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Research Report

NONVERBAL COUNTING IN HUMANS: The Psychophysics of Number Representation

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Abstract—In a nonverbal counting task derived from the animal literature, adult human subjects repeatedly attempted to produce target numbers of key presses at rates that made vocal or subvocal counting difficult or impossible. In a second task, they estimated the number of flashes in a rapid, randomly timed sequence. Congruent with the animal data, mean estimates in both tasks were proportional to target values, as was the variability in the estimates. Converging evidence makes it unlikely that subjects used verbal counting or time durations to perform these tasks. The results support the hypothesis that adult humans share with nonverbal animals a system for representing number by magnitudes that have scalar variability (a constant coefficient of variation). The mapping of numerical symbols to mental magnitudes provides a formal model of the underlying nonverbal meaning of the symbols (a model of numerical semantics).

Animal subjects represent both number and duration by mental magnitudes (Church, 1984; Gibbon, Church, & Meck, 1984; Meck & Church, 1983). These representations are formally analogous to points on the real number line. Meck and Church (1983) proposed such a representation to account for animals' nonverbal counts of objects or events (Fig. 1). According to their theory, each item is enumerated by an impulse of activation that is added to an accumulator. The magnitude in the accumulator at the end of the count is read into memory, where it represents the number of the counted set. The noise (trial-to-trial variability) in remembered magnitude is proportional to the magnitude, a property that Gibbon (1977) called scalar variability.¹ Mathematical modeling of psychophysical data from a variety of tasks indicates that memory, rather than the process of accumulation or comparison, is the dominant source of trial-to-trial variability (Gibbon, 1992).

Animal experiments give direct evidence for scalar variability in the representation of numerosity. For example, Platt and Johnson (1971) varied the number of bar presses required for rats to arm a feeder that was activated by the interruption of a light beam. They plotted the distributions of bar presses before the rats tried the feeder, and found that the actual number of presses was roughly normally distributed about the required number for the reward, and that the standard deviation of the distribution increased in direct proportion to the number of presses required (see Fig. 2a). Although these experiments did not attempt to deconfound discriminations based on number or time duration, pigeons and rats have been shown to use both measures

within a given trial (Fetterman, 1993; Wilkie, Webster, & Leader, 1979).

Platt and Johnson's (1971) data have been redrawn in Figure 2b to reveal the mean response, standard deviation, and coefficient of variation. The constant coefficient of variation across target sizes in Figure 2b reflects the direct relation between the magnitude of the target number and the response variation.

It has been suggested repeatedly that human numerical reasoning also depends on a mapping of Arabic numerals and number words into mental magnitudes that obey Weber's law (Dehaene & Akhavein, 1995; Moyer & Landauer, 1967). This hypothesis explains the surprising but robust effects of numerical magnitude on the reaction time to judge numerical inequality in well-practiced human subjects. The bigger the arithmetic difference between two numbers, the more rapidly subjects judge which is greater. And for a fixed difference, the bigger the two numbers, the more slowly subjects judge their numerical ordering (Moyer & Landauer, 1967).

One explanation of these reaction time effects in humans assumes that humans share with nonverbal subjects a representation of number by means of mental magnitudes with scalar variability (Gallistel & Gelman, 1992). When mental magnitudes have scalar variability, the discriminability of the values obeys Weber's law, because the degree of overlap between representations remains constant when the ratio of the means is held constant. Thus, the amount by which two scalar quantities (two weights, two durations, two numbers, etc.) must differ in order to meet a constant criterion of discriminability is proportional to their magnitude (Weber's law).

The present series of experiments was designed to provide a direct, quantitative evaluation of the hypothesis that humans also use noisy magnitudes to represent number, by approximately replicating animal paradigms with humans. Our results provide evidence that human subjects have a nonverbal, magnitude-generating counting process and a bidirectional mapping between verbal number symbols and the magnitudes generated by this process. The results also provide evidence of scalar variability in the magnitudes.

KEY-PRESS EXPERIMENT

This experiment was designed to approximately replicate experiments in which animals were required to press a key a specific number of times (Mechner, 1958; Platt & Johnson, 1971). We sought to determine if human performance would reveal the same psychophysical properties as the animal data if the task limited humans to the use of nonverbal counting. (We assume that humans can produce the exact number of key presses by verbally counting "one, two, three. . ." Thus, verbal counting will not exhibit scalar variability.) This experiment tested for three things: (a) a nonverbal counting process, (b) the subject's ability to map from a written numerical symbol to a remembered mental magnitude, and (c) scalar variability in the remembered mental magnitudes.

