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Children's mapping between symbolic and nonsymbolic representations of number

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ABSTRACT

When children learn to count and acquire a symbolic system for representing numbers, they map these symbols onto a preexisting system involving approximate nonsymbolic representations of quantity. Little is known about this mapping process, how it develops, and its role in the performance of formal mathematics. Using a novel task to assess children's mapping ability, we show that children can map in both directions between symbolic and nonsymbolic numerical representations and that this ability develops between 6 and 8 years of age. Moreover, we reveal that children's mapping ability is related to their achievement on tests of school mathematics over and above the variance accounted for by standard symbolic and nonsymbolic numerical tasks. These findings support the proposal that underlying nonsymbolic representations play a role in children's mathematical development.

Introduction

What drives development in learning mathematics? Children learn a great deal about symbolic representations of number over the first few years of mathematics schooling. But they also bring to school remarkable intuitive skills about numbers and quantities. To what extent do these nonsymbolic abilities contribute to children's ability to learn school mathematics?

Symbolic–nonsymbolic number mappings

We now have considerable evidence that infants, children, and adults have a system for representing and manipulating numerical information without using symbols. Children and adults can

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compare, add, and subtract sets on the basis of number when these sets are represented by dot arrays or sound sequences (e.g., Barth, Kanwisher, & Spelke, 2003; Barth, La Mont, Lipton, & Spelke, 2005; Cordes, Gelman, Gallistel, & Whalen, 2001; McCrink & Wynn, 2004; Pica, Lemer, Izard, & Dehaene, 2004; Whalen, Gallistel, & Gelman, 1999). Performance on these tasks is characterized by an effect of the ratio, or distance, between the items being compared. Accuracy falls when the quantities being compared are closer together or the ratio between them approaches 1. This effect is thought to stem from the approximate nature of representations within this system. These noisy representations overlap if the quantities that are being compared are close together, leading to slower and less accurate performance. The ratio at which individuals are able to distinguish items is 2:3 at 6 months of age (Jordan, Suanda, & Brannon, 2008) and 7:8 by adulthood (van Oeffelen & Vos, 1982). This suggests that the precision of these nonsymbolic representations changes over development.

When children learn to count and start to learn mathematics in school, they acquire a new symbolic system for representing numbers. This system involves precise representations of quantity and also allows quantities to be compared and manipulated. The symbolic system does not, however, replace the preexisting nonsymbolic system; rather, these systems become mapped onto one another. Evidence for this mapping comes primarily from the well-established numerical distance effect for symbolic number comparison (e.g., Moyer & Landauer, 1967; Temple & Posner, 1998; however, see Zorzi & Butterworth, 1999, for an alternative view). When adults or children are asked to compare numerical digits, their reaction times are affected by the numerical distance between the digits. Performance is slower when the digits are closer numerically than when they are more distant, mirroring the distance and ratio effects shown for comparison of nonsymbolic quantities. This effect arises because the symbolic representations are mapped onto underlying nonsymbolic representations and the approximate nature of the nonsymbolic representations affects individuals' ability to compare the symbolic representations. Children have been found to show a numerical distance effect from 5 years of age, and the size of this effect reduces over development (Holloway & Ansari, 2008a; Sekuler & Mierkiewicz, 1977). The decrease in the size of the distance effect may represent an increase in the precision of children's nonsymbolic representations or may represent an increase in the precision of the mapping between symbolic and nonsymbolic representations.

Furthermore, children can use the nonsymbolic system to perform arithmetic with symbolic representations before they have learned symbolic arithmetic (Gilmore, McCarthy, & Spelke, 2007), and the nonsymbolic system also affects adults' ability to perform symbolic arithmetic (Gilmore & Inglis, 2009). The symbolic system, therefore, appears to be mapped onto the preexisting nonsymbolic system and plays a role when individuals both compare and manipulate symbolic representations.

To date, however, there has been little direct investigation into children's ability to map between representations in each of these systems. Typically, the mapping between symbolic and nonsymbolic representations has been indexed by the numerical distance effect for symbolic number comparison tasks (Holloway & Ansari, 2008b; Rousselle & Noël, 2007). However, this measure does not directly assess an individual's ability to map between these representations; rather, it represents the extent to which approximate nonsymbolic representations interfere with the ability to compare precise symbolic representations.

Work involving adult participants has begun to investigate the mapping between nonsymbolic and symbolic representations more directly. Izard and Dehaene (2008) presented participants with nonsymbolic representations (dot arrays) and asked them to produce a symbolic estimate of the number of dots. Adults' estimates were generally inaccurate and showed a tendency to underestimate, although they did increase with the numerosity of the set. When participants were provided with a single reference point, estimates improved significantly across the whole range and not just locally around the reference point. This work suggests that although adults can map from nonsymbolic to symbolic representations, some calibration is required for these estimates to be accurate.

Izard and Dehaene (2008) demonstrated that adults can map between symbolic and nonsymbolic representations of number, but we currently know little about how and when this ability develops. Only one study has directly tested children's mapping ability. Lipton and Spelke (2005) examined 5-year-olds' mapping on three tasks. On a free estimation task where children were asked to estimate the number of items in a set, 5-year-olds who were skilled counters, but not those who were unskilled counters, produced estimates that were linearly related to numerosity across the range of arrays.

