Optimal Control of Twin Rotor MIMO System Using LQR Technique

Sumit Kumar Pandey and Vijaya Laxmi

Abstract In this paper, twin rotor multi input multi output system (TRMS) is considered as a prototype laboratory set-up of helicopter. The aim of studying the model of TRMS and designing the controller for it is to provide a platform for controlling the flight of helicopter. An optimal state feedback controller based on linear quadratic regulator (LQR) technique has been designed for twin rotor multi input multi output system. TRMS is a nonlinear system with two degrees of freedom and cross couplings. The mathematical modeling of TRMS has been done using MATLAB/SIMULINK. The linearised model of TRMS is obtained from the nonlinear model. The simulation results of optimal controller are compared with the results of conventional PID controller. The appropriateness of proposed controller has been shown both in terms of transient and steady state response.

Keywords Twin rotor MIMO system · Linear quadratic regulator (LQR) · Unmanned air vehicle (UAV)

1 Introduction

Recent times the development of several approaches for controlling the flight of air vehicle such as helicopter and unmanned air vehicle (UAV) has been studied frequently. The modeling of the air vehicle dynamics is a highly challenging task due to the complicated nonlinear interactions among the various variables and also there are certain states which are not accessible for the measurement. The twin rotor multi input multi output system (TRMS) is an experimental set-up that resembles with the helicopter model. The TRMS consist of two rotors at each ends of the

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horizontal beam known as main rotor and tail rotor which is driven by a DC motor and it is counter balanced by a pivoted beam [1]. The TRMS can rotate in both horizontal and vertical direction. The main rotor generates a lift force due to this the TRMS moves in upward direction around the pitch axis. While, due to the tail rotor TRMS moves around the yaw axis. However TRMS resembles with the helicopter but there is some significant differences between helicopter and TRMS. In helicopter, by changing the angle of attack controlling has been done, while in TRMS it has been done by changing the speed of rotors. Several techniques have been implemented for the modeling and control purpose of TRMS.

In [2] authors provide the detail description of dynamic modeling of twin rotor MIMO system and investigate the open loop control along longitudinal axis. In [3] the model decouples method and implementation of optimal controller has been proposed for two independent SISO systems for TRMS. The controller has been designed to tolerate some changes in system parameter. In [4] the time optimal control method has been proposed for twin rotor MIMO system. In [5] the author discuss about the sliding mode state observer controller for TRMS system. Here the Lyapunov method is used to derive the asymptotic stability conditions for robust and global sliding mode control. In [6] dynamic model is proposed to a one degree of freedom (DOF) twin rotor MIMO system (TRMS) based on a black box system identification technique. This extracted model is connected with a feedback LQG regulator. The authors describe how the system performance has been improved by using artificial non-aerodynamic forces.

In this work, dynamic and linear model for TRMS have been developed. A PID controller and an optimal state feedback controller based on LQR technique has been designed separately. The transient and steady state performance of the system has been analyzed for step input.

The paper is organized as follows. Next section deals with the modeling of the system, followed by the control technique. Section 4 deals with the results obtained and last section consists of conclusion.

2 Mathematical Modeling

According to the diagram presented in Fig. 1, the non linear equation has been derived [7, 8] and the parameters of TRMS are shown in Table 1.

$$I_1 \cdot \dot{\psi} = M_1 - M_{FG} - M_{B\psi} - M_G \tag{1}$$

where, M_1 is the nonlinearity caused by the rotor and can be estimated as second order polynomial and due to this the torque is induced to the TRMS as given below.

