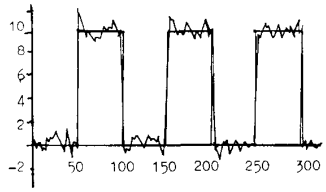

Fig. 1. System output and reference input for the adaptive scheme of [8].

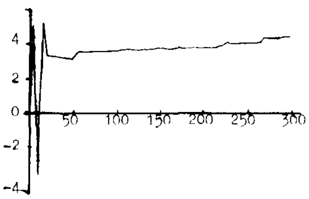

Fig. 2. Variation of Q for the adaptive scheme of [8].

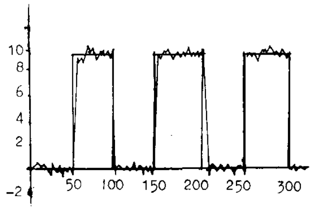

Fig. 3. System output and reference input for the indirect scheme proposed.

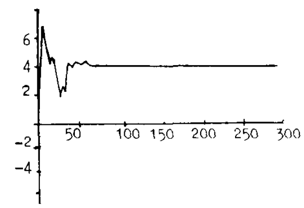

Fig. 4. Variation of Q for the indirect scheme presented.

The aim is to have the closed-loop poles defined by $T = 5(1 - 0 \cdot 5z^{-1})$. For this known system, values of $P = p_0 = 1$, $Q = q_0 = 4$ provide the required pole location. Set $R = P$.

The reference signal $w(t)$ is a rectangular wave with amplitude 10 and a half period 50. The initial values are all zero except $c_1(0) = -1$, $P = 1$, and $R = 1$. Fig. 1 shows the system output reference signal and Fig. 2 shows the manner in which the weighting polynomial Q approaches the required value when the generalized self-tuning controller of [8] is used. In Figs. 3 and 4 the results of the same simulation are presented using the indirect adaptive algorithm. Obviously, Q converges to its final value much more slowly when the algorithm of [8] is applied than in the case when the indirect scheme is used. The performance of the indirect algorithm is satisfactory.

VI. Conclusions

The indirect adaptive control scheme presented not only minimizes a performance-cost function but also achieves pole assignment. It adopts the technique of on-line choice of weighting polynomials, so that no *a priori* knowledge of system parameters is required and the difficulty of *a priori* choice of weighting polynomials is overcome. This scheme also has the advantages that much less parameters to be estimated and memory space are required when the system time delay is large. Since updating of weighting polynomials is separated from the control law calculation, the weighting polynomials converge to their final value much faster. It is shown that under certain conditions this indirect algorithm is stable and convergent even for nonminimum phase systems.

References

[1] G. C. Goodwin, P. J. Ramadge, and P. E. Caines, "Discrete stochastic adaptive control," *SIAM J. Contr. Optimiz.*, vol. 19, pp. 829–853, 1981.

[2] G. C. Goodwin, K. S. Sin, and K. K. Saluja, "Stochastic adaptive control and prediction: The general-delay-colored noise case," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 946–950, 1980.

[3] J. J. Fuchs, "Indirect stochastic adaptive control: The general delay-white noise case," *IEEE Trans. Automat. Contr.*, vol. AC-27, pp. 219–223, 1982.

[4] —, "Indirect stochastic adaptive control: The general delay-colored noise case," *IEEE Trans. Automat. Contr.*, vol. AC-27, pp. 470–472, 1982.

[5] D. W. Clarke and P. J. Gawthrop, "Self-tuning controller," *Proc. IEE*, vol. 122, pp. 929–934, 1975.

[6] C. A. Tsiligiannis and S. A. Svoronos, "Deterministic convergence of a Clarke-Gawthrop self-tuning controller," *Automatica*, vol. 22, pp. 193–197, 1986.

[7] T. Y. Chai, "Globally convergent self-tuning controllers," *Int. J. Contr.*, vol. 48, no. 2, pp. 417–434, 1988.

