2-D NON-SEPARABLE GENLOT WITH TREND VANISHING MOMENTS

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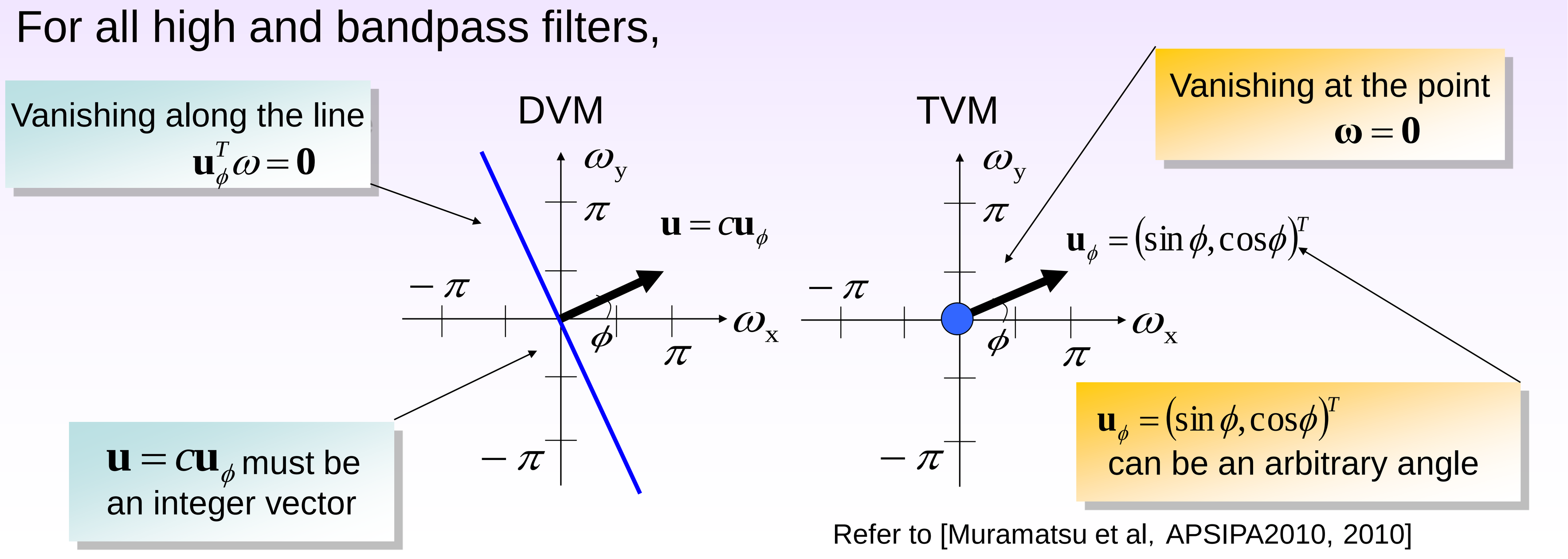
ABSTRACT

A novel design method of 2-D non-separable GenLOT (2-D NS-GenLOT) is proposed.

The 2-D NS-GenLOT is a 2-D DCT based linear-phase paraunitary filter bank (LPPUFB) constructed by a lattice structure.

- 2-D NS-GenLOT
  - Guarantees orthogonality, symmetry, overlap property, variability, directionality[1].
  - Boundary operation is possible[1].
  - DWT construction is possible.
- Contributions
  - The TVM condition is newly derived for parameters on the lattice structure
  - Smooth reconstruction of a diagonal edge is obtained.
  - The TVM condition can be an alternative of the directional vanishing moment (DVM).

For all high and bandpass filters,



Refer to [Muramatsu et al, APSIPA2010, 2010]

Introduction

- Problems
  - JPEG, MPEG-2 and H.264 employ 2-D block DCT and JPEG2000 adopts DWT.