Following this, children were shown two nonsymbolic representations and asked to choose the set with a given number of items. Again, skilled counters, but not unskilled counters, were able to complete this task at above-chance levels, but many children failed to complete the task. Finally, children were shown two arrays, told how many items one set contained, and were asked to estimate how many items were in the second array. Skilled counters, but not unskilled counters, were able to produce estimates in the right direction, but again many children failed to answer. This study demonstrated that many 5-year-olds were unable to map between symbolic and nonsymbolic representations and that the ability to map was related to knowledge of the symbolic system. However, we do not know how the ability to map develops after 5 years of age. Furthermore, the free response tasks used by Lipton and Spelke proved to be difficult, with many children failing to produce estimates at all.

A further gap in the literature exists in that previous investigations have tended to examine mapping in a single direction—producing a symbolic label for a given nonsymbolic representation (e.g., Hollingsworth, Simmons, Coates, & Cross, 1991; Izard & Dehaene, 2008). However, mapping can occur in either direction—from nonsymbolic to symbolic or from symbolic to nonsymbolic. We do not know whether individuals are equally proficient in mapping in either direction or whether the direction of mapping affects performance. The first aim of the current research, therefore, was to directly examine mapping between nonsymbolic and symbolic representations in children, to observe whether it develops over middle childhood, and to test whether the direction of mapping has any effect on this ability.

Relationship with formal mathematics skills

Although it has been frequently suggested that children's nonsymbolic abilities are related to their ability to learn formal mathematics (Butterworth, 1999; Dehaene, 1997), it is only recently that this relationship has started to be tested empirically. Booth and Siegler (2008) found that children's ability to place symbolic representations onto a number line was related to both mathematics achievement and accuracy at solving addition problems as well as future arithmetic gains. It is not clear, however, how representations on number line tasks are related to general nonsymbolic representations of quantity. Number line estimation tasks assess only one aspect of children's numerical representations, namely, the linearity of children's symbolic representations. Tasks involving more general nonsymbolic representations are necessary to fully investigate the role of this system in mathematics learning.

Holloway and Ansari (2008b) examined 6- to 8-year-olds' performance on tests of symbolic and nonsymbolic comparison. The size of the numerical distance effect for symbolic comparison (an index of the connection between nonsymbolic and symbolic representations) was found to be significantly related to children's scores on the Woodcock–Johnson standardized mathematics test. Children who scored lower on the mathematics test tended to have larger distance effects, indicating less precise mapping between nonsymbolic and symbolic representations. However, there was no relationship between performance on the nonsymbolic comparison task and mathematics achievement. This work suggests that children's nonsymbolic representations do affect their ability to learn formal mathematics, but only in terms of the influence of nonsymbolic representations on symbolic representations. Children's ability to compare nonsymbolic representations themselves did not appear to be an important factor. This study did not directly assess mapping between nonsymbolic and symbolic representations, and so it remains to be established whether this ability plays a role in learning mathematics.

Further evidence for the potential importance of mapping between representations for the learning of mathematics comes from individuals with dyscalculia. Rousselle and Noël (2007) demonstrated that children with mathematical learning difficulties were slower and less accurate than a control group on a symbolic comparison task. But they showed no deficit on a task of nonsymbolic comparison. The authors concluded that children with mathematical learning difficulties have deficits in accessing nonsymbolic information from symbols. This suggests that mapping between the two systems is important for learning formal symbolic mathematics, but again children's ability to directly map between symbolic and nonsymbolic representations was not assessed. The second aim of the current research, therefore, was to investigate whether children's ability to map between symbolic and nonsymbolic representations of number is related to performance of school mathematics.

This article describes two studies investigating the development of children's numerical cognition and mathematics. In the first study, we presented a task to assess mapping between symbolic and nonsymbolic representations in 6- to 8-year-olds. The numerical estimation task used with adults (Izard & Dehaene, 2008) is not suitable for use with young children, who may have difficulties with tasks involving the production of free responses (Lipton & Spelke, 2005). Therefore, a more structured two-alternative forced-choice task was developed. In this task, children were shown a target representation of one quantity (symbolic or nonsymbolic) and needed to choose which of two alternative representations (nonsymbolic or symbolic) matched this. Because this task can be bi-directional, it can be used to investigate whether direction of mapping has any effect on performance. Difficulty was manipulated by varying the ratio between the two alternative choices. In the second study, we examined how mapping ability indexed by this task and performance on standard symbolic and nonsymbolic comparison tasks relate to performance of school mathematics. These studies allowed us to investigate the role of symbolic and nonsymbolic representations, and the mapping between them, in the development of mathematical abilities.

Study 1

Method

Participants

A total of 52 children (24 boys and 28 girls) took part in the study. Children in Year 2 ($n = 25$) had a mean age of 6 years 9 months (range = 6 years 4 months to 7 years 3 months), and children in Year 4 ($n = 27$) had a mean age of 8 years 9 months (range = 8 years 5 months to 9 years 3 months). All children spoke English fluently, and none had a statement of special educational needs. All participants were recruited through schools and received a sticker to thank them for taking part.