[8] A. Y. Allidina and F. M. Hughes, "Generalized self-tuning controller with pole assignment," *Proc. IEE*, vol. 127, pp. 13–18, 1980.

[9] T. Y. Chai, S. J. Lang, and X. Y. Gu, "A generalized self-tuning feedforward controller and multivariable application," in *Proc. 24th IEEE Conf. Decision Contr.*, Fort Lauderdale, FL, 1985.

[10] G. C. Goodwin and K. S. Sin, "Adaptive control of nonminimum phase systems," *IEEE Trans. Automat. Contr.*, vol. AC-26, pp. 478–483, 1981.

[11] M. A. Hersh and M. Zarrop, "Stochastic adaptive control of nonminimum phase systems," *Optimiz. Contr. Appl. Meth.*, vol. 7, pp. 153–161, 1986.

[12] G. C. Goodwin and K. S. Sin, *Adaptive Filtering, Prediction and Control*. Englewood Cliffs, NJ: Prentice-Hall, 1984.

Adaptive LQG Regulator via the Separation Principle

HIROAKI MORIMOTO

Abstract—**The continuous-time linear quadratic adaptive control problem with average cost per unit time is studied. It is shown that a consistent estimator of an unknown parameter and the optimal control can be obtained via the separation principle.**

I. Introduction

We are concerned with the continuous-time adaptive LQG regulator problem defined as follows. The evolution of the state is described by the linear stochastic differential equation with an unknown parameter taking values in a finite set. The cost function is given by the quadratic average form per unit time. The aim of adaptive control is finding the optimal control minimizing the cost, together with the estimation of the unknown parameter. This problem without estimation is studied in [4], [5], and [10]. Also, the asymptotic behavior of the estimator is discussed by [2] and [8] without the optimal control problem.

In the present note, we give a precise solution to the adaptive control problem by the separation principle, which decomposes it into two parts: i) to find the consistent estimator in the case without controls; and ii) to solve the Riccati equation in the LQ regulator problem with the parameter estimate replacing the unknown parameter. We use the strong law of large numbers for local martingales [7] to show the optimality over the set U of admissible controls, which satisfies weaker conditions than those of [9]. While our formulation is more restrictive than some results in the literature [3], [4], we can prove the consistency of the estimator, obtain an explicit representation of the optimal control, and a method of finding its minimum. We refer to [3] and [6] for the discrete-time adaptive control problem.

Manuscript received July 20, 1988; revised October 25, 1988, January 17, 1989, and January 24, 1989.

The author is with the Department of Mathematics, Faculty of General Education, Ehime University, Matsuyama 790, Japan.

IEEE Log Number 8931896.

0018-9286/90/0100-0085$01.00 © 1990 IEEE

II. Problem Formulation

We consider the linear stochastic system of the form

$$dx_t = (\theta A x_t + B u_t)\,dt + dW_t, \qquad \theta\text{: unknown, } x_0\text{: given,} \tag{1}$$

with the quadratic average cost criterion

$$J(u) = \limsup_{T\to\infty} \frac{1}{T}\int_0^T (x_t^T Q x_t + u_t^T R u_t)\,dt, \qquad \text{a.s.,} \tag{2}$$

where (x_t) is an n-dimensional state process, $u = (u_t)$ is an m-dimensional control, (W_t) is an n-dimensional Brownian motion on a complete probability space $(\Omega, \mathfrak{F}, P; \{\mathfrak{F}_t\}_{t\geq 0})$ with $W_0 = 0$, A, B are $n \times n$, $n \times m$ matrices, and Q, R are $n \times n$, $m \times m$ symmetric positive definite matrices.

