# Image Denoising with Union of Directional Orthonormal DWTs

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## ABSTRACT


- A novel image denoising technique is proposed by using directional lapped orthogonal transforms (DirLOTs).
- A redundant dictionary is constructed with multiple hierarchical DirLOTs and it is applied to solve the basis pursuit denoising (BPDN) problem.
- The denoising performance is evaluated for several images through the heuristic shrinkage and block-coordinate-relaxation algorithm.


**Key words**– DirLOT, Basis pursuit, Wavelet denoising, Heuristic shrinkage, Block-coordinate-relaxation (BCR) algorithm

## Introduction

- A common problem of image denoising, i.e. removal of additive white Gaussian noise (AWGN) from an observed image is dealt with.
- Purpose is to find a good candidate image $\hat{\mathbf{x}}$ of $\mathbf{x}^*$ only from $\mathbf{x}$.

$$\underbrace{\mathbf{x}}_{\text{Observed image}} = \underbrace{\mathbf{x}^*}_{\text{Original image}} + \underbrace{\mathbf{w}}_{\text{White Gaussian noise}}$$

- Popular denoising approaches include solving the basis pursuit denoising (BPDN) problem.
- BPDN assumes that the candidate $\hat{\mathbf{x}}$ is expressed by a linear-combination of image prototypes (atoms) in a dictionary $\mathbf{D}$:

$$\underbrace{\hat{\mathbf{x}}}_{\text{Candidate image}} = \underbrace{\mathbf{D}}_{\text{Dictionary}} \underbrace{\hat{\mathbf{y}}}_{\text{Coefs.}},$$

where $\hat{\mathbf{y}}$ refers to the solution of

$$\hat{\mathbf{y}} = \arg\min_{\mathbf{y}} \underbrace{\frac{1}{2}\|\mathbf{x}-\mathbf{D}\mathbf{y}\|_2^2}_{\text{Fidelity}} + \boldsymbol{\lambda}^T \underbrace{|\mathbf{y}|}_{\text{Sparsity}}.$$

- In this work, we suggest to adopt

$\boxed{\text{Union of Directional Symmetric Orthonormal DWTs as a Dictionary } \mathbf{D}}$

## Review of Directional LOTs (DirLOTs)

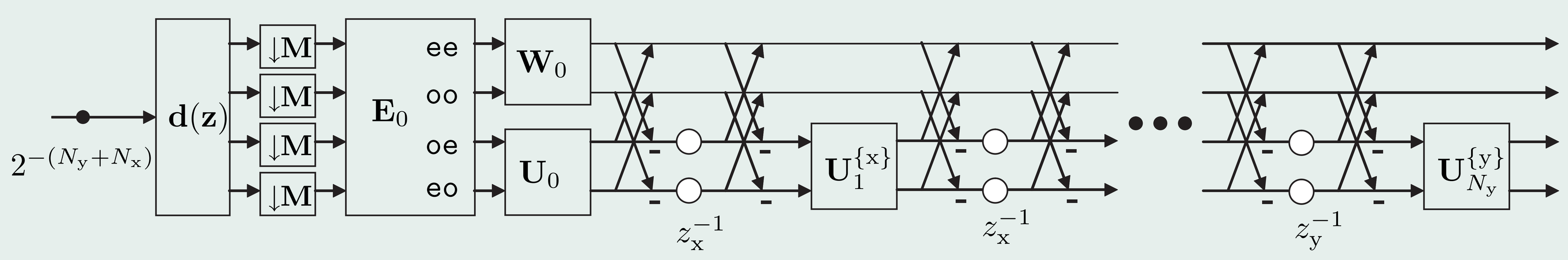
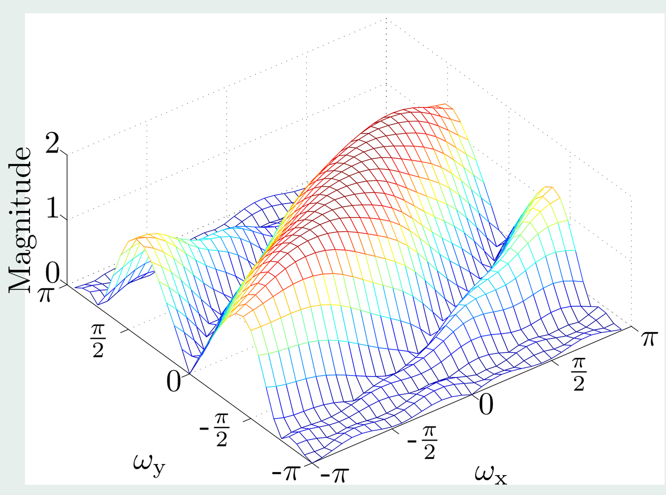
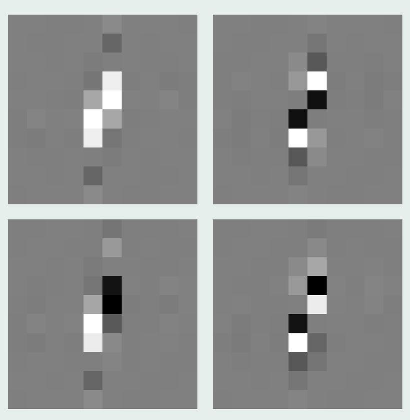