Materials

The mapping task was presented on a laptop computer. It consisted of 24 experimental trials and 6 training trials. On each experimental trial, a target quantity was presented, followed by two alternative choices. On half of the trials, the target quantity was an Arabic symbol with prerecorded spoken label and the alternative choice quantities were dot arrays (symbolic to nonsymbolic version). On the other half of the trials, the target quantity was a set of dots and the alternative choice quantities were Arabic symbols with spoken labels (nonsymbolic to symbolic version). The target quantities varied from 20 to 50, and the alternative choices consisted of the correct quantity and a distractor. On half of the trials, the ratio between the correct quantity and the distractor was .50 (easy ratio). On the other half of the trials, the ratio between the correct quantity and the distractor was .67 (difficult ratio). The

Table 1
Numerosities used in mapping task for both symbolic to nonsymbolic and nonsymbolic to symbolic problems.

Ratio	Target	Distractor
.50	21	42
	26	13
	30	60
	34	17
	40	80
	44	22
.67	20	30
	25	17
	31	46
	34	22
	40	60
	47	32

correct quantity was the larger or smaller amount an equal number of times. The two versions of the task involved the same numerosities (see Table 1).

The method of Pica and colleagues (2004) was used to create the dot array stimuli. On half of the trials, dot size and total enclosure area decreased with numerosity; thus, density increased with numerosity. On the other half of the trials, dot size and total enclosure area increased with numerosity; thus, density decreased with numerosity. This prevented children from consistently using perceptual features to compare the dot arrays.

Procedure

Children were tested individually in a single session in a quiet environment outside the classroom. Initially, they were given a brief counting assessment to measure knowledge of symbolic numbers. First, children were asked to count on from "35, 36, 37, . . ." and then from "75, 76, 77, . . ." to ensure that they could correctly cross the decade boundary (i.e., to 40 and 80, respectively). Finally, they were asked to count backward from 25 until they reached 19. Each participant was given a score out of 3.

Following the counting assessment, the children completed the mapping task. Children completed one block of symbolic to nonsymbolic trials and one block of nonsymbolic to symbolic trials. The order in which the blocks were completed was counterbalanced across participants. Before each block of 12 experimental trials, participants were presented with two reference dot arrays for 3 s and were told how many dots were present (numerosities involved: 60 and 17, 80 and 19). These numerosities were outside the range of quantities used in the experimental trials and helped to calibrate children's responses (following Lipton & Spelke, 2005). Three training trials preceded both experimental blocks. The training trials involved matching a colored piece of fruit with a block of color. These nonnumerical training trials allowed participants to learn the rules of the task and gain confidence before the numerical experimental trials. The mapping task was presented as a game that incorporated the experimental trials into a fun, yet simple, narrative about exploring a desert island looking for treasure. The dot arrays represented pieces of treasure, and children were asked to help a character to guess how many there were hidden in a treasure chest.

Results and discussion

Children were highly accurate at completing the counting assessment (Year 4: 25 scored 3/3 and 2 scored 2/3; Year 2: 16 scored 3/3, 8 scored 2/3, and 1 scored 1/3). Children who made errors were nevertheless able to recognize symbols from across the range used in the mapping task. Thus, all children were considered to have adequate knowledge of symbolic numbers to complete the mapping task.

Children's accuracy on the mapping task demonstrated that they were able to map between the two representations of quantity. Children performed above chance (50%) on this task in both the Year 2 age group ($M = 58.8\%$, $SD = 8.2$), $t(24) = 5.40$, $p < .001$, and the Year 4 age group ($M = 68.4\%$, $SD = 10.5$), $t(26) = 9.10$, $p < .001$. Overall, children performed above chance for problems at both the easier ratio ($M = 71.3\%$, $SD = 14.0$), $t(51) = 10.95$, $p < .001$, and the more difficult ratio ($M = 56.3\%$, $SD = 15.2$), $t(51) = 2.96$, $p = .005$, as well as for both nonsymbolic to symbolic mapping problems ($M = 65.7\%$, $SD = 13.1$), $t(51) = 5.86$, $p < .001$, and symbolic to nonsymbolic mapping problems ($M = 61.9\%$, $SD = 14.6$), $t(51) = 8.67$, $p < .001$.

The effect of different task factors was examined using analysis of variance (ANOVA). Initial analysis found that there was no effect of block order, and so this factor was removed from the analysis. A three-way mixed-design ANOVA was performed with problem difficulty (.50 ratio or .67 ratio) and direction of mapping (nonsymbolic to symbolic or symbolic to nonsymbolic) as repeated-measures factors and age group (Year 2 or Year 4) as a between-groups factor.

There was a main effect of age group, $F(1,50) = 13.2$, $p = .001$, and a main effect of problem difficulty, $F(1,50) = 28.2$, $p < .001$. As predicted, children in Year 4 performed more accurately than children in Year 2, and problems with a .50 ratio were solved more accurately than problems with a .67 ratio. Although there was no main effect of direction of mapping, there was an interaction between problem difficulty and direction of mapping, $F(1,50) = 4.0$, $p = .05$ (see Fig. 1A). Simple main effects analysis revealed that the direction of mapping had no effect on performance at the easier ratio ($F < 1$) but did affect performance at the harder ratio, $F(1,51) = 6.1$, $p = .017$. Children were more