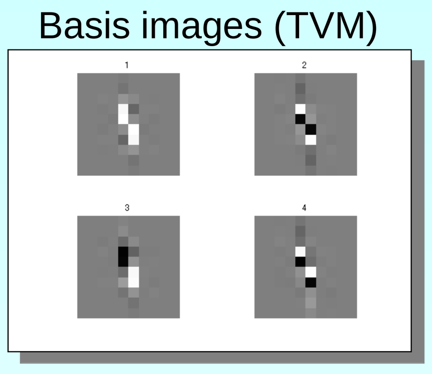
  All of these are separable and week for diagonal edges and texture.

  - Non-separable transforms have been developed. e.g.
    - 2-D NS-GenLOT [Muramatsu et al, IEEE Trans. SP 1999]
    - Contourlet (The DVM condition is employed.) [Minh et al, IEEE Trans. IP 2005]
  - As a previous work, we have proposed
    - the block-wise implementation and directional design approach[1]
    - Two-degree vanishing moment (VM2) constraints[2]
    - It is observed the VM2 constraint disturbs filter directionality.
- Purpose
  - To achieve smooth reconstruction of a diagonal texture image.
    - Define the trend vanishing moment (TVM) condition.
    - Derive the TVM constraint for the lattice structure.

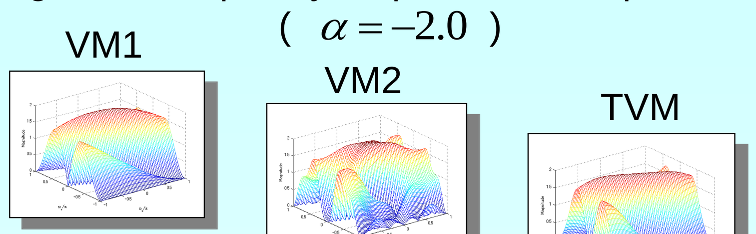
Experimental Results

- In order to verify the significance of the TVM constraints, coding results with design for $M_y = M_x = 2,\ N_y = N_x = 4$ are shown.

Basis images (TVM)