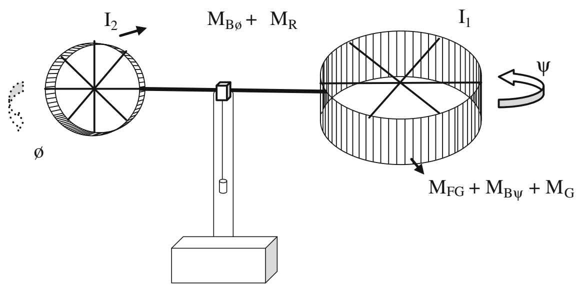

Fig. 1 Twin rotor MIMO system

Table 1 Physical parameters of TRMS

Symbol	Parameter	Value	Unit
I_1	Vertical rotor moment of inertia	6.8×10^{-2}	kg m^2
I_2	Horizontal rotor moment of inertia	2×10^{-2}	kg m^2
a_1	Parameter of static characteristic	0.0135	N/A
a_2	Parameter of static characteristic	0.0924	N/A
b_1	Parameter of static characteristic	0.02	N/A
b_2	Parameter of static characteristic	0.09	N/A
m_g	Gravity momentum	0.32	N m
$B_{1\psi}$	Parameter of friction momentum	6×10^{-2}	N m s/rad
$B_{2\psi}$	Parameter of friction momentum	1×10^{-3}	N m s/rad
$B_{1\phi}$	Parameter of friction momentum	1×10^{-1}	N m s/rad
$B_{2\phi}$	Parameter of friction momentum	1×10^{-2}	N m s/rad
K_{gy}	Parameter of gyroscopic momentum	0.05	s/rad

$$M_1 = a_1 \cdot \tau_1^2 + b_1 \cdot \tau_1 \tag{2}$$

Considering the Fig. 1, the weight of the helicopter produces the gravitational torque about the pivot point, which is described by the following Eq. 3.

$$M_{FG} = M_g \cdot \sin\psi \tag{3}$$

The frictional torque can be estimated as following equation.

$$M_{B\psi} = B_{1\psi} \cdot \dot{\psi} + B_{2\psi} \cdot \mathrm{sign}(\dot{\psi}) \tag{4}$$

The gyroscopic torque occurs due to coriolis force. This torque is resulted when moving main rotor changes its position in azimuth direction, and describes as the Eq. 5 given below.

$$M_G = K_{gy} \cdot M_1 \cdot \dot{\phi} \cdot \cos\psi \tag{5}$$

Here, the motor and electrical control circuit is considered as transfer function of first order. Hence the motor momentum is described in Laplace domain is as below.

$$\tau_1 = \frac{K_1}{T_{11s} + T_{10}} \cdot u_1 \tag{6}$$

Similar equation is developed for the horizontal plane motion. The net torques produced in horizontal plane motion is described by the following Eq. 7

$$I_2 \cdot \ddot{\phi} = M_2 - M_{B\phi} - M_R \tag{7}$$

where, M_2 is nonlinear static characteristic similar as main rotor.

$$M_2 = a_2 \cdot \tau_2^2 + b_2 \cdot \tau_2 \tag{8}$$

Frictional torque is calculated same as the main rotor dynamics.

$$M_{B\psi} = B_{1\phi} \cdot \dot{\psi} + B_{2\phi} \cdot \text{sign}\ (\dot{\Phi}) \tag{9}$$

M_R is the cross reaction momentum estimated by first order transfer function described by the following equation.

$$M_R = \frac{K_c \cdot (T_{0S} + 1)}{\left(T_{ps} + 1\right)} \cdot \tau_1 \tag{10}$$

Again, the D.C. motor with electrical circuit is estimated as the first order transfer function and given by the following equation.

$$\tau_2 = \frac{k_2}{T_{21}S + T_{20}} \cdot u_2 \tag{11}$$

The above mathematical model given by Eqs. 1–11, is linearized [6, 7] across equilibrium point X0 given as

$$X0 = [\,0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\,]$$

The state and output vector here is given by

$$X = [\psi, \dot{\psi}, \Phi, \dot{\Phi}, \tau_1, \tau_2, M_R]^T$$
$$Y = [\,\psi \quad \Phi\,]^T$$

Here the TRMS plant is represented as below.

$$\left.\begin{aligned}\dot{x} &= Ax + Bu\\ y &= C\,x\end{aligned}\right\} \tag{12}$$

The system considered here consists of 7 states, there are two control input and two output state namely pitch and yaw. The system matrix can be obtained by linearizing is as below.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -4.34 & 0 & -0.0882 & 0 & 1.24 & 0 & 0 \\ 0 & 0 & 0 & -5 & 1.4823 & 3.6 & 18.75 \\ 0 & 0 & 0 & 0 & -0.8333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -0.0169 & 0 & -0.5 \end{bmatrix}$$

$$B = \begin{bmatrix} 00 \\ 00 \\ 00 \\ 00 \\ 10 \\ 01 \\ 00 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3 Proposed Control Techniques of TRMS

This section presents the control techniques using PID controller and optimal state feedback controller using LQR technique.

3.1 PID Control Technique

The conventional PID controller is used to control the horizontal and vertical movements separately. The outputs are compared with desired output value and then the error is processed to conventional PID controller as shown in Fig. 2 [8].