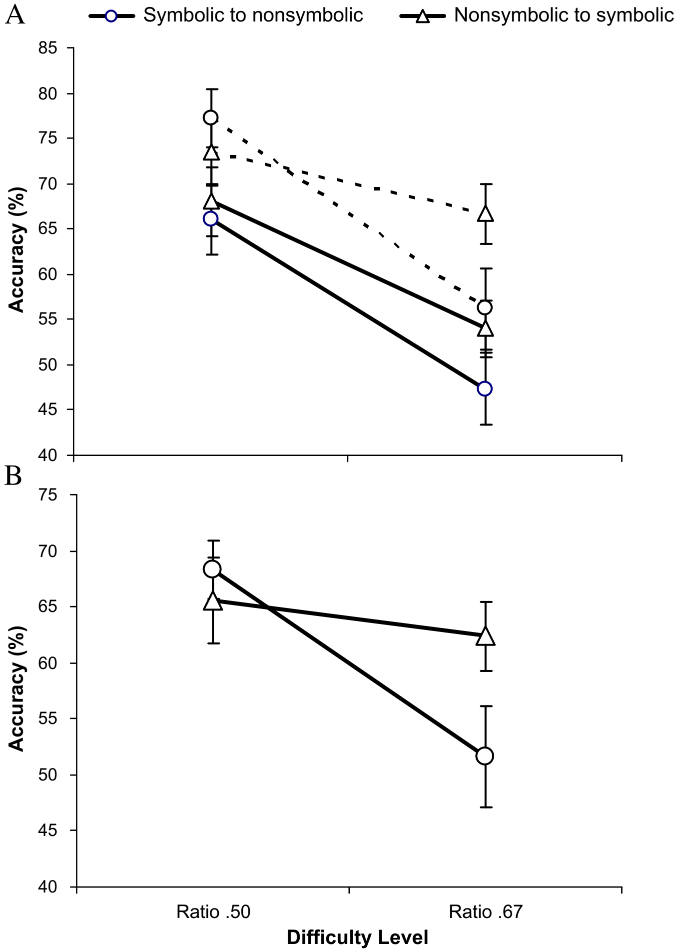

Fig. 1. Children's accuracy on the mapping task by ratio and direction of mapping in Study 1 (A) and Study 2 (B). Solid lines indicate data from Year 2 children, and dashed lines indicate data from Year 4 children.

accurate on problems that involved mapping from a given nonsymbolic representation to two alternative symbols (M = 60.6%, SD = 17.5) than on problems that involved mapping from a given symbol to two alternative nonsymbolic representations (M = 51.9%, SD = 21.8). In fact, children performed at chance level on these symbolic to nonsymbolic problems with the more difficult .67 ratio, $t(51) < 1$, *ns.*

The group effect of age on performance was replicated when results were considered on an individual basis. According to a binomial distribution, the probability of an individual child scoring 17 or more correctly out of 24 trials is $p = .03$. Using this conservative criterion to consider children as performing above chance on an individual basis, 3 of 25 children in Year 2 met this criterion, whereas 14 of 27 children in Year 4 met this criterion. These proportions are significantly different (Fisher's exact test, $p = .003$). Thus, development in the ability to map between nonsymbolic and symbolic representations of number can be seen at both a group level and an individual level between 6 and 8 years of age.

A final set of analyses examined whether children were affected by the perceptual features of the nonsymbolic representations. Children's performances for the symbolic to nonsymbolic mapping problems were compared for stimuli with different continuous quantity controls. Children were more accurate when dot size and total occupied area were positively correlated and density was negatively correlated with numerosity than when density was positively correlated with numerosity and dot size and total occupied area were negatively correlated with numerosity, $t(51) = 7.14$, $p < .001$. This suggests that children can map between symbolic and nonsymbolic representations of number more accurately when they can use the features of dot size and total occupied area to distinguish between alternative representations. This replicates findings from previous research (e.g., Barth et al., 2005).

Study 2

Study 1 revealed that children can map between symbolic and nonsymbolic representations of number and that this ability develops with age. In Study 2, we investigated how this ability and performance on tests of symbolic and nonsymbolic comparison are related to achievement in school mathematics.

Method

Participants

Participants were a new group of 33 children in Year 2 (16 girls and 17 boys) with a mean age of 7 years 4 months (range = 6 years 11 months to 7 years 11 months). Two children did not complete the mapping task or mathematics test due to absence. All children spoke English fluently, and none had a statement of special educational needs. All participants were recruited through school and received stickers to thank them for taking part.

Materials and procedure

Children were tested in two sessions. In the first session, children were tested individually and completed the nonsymbolic comparison task, the symbolic comparison task, and finally the mapping task. In the second session, children were tested in pairs and completed the mathematics test worksheet.

The nonsymbolic and symbolic comparison tasks and mapping task were all presented on a laptop computer. The mapping task was the same as that described in Study 1. The two comparison tasks both consisted of 72 trials. On each trial, two stimuli were presented in the center of the screen and children needed to choose the larger quantity. In the nonsymbolic comparison task, the quantities were presented as dot arrays. In the symbolic comparison task, the quantities were presented as Arabic symbols. In each task, all combinations of the numbers 1 to 9 (except ties) were presented twice. On each trial, children responded by indicating on a keyboard which quantity was larger using two colored keys. Children were asked to respond as quickly and as accurately as possible. In the symbolic version of the task, the digits remained on the screen until children responded. In the nonsymbolic version of the task, the dots remained either until children responded or for 840 ms (whichever was sooner) to prevent children using slow counting procedures to compare the dot arrays. As in Study 1, two sets of dot stimuli were again used to prevent children from consistently using perceptual features to compare the arrays. Three practice trials preceded each task.

