Let us write $G(2,4) = \{$2-dimensional (vector) subspaces of $\mathbb{R}^4\}$ A plane $V \subseteq \mathbb{R}^4$ is specified by 4×2 matrix $A = [a_1, a_2] \in Mat_{4\times2}$ where $\{a_1, a_2\}$ is a basis for V. I.e., given $A \in Mat_{4\times2}$, $rkA = 2$, we get a 2-plane $V \subseteq \mathbb{R}^4$ by $V =$span $\{a1, a2\}$. Conversely, given any 2-dim subspace $V \subseteq \mathbb{R}^4$, there is a 4×2 matrix A with $rk(A) = 2$, from which V is obtained in the above way. Two matrices, A and B, determine the same subspace $V \iff \exists g \in GL_2(\mathbb{R})$, such that $B = Ag$.
We thus have the following setup. Let

$$F(2,4) = \{A|rkA = 2\} \subseteq Mat_{2\times4}(\mathbb{R}) \simeq \mathbb{R}^8$$

and consider on it the equivalence relation.

$$B \sim A \text{ if } \exists g \in GL_2(\mathbb{R}), \text{ s.t. } B = Ag$$

We have described a bijection of sets

$$F(2,4)/\sim \simeq G(2,4)$$

In this problem we argue that $G(2,4)$ can be equipped with a natural smooth structure.

Bibliography