We define the set U of progressively measurable processes $u = (u_t)$ such that

$$\int_0^t \|u_s\|^2\,ds < \infty \qquad \text{a.s. for each } t \geq 0, \tag{3}$$

and the solution (x_t) of (1) satisfies

$$\liminf_{t\to\infty} \|x_t\|^2/t = 0 \qquad \text{a.s.,} \tag{4}$$

$$\int_0^\infty \|x_t\|^2/(1+t)^2\,dt < \infty \qquad \text{a.s.} \tag{5}$$

We make the following assumptions on θ and (x_t):

$$\theta \text{ takes values in the finite subset } G \text{ of } R, \tag{6}$$

$$(x_t) \text{ is measured for } u = 0 \text{ and each } u \in U. \tag{7}$$

The adaptive control problem is to find a pair $\{(\theta_t), (u_t^*)\}$ such that

$$(\theta_t)\text{: right continuous,} \quad \mathfrak{F}_t - \text{adapted,} \quad \theta_t \to \theta \quad \text{a.s. } (t\to\infty), \tag{8}$$

$$u^* = (u_t^*) \in U, \qquad J(u^*) \leq J(u) \quad \text{a.s. for all } u \in U. \tag{9}$$

III. The Main Result

We decompose the adaptive control problem as follows:

i) to find the estimator (θ_t) of θ from the observations (y_t) given by

$$dy_t = \theta A y_t\,dt + dW_t, \qquad \theta \in G, \tag{10}$$

ii) to solve the Riccati equation

$$D^+ L_t + \theta_t A^T L_t + \theta_t L_t A - L_t B R^{-1} B^T L_t + Q = 0$$

$$L_t\text{: continuous,} \qquad \mathfrak{F}_t - \text{adapted,}$$

$$\text{symmetric nonnegative definite,} \tag{11}$$

denoting the right-derivative by D^+, that arises in the LQ regulator problem in minimizing the cost functional

$$\bar{J}(u) = \int_0^S [z_t^T Q z_t + u_t^T R u_t]\,dt \text{ a.s. } (S \geq 0\text{: fixed}) \tag{12}$$

subject to

$$dz_t = (\theta_t A z_t + B u_t)\,dt, \qquad z_0\text{: given.} \tag{13}$$

Now, we shall state the main result.

Theorem 1: a) We assume (6), (7), and

$$\int_0^\infty \|A y_t\|^2\,dt = \infty \qquad \text{a.s.} \tag{14}$$

Then the right continuous $\mathfrak{F}_t$-adapted estimator (θ_t) is given by

$$\theta_t = \sum_{i\in G} i I(\xi_n \in G_i) \qquad \text{if } n \leqq t < n+1,$$

$$n = 0, 1, 2, \cdots, \tag{15}$$

where $\{G_i : i \in G\}$ is a family of disjoint compact intervals of R with its middle point $i \in G$, and

$$d\xi_t = -\gamma_t \xi_t \|A y_t\|^2\,dt + \gamma_t A y_t\,dy_t, \qquad \xi_0 = 1$$

$$d\gamma_t = -\gamma_t^2 \|A y_t\|^2\,dt, \qquad \gamma_0 = 1. \tag{16}$$

b) Furthermore, we assume that

$$(\theta_t A, B) \text{ is completely controllable a.s.} \tag{17}$$

Then L_t converges a.s. to the solution L of

$$\theta A^T L + \theta L A - L B R^{-1} B^T L + Q = 0$$

$$L\text{: symmetric nonnegative definite.} \tag{18}$$

In addition, if L satisfies the following condition:

the LDS $\dot{x}_t = \Pi x_t$, $\Pi = (\theta A - B R^{-1} B^T L)$

is exponentially stable; i.e.,

$$\exists C > 0, \alpha > 0;\ \|e^{\Pi(t-s)}\| \leqq C e^{-\alpha(t-s)}, \forall t \geqq s, \tag{19}$$

then the optimal control and the minimum of the cost functional are, respectively, given by

$$u_t^* = -R^{-1} B^T L_t x_t, \tag{20}$$

$$J(u^*) = \mathrm{Tr}(L). \tag{21}$$

IV. Proofs

In order to prove Theorem 1, we need the following two lemmas.

