Design Method of Directional GenLOT with Trend Vanishing Moments

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Outline

- Background
- Review of Symmetric Orthogonal Transforms
- Contribution of This Work
- Review of GenLOT and Trend Vanishing Moment (TVM)
- Two-Order TVM Condition for 2-D GenLOT
- Design Procedure, Design Example and Evaluation
- Conclusions

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Background

Transforms play a crucial role in lots of applications such as

- Coding/Denoising

$$\mathbf{x} \to [\boldsymbol{\Psi}] \to \mathbf{y} \to \left[\begin{array}{c}\text{Quantization}\\ \text{or Shrinkage}\end{array}\right] \to \hat{\mathbf{y}} \to [\boldsymbol{\Psi}^{-1}] \to \hat{\mathbf{x}}$$

- Modeling (e.g. Compressive Sensing)

$$\begin{array}{l}\mathbf{x} \to [\boldsymbol{\Phi}] \to \mathbf{v} \to [\text{Estimation}] \to \hat{\mathbf{y}} \to [\boldsymbol{\Psi}^{-1}] \to \hat{\mathbf{x}} \\ \quad \searrow [\boldsymbol{\Psi}] \to \mathbf{y} \ \text{(modeled to be sparse)}\end{array}$$

- Feature extraction

$$\mathbf{x} \to [\boldsymbol{\Psi}] \to \mathbf{y} \to \left[\begin{array}{c}\text{Classification}\\ \text{or Regression}\end{array}\right] \to \omega$$

Orthogonality, i.e. $\boldsymbol{\Psi}^T\boldsymbol{\Psi} = \mathbf{I}$, makes things simple because of Perseval's theorem $\|\mathbf{x}\|_2^2 = \|\mathbf{y}\|_2^2$.

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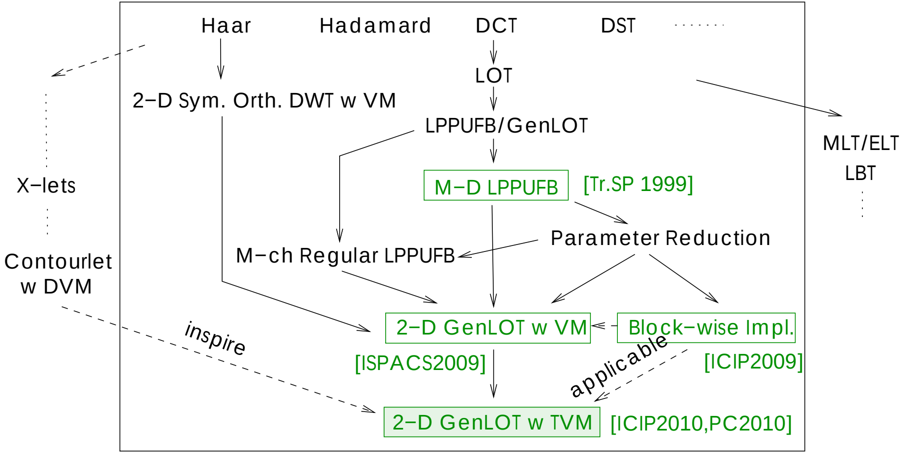
Symmetric Orthogonal Transforms

Symmetric bases are preferably adopted in image processing.

Family of Symmetric Orthogonal Transforms



LPPUFB: Linear-Phase Paraunitary Filter Banks

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Contribution

Experimental Results of ECSQ at 0.5bpp [ICIP2010]

Original
(PSNR)

8×8 DCT
(28.75[dB])

3-Lv. 9/7 DWT
(28.85[dB])

3-Lv. GenLOT w VM2
(28.68[dB])

3-Lv. GenLOT w TVM
(29.82[dB])

- Current issues on 2-D non-separable GenLOT
    - Adaptive control of bases
    - Efficient implementation
    - Design procedure
- This work contributes to
    - Clarify the relation between TVM direction and overlapping factor
    - Generalize the design procedure in terms of overlapping factor

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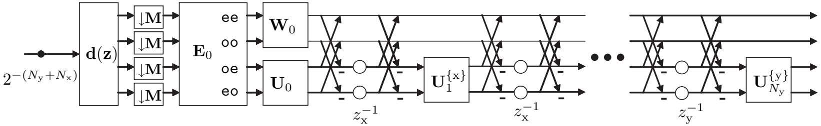
What is GenLOT?