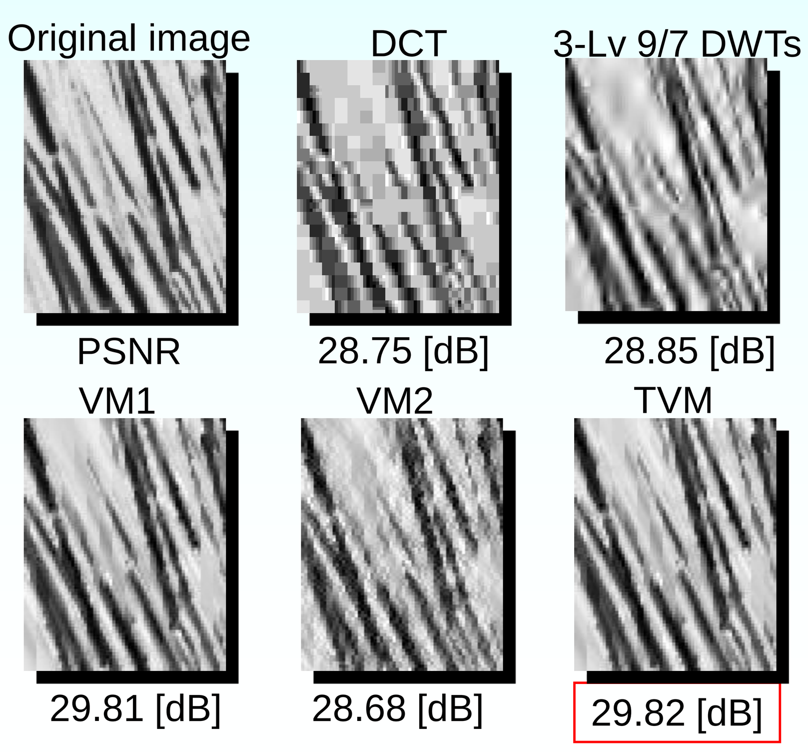


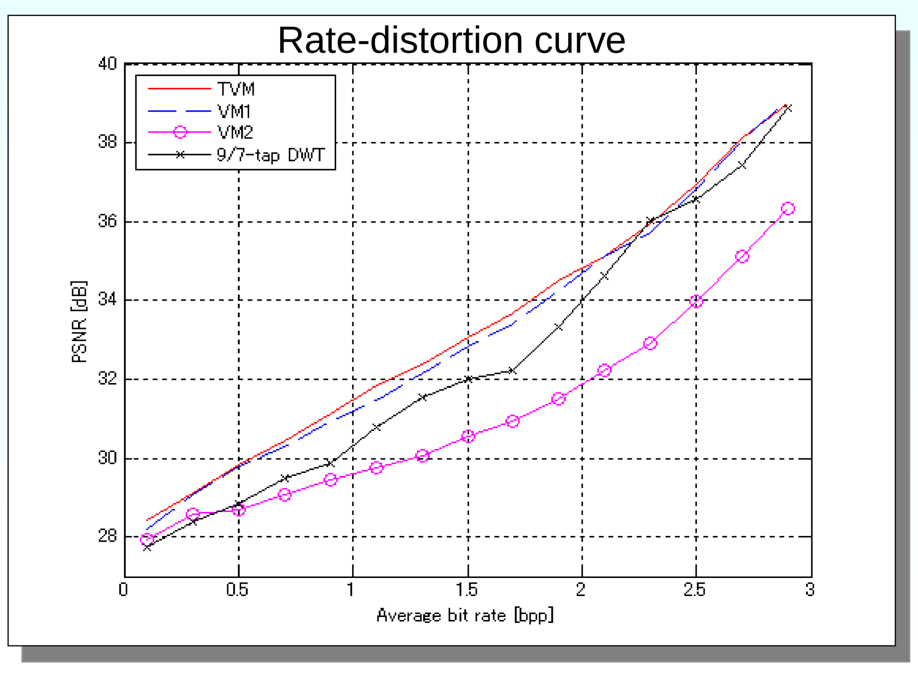
Magnitude frequency responses of lowpass filters ($\alpha = -2.0$)



$\phi = \tan^{-1}(-1/2)$

- Experimental results
  - ECSQ at 0.5 bpp for 8-bit grayscale image of size 80x64 where 3-Lv DWTs of 2-D NS-GenLOT ware used.





Review of 2-D non-separable GenLOT

- Product form of 2-D NS-GenLOT with 2-D DCT $\mathbf{E}_0$:

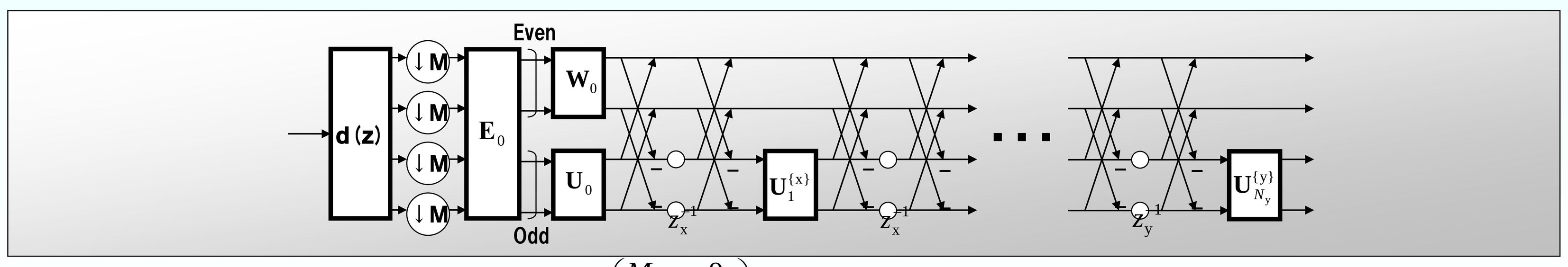
$$\mathbf{E}(\mathbf{z}) = \prod_{n_y=1}^{N_y}\left\{\mathbf{R}_{n_y}^{\{y\}}\mathbf{Q}(z_y)\right\}\prod_{n_x=1}^{N_x}\left\{\mathbf{R}_{n_x}^{\{x\}}\mathbf{Q}(z_x)\right\}\mathbf{R}_0\mathbf{E}_0$$

