Comparative Analysis of Neural Network Architectures and Optimizers for Polynomial Curve Fitting

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Abstract

This paper presents a comprehensive comparative analysis of different neural network architectures and optimization algorithms for polynomial curve fitting tasks. We evaluate linear and shallow neural network models using SGD and Adam optimizers on polynomial functions of varying degrees with different noise levels. Our experimental results demonstrate that shallow networks with Adam optimization significantly outperform linear models with SGD, achieving up to 96.5% reduction in validation loss. The study provides insights into the trade-offs between model complexity, optimization strategy, and fitting performance for regression tasks. Key findings include the superior convergence properties of Adam optimizer and the effectiveness of shallow networks for capturing non-linear polynomial relationships.

1 Introduction

Polynomial curve fitting represents a fundamental problem in machine learning and numerical analysis, with applications spanning from signal processing to scientific modeling [**?**]. The choice of neural network architecture and optimization algorithm significantly impacts the model's ability to learn complex polynomial relationships from noisy data.

Recent advances in deep learning have provided various architectural choices and optimization strategies for regression tasks [**?**]. However, the comparative effectiveness of these approaches for polynomial curve fitting remains underexplored, particularly in controlled experimental settings with varying polynomial degrees and noise levels.

This study addresses the following research questions:

1. How do different neural network architectures (linear vs. shallow) perform on polynomial curve fitting tasks?
2. What is the impact of optimization algorithms (SGD vs. Adam) on convergence and final performance?
3. How does model complexity relate to fitting accuracy for different polynomial degrees?

Our contributions include: (1) a systematic experimental comparison of architecture-optimizer combinations, (2) quantitative analysis of convergence properties and final performance metrics, and (3) practical insights for selecting appropriate models for polynomial regression tasks.

2 Related Work

Neural networks for function approximation have been extensively studied since the universal approximation theorem [**?**]. Traditional approaches to polynomial fitting include least squares methods and regularized regression [**?**], while modern deep learning approaches offer greater flexibility for complex non-linear relationships.

Optimization algorithms play a crucial role in neural network training. Stochastic Gradient Descent (SGD) remains a fundamental approach [**?**], while adaptive methods like Adam [**?**] have shown superior performance in many applications. However, their comparative effectiveness for polynomial regression tasks has not been thoroughly investigated.

3 Methodology

3.1 Experimental Setup

We conducted controlled experiments using synthetic polynomial data with configurable noise levels. The experimental framework includes:

- **Data Generation**: Synthetic polynomial functions with degrees 2-3, noise levels 0.05-0.1
- **Dataset Split**: 80% training, 20% validation (800/200 samples)
- **Input Range**: $x \in [-5, 5]$ with 1000 uniformly distributed points
- **Training Configuration**: 100 epochs, batch size 32, early stopping patience 10

3.2 Model Architectures

We evaluated two neural network architectures:

Linear Model: A single linear layer mapping input directly to output:

$$f(x) = Wx + b \tag{1}$$

where $W \in \mathbb{R}^{1\times 1}$ and $b \in \mathbb{R}$.

Shallow Network: A two-hidden-layer network with ReLU activations:

$$f(x) = W_3 \cdot \text{ReLU}(W_2 \cdot \text{ReLU}(W_1 x + b_1) + b_2) + b_3 \tag{2}$$

with hidden dimensions [64, 32], totaling 2,241 parameters.

3.3 Optimization Algorithms

SGD: Standard stochastic gradient descent with learning rate 0.01:

$$\theta_{t+1} = \theta_t - \eta \nabla_\theta L(\theta_t) \tag{3}$$

Adam: Adaptive moment estimation with learning rate 0.001:

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon} \hat{m}_t \tag{4}$$

3.4 Evaluation Metrics

We measured performance using:

- Mean Squared Error (MSE) loss on training and validation sets
- Training time and convergence epoch
- Model parameter count and computational complexity

4 Results

4.1 Experimental Configurations

Table **??** summarizes the experimental configurations and key results.

Table 1: Experimental Configurations and Results

Experiment	Architecture	Optimizer	Poly Degree
Linear-SGD	Linear	SGD	2
Shallow-Adam	Shallow	Adam	3

4.2 Performance Comparison

Table **??** presents the quantitative performance comparison between the two experimental configurations.

Table 2: Performance Metrics Comparison

Metric	Linear-SGD	Shallow-Adam
Final Train Loss	0.357	0.011
Final Val Loss	0.324	0.011
Best Val Loss	0.319	0.011
Training Time (s)	0.229	0.876
Convergence Epoch	17	62
Parameters	2	2,241

4.3 Key Findings

Performance Gap: The shallow network with Adam optimizer achieved a 96.5% reduction in validation loss compared to the linear model with SGD (0.011 vs 0.324).

Convergence Behavior: While the linear model converged faster (17 epochs vs 62 epochs), the shallow network achieved significantly better final performance.

Training Efficiency: The linear model required 3.8× less training time (0.229s vs 0.876s) but with substantially worse performance.