Generalized Lapped Orthogonal Transform

- Orthogonality, symmetry and variability of basis
- Compatibility w block DCT
- Constructed by a lattice structure



Lattice structure of a 2-D non-separable GenLOT (forward transform)

$$\mathbf{E}(\mathbf{z}) = \prod_{n_y=1}^{N_y} \left\{ \mathbf{R}_{n_y}^{\{y\}} \mathbf{Q}(z_y) \right\} \cdot \prod_{n_x=1}^{N_x} \left\{ \mathbf{R}_{n_x}^{\{x\}} \mathbf{Q}(z_x) \right\} \cdot \mathbf{R}_0 \mathbf{E}_0,$$

where $\mathbf{W}_0$, $\mathbf{U}_0$ and $\mathbf{U}_{n_d}^{\{d\}}$ are parameter matrices.

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What is TVM?

Trend vanishing moment is an extention of 1-D VM to 2-D case.

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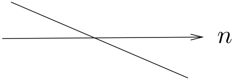

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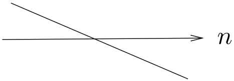

Every wavelet filters with VM
annhilate piece-wise polynomials

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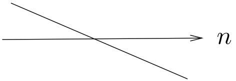
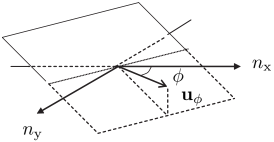


Every wavelet filters with VM annhilate piece-wise polynomials



Every wavelet filters with TVM annhilate piece-wise polynomial surfaces in the direction ϕ

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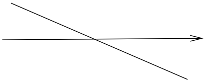

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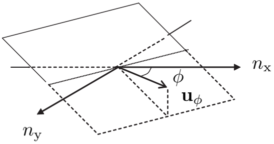


Every wavelet filters with TVM annhilate piece-wise polynomial surfaces in the direction ϕ

- 2-D VM – [Stanhill *et al.*, IEEE Trans. on SP 1996]
- Directional VM (DVM) – [Do *et al.*, IEEE Trans. on IP 2005]
- Trend VM (TVM) – [ICIP2010, PCS2010]

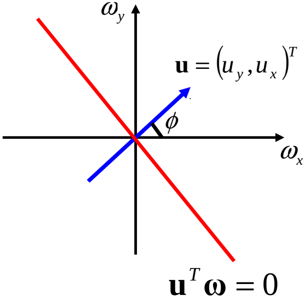
DVM

Every wavelet filters annihilate piece-wise polynomials along every directed lines.

TVM

Every wavelet filters annihilate directed piece-wise polynomial surfaces.

Freq. Domain



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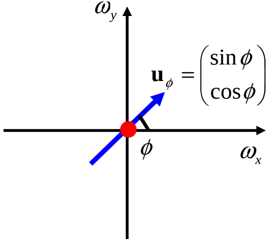

NOTE: $\mathbf{u}$ must be INTEGER,
while $\mathbf{u}_\phi$ is STEERABLE FLEXIBLY.

Trend Vanishing Moment (TVM) Condition

We say that a filter bank has P-order TVM along the direction $\mathbf{u}_\phi = (\sin\phi, \cos\phi)^T$ if and only if the following condition holds:

- For wavelet filters $(k = 1, 2, \cdots, M-1,\ p = 0, 1, 2, \cdots, P-1)$

$$0 = (-j)^p \sum_{q=0}^{p} \binom{p}{q} \sin^{p-q}\phi \cos^q\phi \frac{\partial^p}{\partial\omega_{\mathrm{y}}^{p-q}\partial\omega_{\mathrm{x}}^{q}} H_k\left(e^{j\boldsymbol{\omega}^T}\right)\Bigg|_{\boldsymbol{\omega}=\mathbf{0}}$$

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An equivalent condition is derived for FIR PU systems

- For a polyphase matrix $(p = 0, 1, 2, \cdots, P-1)$

$$c_p \mathbf{a}_M = \sum_{q=0}^{p} \binom{p}{q} \sin^{p-q}\phi \cos^q\phi \frac{\partial^p}{\partial z_{\mathrm{y}}^{p-q}\partial z_{\mathrm{x}}^{q}} \mathbf{E}\left(\mathbf{z}^{\mathbf{M}}\right)\mathbf{d}(\mathbf{z})\Big|_{\mathbf{z}=\mathbf{1}},$$

where c_p is a constant, $\mathbf{1} = (1, 1, \cdots, 1)^T$ and $\mathbf{a}_m = (1, 0, \cdots, 0)^T$ [PCS2010].