where $\mathbf{R}_0 = \begin{pmatrix}\mathbf{W}_0 & \mathbf{0}\\ \mathbf{0} & \mathbf{U}_0\end{pmatrix}$, $\mathbf{R}_n^{\{d\}} = \begin{pmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{U}_n^{\{d\}}\end{pmatrix}$ and $\mathbf{Q}_n^{\{d\}} = \begin{pmatrix}\mathbf{I} & \mathbf{I}\\ \mathbf{I} & -\mathbf{I}\end{pmatrix}\begin{pmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & z_d^{-1}\mathbf{I}\end{pmatrix}\begin{pmatrix}\mathbf{I} & \mathbf{I}\\ \mathbf{I} & -\mathbf{I}\end{pmatrix}$ for $d \in \{y, x\}$.

$\mathbf{W}_0$, $\mathbf{U}_0$ and $\mathbf{U}_{n_d}^{\{d\}}$ are parameter orthonormal matrices.

- Lattice structure of 2-D NS-GenLOT in 4-channel case

[Muramatsu et al, IEEE Trans. SP 1999] [Lu Gan et al, IEEE SP Letters 2001]

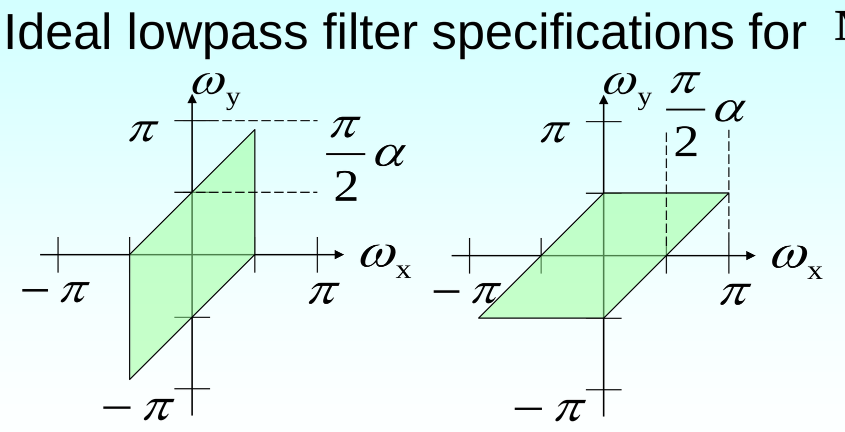


where $\mathbf{d}(\mathbf{z})$ is delay chain and $\mathbf{M} = \begin{pmatrix}M_y & 0\\ 0 & M_x\end{pmatrix}$, where M_y and M_x are vertical and horizontal decomposition factor.

- Orthogonality, symmetry, overlap property, changeability, directivity are guaranteed.

