Assignment 3

Veton Abazovic — Michael Belmont

April 23, 2019

Question 1

Consider the following Bayesian network, where variables A through E are all Boolean valued. Note: there is a typo in the image, it should be P(A = true) = 0.2 instead of P(D = true) = 0.2.

A)

$P(A) * P(B) * P(C) * P(D \mid A, B) * P(E \mid B, C) = .2 * .5 * .8 * .1 * .3 = 0.0024$

B)

$P(\neg A, \neg B, \neg C, \neg D, \neg E) = \neg P(A) * \neg P(B) * \neg P(C) * \neg P(D \mid A, B) \neg P(E \mid B, C)$
= 0.8 * 0.5 * 0.2 * 0.1 * 0.8 = 0.0064

C)

$$P(\neg A \mid B, C, D, E) = \frac{P(\neg A) * P(B) * P(C) * P(D \mid \neg A, B) * P(E \mid B, C)}{P(A, B, C, D, E) + P(\neg A) * P(B) * P(C) * P(D \mid \neg A, B) * P(E \mid B, C)}$$
$$= \frac{0.8 * 0.5 * 0.8 * 0.6 * 0.3}{0.0024 + 0.8 * 0.5 * 0.8 * 0.6 * 0.3} = \frac{0.0576}{0.06}$$
$= 0.96$

Question 2

A)

$P(B), P(E), P(A \mid B, E), P(J \mid A), P(M \mid A)$
$\sum_B P(B) * \sum_E P(E) * \sum_A P(A \mid B, E) * P(J \mid A) * P(M \mid A)$
$F1(B) * \sum_E F2(E) * \sum_A F3(A, B, E) * F3(A) * F4(A)$
F1(B) = P(B) = $\binom{P(b)}{P(\neg b)} = \binom{0.9}{0.05}$
F2(E) = P(E) = $\binom{P(e)}{P(\neg e)} = \binom{0.002}{0.998}$
F4(A) = $P(J \mid A) = \binom{P(j|a)}{P(j|\neg a)} = \binom{0.90}{0.05}$
F5(A) = $P(M \mid A) = \binom{P(m|a)}{P(m|\neg a)} = \binom{0.70}{0.01}$
F3(A,B,E) = $P(A \mid B, E) = \begin{pmatrix} P(a|b,e) & P(a|b,\neg e) \\ P(a|\neg b,e) & P(a|\neg b,\neg e) \end{pmatrix} = \begin{pmatrix} 0.95 & 0.94 \\ 0.29 & 0.001 \end{pmatrix}$
F3($\neg$A,B,E) = $P(\neg A \mid B, E) = \begin{pmatrix} P(\neg a|b,e) & P(\neg a|b,\neg e) \\ P(\neg a|\neg b,e) & P(\neg a|\neg b,\neg e) \end{pmatrix} = \begin{pmatrix} 0.05 & 0.06 \\ 0.71 & 0.999 \end{pmatrix}$
F6(B,E) = $\sum_A P(A \mid B, E) * P(J \mid A) P(M \mid A)$
F6(B,E) = F3(A,B,E) * F4(A) * F5(A)
F6(B,E) = F3(A,B,E) * F4(A) * F5(A) + F3(A,B,E) * F4(A) * F5(A)
F7(B) = F2(B) * F6(B,E) + F2(B) * F6(B,E)
$P(B \mid J, M) = \propto * F1(B) * F7(B)$
F1(B)*F7(B) = $\binom{0.00059224}{0.001491857}$
$\propto$= 1/(0.00059224 + 0.001491857) = 479.8233
F1(B) * F7(B)(Normalizing with $\propto$) = $\binom{0.2841718}{0.7158281}$

B)

Enumeration = $O(n2^{n-2})$
Variable Elimination = O(n)

Question 3

1)

First we find the probability the rover is in a hot location.

Table 1: Probability of Rover, Hot

X_1	$\mathbf{P}(X_1 \parallel E_1 = \textbf{Hot})$
A	1.0
B	0.0
C D E F	0.0

Then we find where P($X_2 \parallel E_1$ = Hot, E_2 = Cold)

Table 2: Probability of Rover, Hot Cold

X_2	$\mathbf{P}(X_2 \parallel E_1 = \textbf{Hot}, E_2 = \textbf{Cold})$
A	0.0
B	1.0
C D E F	0.0

Then we find the X_3 value and normalize it.