Lemma 2: Under (6), (7), and (14), we have $\theta_t \to \theta$ a.s. More precisely, we can find $\Omega_0 \in \mathfrak{F}$ with $P(\Omega_0) = 1$ such that for each $\omega \in \Omega_0$,

$$\exists \tau = \tau(\omega) \in R; \qquad \theta_t = \theta \qquad \text{for all } t \geqq \tau. \tag{22}$$

Proof: We shall show that (16) has a solution (ξ_t) given by

$$\xi_t = \left(1 + \int_0^t A y_s\,dy_s\right) \Big/ \left(1 + \int_0^t \|A y_s\|^2\,ds\right). \tag{23}$$

It is easy to see that

$$\gamma_t = 1 \Big/ \left(1 + \int_0^t \|A y_s\|^2\,ds\right).$$

By Ito's formula, we have

$$\xi_t \gamma_t^{-1} = 1 + \int_0^t \gamma_s^{-1}\,d\xi_s + \int_0^t \xi_s\,d(\gamma_s^{-1})$$

$$= 1 + \int_0^t A y_s\,dy_s.$$

This implies (23).

Now, we define the continuous local martingales (M_t) and (m_t) by

$$M_t = \int_0^t A y_s\,dW_s,$$

$$m_t = \int_0^t (1 + \langle M\rangle_s)^{-1}\,dM_s,$$

where $\langle\cdot\rangle$ denotes the predictable quadratic variation process of the local square integrable martingale, i.e.,

$$\langle M\rangle_t = \int_0^t \|A y_s\|^2\,ds.$$

Under (14) we can apply the strong law of large numbers for local martingales (see the Appendix).

Hence,

$$M_t/(1+\langle M\rangle_t) \to 0 \qquad \text{a.s. } (t\to\infty)$$

and then

$$\xi_t = (1+\theta\langle M\rangle_t + M_t)/(1+\langle M\rangle_t) \to \theta \qquad \text{a.s.}$$

It is clear that (θ_t) is right continuous and $\mathfrak{F}_t$-adapted. By (6) and (15), we deduce (22) and

$$\theta_t \to \theta \qquad \text{a.s.}$$

Thus, the lemma is proved.

Lemma 3: Under the assumptions of Theorem 1, we have $L_t \to L$ a.s. and

$$L_t = L \qquad \text{for } t \geqq \tau \qquad \text{on } \Omega_0. \tag{24}$$

Proof: Let us consider the Riccati equation

$$\dot{K}_t + \theta A^T K_t + \theta K_t A - K_t BR^{-1}B^T K_t + Q = 0$$

$$K_t\text{: symmetric nonnegative definite.} \tag{25}$$

Recall the results on the deterministic infinite-horizon regulator (see [1]). Then we see that the Riccati equation (25) with the boundary condition $K_T = 0$ admits a unique solution, denoted by $K(t; 0, T)$. We notice by Lemma 2 and (17) that $(\theta A, B)$ is completely controllable, and then the mapping $T \to K(t; 0, T)$ is bounded and increasing. Thus, we can obtain the solution (K_t) of (25) by $K_t = \lim_{T\to\infty} K(t; 0, T)$. Moreover, since the coefficients of (25) are constant matrices, we get the solution L of (18) by setting

$$L = K_t = \lim_{t\to-\infty} K(0; 0, T-t) = \lim_{t\to-\infty} K(t; 0, T).$$

Next, we consider the Riccati equation (11). By using an argument similar to the above, we can obtain the unique solution with the boundary condition $L_T = 0$, denoted by $L(t; 0, T)$. Under (17), the solution L_t of (11) is given by $L_t = \lim_{T\to\infty} L(t; 0, T)$. From (22), it follows that

$$L(t; 0, T) = K(t; 0, T) \qquad \text{for } t \geqq \tau \qquad \text{on } \Omega_0.$$

Thus, letting $T\to\infty$, we deduce (24).

Proof of Theorem 1: The proof is divided into three steps.