Directional specification in design

- Ideal lowpass filter specifications for $\mathbf{M}_y = \mathbf{M}_x = 2$

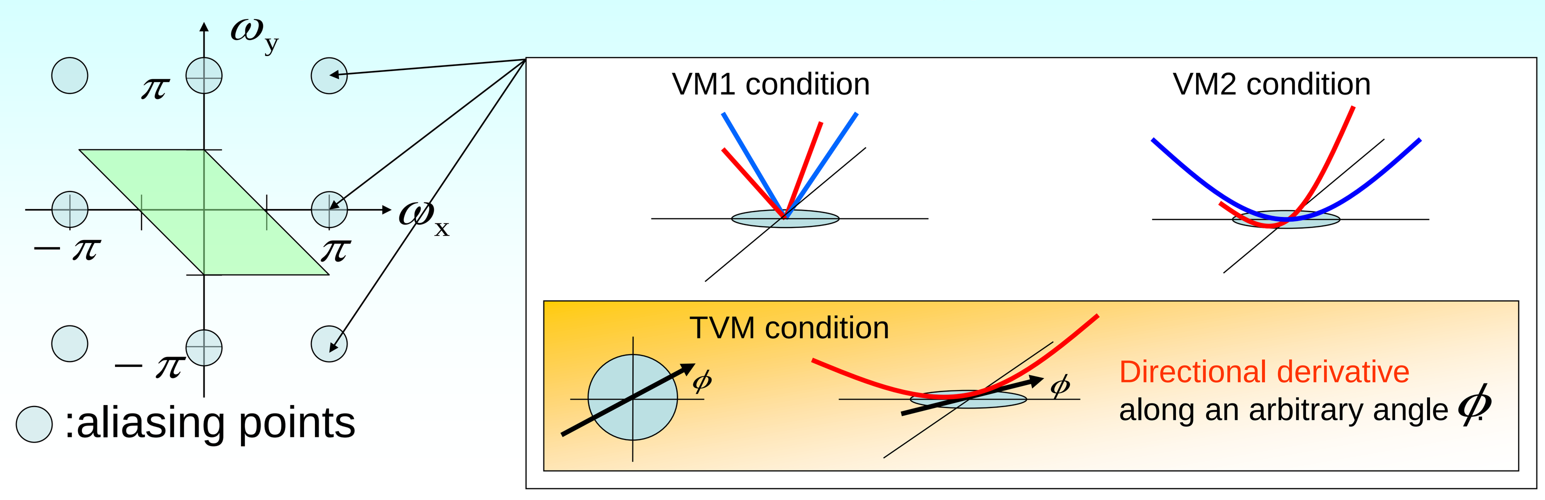


Optimization of parameters:
$\mathbf{W}_0$, $\mathbf{U}_0$ and $\mathbf{U}_{n_d}^{\{d\}}$

where α controls the passband shape.

Comparison with classical VM condition

- Moments are imposed to vanish at all aliasing points in lowpass filter.



The TVM condition

- The one-degree vanishing moment (VM1) constraint for parameter $\mathbf{W}_0$.

$$\mathbf{E}(\mathbf{z}^{\mathbf{M}})\mathbf{d}(\mathbf{z})\Big|_{\mathbf{z}=\mathbf{1}} = \mathbf{E}(\mathbf{1})\mathbf{1} = c_0\mathbf{a}_M \quad\Rightarrow\quad \mathbf{W}_0 = \begin{pmatrix}1 & \mathbf{0}^T\\ \mathbf{0} & \hat{\mathbf{W}}_0\end{pmatrix}$$

where $\mathbf{1} = (1, 1, \cdots, 1)^T$, $\mathbf{a}_M = (1, 0, \cdots, 0)^T$ and $\mathbf{z}^{\mathbf{M}} = (z_y^{M_y} z_x^{M_x})$.

where $\hat{\mathbf{W}}_0$ is orthonormal matrix.

- The TVM condition for direction $(\sin\phi, \cos\phi)^T$.

$$\left\{\frac{\partial}{\partial z_y}\sin\phi + \frac{\partial}{\partial z_x}\cos\phi\right\}\mathbf{E}(\mathbf{z}^{\mathbf{M}})\mathbf{d}(\mathbf{z})\Big|_{\mathbf{z}=\mathbf{1}} = c_1\mathbf{a}_M$$

- The TVM constraint for parameters on lattice structure

$$\mathbf{a}_{\frac{M}{2}} + \left(\sum_{k_y=1}^{N_y-1}\prod_{n_y=k_y}^{N_y-1}\mathbf{U}_{n_y}^{\{y\}} + \frac{M_x}{M_y}\cot\phi\prod_{n_y=1}^{N_y-1}\mathbf{U}_{n_y}^{\{y\}}\sum_{k_x=1}^{N_x}\prod_{n_x=k_x}^{N_x}\mathbf{U}_{n_x}^{\{x\}}\right)\mathbf{a}_{\frac{M}{2}} + \prod_{n_y=1}^{N_y-1}\mathbf{U}_{n_y}^{\{y\}}\prod_{n_x=1}^{N_x}\mathbf{U}_{n_x}^{\{x\}}\mathbf{U}_0\bar{\mathbf{b}}_\phi = \mathbf{0}$$