Table 3: Probability of Rover, Hot Cold Cold

X_3	$\mathbf{P}(X_3 \parallel E_1 = \textbf{Hot}, E_2 = \textbf{Cold}, E_3 = \textbf{Cold})$
A	0.0
B	0.2
C	0.8
D E F	0.0

2)

The probability of position at the end of day 2 has to be B because if day 1 is hot, it must stay at A, and if day 2 is cold and it moves a mile a day, it must be at B.

Table 4: Probability of Position at End of Day 2

X_2	$\mathbf{P}(X_2 \parallel Hot_1, Cold_1, Cold_2)$
A	0.0
B	1.0
C D E F	0.0

3)

The most likely sequence of positions would be day 1 is A, day 2 is B, day 3 is B.

4)

Because we already know hot_1, $cold_2$, and $cold_3$, we know that we have to be at C so we could have hot_4. The probability of being hot_4 would be 0.64 * 0.8, as we would have to be at C (0.64) to move to D and there is a 0.8 chance of moving to D instead of staying at C, so 0.8 times 0.64 is equal to 0.512.

Next, we have hot_5. We have a 0.2 chance of staying at D instead of moving to E, so it is 0.2 times 0.512, or 0.1024.

Finally, to move to $cold_6$, we have a 0.8 chance. So 0.8 times 0.1024 would be 0.08192, or a 8.192% chance of this sequence.

5)

We start with the following probability distribution.

Table 5: Probability of Rover, Hot Cold Cold

X_3	$\mathbf{P}(X_3 \parallel E_1 = \textbf{Hot}, E_2 = \textbf{Cold}, E_3 = \textbf{Cold})$
A	0.0
B	0.2
C	0.8
D E F	0.0

Then we have the possibility of either moving to D or staying at C. If we're still at B on day 3, then we could either stay at B or move to C. If we're at C, then we could either stay at C or move to D. So we find the probability of location on day 4 as:

Table 6: Probability of Rover, Day 4 Unknown

X_4	$\mathbf{P}(X_3 \parallel E_1 = \textbf{Hot}, E_2 = \textbf{Cold}, E_3 = \textbf{Cold}, E_4 = \textbf{Unknown})$
A	0.0
B	0.04
C	0.32
D	0.64
E F	0.0

Moving on to day 5, we have a similar situation. We could stay at any of the locations or move forward.

Table 7: Probability of Rover, Day 5 Unknown

X_5	$\mathbf{P}(X_3 \parallel E_1 = \textbf{Hot}, E_2 = \textbf{Cold}, E_3 = \textbf{Cold}, E_4, E_5 = \textbf{Unknown})$
A	0.0
B	0.008
C	0.096
D	0.384
E	0.512
F	0.0

Question 4

a

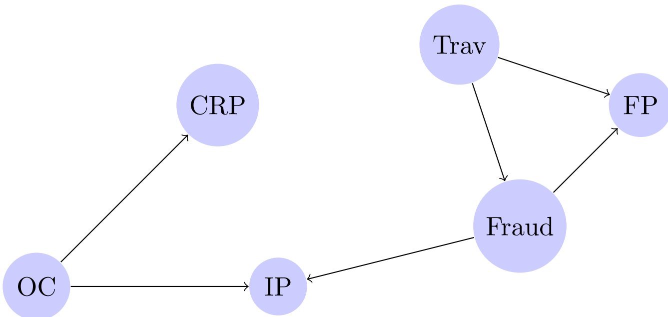

Table 8: OC

True	False
0.75	0.25

Table 9: CRP

OC	True	False
True	0.10	0.90
False	0.001	0.999

Table 10: Trav

True	False
0.05	0.95

Table 11: Fraud

Trav	True	False
True	0.01	0.99
False	0.004	0.996

Table 12: FP

Trav	Fraud	True	False
True	True	0.90	0.10
True	False	0.10	0.90
False	True	0.90	0.10
False	False	0.01	0.99

b

P(Fraud = true $\|$ CRP = false, IP = false, FP = false, Trav = false) OR
P(Fraud = true $\|$ CRP = false, IP = false, FP = false, Trav = true)
= 0.01 + 0.004 = 0.0104 or 1.04%

Table 13: IP

OC	Fraud	True	False
True	True	0.02	0.98
True	False	0.01	0.99
False	True	0.011	0.989
False	False	0.001	0.999

P(Fraud = true ‖ FP = true, IP = false, CRP = true)
= 0.9 * (0.10 * 0.02) = 0.0018 or 0.18%