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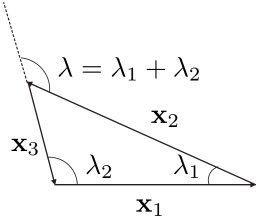
Two-Order TVM Conditions for Lattice Parameters

$$\mathbf{o} = M_{\mathrm{y}} \sin\phi \sum_{k_{\mathrm{y}}=1}^{N_{\mathrm{y}}} \prod_{n_{\mathrm{y}}=k_{\mathrm{y}}}^{N_{\mathrm{y}}} \mathbf{U}_{n_{\mathrm{y}}}^{\{\mathrm{y}\}} \cdot \mathbf{a}_{\frac{M}{2}} + M_{\mathrm{x}} \cos\phi \prod_{n_{\mathrm{y}}=1}^{N_{\mathrm{y}}} \mathbf{U}_{n_{\mathrm{y}}}^{\{\mathrm{y}\}} \cdot \sum_{k_{\mathrm{x}}=1}^{N_{\mathrm{x}}} \prod_{n_{\mathrm{x}}=k_{\mathrm{x}}}^{N_{\mathrm{x}}} \mathbf{U}_{n_{\mathrm{x}}}^{\{\mathrm{x}\}} \cdot \mathbf{a}_{\frac{M}{2}} + \prod_{n_{\mathrm{y}}=1}^{N_{\mathrm{y}}} \mathbf{U}_{n_{\mathrm{y}}}^{\{\mathrm{y}\}} \cdot \prod_{n_{\mathrm{x}}=1}^{N_{\mathrm{x}}} \mathbf{U}_{n_{\mathrm{x}}}^{\{\mathrm{x}\}} \cdot \mathbf{U}_0 \mathbf{b}_{\phi},$$



The same approach as [Oraintara *et al.*, IEEE Trans. SP 2001] is applicable to obtain the design constraint.

Conditions for Polyphase Order

Theorem (Necessary Condition for the Polyphase Order)

2-D GenLOT requires polyphase order $[N_{\mathrm{y}}, N_{\mathrm{x}}]$ *such that* $(N_{\mathrm{y}} + N_{\mathrm{x}}) > 1$ *to hold the two-order TVM except for some singular angles.*

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Theorem (Necessary Condition for the Polyphase Order)

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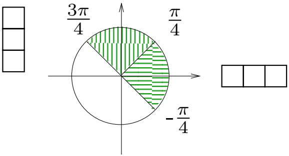
Theorem (Sufficient Condition for the Polyphase Order)

2-D GenLOT can hold the two-order TVM for any angle in the following specified range when the corresponding condition is satisfied:

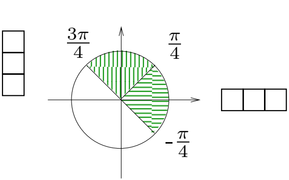
1. *For* $\phi \in [-\pi/4, \pi/4]$*, the horizontal polyphase order* $N_\mathrm{x} \geq 2$.
2. *For* $\phi \in [\pi/4, 3\pi/4]$*, the vertical polyphase order* $N_\mathrm{y} \geq 2$.

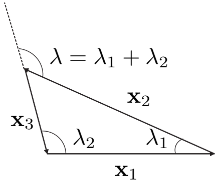
Vertical overlapping factor ≥ 2



Horizontal overlapping factor ≥ 2

Design Procedure with Two-Order TVM

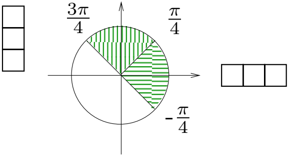


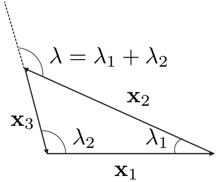


For $\phi \in [\pi/4, 3\pi/4]$ and $N_\mathrm{y} \geq 2$

1. Give a direction ϕ and let $\bar{\mathbf{x}}_3$ as given in Tab. II.
2. Impose parameter matrices $\mathbf{U}^{\{\mathrm{y}\}}_{N_\mathrm{y}-2}$ and $\mathbf{U}^{\{\mathrm{y}\}}_{N_\mathrm{y}-1}$ to constitute a triangle.
3. Optimize parameter matrices for minimizing a given cost function under the constraint $\|\bar{\mathbf{x}}_3\| \leq 2$.