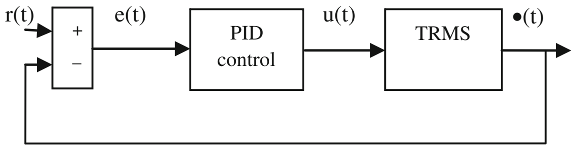

Fig. 2 PID control scheme for TRMS

Here, r(t) is the reference input, e(t) is error signal, u(t) is the control force and α(t) is the output of plant.

The controller output of conventional PID controller [9] is given as in Eq. 13

$$u(t) = K_p e(t) + K_i \int e(t)\,dt + K_d \frac{de(t)}{dt} \tag{13}$$

where,

K_p Proportional gain,
K_i Integral gain,
K_d Derivative gain.

3.2 Optimal Control Technique

Here the plant taken is time varying because the optimal control problem is formulated for the time varying system. Control input of the plant is described as shown in Fig. 3.

$$u(t) = -K(t)\,x(t) \tag{14}$$

The control input here is linear and the control energy is given by $u^T(t)R(t)u(t)$, where R(t) is the square matrix known as control cost matrix. Control energy expression is in quadratic form because the equation contains $u^T(t)R(t)u(t)$ quadratic function of u(t). The transient energy can be expressed as $x^T(t)\,Q(t)\,x(t)$,

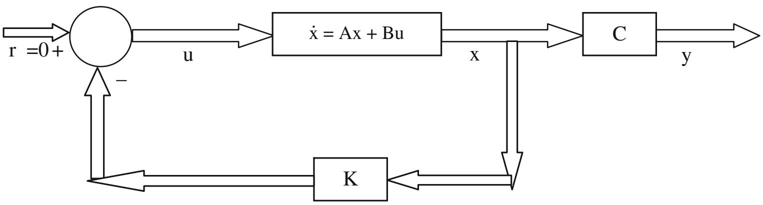

Fig. 3 Optimal control scheme of TRMS

where Q(t) is square symmetric matrix called state weighing matrix [10]. Hence the objective function is

$$J(t, t_f) = \int_{t}^{t_f} (x^T(t)Q(t)\,x(t) + u^T(t)\,R(t)\,u(t))dt \tag{15}$$

where t and t_f are initial and final time respectively. The main objective here is to minimize the objective function as described by Eq. 15 by choosing an optimal value of gain matrix K(t). By considering Eqs. 12 and 14.

$$\dot{x}(t) = (A - BK(t))x(t) \tag{16}$$

$$\dot{x}(t) = A_K\,x(t) \tag{17}$$

where $A_K = (A - BK(t))$ is close loop state dynamics matrix. The solution of Eq. 16 is

$$x(t) = \theta_K(t,\ t_0)x(t_0) \tag{18}$$

where $\theta_K(t,\ t_0)$ is the state transition matrix of closed loop system, on substituting Eq. 18 in 15 the given objective function is as

$$J(t,\ t_f) = \int_{t}^{t_f} (x^T(t)\theta_K^T(\tau, t)(Q(\tau) + K^T(\tau)R(\tau)K(\tau))\theta_K(\tau, t)x(t)d\tau \tag{19}$$

This can be written as

$$J(t, t_f) = x^T(t)M(t, t_f)x(t) \tag{20}$$

where

$$M(t, t_f) = \int_{t}^{t_f} \theta_K(\tau, t)(Q(\tau) + K^T(\tau)R(\tau)K(\tau))\theta_K(\tau, t)d\tau \tag{21}$$

By Eqs. 18 and 19

$$(t, t_f) = \int_{t}^{t_f} (x^T(\tau)(Q(\tau) + K^T(\tau)\,R(\tau)\,K(\tau))\,x(\tau)\,d\tau \tag{22}$$

Now differentiating Eq. 22 with respect to time ‘t’

$$\frac{\partial J(t, t_f)}{\partial t} = -x^T(t)(Q(t) + K^T(t)R(t)K(t))x(t) \tag{23}$$

Also partially differentiating Eq. 19 with respect to time 't'

$$\frac{\partial J(t, t_f)}{\partial t} = \dot{x}(t)^T M(t, t_f)\, x(t) + x^T(t)(\frac{\partial M(t, t_f)}{\partial t})\, x(t) + x^T(t)\, M(t, t_f)\dot{x}(t) \tag{24}$$