The mathematics test was presented on a worksheet that children completed independently. They were given up to 25 min to complete the worksheet. It consisted of 63 questions testing different aspects of school mathematics knowledge. The first half of the test assessed knowledge of symbolic numbers, including identifying the smallest or largest number in a set, ordering numbers, and completing missing items in ascending or descending number lines. The second half of the test assessed calculation skills, including solving addition, subtraction, and multiplication two- and three-term missing number problems, completing calculation pyramids, counting in tens, and solving simple word problems involving doubling or halving quantities. All of these tasks are typical of the mathematics curriculum for children of this age group. The researcher gave simple instructions for each question to children individually.

Results and discussion

As in Experiment 1, children performed above chance on the mapping task ($M = 62.0\%$, $SD = 10.7$), $t(30) = 6.23$, $p < .001$. A two-way repeated-measures ANOVA was conducted on children's accuracy scores with problem difficulty (.50 ratio or .67 ratio) and direction of mapping (nonsymbolic to symbolic or symbolic to nonsymbolic) as factors. There was a significant effect of problem difficulty, $F(1,30) = 8.90$, $p = .006$, and a marginal interaction between problem difficulty and direction of mapping, $F(1,30) = 3.35$, $p = .077$ (see Fig. 1B), replicating the findings of Study 1. The direction of mapping had no effect on performance at the easier ratio ($F < 1$) but did marginally affect performance at the harder ratio, $F(1,30) = 3.91$, $p = .057$. Children were more accurate on problems that involved mapping from a given nonsymbolic representation to two symbols ($M = 62.4\%$, $SD = 17.2$) than on problems that involved mapping from a given symbol to two alternative nonsymbolic representations ($M = 51.6\%$, $SD = 25.2$). As in Study 1, children performed at chance level on symbolic to nonsymbolic problems with the more difficult .67 ratio, $t(30) < 1$, *ns.*

Children showed high, but not ceiling level, accuracy on both the symbolic and nonsymbolic comparison tasks. Accuracy and mean reaction time (RT) for correct trials on the two tasks were compared. Children were more accurate on the symbolic comparison task than on the nonsymbolic comparison task, $t(32) = 2.55$, $p = .016$ (symbolic $M = 90.2\%$, $SD = 5.5$, and nonsymbolic $M = 87.3\%$, $SD = 6.5$), but there was no difference in mean RT for each task, $t(32) = 0.78$, *ns.*

The effect of numerical distance between the comparison items on children's RTs for the symbolic and nonsymbolic comparison tasks was examined. Mean RT for trials answered correctly was plotted against the numerical distance between the items (e.g., comparison trial 4 vs. 9 has a numerical distance of 5). Fig. 2 shows the group mean RTs for each distance for both the symbolic and nonsymbolic comparison tasks. This reveals the characteristic numerical distance effect demonstrated in previous research; RTs were longer when the numerical distance between the items was small, with repeated-measures ANOVA showing the effect of distance on RT for both symbolic tasks, $F(7,26) = 12.87$, $p < .001$, and nonsymbolic tasks, $F(7,26) = 14.59$, $p < .001$. A numerical distance effect (NDE) score was calculated for each child following the method of Holloway & Ansari, 2008b. The NDE was given by calculating $\frac{RT_{small} - RT_{large}}{RT_{large}}$, where RT_{small} was the mean RT for trials with a numerical distance of 1 or 2 and RT_{large} was the mean RT for trials with a numerical distance of 5 or 6. Children's symbolic NDE scores varied from .01 to .54 ($M = .21$), and their nonsymbolic NDE scores varied from .01 to .71 ($M = .26$). There was no difference in the size of the NDE for symbolic and nonsymbolic stimuli, $t(32) = 1.37$, *ns.* The symbolic NDE was significantly correlated with performance on the test of school mathematics ($r = -.52$, $p = .003$) (see Fig. 3), but there was no corresponding relationship between the test of school mathematics and nonsymbolic NDE ($r = .02$, *ns*). The same pattern was also found when only the second half of the mathematics test involving calculation problems was considered (symbolic

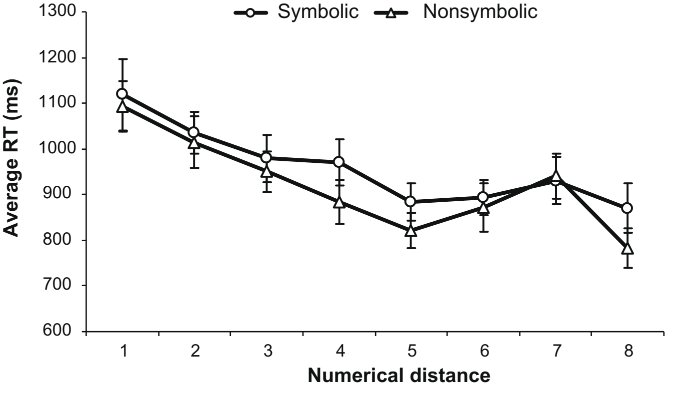

Fig. 2. Mean RTs by numerical distance for symbolic and nonsymbolic comparisons. Error bars represent standard errors.

