SURE-LET Image Denoising with Multiple Directional LOTs

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ABSTRACT

- A novel non-iterative image denoising technique is proposed by using directional lapped orthogonal transforms (DirLOTs).
- A redundant transform is constructed with multiple DirLOTs and applied to the SURE-LET image denoising.
- The denoising performance is evaluated for several images and compared with the traditional single orthonormal wavelet approach.

Key words– DirLOT, Wavelet shrinkage, SURE-LET image denoising

Introduction

- A common problem of image denoising, i.e. removal of additive white Gaussian noise (AWGN) from an observed image is dealt with.
- Purpose is to find a good candidate image $\hat{\mathbf{x}}$ of $\mathbf{x}$ only from $\mathbf{v}$.

$$\underbrace{\mathbf{v}}_{\text{Observed image}} = \underbrace{\mathbf{x}}_{\text{Original image}} + \underbrace{\mathbf{w}}_{\text{White Gaussian noise}}$$

- Popular denoising approaches include wavelet shrinkage techniques [e.g., Donoho and Johnstone, 1994].
- Luisier *et al.* proposed a linear optimization technique to determine the shape of shrinkage function [Luisier *et al.*, 2007].
  - Referred to as the SURE-LET approach, which minimizes the Stein's unbiased risk estimator (SURE) with linear expansion of thresholds (LET).
- Main issue of the SURE-LET approach is to improve the quality for diagonal edges and textures.
- In this work, we suggest to introduce a redundant transform:

$$\boxed{\text{Union of Directional Symmetric Orthonormal DWTs}}$$

Review of SURE-LET Image Denoising [Luisier *et al.*, IEEE TIP 2007]

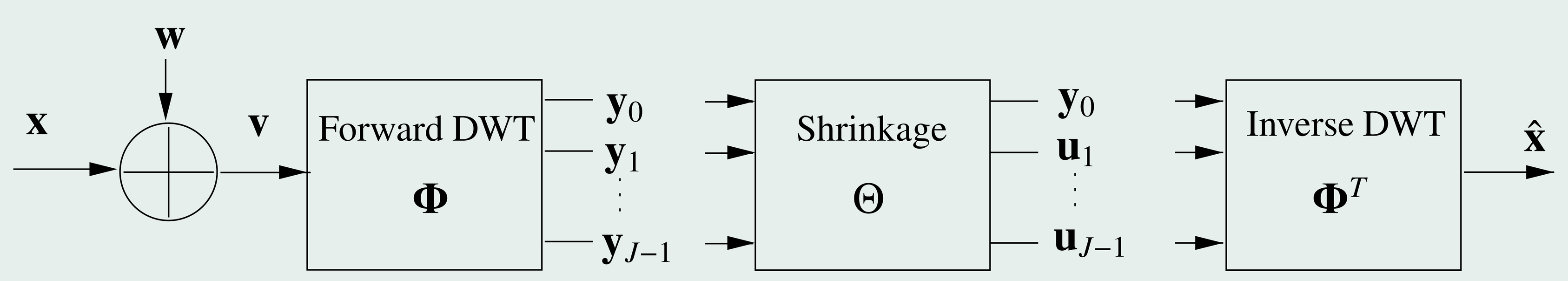


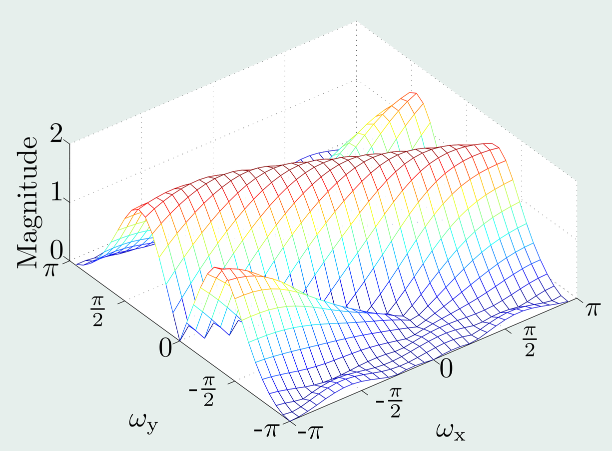
Figure: Principal of orthonormal wavelet denoising, where $\mathbf{w}$ is an AWGN.

- The denoising quality depends on the choice of the orthonormal transform $\mathbf{\Phi}$ and the shrinkage function $\Theta(\cdot)$, where $\hat{\mathbf{x}} = \mathbf{\Phi}^T\Theta(\mathbf{\Phi}\mathbf{v})$.
- The SURE-LET approach is a technique to realize function $\Theta(\cdot)$, which avoids any a priori hypotheses on the noiseless picture $\mathbf{x}$.
- Function $\Theta(\cdot)$ is point-wisely defined and completely characterized by a set of parameters a_k and b_k:

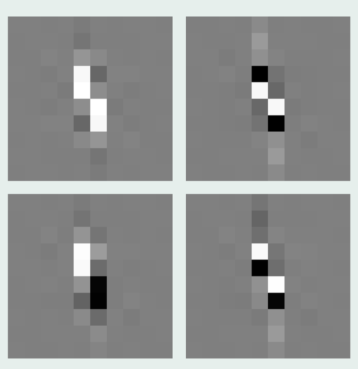
$$\theta(y, y_p; \mathbf{a}, \mathbf{b}) = e^{-\frac{y_p^2}{12\sigma^2}} \sum_{k=1}^{K} a_k y e^{-(k-1)\frac{y^2}{12\sigma^2}} + \left(1 - e^{-\frac{y_p^2}{12\sigma^2}}\right) \sum_{k=1}^{K} b_k y e^{-(k-1)\frac{y^2}{12\sigma^2}},$$

where y and y_p are a wavelet coefficient and interscale prediction of y obtained from the wavelet parent-child relationship, respectively. ($K = 2$ and $T = \sqrt{6}\sigma$ are suggested.)
- The parameters a_k and b_k are linearly solved for minimizing SURE.

