Image Restoration with Union of Directional Orthonormal DWTs

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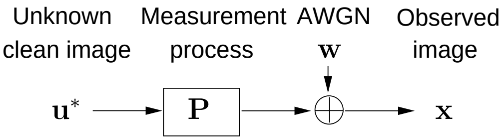
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Image Restoration with Sparse Representation

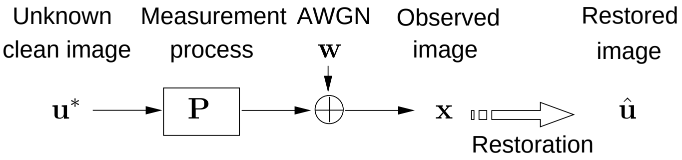
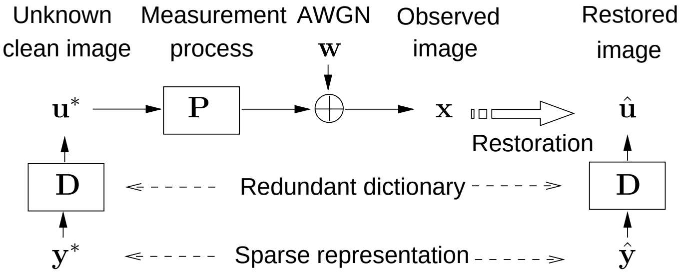


Image Restoration with Sparse Representation



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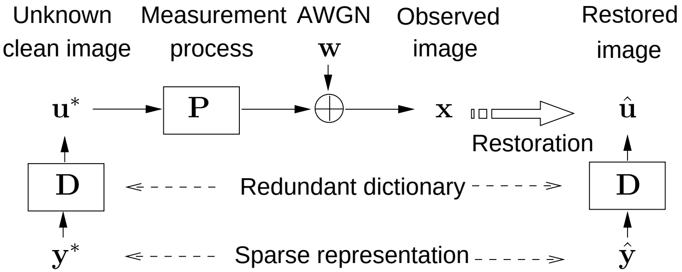
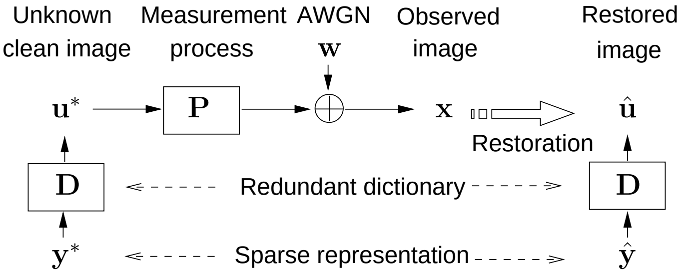


Assumption

$$\mathbf{x} = \mathbf{P}\mathbf{u}^* + \mathbf{w}$$
$$\mathbf{u}^* = \mathbf{D}\mathbf{y}^*$$



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$$\mathbf{u}^* = \mathbf{D}\mathbf{y}^*$$

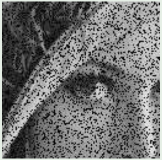
Example of Problem Setting

$$(Q_1^\lambda) \quad \hat{\mathbf{y}} = \arg\min_{\mathbf{y}} \|\mathbf{x} - \mathbf{P}\mathbf{D}\mathbf{y}\|_2^2 + \lambda\|\mathbf{y}\|_1$$
$$\hat{\mathbf{u}} = \mathbf{D}\hat{\mathbf{y}}$$

Selection of Dictionary

Selection of an appropriate dictionary is a KEY STEP in (Q_1^λ).

Original



Observed

ISTA: Iterative Shrinkage/Thresholding Algorithm

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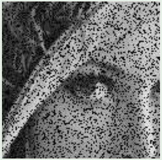
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NS-Haar WT

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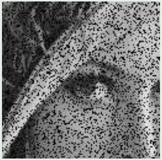
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Proposal

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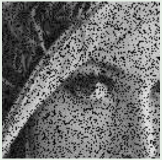

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What kind of properties are preferable?

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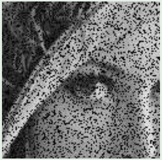

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What kind of properties are preferable?

- Redundant (sparse representation)

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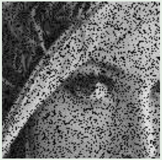

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Proposal

What kind of properties are preferable?

- Redundant (sparse representation)
- Overlapping (smoothness)

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Original　Observed　NS-Haar WT　Proposal

What kind of properties are preferable?

- Redundant (sparse representation)
- Overlapping (smoothness)
- Symmetric (anti-phase-distortion)

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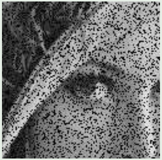

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Proposal

What kind of properties are preferable?

- Redundant (sparse representation)
- Overlapping (smoothness)
- Symmetric (anti-phase-distortion)
- Tight (energy preservation)

ISTA: Iterative Shrinkage/Thresholding Algorithm

As well, the DIRECTIONAL property is of interest.

