We will show that S^2 is a smooth manifold. Let's denote a sphere as:

$$M = S^2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1^2 + x_2^2 + x_3^2 = 1\}$$

And let's descrbe the atlas covering the sphere as:

$$A = \{(u_i^{\pm}, \varphi_i^{\pm}) : i \in I\}, \quad I = \{1, 2, 3\}$$

Also, we define a disk with a center at (x_0, y_0) and radius ϵ as:

$$D_\epsilon(x_0, y_0) = \{(x, y) \in \mathbb{R}^2 : (x - x_0)^2 + (y - y_0)^2 < \epsilon^2\}$$

Thus we can cover the sphere with 6 charts and 6 functions, described as follows:

$$u_i^+ = S^2 \cap \{x_i > 0\}, \quad u_i^- = S^2 \cap \{x_i < 0\}$$

$$\varphi_i^+ : u_i^+ \to D_1(0,0), \quad \varphi(x_1, x_2, x_3) = (\dots \hat{x_i} \dots)$$

$$\varphi_i^- : u_i^- \to D_1(0,0), \quad \varphi(x_1, x_2, x_3) = (\dots \hat{x_i} \dots)$$

For example $\varphi_1^+ : u_i^+ \to \mathbb{R}^2, \quad \varphi_1^+(x_1, x_2, x_3) = (\hat{x_1}, x_2, x_3) = (x_2, x_3)$
We first show that a function $\varphi_3^+ : u_3^+ \to \mathbb{R}$ is injective. Assume $\exists P_1$ and P_2 s.t. $P_1 = (p_1, p_2, p_3)$, and $P_2 = (p_1\prime, p_2\prime, p_3\prime)$, $P_1 \neq P2$, and $f(P_1) = f(P_2)$ Since u_3^+ has only positive numbers for a third coordinate $p_3 = p_3\prime \implies P_1 = P_2$, and we can conclude that φ_3^+ is injective.
To show that $\varphi_i^{\pm}$ is injective, we similarly assume that $\exists P_1 and P_2$, $P_1 \neq P_2$ and $\varphi_i^{\pm}(P_1) = \varphi_i^{\pm}(P_2)$. Let $K = \{1, 2, 3\}/i$. Then $\forall k, a_k = a_k\prime \quad a_i = \sqrt{\sum(a_k^2)} = a_i\prime$. Thus $P_1 = P_2$, and $\varphi_i^{\pm}$ is injective.
To prove surjectivity, consider $(\varphi_i^{\pm})^{-1}$ componentwise. It sends an arbitary point $(b_1, b_2) \in \mathbb{R}^2$ to a point $(a_1, a_2, a_3) \in S^2$, or componentwise

$$a_j = \begin{cases} b_j & j < i \\ \sqrt{\sum_{m=1}^{2} b_m^2} & j = i \\ b_{j-1} & j > i \end{cases}$$

Since each component in both S^2 and the disk varies from 0 to 1, each point in the codomain is mapped to, and we have surjectivity for $\varphi_i^{\pm}$

Bibliography