Congestion control in cell-free massive MIMO

$C_i > 0$: LTE

$C_i = 0$: W

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Price: c — 1 LTE: DR: S_L

Free: — K APs: DR: S_W

N: users

1 System Models and Optimization Problem Formulation

Money: C_i

We consider a system architecture for coexistence and spectrum sharing between Wi-Fi and LTE networks. In which, one LTE base station (BS) and K APs are deployed to provide services in the considered area. There are N users in this area requesting for accessing network. A user can select providers between LTE BS and WiFi APs. In detail, if the user selects LTE BS, the i-th user must pay some price per unit of data rate, denote as c. In contrast, the user can select WiFi for free. However, the satisfaction coefficient per unit of the data rate of LTE, S_L, is higher than that of WiFi, S_W. We assume that the i-th user has a particular affordability, C_i, representing the amount of money that the user is willing to pay. Therefore, the user only can access WiFi if the affordability is zero. Mathematically, we define the AP-user association variable by x_{ik}

$$x_{ik} = \begin{cases} 1 & \text{If user } i \text{ selects the } k\text{-th WiFi AP;} \\ 0 & \text{otherwise.} \end{cases}$$

Denote x_{iL} as the binary variable indicating the association between the i-th user and the LTE BS

$$x_{nL} = \begin{cases} 1 & \text{If user } i \text{ selects LTE BS;} \\ 0 & \text{otherwise.} \end{cases}$$

Since a user can select only one AP or the LTE BS, thus

$$x_{iL} + \sum_{k=1}^{K} x_{ik} = 1 \tag{1}$$

The unlicensed spectrum is divided into F non-overlap sub-bands to allocate for both WiFi and LTE. Assume that the number of users is larger than the number of sub-bands. Thus, a sub-band can be used for multiple users. The received signal at the k-th AP on the f-th sub-band is calculated as:

$N > F$

$$y_{kf} = \sum_{i=1}^{N} \psi_{if} x_{kn} \sqrt{P_i} s_i + i_{kf} \tag{2}$$

Similarly, the received signal at the LTE BS at the f-th sub-band is given by:

$$y_{Lf} = \sum_{i=1}^{N} \psi_{if} x_{Li} \sqrt{P_i} s_i + n_{Lf}, \tag{3}$$

Accordingly, the uplink signal-interference-noise-ratio (SINR) of the i-th user is calculated as

$$\mathrm{SINR}_i(\mathbf{p},\mathbf{X},\mathbf{\Psi}) = \sum_{f=1}^{F} \psi_{if} \frac{x_{iL}P_i h_{iL} + \sum_{k=1}^{K} x_{ik}P_n|h_{ik}|^2}{\sum\limits_{i'\neq i}^{N} \psi_{i'f}(x_{iL}P_{i'}|h_{i'L}|^2 + \sum_{k=1}^{K} x_{ik}P_{i'}|h_{i'k}|^2) + \sigma^2}, \tag{4}$$

The achievable rate of the i-th user is calculated as

$$r_i(\mathbf{p},\mathbf{X},\mathbf{\Psi}) = \log_2\left(1 + \mathrm{SINR}_i(\mathbf{p},\mathbf{X},\mathbf{\Psi})\right), \tag{5}$$

The satisfaction of the n-th user is given by

$$L_i(\mathbf{p},\mathbf{X},\mathbf{\Psi}) = (x_{iL}S_L + \sum_{k=1}^{K} x_{ik}S_W)r_i, \tag{6}$$

where $\mathbf{p} = [P_1, \ldots, P_N]^T \in \mathbb{R}^N$, $\mathbf{X} = [\mathbf{x}_{iL}, \mathbf{x}_{i1}, \ldots, \mathbf{x}_{iK}]$

In the considered network, a centralized server is utilized to calculate the resource allocation based on the gathered information consisting of the users' requirements and network resources. The centralized server calculates the best solution for the association (LTE or WiFi association), power allocation, and spectrum allocation.