Good approximation for diagonal edges and textures.

Real-valued and compact-support dictionaries

Dictionaries	Dir.	Red.	Ovl.	Sym.	Tight
DCT	No	No	No	Yes	Yes
Haar DWT	No	No	No	Yes	Yes
5-3/9-7 DWT	No	No	Yes	Yes	No
NS-DCT	No	Yes	Yes	Yes	Yes
NS-Haar DWT	No	Yes	Yes	Yes	Yes
DirSOWT	Yes	No	Yes	Yes	Yes
Contourlet	Yes	Yes	Yes	Restricted	
Union of DirSOWTs	Yes	Yes	Yes	Yes	Yes

Contourlet: [Do et al., IEEE TIP 2005]
DirSOWT: [Muramatsu et al., IEEE TIP 2012]

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Purpose

- A union of DirSOWTs was applied to image denoising as a redundant dictionary $\mathbf{D}$ and shown to be effective.

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  - S. Muramatsu *et al.*: "Directional lapped orthogonal transform: Theory and design," IEEE TIP, 2012.
  - S. Muramatsu *et al.*: "Image denoising with union of directional orthonormal DWTs," IEEE Proc. ICASSP, 2012.
  - S. Muramatsu *et al.*: "SURE-LET image denoising with multiple directional LOTs," Proc. PCS, 2012.

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Extend the application of a union of DirSOWTs to image restoration.

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Outline

Review of 2-D Directional LOT

- A DirSOWT, directional symmetric orthonormal WT, is a hierarchical tree construction of a directional LOT (DirLOT).

S. Muramatsu et al.: "Directional lapped orthogonal transform: Theory and design," IEEE TIP, *May 2012.*

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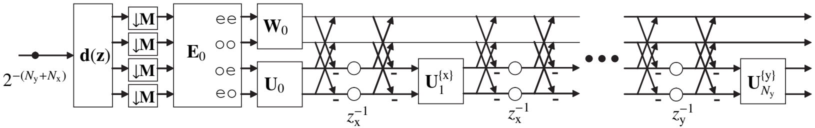
  - Capable to hold trend vanishing moments (TVMs)
- Constructed by a lattice structure



  - $\mathbf{d}(\mathbf{z})$ is a 2-D delay chain.
  - $\mathbf{E}_0$ is a symmetric orthonormal matrix given by the 2-D DCT.
  - $\mathbf{W}_0$, $\mathbf{U}_0$ and $\mathbf{U}_{n_d}^{\{d\}}$ are parameter orthonormal matrices.

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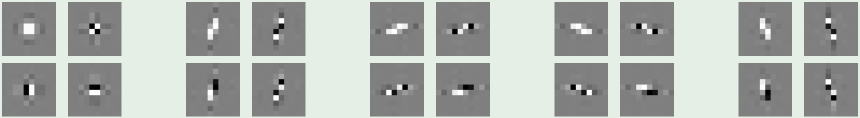
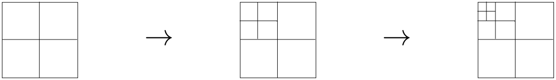
Construction of Redundant Dictionary

Examples of 2×2-decomposition NS GenLOT/DirLOT bases



Iterative decomposition yields SOWT



$$\mathbf{D} = \begin{bmatrix} \mathbf{\Phi}^T_{0\cup\frac{\pi}{2}} & \mathbf{\Phi}^T_{\phi_1} & \mathbf{\Phi}^T_{\phi_2} & \mathbf{\Phi}^T_{\phi_3} & \cdots & \mathbf{\Phi}^T_{\phi_{K-1}} \end{bmatrix}$$

- $\mathbf{\Phi}_{0\cup\frac{\pi}{2}}$ is a nondirectional SOWT with the classical two-order VMs.
- $\mathbf{\Phi}_{\phi}$ is a DirSOWT with the two-order TVMs for the direction $\mathbf{u}_\phi = (\sin\phi, \cos\phi)^T$.

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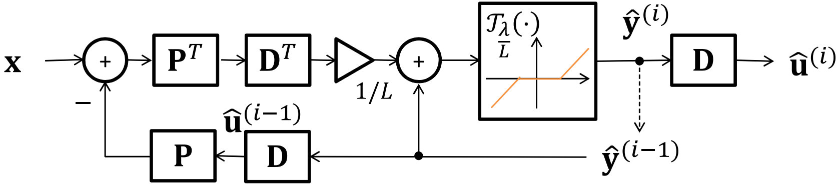
ISTA with a Tight Frame

- ISTA solves (Q_1^λ) exactly.