Figure: Lattice structure of a 2-D non-separable lapped orthogonal transform.




(a) $|H_0(e^{j\omega_y}, e^{j\omega_x})|$



(b) Basis images

Figure: A design example of $2 \times 2$-Ch. DirLOT with the 2-order trend vanishing moments (TVMs) of $\phi = \frac{\pi}{6}$, where $[N_y, N_x]^T = [4, 4]^T$, i.e. the basis size is $10 \times 10$.

- DirLOTs are 2-D non-separable lapped orthogonal transforms with directional characteristics [ICIP2010,PCS2010,APSIPA2010].
- Bases are symmetric and real-valued, and have compact-support.

## Union of Directional Symmetric Orthonormal DWTs (DirSOWTs)

- Suggestion of dictionary $\mathbf{D}$

$$\mathbf{D} = \left[\boldsymbol{\Phi}_{0\cup\frac{\pi}{2}}^T\ \boldsymbol{\Phi}_{\phi_1}^T\ \boldsymbol{\Phi}_{\phi_2}^T\ \boldsymbol{\Phi}_{\phi_3}^T \cdots \boldsymbol{\Phi}_{\phi_{K-1}}^T\right]$$

  - $\boldsymbol{\Phi}_{0\cup\frac{\pi}{2}}$: Nondirectional SOWT.
  - $\boldsymbol{\Phi}_{\phi}$: Directional SOWT for TVM direction $\phi$.
- $\mathbf{D}$ becomes a normalized tight frame and satisfies

$$\mathbf{D}\mathbf{D}^T = \sum_{k=0}^{K-1} \boldsymbol{\Phi}_k^T \boldsymbol{\Phi}_k = K\mathbf{I}$$

where $K$ is the redundancy.

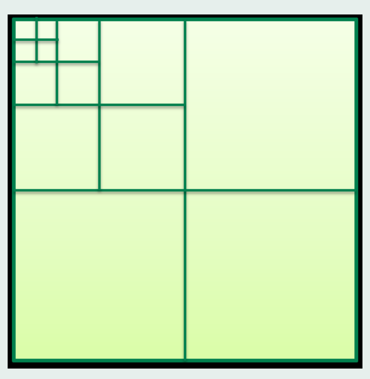


Figure: DWT decomposition

## Heuristic Shrinkage and BCR Iterative-shrinkage

- Heuristic shrinkage is available for tight frames and simply calculates

$$\hat{\mathbf{y}} = \frac{1}{K}\mathrm{Shrink}\left(\mathbf{D}^T\mathbf{x}\right)$$

- BCR Iterative-shrinkage Alg. is available for union of orthogonal transforms and iteratively solves

$$\hat{\mathbf{y}}_k = \arg\min_{\mathbf{y}_k} \frac{1}{2}\|(\tilde{\mathbf{y}}_k + \boldsymbol{\Phi}_k(\mathbf{x}-\tilde{\mathbf{x}})) - \mathbf{y}_k\|_2^2 + \boldsymbol{\lambda}_k^T|\mathbf{y}_k|$$

where $\tilde{\mathbf{y}}_k$ and $\tilde{\mathbf{x}}$ are the estimations of $\mathbf{y}_k$ and $\mathbf{x}^*$ in the previous step, respectively. $\mathbf{y}_k$, $\hat{\mathbf{y}}_k$ and $\boldsymbol{\lambda}_k$ are the $k$-th subvector of $\mathbf{y}$, $\hat{\mathbf{y}}$ and $\boldsymbol{\lambda}$, respectively.