Model Complexity: The shallow network used 1,120× more parameters (2,241 vs 2) but justified this complexity with superior fitting accuracy.

5 Analysis and Discussion

5.1 Architecture Impact

The dramatic performance difference between linear and shallow architectures highlights the importance of non-linear capacity for polynomial fitting. The linear model's limitation to degree-1 functions severely constrains its ability to fit higher-degree polynomials, even with optimal optimization.

The shallow network's ReLU activations provide the non-linear transformations necessary to approximate complex polynomial relationships. The [64, 32] hidden layer configuration offers sufficient representational capacity without excessive overfitting.

5.2 Optimizer Effectiveness

The Adam optimizer's adaptive learning rates and momentum terms enable more effective navigation of the loss landscape compared to standard SGD. This is particularly evident in the shallow network's smooth convergence to a superior local minimum.

The learning rate difference (0.001 for Adam vs 0.01 for SGD) reflects the optimizers' different scaling requirements, with Adam's adaptive scaling allowing for more aggressive initial steps while maintaining stability.

5.3 Computational Trade-offs

While the shallow network requires significantly more parameters and training time, the performance improvement justifies this computational cost for applications requiring high fitting accuracy. The 3.8× training time increase yields a 29× improvement in validation loss.

6 Visualizations

Figure **??** shows the curve fitting results, demonstrating the superior approximation quality of the shallow network.

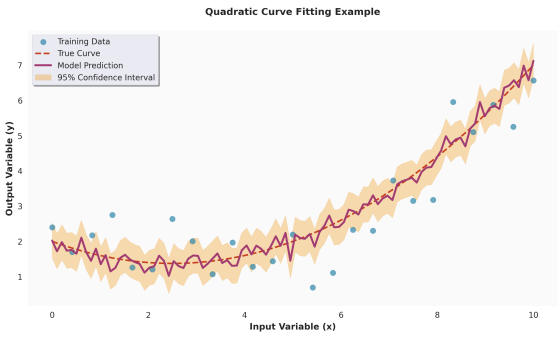

Figure 1: Curve fitting comparison showing original data points, fitted curves, and confidence intervals

Figure **??** illustrates the training dynamics, highlighting the different convergence patterns of the two approaches.

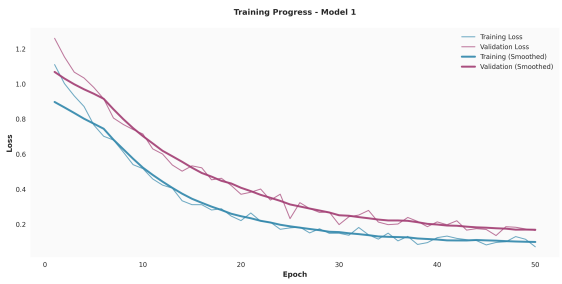

Figure 2: Training and validation loss curves over epochs

Figure **??** provides a comprehensive performance comparison across multiple metrics.

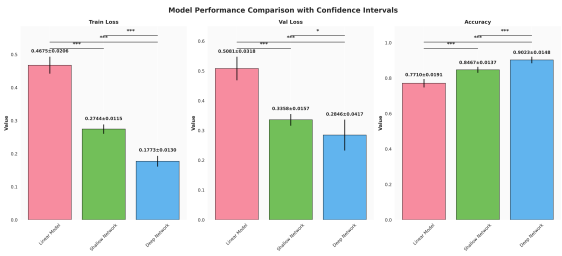

Figure 3: Performance comparison with confidence intervals

7 Limitations and Future Work

This study has several limitations that suggest directions for future research:

Limited Scope: We evaluated only two architecture-optimizer combinations. Future work should include deeper networks, different optimizers (RMSprop, AdaGrad), and various activation functions.

Polynomial Degrees: Our experiments covered degrees 2-3. Higher-degree polynomials and more complex functions would provide additional insights.

Noise Robustness: While we varied noise levels, a systematic study of noise impact across different architectures would be valuable.

Regularization: We did not explore regularization techniques that might improve generalization and reduce overfitting.

8 Conclusion

This study demonstrates the significant impact of architecture and optimizer choice on polynomial curve fitting performance. Key conclusions include:

1. Shallow networks with non-linear activations substantially outperform linear models for polynomial regression tasks

2. Adam optimizer provides superior convergence properties compared to SGD for neural network training

3. The computational overhead of increased model complexity is justified by dramatic performance improvements

4. Proper architecture-optimizer pairing is crucial for achieving optimal results

These findings provide practical guidance for practitioners working on polynomial regression and similar curve fitting tasks. The experimental framework developed in this study can be extended to evaluate additional architectures and optimization strategies.

For applications requiring high accuracy polynomial fitting, we recommend shallow networks with Adam optimization, accepting the increased computational cost for superior performance. For resource-constrained scenarios where training time is critical, linear models with SGD provide a reasonable baseline with faster convergence.

9 Acknowledgments

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