By combining Eqs. 17 and 24

$$\frac{\partial J(t, t_f)}{\partial t} = x^T(t)(A_K(t)M(t, t_f) + (\frac{\partial M(t, t_f)}{\partial t})x(t) + M(t, t_f)A_K(t))x(t) \tag{25}$$

Now considering Eqs. 23 and 25

$$-\frac{\partial M(t, t_f)}{\partial t} = A_K(t)\, M(t, t_f) + A_K^T(t)\, M(t, t_f) + (Q(t) + K^T(t)R(t)K(t)) \tag{26}$$

Above equation describe the matrix Riccati equation for finite time duration. Optimal control gain matrix K(t) is obtained by solving Eq. 26.

$$K(t) = R^{-1}(t)\, B^T(t)M \tag{27}$$

By considering the closed loop system as asymptotically stable, M is a optimal matrix, Q(t) and R(t) are positive definite matrix and positive semi definite matrix are time independent. The value of Q and R are randomly chosen and varied until the output of system does not get the desired value.

The control gain matrix K has been calculated here is as below.

$$K = \begin{bmatrix} -0.1510 & 0.0044 & 0.0352 & 0.0011 & -0.54 & 0.0024 & 0.0226 \\ -0.0053 & 1 & -0.0046 & 1 & 0.0421 & -0.0056 & -2.0037 \end{bmatrix}$$

4 Simulation Results

To implement the above control techniques, the TRMS is designed using Simulink. The responses of reference inputs of the LQR controller are presented in this section which is compared with the results of the conventional PID controller. In simulation the transient and steady state response of the system is investigated such as overshoot, settling time, steady state error. The reference value for step input is taken here as 0.5 for horizontal plane and 0.2 for vertical plane. Figure 4a, b shows the

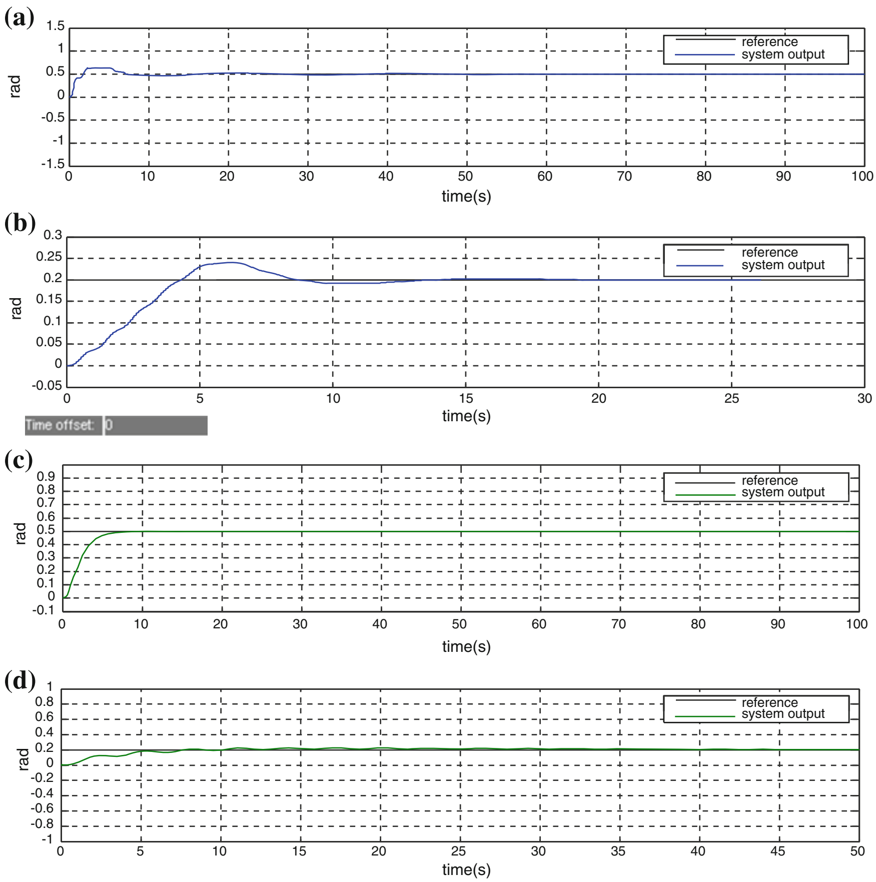

Fig. 4 **a** Step response in horizontal plane using PID control. **b** Step response in vertical plane using PID control. **c** Step response in horizontal plane using optimal control. **d** Step response in vertical plane using optimal control

response of the TRMS in horizontal and vertical plane using PID control technique. Figure 4c, d shows the response of the TRMS in horizontal and vertical plane using optimal control technique. Figure 5a, b shows the control effort of the TRMS in horizontal and vertical plane using PID control technique. Figure 5c, d shows the control effort of the TRMS in horizontal and vertical plane using optimal control technique.