We define $\mathbf{x}_1 = \mathbf{a}_{M/2}, \mathbf{x}_2 = \mathbf{U}_{N_y-1}^{\{y\}}\mathbf{a}_{M/2}, \mathbf{x}_3 = \mathbf{U}_{N_y-1}^{\{y\}}\mathbf{U}_{N_y-2}^{\{y\}}\bar{\mathbf{x}}_3$, where $\mathbf{U}_0^{\{y\}} = \begin{cases}\mathbf{U}_{N_x}^{\{x\}} & , N_x > 0\\ \mathbf{U}_0 & , N_x = 0\end{cases}$

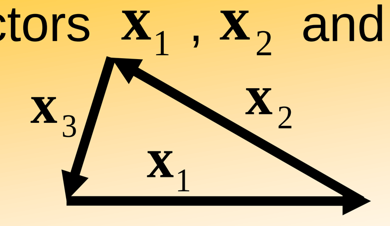
$$\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3 = \mathbf{0}$$

$$\|\mathbf{x}_1\| = \|\mathbf{x}_2\| = 1$$

$$\bar{\mathbf{x}}_3 = \begin{cases}\frac{M_x}{M_y}\cot\phi\left(\mathbf{I} + \sum_{k_x=1}^{N_x-1}\prod_{n_x=k_x}^{N_x-1}\mathbf{U}_{n_x}^{\{x\}}\right)\mathbf{a}_{M/2} + \prod_{n_x=1}^{N_x-1}\mathbf{U}_{n_x}^{\{x\}}\mathbf{U}_0\bar{\mathbf{b}}_\phi & , N_y = 2\\ \left(\mathbf{I} + \frac{M_x}{M_y}\cot\phi\sum_{k_x=1}^{N_x}\prod_{n_x=k_x}^{N_x}\mathbf{U}_{n_x}^{\{x\}}\right)\mathbf{a}_{M/2} + \prod_{n_x=1}^{N_x}\mathbf{U}_{n_x}^{\{x\}}\mathbf{U}_0\bar{\mathbf{b}}_\phi & , N_y = 3\\ \left(\mathbf{I} + \sum_{k_y=1}^{N_y-3}\prod_{n_y=k_y}^{N_y-3}\mathbf{U}_{n_y}^{\{y\}} + \frac{M_x}{M_y}\cot\phi\sum_{k_y=1}^{N_y-3}\mathbf{U}_{n_y}^{\{y\}}\sum_{k_x=1}^{N_x}\prod_{n_x=k_x}^{N_x}\mathbf{U}_{n_x}^{\{x\}}\right)\mathbf{a}_{M/2} + \prod_{n_y=1}^{N_y-3}\mathbf{U}_{n_y}^{\{y\}}\prod_{n_x=1}^{N_x}\mathbf{U}_{n_x}^{\{x\}}\mathbf{U}_0\bar{\mathbf{b}}_\phi & , N_y > 3\end{cases}$$

TVM constraint

Three vectors $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ should construct a bilateral triangle.



$\|\mathbf{x}_3\| = \|\bar{\mathbf{x}}_3\| \leq 2$ have to be satisfied.

Conclusions

- The TVM condition was proposed and achieved smooth reconstruction of a diagonal texture image.
- Future works include adaptive control and fast implementation.
- Compressive sensing, denoising & feature extraction are prospective Apps.

References

[1] Shogo Muramatsu and Minoru Hiki, **Block-Wise Implementation of Directional GenLOT**, *IEEE Proc. of ICIP2009*, pp.3977-3980, Cairo, Egypt, Nov. 7-11 2009.
[2] Tomoya Kobayashi, Shogo Muramatsu and Hisakazu Kikuchi, **Two-degree vanishing moments on 2-D non-separable GenLOT**, in IEEE Proc. of ISPACS, Dec. 2009, pp.248-251.