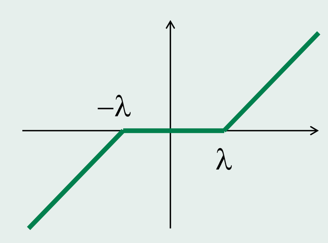



Figure: Soft-shrinkage function

## Experimental Results

Table: Adopted transforms and the features.

| Abrv. | Features | $K$ |
|---|---|---|
| DB5 | Daubechies' least asymmetric compactly-supported wavelet with five VMs, separable, orthonormal | 1 |
| SON4 | Symmetric orthonormal DWT with two VMs of $[N_y, N_x]^T = [4,4]^T$, nonseparable, nondirectional | 1 |
| NSCT | Nonsubsampled contourlet with $2^3, 2^3, 2^4, 2^4$ directions in the scales from coarser to finer, near tight, symmetric | 49 |
| UDN4 | Union of SON4 and DirSOWTs with two TVMs of $[N_y, N_x]^T = [4,4]^T$, multidirectional, tight, symmetric | 19 |

- UDN4: TVM direction $\phi_k \in \{-\frac{4\pi}{18}, -\frac{3\pi}{18}, -\frac{2\pi}{18}, \cdots \frac{13\pi}{18}\}$, $K = 1 + 18 = 19$
- Sweeping scalar $\boldsymbol{\lambda} = \lambda\mathbf{1}$

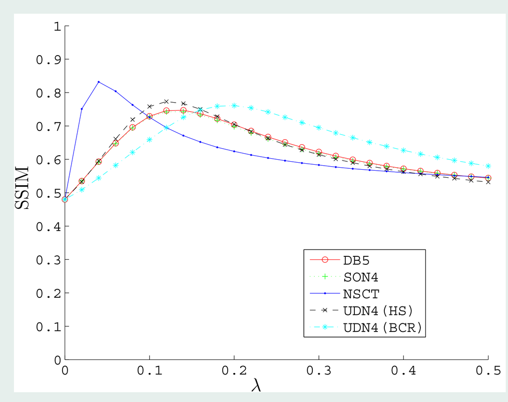
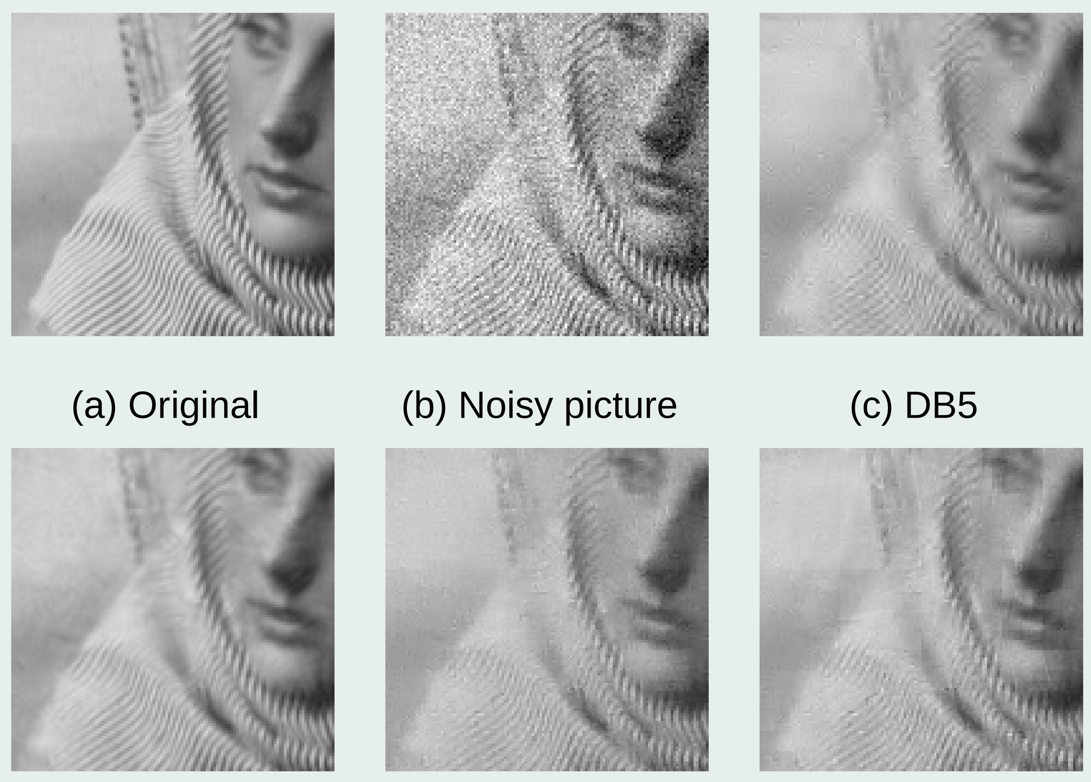