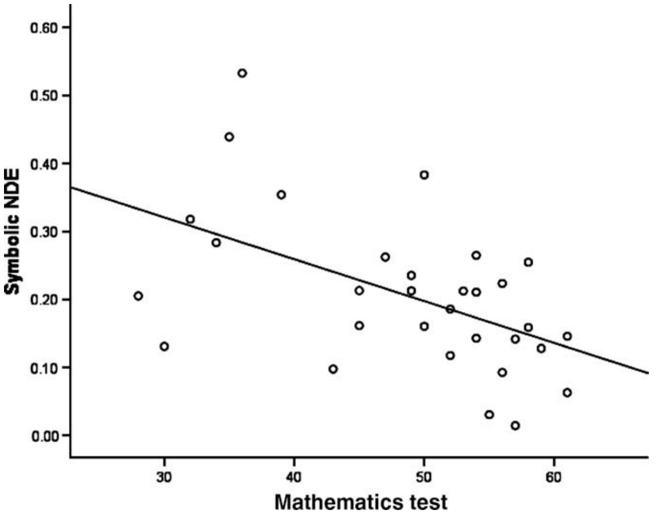

Fig. 3. Relationship between NDE for symbolic comparison task and performance on test of school mathematics.

NDE $r = -.54$, $p = .002$, and nonsymbolic NDE $r = .03$, *ns*). Children with a smaller symbolic NDE, and thus a shallower distance effect curve, scored higher on the test of school mathematics, replicating the finding of Holloway & Ansari, 2008b.

When overall accuracy was considered rather than the size of the NDE, children's performance on the test of school mathematics was correlated with performance on both the symbolic and nonsymbolic comparison tasks ($r = .52$, $p = .003$, and $r = .35$, $p = .05$, respectively) (see Fig. 4). Again these correlations held up when only the calculation part of the mathematics test was considered (symbolic

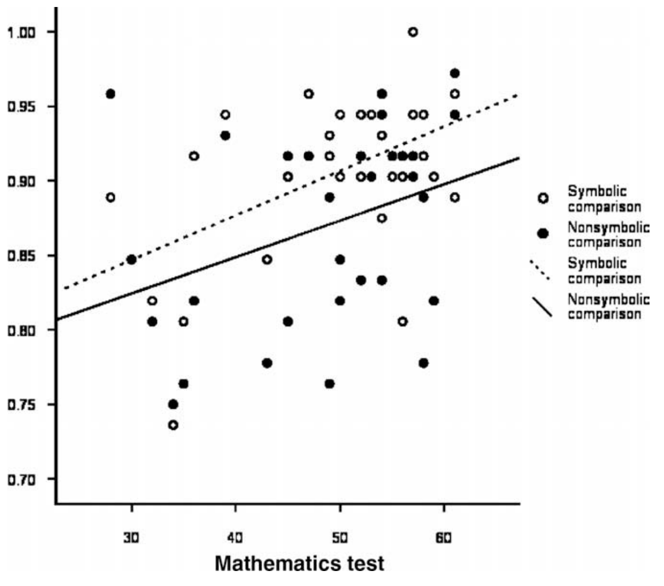

Fig. 4. Relationship between accuracy on symbolic and nonsymbolic comparison tasks and performance on test of school mathematics.

Table 2
Hierarchical regression models predicting performance on test of school mathematics.

Model	Predictor variables	β	Change in R^2	Significance of R^2 change
1	Nonsymbolic comparison accuracy	.349	.122	$p = .047$
	Symbolic comparison accuracy	.434	.155	$p = .017$
	Mapping accuracy	.325	.092	$p = .049$
2	Symbolic comparison accuracy	.504	.254	$p = .003$
	Nonsymbolic comparison accuracy	.167	.023	$p = .335$
	Mapping accuracy	.325	.092	$p = .049$

Note. Outcome variable = accuracy on mathematics test.

accuracy $r = .53$, $p = .002$, and nonsymbolic accuracy $r = .37$, $p = .041$). However, scores on the mapping task were not correlated with performance on the test of school mathematics either when overall mapping score was considered or when symbolic to nonsymbolic and nonsymbolic to symbolic problems were considered separately (overall mapping score $r = .167$, *ns*, difficult ratio only $r = .138$, *ns*, symbolic to nonsymbolic problems $r = .01$, *ns*, and nonsymbolic to symbolic problems $r = .21$, *ns*). Neither was performance on the mapping task related to performance on the comparison tasks (symbolic comparison $r = -.17$, *ns*, and nonsymbolic comparison $r = -.04$, *ns*).

A hierarchical regression was carried out to further examine the relationships among the measures of nonsymbolic and symbolic representations, mapping, and school mathematics. With scores on the mathematics test as the outcome variable, nonsymbolic comparison accuracy score was entered into the model first, followed by symbolic comparison accuracy score and finally mapping score. Performance on the difficult problems for the mapping task was used because they were a more sensitive test of children's mapping ability. All three predictors significantly improved the fit of the model when added in turn (see Table 2). Children's performance on the school mathematics test, therefore, was related to their abilities to compare nonsymbolic representations, to compare symbolic representations, and to map between nonsymbolic and symbolic representations. To assess the relative contribution of scores on the two comparison tasks, a second model was run with symbolic comparison score entered first, followed by nonsymbolic comparison score and finally mapping score. For this model, nonsymbolic comparison no longer made a significant improvement to the fit of the model. Thus, children's ability to compare symbolic representations captures the variance in their ability to compare nonsymbolic representations as well as additional variance perhaps related to knowledge of symbolic representations.