Step 1: We shall show

$$J(u) \geqq \operatorname{Tr}(L) \qquad \text{for all } u \in U. \tag{26}$$

Applying Ito's formula to $x^T Lx$, we have

$$\begin{aligned}
x_t^T Lx_t &= x_0^T Lx_0 + \int_0^t [x_s^T L\,dx_s + dx_s^T Lx_s] + \frac{1}{2}\int_0^t 2\operatorname{Tr}(L)\,ds \\
&= x_0^T Lx_0 + \int_0^t [x_s^T L\,dW_s + dW_s^T Lx_s] \\
&\quad + \int_0^t [\{x_s^T(\theta A^T L + \theta LA)x_s\} \\
&\quad + (x_s^T LBu_s + u_s^T B^T Lx_s) + \operatorname{Tr}(L)]\,ds.
\end{aligned}$$

By (18) we get

$$\begin{aligned}
&x^T Qx + u^T Ru + [\{x^T(\theta A^T L + \theta LA)x\} + (x^T LBu + u^T B^T Lx)] \\
&\quad = x^T LBR^{-1}B^T Lx + (x^T LBu + u^T B^T Lx) + u^T Ru \\
&\quad = (Ru + B^T Lx)^T R^{-1}(Ru + B^T Lx) \geqq 0.
\end{aligned}$$

Hence,

$$\begin{aligned}
J(u) = \limsup_{T\to\infty} \Bigg\{ &(x_0^T Lx_0 - x_T^T Lx_T)/T \\
&+ \frac{1}{T}\int_0^T [x_t^T L dW_t + dW_t^T Lx_t] + \operatorname{Tr}(L) \\
&+ \frac{1}{T}\int_0^T (Ru_t + B^T Lx_t)^T R^{-1}(Ru_t + B^T Lx_t)\,dt \Bigg\}.
\end{aligned} \tag{27}$$

Define the continuous local martingale (N_t) by

$$N_t = \int_0^t (1+s)^{-1}[x_s^T L dW_s + dW_s^T Lx_s].$$

By (5) we have $\langle N\rangle_\infty < \infty$ a.s. and then we deduce, by the strong law of large numbers,

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t [x_s^T L dW_s + dW_s^T Lx_s] = 0.$$

Thus, (26) follows from (4) and (27).

Step 2: We shall check that u^* defined by (20) belongs to U. It is clear that u^* satisfies (3). Let us consider the closed-loop system with random coefficients:

$$dx_t = \Pi_t x_t dt + dW_t, \qquad \Pi_t = \theta A - BR^{-1}B^T L_t. \tag{28}$$

By Ito's formula,

$$\begin{aligned}
\Phi_t x_t &= x_0 + \int_0^t [\Phi_s dx_s + d\Phi_s x_s] \\
&= x_0 + \int_0^t [\Phi_s \Pi_s x_s + \dot{\Phi}_s x_s]\,ds + \int_0^t \Phi_s dW_s \\
&= x_0 + \int_0^t \Phi_s dW_s,
\end{aligned}$$

where (Φ_t) denotes the continuous $\mathfrak{F}_t$-adapted solution of

$$\dot{\Phi}_t = -\Phi_t \Pi_t, \qquad \Phi_0 = I,$$

$$\Phi_t \text{ is nonsingular for all } t.$$

Hence, (28) admits a continuous solution (x_t) of the form

$$x_t = \Phi_t^{-1}\left[\int_0^t \Phi_s dW_s + x_0\right].$$

Now, we consider the equation

$$df_t = \Pi f_t dt + dW_t,$$

which admits a solution (f_t) of the form

$$f_t = \int_0^t e^{\Pi(t-s)} dW_s + e^{\Pi t} f_0.$$

Then, by (19),

$$\begin{aligned}
E[\|f_t\|^2] &= \int_0^t \|e^{\pi(t-s)}\|^2 ds + \|e^{\pi t}\|^2 \|f_0\|^2 \\
&\leqq C\left[\int_0^t e^{-2\alpha(t-s)} ds + e^{-2\alpha t}\|f_0\|^2\right]
\end{aligned}$$