1.1 Problem Formulation

1.1.1 The total satisfaction maximization problem

$$\max_{\mathbf{p},\mathbf{X},\mathbf{\Psi}} \quad \sum_{i=1}^{N} L_i(\mathbf{p},\mathbf{X},\mathbf{\Psi}) \tag{7a}$$

$$\text{s.t.} \quad P_i \leq P_i^{\max}, \forall i \tag{7b}$$

$$x_{iL} c r_i(\mathbf{p},\mathbf{X},\mathbf{\Psi}) \leq C_i, \forall i \tag{7c}$$

$$x_{iL} + \sum_{k=1}^{K} x_{ik} = 1, \forall i \tag{7d}$$

$$\sum_{f}^{F} \psi_{if} \leq 1, \forall i \tag{7e}$$

$$L_i(\mathbf{p},\mathbf{X},\mathbf{\Psi}) \geq L_i^{\mathrm{thr}}, \forall i \tag{7f}$$

1.1.2 The dual-objective problem for the power allocation and the AP association under infeasible circumstances

$$
\begin{aligned}
\max_{\mathbf{p},\mathbf{X},\mathbf{\Psi}} \quad & [|\mathcal{Q}(\mathbf{p},\mathbf{X},\mathbf{\Psi})|, \sum_{i=1}^{N} L_i(\mathbf{p},\mathbf{X},\mathbf{\Psi})] && \text{(8a)}\\
\text{s.t.} \quad & P_i \le P_i^{\max}, \forall i && \text{(8b)}\\
& x_{iL} c r_i(\mathbf{p},\mathbf{X},\mathbf{\Psi}) \le C_i, \forall i && \text{(8c)}\\
& x_{iL} + \sum_{k=1}^{K} x_{ik} = 1, \forall i && \text{(8d)}\\
& \sum_{f}^{F} \psi_{if} \le 1, \forall i && \text{(8e)}\\
& L_i \ge L_i^{\text{thr}}, \forall i \in \mathcal{Q}(\mathbf{p},\mathbf{X},\mathbf{\Psi}) && \text{(8f)}
\end{aligned}
$$

$$
\begin{aligned}
\max_{\mathbf{p},\ \mathbf{X},\ \mathbf{\Psi},\ \mathbf{q}} \quad & \sum_{i=1}^{N} L_i(\mathbf{p},\mathbf{X},\mathbf{\Psi}) && \text{(9a)}\\
\text{s.t.} \quad & P_i \le P_i^{\max}, \quad \forall i && \text{(9b)}\\
& x_{iL} \cdot c \cdot r_i(\mathbf{p},\mathbf{X},\mathbf{\Psi}) \le C_i, \quad \forall i && \text{(9c)}\\
& x_{iL} + \sum_{k=1}^{K} x_{ik} = 1, \quad \forall i && \text{(9d)}\\
& \sum_{f=1}^{F} \psi_{if} \le 1, \quad \forall i && \text{(9e)}\\
& L_i(\mathbf{p},\mathbf{X},\mathbf{\Psi}) \ge L_i^{\text{thr}} \cdot q_i, \quad \forall i && \text{(9f)}\\
& \sum_{i=1}^{N} q_i \ge \varepsilon && \text{(9g)}\\
& q_i \in \{0,1\}, \quad \forall i && \text{(9h)}
\end{aligned}
$$

2 Proposed Solution Methodology

To solve the dual-objective problem defined in (8), we adopt the ε-constraint method to generate the Pareto frontier. Specifically, we fix the number of satisfied users $\varepsilon \in \{1, 2, \dots, N\}$ and solve a sequence of single-objective problems maximizing the total satisfaction $\sum_i L_i$, while ensuring that at least ε users meet their satisfaction threshold. The reformulated problem is given in (33).

2.1 Overall Framework

We propose an alternating optimization (AO) algorithm, which iteratively updates the power allocation, user association, and frequency selection until convergence. For each fixed ε, the algorithm proceeds as follows:

1. **Initialize** user association $\mathbf{X}$, frequency allocation $\boldsymbol{\Psi}$, and satisfaction indicators $\mathbf{q}$.