General discussion

The studies reported here demonstrate the importance of basic numerical processes in the development of mathematics. We showed that children are able to map between symbolic and nonsymbolic representations of number and that this ability develops with age, is not symmetrical across direction of mapping, and is related to the performance of school mathematics.

Using a novel task, we found that children can map in both directions between symbolic and nonsymbolic representations of number when the ratio involved is .50. Previous work has assessed this ability in adults, but no prior studies have directly examined whether children are able to do so. We demonstrated that children from 6 years of age are able to choose the correct symbolic representation for a nonsymbolic array and vice versa and that this ability develops with age. A connection between symbolic and nonsymbolic representations of number in children had previously been inferred from the presence of a numerical distance effect on children's ability to compare symbolic representations. However, this effect demonstrates only that nonsymbolic representations interfere with children's processing of symbolic representations. We demonstrated that children can directly access the mapping between representations when required.

We found development in children's ability to directly map between symbolic and nonsymbolic representations between 6 and 8 years of age. Previous work has found that there is change in both the symbolic and nonsymbolic distance effects over this period (Holloway & Ansari, 2008a, 2008b; Sekuler & Mierkiewicz, 1977). The increased precision of children's nonsymbolic representations (demonstrated by change in the nonsymbolic distance effect) may lead to both development in the symbolic distance effect and development in mapping ability. If nonsymbolic representations are more precise, then the effect of numerical distance on symbolic comparison will also be reduced and nonsymbolic representations can be more precisely mapped onto symbolic representations. This does not necessitate a direct relationship between change in the symbolic distance effect and mapping ability. However, work with adult participants has suggested that variability in nonsymbolic comparison tasks can account for only a small proportion of the variance in mapping tasks (Krueger, 1984). The causal relationships among development in symbolic comparison, nonsymbolic comparison, and mapping ability require further investigation.

The novel task we developed also allowed children's mapping ability to be assessed in two directions: from a symbolic target to two alternative nonsymbolic options and from a nonsymbolic target to two alternative symbolic options. Previous studies have typically examined mapping in only one direction by asking participants to produce a symbolic label for a given nonsymbolic representation. We found that children were more accurate at choosing a symbolic label to match a nonsymbolic target than vice versa. In fact, children were unable to map from a symbol to nonsymbolic representations when the ratio between the alternative nonsymbolic representations was .67, but they were able to make the mapping in the opposite direction at the same ratio. This asymmetry in mapping ability might stem from the precision of each of the representations. Izard and Dehaene (2008) proposed a model of the mapping between symbolic and nonsymbolic representations. They suggested that nonsymbolic representations are encoded on an internal number line and that symbolic representations are mapped onto this number line by means of a response grid; the number line is divided into segments, with each one being associated with a different verbal label. A symbolic stimulus is associated with a precise point on the number line, but nonsymbolic representations are approximate; therefore, a given nonsymbolic stimulus probabilistically activates a region of the number line according to a Gaussian distribution. The mapping task used in the current studies involves three quantities: the target and two alternatives. The symbolic to nonsymbolic version of the mapping task involves one precise and two approximate regions of the number line, whereas the nonsymbolic to symbolic version of the mapping task involves one approximate and two precise regions of the number line. As a result, children may be more accurate at the nonsymbolic to symbolic version, which involves fewer approximate representations. The disadvantage of dealing with two approximate representations will be greater when there is greater overlap of the distributions of activation. This would lead to the interaction between direction of mapping and difficulty ratio observed in both Studies 1 and 2.

We found that children's ability to compare both nonsymbolic and symbolic representations was related to their knowledge of school mathematics. This extends previous work that found that the symbolic distance effect, but not the nonsymbolic distance effect, predicted performance on standardized tests of mathematics (Holloway & Ansari, 2008b). In the current studies, we also found that the symbolic distance effect, but not the nonsymbolic distance effect, was related to children's performance on a test of school mathematics. However, when children's overall accuracy on the comparison tasks was considered rather than the distance effect, both symbolic comparison accuracy and nonsymbolic comparison accuracy were related to mathematics performance. The difference in these patterns of relationships might stem from the different measures used. Whereas accuracy on comparison tasks assesses children's abilities in making comparisons between symbolic and nonsymbolic representations, the distance effect indexes children's difficulties in making certain comparisons. Although the distance effect is an important element of some aspects of children's representations, it does not fully capture their ability at processing these representations. It is important, therefore, that multiple indexes of children's nonsymbolic system be used in future research because a single measure such as the distance effect does not give the full picture of the role of the nonsymbolic system in mathematics.

We found that children's ability to map between symbolic and nonsymbolic representations was related to their performance on the school mathematics test over and above the influence of performance on the two comparison tasks. The ability to map directly between representations is an important basic numerical skill and is not fully captured by performance on a symbolic comparison task. Therefore, performance on symbolic comparison tasks should not be used as a sole index of an individual's ability to map between these representations. The influence of children's mapping scores on their mathematics test performance was found in the hierarchical regression models despite the lack of a simple correlation between the mathematics test and the mapping task. Mapping score was a significant predictor only after performance on the comparison tasks was controlled. This suggests that there is a complex relationship among these variables such that mapping score interacts with performance on the comparison tasks. It is possible that children's differing abilities to compare symbolic and nonsymbolic stimuli mask the relationship between mapping skills and mathematics test results because the ability to distinguish between nonsymbolic stimuli is an important prerequisite to the ability to accurately map those stimuli onto symbolic representations.

