SURE-LET IMAGE DENOISING WITH DIRECTIONAL LOTS

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ABSTRACT

It is proposed to adopt directional lapped orthogonal transforms (DirLOTs) in hierarchical wavelet structure to image denoising. So far, the orthonormal wavelet image denoising techniques have shown a disadvantage in the restoration of diagonal textures and edges because of the separability of the adopted transforms. This work shows through some experimental results that the SURE-LET approach with DirLOTs overcomes the geometric problem.

Key words– Directional transforms, Image denoising

Introduction

- A common problem of image denoising, i.e. removal of additive white Gaussian noise (AWGN) from a given image is dealt with.
- One of the most popular techniques is the orthonormal wavelet shrinkage with soft-thresholding [Donoho and Johnstone, 1994].
- Luisier *et al.* proposed a linear optimization technique to determine the shape of shrinkage function [Luisier *et al*, 2007].
  - It is called the SURE-LET approach, which minimizes the Stein's unbiased risk estimator (SURE) with linear expansion of thresholds (LET).
- Main issue of orthonormal wavelet shrinkage for images is to improve the quality for diagonal edges and textures.

Let us introduce directional orthonormal discrete wavelet transforms.

Review of Orthonormal Wavelet Image Denoising

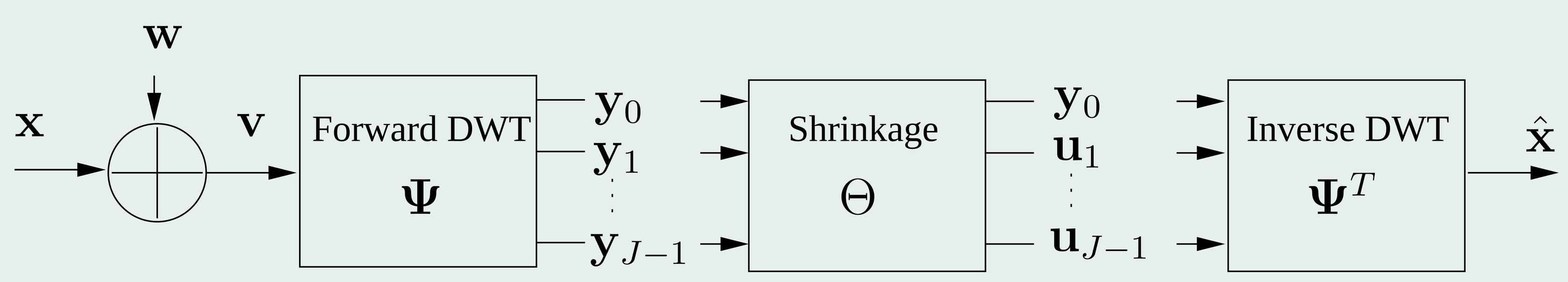


Figure: Principal of orthonormal wavelet denoising, where **w** is an AWGN.

- A wavelet denoising process is summarized as follows:
  i) Perform a forward DWT of the noisy picture $\mathbf{v} = \mathbf{x} + \mathbf{w}$. Then, obtain the transform coefficients
  $$\mathbf{y} = \left(\mathbf{y}_0^T\ \mathbf{y}_1^T\ \cdots\ \mathbf{y}_{J-1}^T\right)^T = \mathbf{\Psi}\mathbf{v}.$$
  ii) Denoise wavelet subimages $\mathbf{y}_j$ for $j \in [1, J-1]$. Then, obtain denoised subimages
  $$\mathbf{u} = \left(\mathbf{y}_0^T\ \mathbf{u}_1^T\ \cdots\ \mathbf{u}_{J-1}^T\right)^T = \Theta(\mathbf{y}).$$
  iii) Perform the inverse DWT of coefficients **u** as
  $$\hat{\mathbf{x}} = \mathbf{\Psi}^{-1}\mathbf{u} = \mathbf{\Psi}^T\mathbf{u}.$$
- Since $\hat{\mathbf{x}} = \mathbf{\Psi}^T\Theta(\mathbf{\Psi}\mathbf{v})$, the denoising quality depends on the choice of the transform $\mathbf{\Psi}$ and the shrinkage function $\Theta(\cdot)$.