Table 1 shows the physical parameters of TRMS and Table 2 depicts the characteristics of step response of TRMS using PID controller and optimal controller.

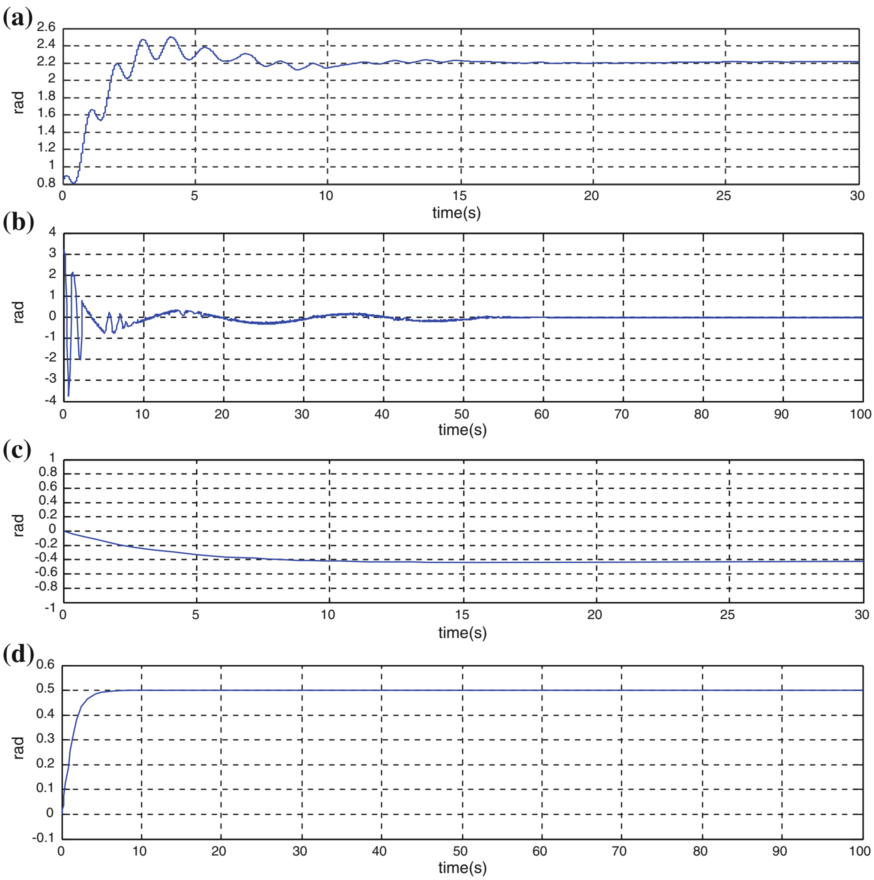

Fig. 5 **a** Control effort in vertical plane using PID control. **b** Control effort in horizontal plane using PID control. **c** Control effort in vertical plane using optimal control. **d** Control effort in horizontal plane using optimal control

Table 2 Characteristics of step response

	Plane	Reference value	Rise time (s)	Settling time (s)	Max. over shoot (%)	Steady state error
PID controller [8]	Horizontal	0.5	2.0	18	25.0	0.0
	Vertical	0.2	6.0	14	20.0	0.0
Optimal controller	Horizontal	0.5	4.4	5.7	0.0	0.0
	Vertical	0.2	5.0	8.0	8.0	0.0

5 Conclusion

In this paper, the TRMS with two degrees of freedom was considered. The mathematical modeling of TRMS has been done in MATLAB/SIMULINK. Here PID and optimal controllers has been designed to control the horizontal and vertical movements of the system. The performance of the designed controllers has been evaluated with step input. The results show that the optimal controller gives better performance in terms of both transient and steady state response as compared to the PID controller. The control effort in case of optimal controller is minimum then the PID controller.

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