2. **Repeat until convergence:**

   (a) **Power allocation step:** Fix $\mathbf{X}$, $\boldsymbol{\Psi}$, and $\mathbf{q}$. Solve a convex optimization problem to determine optimal power values $\mathbf{p}$ that maximize the total satisfaction:

   $$\max_{\mathbf{p}} \sum_{i=1}^{N} L_i = \left(x_{iL} S_L + \sum_{k=1}^{K} x_{ik} S_W \right) \cdot \log_2 \left(1 + \mathrm{SINR}_i(\mathbf{p})\right)$$

   subject to:

   - Power budget: $P_i \leq P_i^{\max}$.
   - Affordability: $x_{iL} \cdot c \cdot r_i(\mathbf{p}) \leq C_i$.
   - Satisfaction: $L_i(\mathbf{p}) \geq L_i^{\mathrm{thr}} \cdot q_i$.

   This is solved using Sequential Convex Approximation (SCA) or WMMSE-based transformations.

   (b) **Association step:** Fix $\mathbf{p}$. Solve for $\mathbf{X}$ by relaxing $x_{ik} \in \{0,1\}$ to $x_{ik} \in [0,1]$ with the constraint $\sum_k x_{ik} + x_{iL} = 1$. After solving the relaxed problem, each user selects the provider with the highest value:

   $$x_{ik^*} = 1, \quad \text{where } k^* = \arg\max_k x_{ik}$$

   (c) **Frequency allocation step:** For each user i, select the sub-band f with the lowest estimated interference. Ensure $\sum_{f=1}^{F} \psi_{if} \leq 1$.

   (d) **Update satisfaction indicators:** For each user i, set:

   $$q_i = \begin{cases} 1, & \text{if } L_i \geq L_i^{\mathrm{thr}} \\ 0, & \text{otherwise} \end{cases}$$

   If $\sum_i q_i < \varepsilon$, reduce ε or adjust association.

3. **Store the result:** Record $(\varepsilon, \sum_i L_i)$ as one point on the Pareto frontier.

4. **Repeat for next** ε.

2.1.1 Power Allocation Step via Sequential Convex Approximation

Given fixed user association $\mathbf{X}$, frequency allocation $\boldsymbol{\Psi}$, and satisfaction indicators $\mathbf{q}$, we aim to optimize the transmit power $\mathbf{p} = [P_1, \ldots, P_N]^T$ to maximize the total user satisfaction.

SINR Model. The effective channel gain from user i to its associated access point is:

$$\bar{h}_{ii} = x_{iL} h_{iL} + \sum_{k=1}^{K} x_{ik} h_{ik}, \tag{10}$$

where h_{iL} is the channel from user i to the LTE BS, and h_{ik} is the channel to the k-th Wi-Fi AP.

The interference coefficient from user j to the receiver of user i is defined as:

$$\bar{h}_{ij} = \sum_{f=1}^{F} \psi_{if} \psi_{jf} \left(x_{iL} h_{jL} + \sum_{k=1}^{K} x_{ik} h_{jk} \right), \tag{11}$$

which captures co-channel interference due to sub-band overlap and AP association.

Then, the SINR of user i is:

$$\mathrm{SINR}_i(\mathbf{p}) = \frac{P_i \bar{h}_{ii}}{\sum_{j \neq i} P_j \bar{h}_{ij} + \sigma^2}. \tag{12}$$

Let us define the satisfaction weight:

$$\alpha_i = x_{iL} S_L + \sum_{k=1}^{K} x_{ik} S_W,$$

and the rate as:

$$r_i(\mathbf{p}) = \log_2 \left(1 + \mathrm{SINR}_i(\mathbf{p})\right).$$

Objective. The power allocation subproblem becomes:

$$\max_{\mathbf{p}} \sum_{i=1}^{N} \alpha_i \cdot \log_2 \left(1 + \frac{P_i \bar{h}_{ii}}{\sum_{j \neq i} P_j \bar{h}_{ij} + \sigma^2} \right), \tag{13}$$

which is non-convex due to the SINR expression in the logarithm.