from which we have

$$\sup_t E[\|f_t\|^2] < \infty.$$

Hence,

$$E\left[\liminf_{t\to\infty} \|f_t\|^2/t\right] \leqq \liminf_{t\to\infty} E[\|f_t\|^2]/t = 0,$$

$$E\left[\int_0^\infty \|f_t\|^2/(1+t)^2\,dt\right] < \infty.$$

Thus, we deduce

$$\liminf_{t\to\infty} \|f_t\|^2/t = 0 \qquad \text{a.s.} \tag{29}$$

$$\int_0^\infty \|f_t\|^2/(1+t)^2\,dt < \infty \qquad \text{a.s.} \tag{30}$$

We set $g_t = x_t - f_t$. Then, we have

$$dg_t = [(\theta A - BR^{-1}B^T L_t)g_t + BR^{-1}B^T(L - L_t)f_t]\,dt.$$

Hence,

$$dg_t = \Pi g_t\,dt \qquad \text{for } t \geqq \tau \qquad \text{on } \Omega_0,$$

and then

$$x_t - f_t = C_\tau e^{\Pi t} \qquad \text{for } t \geqq \tau \qquad \text{on } \Omega_0, \tag{31}$$

where C_τ is a constant depending on τ and $\omega \in \Omega_0$. Therefore, u^* satisfies (4) and (5) by (19), (29), (30), and (31).

Step 3: To complete the proof of Theorem 1, it suffices to show (21). By Ito's formula, we have

$$\begin{aligned} x_t^T L_t x_t &= x_0^T L_0 x_0 + \int_0^t x_s^T(D^+ L_s)x_s ds \\ &\quad + \int_0^t [x_s^T L_s dx_s + dx_s^T L_s x_s] + \frac{1}{2}\int_0^t 2\mathrm{Tr}(L_s)ds \\ &= x_0^T L_0 x_0 + \int_0^t [x_s^T L_s dW_s + dW_s^T L_s x_s] + \int_0^t \mathrm{Tr}(L_s)ds \\ &\quad + \int_0^t [\{x_s^T(D^+ L_s + \theta A^T L_s + \theta L_s A)x_s\} \\ &\quad + (x_s^T L_s B u_s^* + u_s^{*T} B^T L_s x_s)]\,ds. \end{aligned}$$

By (11) and (20), we get

$$\begin{aligned} & x_s^T Q x_s + u_s^{*T} R u_s^* + \{x_s^T(D^+ L_s + \theta_s A^T L_s + \theta_s L_s A)x_s\} \\ &\quad + (x_s^T L_s B u_s^* + u_s^{*T} B^T L_s x_s) \\ &= x_s^T L_s B R^{-1} B^T L_s x_s \\ &\quad + (x_s^T L_s B u_s^* + u_s^{*T} B^T L_s x_s) + u_s^{*T} R u_s^* \\ &= (R u_s^* + B^T L_s x_s)^T R^{-1}(R u_s^* + B^T L_s x_s) = 0. \end{aligned}$$

Hence,

$$\begin{aligned} J(u^*) = \limsup_{T\to\infty} \{ & (x_0^T L_0 x_0 - x_T^T L_T x_T)/T \\ & + \frac{1}{T}\int_0^T [x_t^T L_t dW_t + dW_t^T L_t x_t] + \frac{1}{T}\int_0^T \mathrm{Tr}(L_t)\,dt \\ & + \frac{1}{T}\int_0^T (\theta - \theta_t)[x_t^T(A^T L_t + L_t A)x_t]\,dt \Big\}. \end{aligned}$$

Thus, taking into account (4) and (5), we deduce (21) by Lemmas 2, 3, and the strong law of large numbers. The proof is complete.