Review of DirLOTs [Muramatsu *et al.*, IEEE TIP 2012]



(a) $|H_0(e^{j\omega_y}, e^{j\omega_x})|$



(b) Basis images

Figure: A design example of 2×2-Ch. DirLOT with the 2-order trend vanishing moments (TVMs) of $\phi = \frac{\pi}{6}$, where $[N_y, N_x]^T = [4, 4]^T$, i.e. the basis size is 10×10.

- DirLOTs are 2-D non-separable lapped orthogonal transforms with directional characteristics.
- Bases are symmetric and real-valued, and have compact-support.
- DirLOTs were shown to improve the performance of SURE-LET image denoising for diagonal edges and textures when the direction is KNOWN and FIXED [Muramatsu *et al.*, APSIPA 2011].

Image Denoising with Multiple DirLOTs

- For pictures with rich amount of geometrical structures, a single DirLOT does not work well.
- To improve the performance, we propose to introduce a redundant transform:

$$\mathbf{T} = \left[\mathbf{\Phi}^T_{0\cup\frac{\pi}{2}}\ \mathbf{\Phi}^T_{\phi_1}\ \mathbf{\Phi}^T_{\phi_2}\ \mathbf{\Phi}^T_{\phi_3} \cdots \mathbf{\Phi}^T_{\phi_{R-1}}\right]^T$$

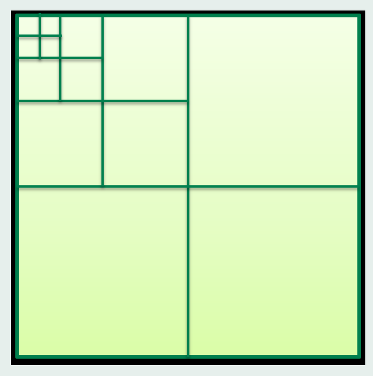


Figure: DWT decomposition

  - $\mathbf{\Phi}_{0\cup\frac{\pi}{2}}$: Nondirectional SOWT.
  - $\mathbf{\Phi}_{\phi}$: Directional SOWT for TVM direction ϕ.
- $\mathbf{T}^T$ constitutes a normalized tight frame and satisfies

$$\mathbf{T}^T\mathbf{T} = \sum_{k=0}^{R-1} \mathbf{\Phi}_k^T\mathbf{\Phi}_k = R\mathbf{I}$$

where R is the redundancy.
- Heuristic shrinkage is available and simply realized by

$$\hat{\mathbf{x}} = \frac{1}{R}\mathbf{T}^T\Theta(\mathbf{T}\mathbf{v}) = \frac{1}{R}\sum_{k=0}^{R-1}\mathbf{\Phi}_k^T\Theta(\mathbf{\Phi}_k\mathbf{v})$$

Experimental Results

- Denoising results for 8-bit grayscale pictures with AWGN ($\sigma = 30$).
  - Sym5 and SON4 denote Symlets of index 5 and symmetric orthonormal WT with the classical two-order VMs, respectively.
  - UDN4 denotes a union of DirLOTs with the two-order TVMs, where the TVM angles are set to $\phi_k \in \{-\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, \frac{4\pi}{6}\}$ ($R = 1 + 6 = 7$).
  - The number of hierarchical levels is set to five for all transforms.

(a) Original (b) Noisy picture (c) Sym5 (d) SON4 (e) UDN4

(f) Original (g) Noisy picture (h) Sym5 (i) SON4 (j) UDN4

- Comparison of PSNRs and SSIM indexes among three transforms.

Test pictures of size 512×512	σ	PSNR			SSIM		
		SYM5	SON4	UDN4	SYM5	SON4	UDN4
goldhill	20	29.40	29.33	29.58	0.760	0.758	0.767
	30	27.84	27.79	27.99	0.699	0.696	0.707
	40	26.74	26.71	26.86	0.653	0.651	0.662
lena	20	31.26	31.26	31.69	0.835	0.832	0.844
	30	29.48	29.48	29.83	0.794	0.790	0.804
	40	28.17	28.16	28.43	0.759	0.754	0.770
barbara	20	27.76	27.53	28.00	0.794	0.786	0.808
	30	25.75	25.63	25.92	0.714	0.707	0.733
	40	24.45	24.39	24.58	0.652	0.646	0.674
baboon	20	25.49	25.40	25.44	0.743	0.742	0.734
	30	23.68	23.61	23.66	0.654	0.651	0.647
	40	22.54	22.48	22.49	0.577	0.573	0.569

Conclusions

- Proposed to apply multiple DirLOTs to SURE-LET image denoising
- Performance is improved from the single transform approach.
- Further improvement is required for fine textures.
- Future works include the extention to color image denoising.

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