Convex Reformulation via SCA. We introduce auxiliary variables v_i and u_i such that:

$$v_i = \frac{P_i \bar{h}_{ii}}{\sum_{j \neq i} P_j \bar{h}_{ij} + \sigma^2}, \tag{14}$$

$$u_i = \log_2(1 + v_i). \tag{15}$$

Now, the objective becomes:

$$\max_{\mathbf{p}, \mathbf{v}, \mathbf{u}} \sum_{i=1}^{N} \alpha_i u_i.$$

The constraint:

$$v_i \leq \frac{P_i \bar{h}_{ii}}{\sum_{j \neq i} P_j \bar{h}_{ij} + \sigma^2}$$

is non-convex. We rewrite it as:

$$P_i \bar{h}_{ii} \geq v_i \cdot \left(\sum_{j \neq i} P_j \bar{h}_{ij} + \sigma^2 \right). \tag{16}$$

This is bilinear and non-convex. We approximate the right-hand side using the first-order Taylor expansion around $(\mathbf{p}^{(t)}, v_i^{(t)})$ at iteration t.

First-order Taylor Approximation. Let:

$$f_i(\mathbf{p}, v_i) = v_i \cdot \left(\sum_{j \neq i} P_j \bar{h}_{ij} + \sigma^2 \right).$$

Then, the linearized approximation at $(\mathbf{p}^{(t)}, v_i^{(t)})$ is:

$$\begin{aligned} \widehat{I}_i^{(t)}(v_i, \mathbf{p}) &= f_i(\mathbf{p}^{(t)}, v_i^{(t)}) + \nabla_{\mathbf{p}} f_i^{(t)} \cdot (\mathbf{p} - \mathbf{p}^{(t)}) + \nabla_{v_i} f_i^{(t)} \cdot (v_i - v_i^{(t)}) && (17) \\ &= v_i^{(t)} \cdot \left(\sum_{j \neq i} P_j^{(t)} \bar{h}_{ij} + \sigma^2 \right) + \sum_{j \neq i} \left(v_i^{(t)} \bar{h}_{ij} \cdot (P_j - P_j^{(t)}) \right) + \left(\sum_{j \neq i} P_j^{(t)} \bar{h}_{ij} + \sigma^2 \right) \cdot (v_i - v_i^{(t)}). && (18) \end{aligned}$$

Final Convex Problem (Per Iteration t) At iteration t, solve the following convex program:

$$\begin{aligned} \max_{\mathbf{p}, \mathbf{v}, \mathbf{u}} \quad & \sum_{i=1}^{N} \alpha_i u_i && (19a) \\ \text{s.t.} \quad & \log_2(1 + v_i) \geq u_i, \quad \forall i && (19b) \\ & P_i \bar{h}_{ii} \geq \widehat{I}_i^{(t)}(v_i, \mathbf{p}), \quad \forall i && (19c) \\ & P_i \leq P_i^{\max}, \quad \forall i && (19d) \\ & x_{iL} \cdot c \cdot \log_2(1 + v_i) \leq C_i, \quad \forall i && (19e) \\ & \alpha_i \cdot \log_2(1 + v_i) \geq L_i^{\text{thr}} \cdot q_i, \quad \forall i && (19f) \end{aligned}$$

This convex problem can be efficiently solved using convex solvers such as MOSEK or CVXPY.

2.1.2 Mixed-Integer Convex Programming (MICP) Formulation

To obtain a near-optimal solution for the joint problem, we formulate it as a Mixed-Integer Convex Program (MICP). This formulation integrates both discrete decision variables (e.g., user association and frequency assignment) and continuous optimization (power control). It allows global or near-global solutions using branch-and-bound techniques.

Decision Variables

- $x_{iL} \in \{0, 1\}$: binary variable indicating user i is associated with LTE.
- $x_{ik} \in \{0, 1\}$: binary variable indicating user i is associated with AP k.
- $\psi_{if} \in \{0, 1\}$: binary variable indicating user i uses sub-band f.
- $q_i \in \{0, 1\}$: binary variable indicating user i is active (QoS enforced).
- $P_i \in [0, P_i^{\max}]$: continuous variable for user i's transmit power.