V. Conclusions

In our system, the unknown scalar parameter assumes values in a finite set and is not contained in the coefficients of control terms. Based on this fact, we can make use of the separation principle in a different sense from that of partially observable LQG problems. This enables us to present an explicit solution to the adaptive LQG regulator problem. Also, one can easily obtain the examples which satisfy (14), (17), and (19) in case of $n = m = 1$. It is desirable to extend these results to more generalized systems.

Appendix

The Strong Law of Large Numbers and Kronecker's Lemma for Local Martingales

Let (M_t) be the local martingale and (A_t) be a nondecreasing predictable process with $M_0 = A_0 = 0$. Define the local martingale (m_t) by the stochastic integral

$$m_t = \int_0^t (1 + A_s)^{-1} dM_s.$$

In the proof of Theorem 1, we base our arguments on the strong law of large numbers for which Kronecker's lemma plays an important role.

Kronecker's Lemma: If $A_\infty = \infty$ a.s. and m_t converges almost surely as $t \to \infty$, then

$$M_t/(1 + A_t) \to 0 \qquad \text{a.s. } (t \to \infty).$$

The Strong Law of Large Numbers: If $A_\infty = \infty$ a.s. and (M_t) is locally square integrable, then we have

$$\langle m \rangle_\infty = \int_0^\infty (1 + A_s)^{-2} d\langle M \rangle_s < \infty \text{ a.s.}$$
$$\Rightarrow M_t/(1 + A_t) \to 0 \text{ a.s. } (t \to \infty).$$

Also, if we take $A_t = \langle M \rangle_t$, then $\langle m \rangle_\infty \leqq 1$ a.s. For the proofs, see [7].

References

[1] B. D. O. Anderson and J. B. Moore, *Linear Optimal Control*. Englewood Cliffs, NJ: Prentice-Hall, 1971.

[2] T. E. Duncan and B. Pasik-Duncan, "A parameter estimate associated with the adaptive control of stochastic systems," in *Lecture Notes in Control and Information Sciences 83*. Berlin and New York: Springer-Verlag, 1986, pp. 508-514.

[3] G. C. Goodwin and K. S. Sin, *Adaptive Filtering Prediction and Control*. Englewood Cliffs, NJ: Prentice-Hall, 1984.

[4] O. Hijab, "The adaptive LQG problem II," *Stochastics*, vol. 16, pp. 25-49, 1986.

[5] —, *Stabilization of Control Systems*. New York: Springer-Verlag, 1987.

[6] P. R. Kumar and P. Varaiya, *Stochastic Systems*. Englewood Cliffs, NJ: Prentice-Hall, 1986.

[7] R. S. Liptser, "A strong law of large numbers for local martingales," *Stochastics*, vol. 3, pp. 217-228, 1980.

[8] P. Mandl, "Some connections between statistics and control theory," in *Mathematical Statistics and Probability Theory*, vol. B. Dordrecht and Boston: Reidel, 1987, pp. 155-168.

[9] H. Morimoto and M. Ohashi, "On linear stochastic differential games with average cost criterions," to appear.

[10] R. Rishel, "An exact formula for a linear quadratic adaptive stochastic optimal control law," *SIAM Contr. Optim.*, vol. 24, pp. 667-674, 1986.

A Stable Iterative Adaptive Control

CISHEN ZHANG AND R. J. EVANS

Abstract—**This paper formulates an iterative adaptive control algorithm which carries out the on-line controller design iteratively. A set of conditions is provided under which the closed-loop system is stable. The presented iteration algorithm is applied to an adaptive LQ regulator.**

I. Introduction

Adaptive controllers based on the certainty-equivalence principle essentially consist of two processes, on-line parameter estimation and on-line controller design. To date, adaptive control algorithms have generally

Manuscript received August 2, 1988; revised November 28, 1988.

The authors are with the Department of Electrical and Computer Engineering, University of Newcastle, Newcastle, N.S.W., Australia.

IEEE Log Number 8931419.

0018-9286/90/0100-0088$01.00