Homework 5c

Question: Prove the impossibility result for leader election for the anonymous ring.

Impossibility result: There is no nonuniform anonymous algorithm for leader election in synchronous rings.

Proof:
Suppose there is a nonuniform anonymous algorithm A for leader election in a ring R of size $n > 1$.
In a synchronous ring, the algorithm proceeds in rounds.
Lemma 3.1 in the book states that for every round k of the admissible execution of A in R, the states of all the processors at the end of round k are the same.
In an anonymous ring, all processors start in the same state. Because they are identical and execute the same program, in every round each processor sends the same messages, therefore they all receive the same messages in each round and change state identically.

By using induction:
Base case: round $k = 0$. All processors start in the same state.
The lemma should hold for round $k - 1$. All processors are still in the same state at the end of round $k - 1$.
In the next round $k - 1 + 1 = k$, all processors will receive the same messages and change state identically, so if a processor enters an elected state, all other processors are also in elected states.

This contradicts with the assumption that A is an algorithm for leader election, which means algorithm A should elect only one leader.