Auxiliary Variables We define:

$$\bar{h}_i = x_{iL} h_{iL} + \sum_{k=1}^{K} x_{ik} h_{ik}, \quad \bar{h}_{ii'} = x_{iL} h_{i'L} + \sum_{k=1}^{K} x_{ik} h_{i'k} \tag{20}$$

$$S_i = S_W \sum_{k=1}^{K} x_{ik} + S_L x_{iL} \tag{21}$$

SINR Expression The SINR of user i is given by:

$$\mathrm{SINR}_i = \sum_{f=1}^{F} \psi_{if} \cdot \frac{P_i \bar{h}_i}{\sum_{i' \neq i} \psi_{i'f} P_{i'} \bar{h}_{ii'} + \sigma^2} \tag{22}$$

Achievable Rate and Satisfaction

$$r_i = \log_2(1 + \mathrm{SINR}_i) \tag{23}$$

$$L_i = S_i \cdot r_i \tag{24}$$

Objective Function We adopt an ϵ-constraint approach to handle the dual objectives. For a given threshold ϵ, the primary objective is to maximize total user satisfaction:

$$\max_{\mathbf{X}, \mathbf{\Psi}, \mathbf{p}} \quad \sum_{i=1}^{N} L_i \tag{25}$$

subject to at least ϵ users being satisfied:

$$\sum_{i=1}^{N} q_i \geq \epsilon$$

Constraints

1. **Unique association constraint:**

$$x_{iL} + \sum_{k=1}^{K} x_{ik} = 1, \quad \forall i \tag{26}$$

2. **Single sub-band per user:**

$$\sum_{f=1}^{F} \psi_{if} \leq 1, \quad \forall i \tag{27}$$

3. **Affordability constraint:**

$$x_{iL} \cdot c \cdot r_i \leq C_i, \quad \forall i \tag{28}$$

4. **QoS threshold for active users:**

$$L_i \geq L_i^{\mathrm{thr}} \cdot q_i, \quad \forall i \tag{29}$$

5. **Transmit power budget:**

$$0 \leq P_i \leq P_i^{\max}, \quad \forall i \tag{30}$$

6. **Binary constraints:**

$$x_{iL}, x_{ik}, \psi_{if}, q_i \in \{0, 1\}, \quad \forall i, k, f \tag{31}$$

Solution Method The problem in (**??**) is a valid MICP with convex substructure. It can be solved using a branch-and-bound solver such as `ECOS_BB`, `MOSEK`, or `Gurobi` (via CVXPY or YALMIP). For small networks (e.g., $N \leq 10$), this yields globally optimal or near-optimal solutions that can be used as benchmarks to evaluate scalable heuristics or approximations.

2.1.3 Relax-and-Round Convex Approximation

To improve scalability, we relax the binary variables to continuous ones in $[0, 1]$ and apply a post-rounding procedure.

We define:

- $x_{iL}, x_{ik}, \psi_{if}, q_i \in [0, 1]$ (relaxed variables),
- All other variables and constraints remain unchanged.

We introduce an entropy regularization to promote sparsity:

$$\mathcal{R} = -\sum_{i=1}^{N} \left(x_{iL} \log x_{iL} + \sum_{k=1}^{K} x_{ik} \log x_{ik} + \sum_{f=1}^{F} \psi_{if} \log \psi_{if} \right)$$

The relaxed problem becomes:

$$\max \lambda \sum_{i=1}^{N} q_i + (1 - \lambda) \sum_{i=1}^{N} L_i(\mathbf{p}, \mathbf{X}, \mathbf{\Psi}) - \tau \cdot \mathcal{R} \tag{32}$$

where $\tau > 0$ controls the regularization weight.

After solving the relaxed problem, we round the results:

- $x_{iL} \leftarrow 1$ if $x_{iL} > \max_k x_{ik}$, else $x_{ik^*} = 1$ where $k^* = \arg\max_k x_{ik}$,
- $\psi_{if^*} = 1$ where $f^* = \arg\max_f \psi_{if}$,
- $q_i = 1$ if $q_i > 0.5$, else 0.