Design Procedure with Two-Order TVM





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For $\phi \in [-\pi/4, \pi/4]$ and $N_\mathrm{x} \geq 2$

1. Give a direction ϕ and let $\bar{\mathbf{x}}_3$ as given in Tab. I.
2. Impose parameter matrices $\mathbf{U}^{\{\mathrm{x}\}}_{N_\mathrm{x}-2}$ and $\mathbf{U}^{\{\mathrm{x}\}}_{N_\mathrm{x}-1}$ to constitute a triangle.
3. ($\leftarrow$ same)

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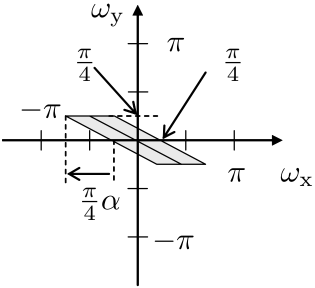
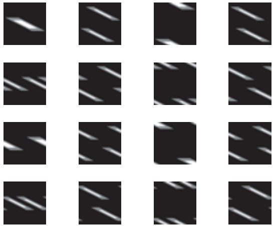
We adopt Passband Error & Passband Energy Criteria.





An example of passband region deformation for ideal lowpass filters, and an example set of magnitude response specification for $M_y = M_x = 4$, $d = x$ and $\alpha = -2.0$ [ICIP2009]

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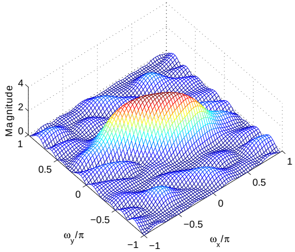
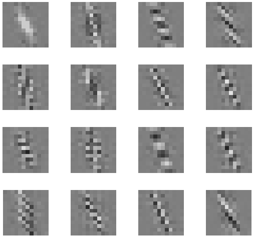
Design Examples with Two-Order TVM

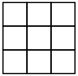


$\left| H_0 \left(e^{j\omega^T} \right) \right|$



Basis images

A design example with two-order TVMs of $\phi = \cot^{-1} \alpha \sim -26.57^\circ$ optimized for the specification given in the previous slide, where $N_\mathrm{y} = N_\mathrm{x} = 2$, i.e. basis images of size 12×12.



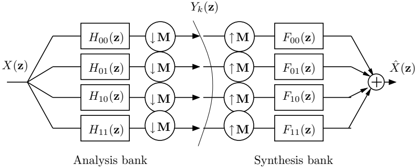
A novel result
more than 2×2 channels.

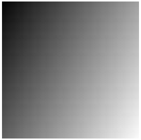
QUESTION!
Do really the obtained systems satisfy the TVM condition?

- Numerically verify
  1. Simulation for Ramp Picture Rotation
  2. Simulation for TVM Rotation
- Sparsity ratio as a fraction of nonzero samples and Coefs.:

$$R_x = \frac{\sum_{k=0}^{M-1} \|y_k[\mathbf{m}]\|_0}{\|x[\mathbf{n}]\|_0},$$

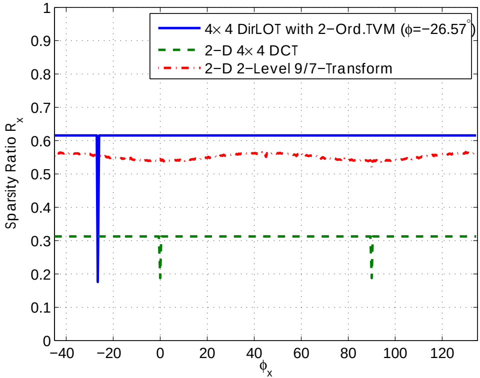
($|x| \leq 10^{-15}$ is regarded as zero.)





A ramp picture of size 128×128, where $\phi_x = 30.00^\circ$

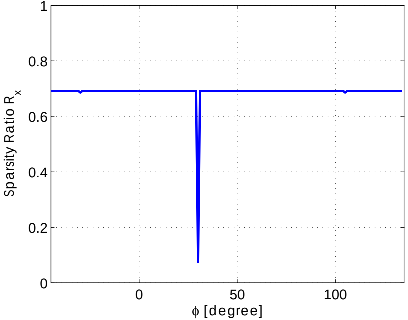
- Direction of TVM is fixed to $\phi = -26.57^\circ$.



Sparsity ratio R_x against for directions of trend surface in ramp pictures

Simulation for TVM Rotation

- Direction of input picture is fixed to $\phi_x = 30.00^\circ$
- 2-D GenLOT w TVM of minimum order is designed for every ϕ
- 3-lv. DWT structure of 2×2-ch 2-D GenLOT is adopted.



Sparsity ratio R_x against for directions of two-order TVMs.

Spiky drop can be seen when $\phi = \phi_x$.

Conclusions

- 2-D GenLOT belongs to symmetric orthogonal transforms
- TVM was introduced
  - An extention of 1-D VM to 2-D case
  - Different from classical VM and DVM
- A novel generalized design procedure was given
- Capability of trend surface annihilation property was shown

Future works

- Design parameter reduction
- Adaptive control of local basis
- Fast hardware-friendly implementation
- Killer application