SURE-LET for Shrinkage Function $\Theta(\cdot)$

- The SURE-LET approach is efficient both in terms of computational complexity and denoising quality.
- The shrinkage function is point-wisely defined and completely characterized by a set of parameters a_k and b_k:
  $$\theta(y, y_p; \mathbf{a}, \mathbf{b}) =$$
  $$e^{-\frac{y_p^2}{12\sigma^2}} \sum_{k=1}^{K} a_k y e^{-(k-1)\frac{y^2}{12\sigma^2}} + \left(1 - e^{-\frac{y_p^2}{12\sigma^2}}\right) \sum_{k=1}^{K} b_k y e^{-(k-1)\frac{y^2}{12\sigma^2}},$$
  where y and y_p are a wavelet coefficient and interscale prediction of y obtained from the wavelet parent-child relationship, respectively.
  - It is suggested to use $K = 2$ and $T = \sqrt{6}\sigma$.
- The parameters a_k and b_k are linearly solved for minimizing SURE.

DirLOTs for Transform $\mathbf{\Psi}$

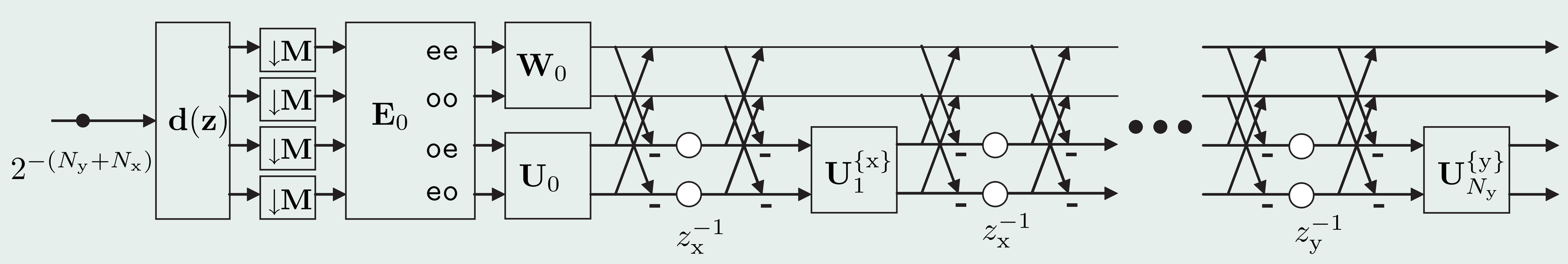
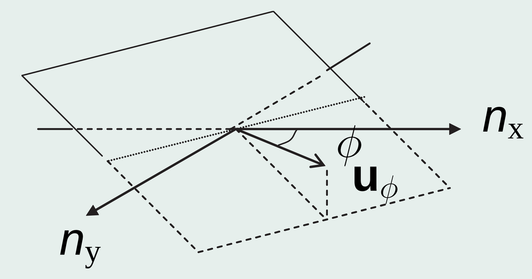


Figure: Lattice structure of a 2-D non-separable lapped orthogonal transform.