This method scales well and achieves near-integer solutions while maintaining tractability.

2.1.4 Evolutionary Metaheuristic: GA + Convex Inner Loop

To address the mixed-integer and highly non-convex nature of the problem, we adopt a genetic algorithm (GA) with an embedded convex power optimization.

Chromosome Encoding: Each solution (chromosome) encodes:

$$\text{Chromosome } = [x_{iL}, x_{ik}, \psi_{if}, q_i] \in \{0, 1\}^{(K+2)N}$$

Fitness Evaluation: For each chromosome:

1. Fix $x_{ik}, x_{iL}, \psi_{if}, q_i$.
2. Solve the power allocation subproblem via Sequential Convex Approximation (SCA) to obtain $\mathbf{p}$.
3. Compute:

$$\text{Fitness} = \left(|\mathcal{Q}|,\ \sum_{i=1}^{N} L_i(\mathbf{p}, \mathbf{X}, \mathbf{\Psi}) \right)$$

Algorithm 1 SCA-Based Power Allocation (Given $\mathbf{X}$, $\mathbf{\Psi}$, $\mathbf{q}$)

Input: Initial power vector $\mathbf{p}^{(0)}$, tolerance δ
for $t = 1$ to max_iter **do**
  Compute $\bar{h}_{ii}$ and $\bar{h}_{ij}$ from $\mathbf{X}$, $\mathbf{\Psi}$
  Evaluate SINR and $v_i^{(t-1)}$ from $\mathbf{p}^{(t-1)}$
  Build linearized approximation $\widehat{I}_i^{(t)}(v_i, \mathbf{p})$
  Solve the convex program (19) to update $\mathbf{p}^{(t)}$
  if $|\mathbf{p}^{(t)} - \mathbf{p}^{(t-1)}| < \delta$ **then**
    break
  end if
end for
Output: Optimal power allocation $\mathbf{p}^*$

Evolution Process: GA proceeds with:

- Tournament selection
- Uniform or one-point crossover
- Mutation with constraint repair (e.g., fix invalid associations)
- Pareto sorting using NSGA-II to maintain diverse non-dominated solutions

This approach balances global exploration of discrete variables with convex precision in the continuous domain. It provides a rich approximation of the Pareto frontier for dual-objective performance.

2.2 Mathematical Formulation for Fixed ε

The ε-constrained formulation is given by:

$$\max_{\mathbf{p},\ \mathbf{X},\ \mathbf{\Psi},\ \mathbf{q}} \quad \sum_{i=1}^{N} L_i(\mathbf{p}, \mathbf{X}, \mathbf{\Psi}) \tag{33a}$$

$$\text{s.t.} \quad P_i \le P_i^{\max}, \quad \forall i \tag{33b}$$

$$x_{iL} \cdot c \cdot r_i(\mathbf{p}, \mathbf{X}, \mathbf{\Psi}) \le C_i, \quad \forall i \tag{33c}$$

$$x_{iL} + \sum_{k=1}^{K} x_{ik} = 1, \quad \forall i \tag{33d}$$

$$\sum_{f=1}^{F} \psi_{if} \le 1, \quad \forall i \tag{33e}$$

$$L_i(\mathbf{p}, \mathbf{X}, \mathbf{\Psi}) \ge L_i^{\text{thr}} \cdot q_i, \quad \forall i \tag{33f}$$

$$\sum_{i=1}^{N} q_i \ge \varepsilon \tag{33g}$$

$$q_i \in \{0, 1\}, \quad \forall i \tag{33h}$$

2.3 Pareto Frontier Construction

By solving the above problem for $\varepsilon = 1, 2, \ldots, N$, we obtain a set of Pareto-optimal points. Each point represents a trade-off between the number of satisfied users and the total satisfaction. The resulting frontier provides a principled way for network designers to balance fairness and overall efficiency based on deployment goals.