How might mapping ability be related to performance of symbolic mathematics? More precise mapping between the symbolic and nonsymbolic systems could allow children to more effectively harness the power of the nonsymbolic system when comparing and manipulating symbolic representations. This in turn may lead to more accurate mathematics performance. Moreover, Booth and Siegler (2008) found that improved mapping (between symbolic and number line representations of quantity) not only was related to children's mathematics performance but also predicted their ability to learn new arithmetic skills. This might suggest that improving the relationship between children's nonsymbolic and symbolic representations more generally might help during the early stages of learning mathematics.

An alternative interpretation of the relationship between mapping ability and performance of symbolic mathematics is that improved knowledge of the symbolic system could lead children to be able to map more precisely between symbolic and nonsymbolic representations. Lipton and Spelke (2005) found that 5-year-olds who had good knowledge of the verbal counting system had better mapping ability than children of the same age whose knowledge of the counting system was poor. The current studies were unable to establish the causal direction of this relationship, and so future research is needed.

Over the early years of school mathematics instruction, children acquire greater knowledge of the symbolic system and also show development in the ability to manipulate and map between representations of quantity. In the current studies, we showed that these improvements are related, but the causal direction of this relationship is as yet undetermined. Discovering whether increased ability to manipulate symbolic representations leads to improved mapping between the symbolic and nonsymbolic systems, or whether improved mapping between these systems leads to improvement in symbolic number skill, is a valuable aim for future research that may have important implications for mathematics education.

References

Barth, H., Kanwisher, N., & Spelke, E. S. (2003). The construction of large number representations in adults. *Cognition, 86*, 201–221.

Barth, H., La Mont, K., Lipton, J., & Spelke, E. S. (2005). Abstract number and arithmetic in preschool children. *Proceedings of the National Academy of Sciences of the United States of America, 102*, 14116–14121.

Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development, 79*, 1016–1031.

Butterworth, B. (1999). *The mathematical brain*. London: MacMillan.

Cordes, S., Gelman, R., Gallistel, C. R., & Whalen, J. (2001). Variability signatures distinguish verbal from nonverbal counting for both large and small numbers. *Psychonomic Bulletin and Review, 8*, 698–707.

Dehaene, S. (1997). *The number sense*. Oxford, UK: Oxford University Press.

Gilmore, C. K., & Inglis, M. (2009). Automatic access to numerosity in symbolic arithmetic, submitted for publication.

Gilmore, C. K., McCarthy, S., & Spelke, E. S. (2007). Symbolic arithmetic knowledge without instruction. *Nature, 447*, 589–591.

Hollingsworth, W. H., Simmons, J. P., Coates, T., & Cross, H. A. (1991). Perceived numerosity as a function of array number, speed of array development, and density of array items. *Bulletin of the Psychonomic Society, 29*, 448–450.

Holloway, I. D., & Ansari, D. (2008a). Domain-specific and domain-general changes in children's development of number comparison. *Developmental Science, 11*, 644–649.

Holloway, I. D., & Ansari, D. (2008b). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement. *Journal of Experimental Child Psychology, 103*, 17–29.

Izard, V., & Dehaene, S. (2008). Calibrating the mental number line. *Cognition, 106*, 1221–1247.

Jordan, K. E., Suanda, S. H., & Brannon, E. M. (2008). Intersensory redundancy accelerates preverbal numerical competence. *Cognition, 108*, 210–221.

Krueger, L. E. (1984). Perceived numerosity: A comparison of magnitude production, magnitude estimation, and discrimination judgments. *Perception & Psychophysics, 35*, 536–542.
Lipton, J. S., & Spelke, E. S. (2005). Preschool children's mapping of number words to nonsymbolic numerosities. *Child Development, 76*, 978–988.
McCrink, K., & Wynn, K. (2004). Large number addition and subtraction by 9-month-old infants. *Psychological Science, 15*, 776–781.
Moyer, R. S., & Landauer, T. K. (1967). Time required for judgements of numerical inequality. *Nature, 215*, 1519–1520.
Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science, 306*, 499–503.
Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition, 102*, 361–395.
Sekuler, R., & Mierkiewicz, D. (1977). Children's judgements of numerical inequality. *Child Development, 48*, 630–633.
Temple, E., & Posner, M. I. (1998). Brain mechanisms of quantity are similar in 5-year-old children and adults. *Proceedings of the National Academy of Sciences of the United States of America, 95*, 7836–7841.
van Oeffelen, M. P., & Vos, P. G. (1982). A probabilistic model for the discrimination of visual number. *Perception and Psychophysics, 32*, 163–170.
Whalen, J., Gallistel, C. R., & Gelman, R. (1999). Nonverbal counting in humans: The psychophysics of number representation. *Psychological Science, 10*, 130–137.
Zorzi, M., & Butterworth, B. (1999). A computational model of number comparison. In M. Hahn & S. C. Stoness (Eds.), *Proceedings of the twenty first annual conference of the Cognitive Science Society*. Mahwah, NJ: Lawrence Erlbaum.