \sin x\, dx = -\cos x + C$

k) $\int \frac{dx}{x^2+4} = \int \frac{dx}{x^2+2^2} = \frac{1}{2} \operatorname{arctg} \frac{x}{2} + C$

l) $\int \frac{dx}{\sqrt{9-x^2}} = \arcsin \frac{x}{3} + C$

SMENA PROMENLJIVE

2\. a) $\int (x+5)^4 dx = \begin{vmatrix} x+5=t \\ dx=dt \end{vmatrix} = \int t^4 dt = \frac{t^5}{5}+C = \frac{(x+5)^5}{5}+C$

b) $\int 2x e^{x^2} dx = \begin{vmatrix} x^2=t \\ 2x dx = dt \end{vmatrix} = \int e^t dt = e^t + C = e^{x^2}+C$

c) $\int x^2 e^{x^3} dx = \begin{vmatrix} x^3=t \\ 3x^2 dx = dt \\ x^2 dx = \frac{dt}{3} \end{vmatrix} = \int e^t \frac{dt}{3} = \frac{1}{3}\int e^t dt = \frac{1}{3} e^{x^3}+C$

d) $\int \frac{\ln^6 x}{x} dx = \begin{vmatrix} \ln x = t \\ \frac{1}{x} dx = dt \end{vmatrix} = \int t^6 dt = \frac{t^7}{7}+C = \frac{\ln^7 x}{7}+C$

e) $\int \sin^5 x \cos x\, dx = \begin{vmatrix} \sin x = t \\ \cos x dx = dt \end{vmatrix} = \int t^5 dt = \frac{t^6}{6}+C = \frac{\sin^6 x}{6}+C$

f) $\int \underbrace{\sin^3 x}_{\sin x \cdot \sin^2 x} \cdot \cos^2 x\, dx = \int \sin x (1-\cos^2 x)\cdot \cos^2 x\, dx = \begin{vmatrix} \cos x = t \\ -\sin x dx = dt \end{vmatrix}$

$= -\int (1-t^2)t^2\, dt = \int (t^4 - t^2) dt = \frac{t^5}{5} - \frac{t^3}{3} + C = \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3}+C$

g) $\int \frac{\operatorname{arctg}^2 x}{1+x^2} dx = \begin{vmatrix} \operatorname{arctg} x = t \\ \frac{1}{1+x^2} dx = dt \end{vmatrix} = \int t^2 dt = \frac{t^3}{3}+C = \frac{\operatorname{arctg}^3 x}{3}+C$

h) $\int \frac{\sqrt[3]{\ln x}}{x} dx = \begin{vmatrix} \ln x = t \\ \frac{1}{x} dx = dt \end{vmatrix} = \int t^{\frac{1}{3}} dt = \frac{t^{\frac{4}{3}}}{\frac{4}{3}}+C = \frac{3}{4}\sqrt[3]{\ln^4 x}+C$

i) $\int \sin 3x\, dx = \begin{vmatrix} 3x=t \\ 3dx = dt \\ dx = \frac{dt}{3} \end{vmatrix} = \int \sin t \frac{dt}{3} = \frac{1}{3}\int \sin t\, dt = -\frac{1}{3}\cos t + C =$

$= -\frac{1}{3}\cos 3x + C$

3. a) $\int x^2 \ln x\,dx = \left| \begin{array}{l} u=\ln x \Rightarrow du = \frac{1}{x}dx \\ dv = x^2dx \rightarrow v = \int x^2 dx = \frac{x^3}{3} \end{array} \right| = \overset{u}{\ln x} \cdot \overset{v}{\frac{x^3}{3}} - \frac{1}{3}\int \overset{v}{x^3} \cdot \overset{du}{\frac{1}{x}}\,dx =$

$= \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx = \frac{x^3}{3}\ln x - \frac{1}{3}\frac{x^3}{3} + C$

$\int P_n(x) \ln x\,dx \qquad \begin{array}{l} u = \ln x \\ dv = P_n(x)dx \end{array}$

b) $\int x \operatorname{arctg} x\,dx = \left| \begin{array}{l} u = \operatorname{arctg} x \Rightarrow du = \frac{1}{1+x^2}dx \\ dv = x dx \Rightarrow v = \int x dx = \frac{x^2}{2} \end{array} \right| = \operatorname{arctg} x \cdot \frac{x^2}{2} - \frac{1}{2}\int x^2 \cdot \frac{1}{1+x^2}dx =$

$= \operatorname{arctg} x \cdot \frac{x^2}{2} - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}dx = \operatorname{arctg} x \cdot \frac{x^2}{2} - \frac{1}{2}\left(\int dx - \int \frac{1}{1+x^2}dx\right) =$

$= \operatorname{arctg} x \cdot \frac{x^2}{2} - \frac{1}{2}\left(x - \operatorname{arctg} x\right) + C$

c) $\int (x^2+1)e^x dx = \left| \begin{array}{l} u = x^2+1 \Rightarrow du = 2x dx \\ dv = e^x dx \Rightarrow v = \int e^x dx = e^x \end{array} \right| = (x^2+1)e^x - 2\int x e^x dx =$

$\left| \begin{array}{l} u_1 = x \Rightarrow du_1 = dx \\ dv_1 = e^x dx \Rightarrow v_1 = \int e^x dx \end{array} \right| = (x^2+1)e^x - \left(xe^x - \int e^x dx\right) = (x^2+1)e^x - xe^x - e^x + C$

$\int P_n(x) e^x dx = \qquad \begin{array}{l} u = P_n(x)dx \\ dv = e^x dx \end{array}$

d) $\int (x-2)\sin x\,dx = \left| \begin{array}{l} u = x-2 \Rightarrow du = dx \\ dv = \sin x dx \Rightarrow v = \int \sin x dx = -\cos x \end{array} \right|$

$= (x-2)(-\cos x) + \int \cos x\,dx = -(x-2)\cos x + \sin x + C$

$\int P_n(x)\sin x\,dx, \quad \int P_n(x)\cos x\,dx \qquad , \quad u = P_n(x)$

e) $\int \sin 2x\, e^x dx = \left| \begin{matrix} u = \sin 2x \Rightarrow du = 2\cos 2x \\ dv = e^x dx \Rightarrow v = \int e^x dx = e^x \end{matrix} \right| =$

$= \sin 2x\, e^x - 2\int e^x \cos 2x\, dx = \left| \begin{matrix} u_1 = \cos 2x \Rightarrow du_1 = -2\sin 2x\, dx \\ dv_1 = e^x dx \Rightarrow v_1 = \int e^x dx = e^x \end{matrix} \right|$

$= \sin 2x \cdot e^x - 2\left(e^x \cos 2x + 2\int \sin 2x\, e^x dx \right)$

$I = \sin 2x\, e^x - 2e^x \cos 2x - 4I$

$5I = e^x(\sin 2x - 2\cos 2x)$

$I = \frac{e^x}{5}(\sin 2x - 2\cos 2x)$

$\int \sin 2x\, e^x dx = \frac{e^x}{5}(\sin 2x - 2\cos 2x) + C$