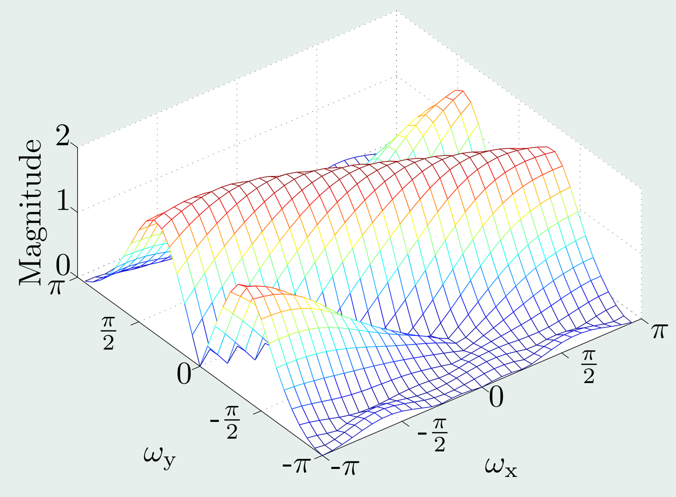
- DirLOTs are 2-D non-separable lapped orthogonal transforms with directional characteristics [ICIP2009,ICIP2010].
- The bases are symmetric, real-valued and have compact-support.

A Design Example of DirLOT

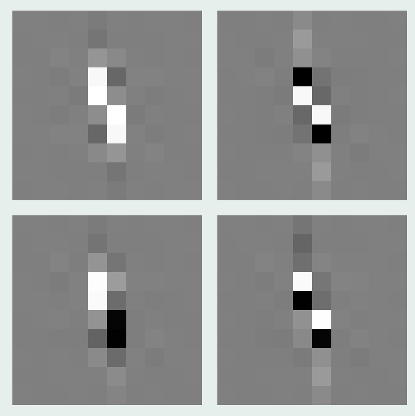
DirLOTs can be constructed under the trend vanishing moment (TVM) constraints, which force wavelet filters to annihilate trend surface components [PCS2010,APSIPA2010].



A trend surface proportional to $(n_y \sin\phi + n_x \cos\phi)$



(a) $|H_0(e^{j\omega^T})|$



(b) Basis images

Figure: A design example with the two-order TVMs of $\phi = \cot^{-1}(-2.0) \sim -0.4636$[rad], where $[N_y, N_x]^T = [4, 4]^T$, i.e. the basis images are of size 10×10.

Experimental Results

(a) Original (b) Noisy Pic. (c) Sym5 (d) VM2 (e) TVM

Figure: Denoising results for an 8-bit grayscale picture of size 128×128 pixels. (a)Original picture, (b)Noisy picture with white Gaussian ($\sigma = 30$). (c),(d) and (e) are denoised results, where Sym5, VM2 and TVM denote Symlets of index 5, DirLOT with the classical VM of order two and DirLOT with the two-order TVMs, respectively. The number of hierarchical levels is three.

Table: Comparison of PSNRs and SSIM indexes among three transforms.

	PSNR			SSIM		
	Sym5	VM2	TVM	Sym5	VM2	TVM
$\sigma = 10$	**29.47**	23.97	28.81	**0.969**	0.907	0.965
$\sigma = 20$	**25.46**	23.11	25.26	0.936	0.902	**0.937**
$\sigma = 30$	**23.22**	21.43	23.17	0.904	0.866	**0.908**
$\sigma = 40$	21.71	20.29	**21.73**	0.871	0.833	**0.878**
$\sigma = 50$	20.50	19.26	**20.59**	0.837	0.797	**0.849**

(a) Original (b) Noisy Pic. (c) Sym5 (d) VM2 (e) TVM

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$\sigma = 10$	24.04	25.70	**27.51**	0.651	0.771	**0.847**
$\sigma = 20$	23.18	23.57	**24.51**	0.605	0.666	**0.728**
$\sigma = 30$	22.28	22.32	**22.89**	0.539	0.571	**0.623**
$\sigma = 40$	21.48	21.49	**21.97**	0.461	0.495	**0.553**
$\sigma = 50$	20.97	20.91	**21.25**	0.409	0.440	**0.490**

Conclusions

- Proposed to adopt the hierarchical tree construction of DirLOTs to image denoising.
- Combination of SURE-LET approach and the hierarchical DirLOT overcomes the diagonal geometric problem.

Acknowledgment

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