Figure: Denoising evaluation in terms of SSIM index for *"barbara"* with AWGN ($\sigma = 20$).



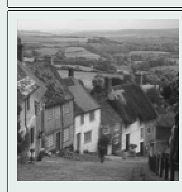
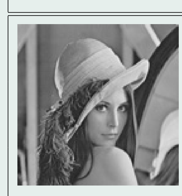
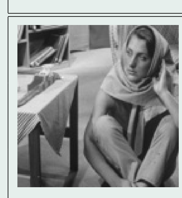
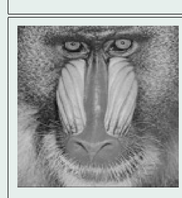
(a) Original (b) Noisy picture (c) DB5

(d) NSCT (e) UDN4(HS) (f) UDN4(BCR)

Figure: Denoising results. (a)Original picture, (b) Noisy picture with AWGN ($\sigma = 20$), (c)-(f) Denoised pictures.

- Adaptive determination of vector $\boldsymbol{\lambda}$

Table: Comparison of SSIM indexes among four transforms for various pictures and noise levels, where BayesShrink was used to determine the parameter $\boldsymbol{\lambda}$.

| | | $\sigma$ | DB5 | SON4 | NSCT | UDN4 | |
|---|---|---|---|---|---|---|---|
| | | | | | (HS) | (HS) | (BCR) |
|  | *goldhill* | 20 | 0.726 | 0.723 | 0.753 | 0.746 | 0.725 |
| | | 30 | 0.668 | 0.664 | 0.666 | 0.684 | 0.664 |
| | | 40 | 0.625 | 0.621 | 0.592 | 0.640 | 0.621 |
|  | *lena* | 20 | 0.794 | 0.793 | 0.780 | 0.814 | 0.792 |
| | | 30 | 0.749 | 0.748 | 0.693 | 0.768 | 0.748 |
| | | 40 | 0.719 | 0.720 | 0.616 | 0.736 | 0.720 |
|  | *barbara* | 20 | 0.748 | 0.760 | 0.767 | 0.778 | 0.750 |
| | | 30 | 0.680 | 0.671 | 0.687 | 0.681 | 0.685 |
| | | 40 | 0.626 | 0.608 | 0.619 | 0.623 | 0.633 |
|  | *baboon* | 20 | 0.668 | 0.714 | 0.757 | 0.698 | 0.730 |
| | | 30 | 0.558 | 0.606 | 0.653 | 0.591 | 0.613 |
| | | 40 | 0.486 | 0.515 | 0.566 | 0.505 | 0.523 |

## Conclusions

- Proposed to apply a union of DirLOTs to image denoising
- Perceptually preferable results were given with a simple computation
- Prospective performance was shown with iterative shrinkage Alg.
- Future works include the application to image restoration.

## Acknowledgment

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