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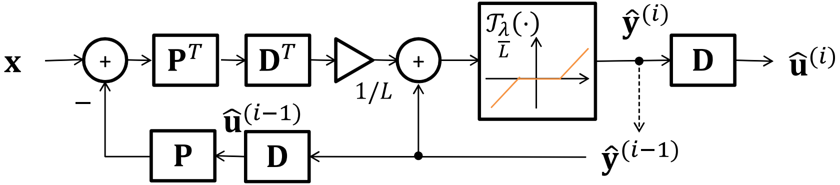

- If $\mathbf{D}$ is a tight frame and $\mathbf{D}\mathbf{D}^T = K\mathbf{I}$

$$\Rightarrow L = K\lambda_{\max}(\mathbf{D}^T\mathbf{P}^T\mathbf{P}\mathbf{D}) = K\lambda_{\max}(\mathbf{P}^T\mathbf{P})$$

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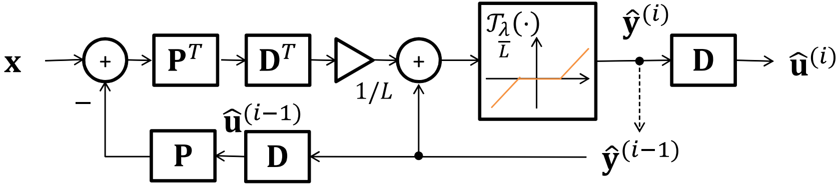

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- Linear measurement process $\mathbf{P}$ is pluggable and applicable to
  - Deblurring
  - Super resolution
  - Inpainting and so on

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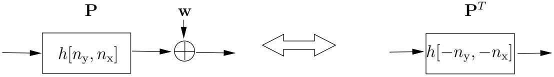
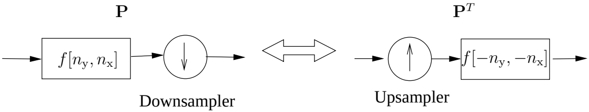
Examples of Measurement Processes

Relation between measurement process and its adjoint

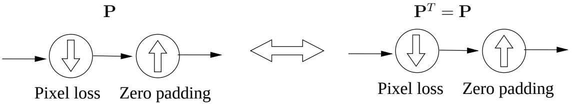
- Deblurring



- Super resolution



- Inpainting



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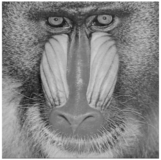
Original Pictures and Adopted Transforms

goldhill

lena

barbara



baboon

Original pictures $\mathbf{u}^*$ of size 512×512, 8-bit grayscale.

Adopted dictionaries

Abrv.	Features
NSHT	Two-level non-subsampled Haar DWT, separable, tight, nondirectional
UDN4	Union of six-level SOWT and DirSOWTs with two TVMs of $[N_{\mathrm{y}}, N_{\mathrm{x}}]^T = [4, 4]^T$, non separable, tight, multidirectional

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Simulation Results in SSIM - Deblurring

Observed

Wiener

NSHT($K = 7$)

UDN4($K = 5$)

Partial results of deblurring for "*lena*."

Picture	Wiener	NSHT(λ)	UDN4(λ)
goldhill	0.633	0.720 (0.0007)	**0.724** (0.0026)
lena	0.666	0.796 (0.0006)	**0.820** (0.0054)
barbara	0.543	0.654 (0.0007)	**0.667** (0.0043)
baboon	0.517	**0.529** (0.0005)	0.528 (0.0000)

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Simulation Results in SSIM - Super Resolution

Observed

Bicubic

NSHT($K = 7$)

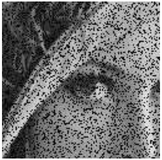
UDN4($K = 5$)

Partial results of super resolution for "*lena*."

Picture	Bicubic	NSHT(λ)	UDN4(λ)
goldhill	0.682[1]	**0.767** (0.0003)	0.759 (0.0004)
lena	0.802	**0.859** (0.0003)	0.854 (0.0004)
barbara	0.646	**0.701** (0.0004)	0.696 (0.0004)
baboon	0.433	**0.551** (0.0002)	0.544 (0.0003)

[1] Wrongly typed in the proceedings

Simulation Results in SSIM - Inpainting



Observed

Median

NSHT($K = 7$)

UDN4($K = 5$)

Partial results of inpainting for "*lena*."

Picture	Median	NSHT(λ)	UDN4(λ)
goldhill	0.632	0.609 (0.0318)	**0.931** (0.0197)
lena	0.655	0.560 (0.0385)	**0.945** (0.0206)
barbara	0.602	0.647 (0.0300)	**0.944** (0.0220)
baboon	0.522	0.638 (0.0324)	**0.907** (0.0251)

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Conclusions

- A novel image restoration technique was proposed by introducing a union of DirSOWTs.
- The significance is verified through the application to
  - Deblurring
  - Super resolution and
  - Inpainting
- The proposed dictionary is shown to be superior to or comparable with the non-subsampled Haar WT.

Thank you very much for your attention!

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