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1. Gibbon's term was actually scalar *variance*, but that term is confusing because it suggests that the variance of the distribution increases in proportion to the mean of the distribution, when in fact it is the standard deviation (square root of the variance) that is scaled to (proportional to) the mean.

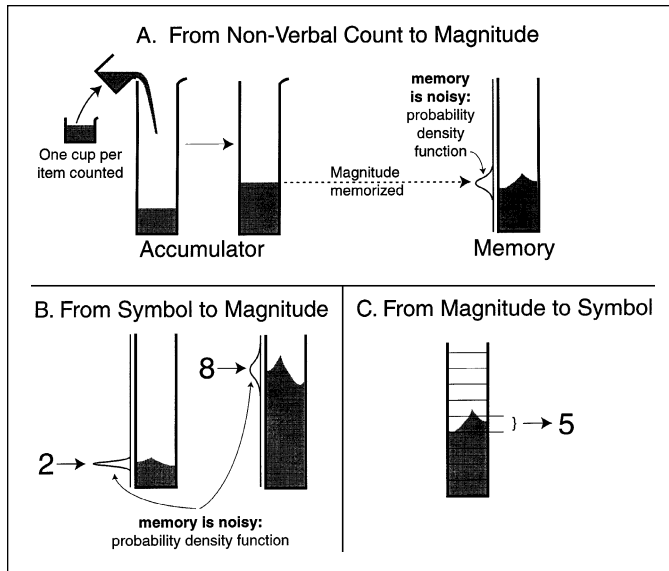


Fig. 1. Cartoon of cognitive processes underlying nonverbal number abilities. Nonverbal counting generates magnitudes, which become the semantic representation of numbers when read into memory (a). The probability density function for the memorized magnitude indicates the distribution of scalar noise. Because of this memory noise, the mapping from a numerical symbol to the corresponding magnitude in semantic memory generates a comparison magnitude that varies from trial to trial (b). Also, the mapping from the magnitude generated by a given nonverbal count to the corresponding symbol does not invariably produce the same symbol, as indicated by the area of the probability density function beyond the decision criteria for a given magnitude-to-symbol mapping (c).

Method

There were two modifications to the animal procedure. First, an Arabic numeral was presented on each trial to indicate the target number of presses for that trial; no repeated selective reinforcement training was required. Second, given that our focus was on nonverbal representations of number, we wanted to limit overt or covert verbal counting—a consideration unique to human subjects. Subjects were required to perform the task under speed pressure, producing key presses as fast as possible. Human adults can press a key about once every 120 ms, significantly faster than typical rates of overt and covert verbal counting, which are reported to be about 240 ms per item (Klahr, 1973; Landauer, 1962). Thus, strong speed pressure was used to make overt or subvocal verbal counting strategies unlikely. No such speed limitations are thought to constrain nonverbal, accumulator mechanisms (pigeon subjects can count key pecks produced at the rate of 6 per second; Rilling & McDiarmid, 1965). Subjects were specifically instructed not to verbally count the number of presses they made, but rather to arrive at the appropriate number of presses “by feel.” No feedback regarding accuracy was provided.

Stimuli and trial description

Trials began with a “Ready?” message in the center of the screen. When the subject pressed the key of a computer joystick, the “Ready?” message was replaced with an odd Arabic numeral from 7 through 25.

The subject was to press a key approximately that number of times as fast as possible. The first key press of the response removed the Arabic numeral from the screen. The subject signaled the completion of his or her response by pressing the second of the two keys on the console of the computer joystick.

Subjects and apparatus

Seven undergraduate and graduate students at the University of California, Los Angeles (2 male, 5 female) volunteered. They were extensively tested in this experiment over eight hourly sessions (which also included other related tasks). All subjects reported normal acquisition of counting and elementary arithmetic abilities when asked about developmental delays and learning disabilities. Each subject provided 40 sets of key presses to each odd number from 7 through 25. Stimuli were presented using an Apple IIvx computer and Psyscope stimulus presentation software (Cohen, MacWhinney, Flatt, & Provost, 1993). Responses were registered using the buttons on a computer joystick connected to a Psyscope Buttonbox, which recorded responses and latencies.

Results

The results are strikingly similar to the animal data. Each individual subject responded to a particular stimulus (e.g., 19) with approximately the correct number of key presses, with the average number of presses increasing linearly with target value. For all subjects, the standard deviation in the number of key presses produced varied in direct proportion to the target magnitude (Fig. 3). For example, observe that Subject 1’s average number of key presses was 9.3 for the target “9,” with a standard deviation of 1.1; for a target of “19,” the mean was 20.4, with a standard deviation of 2.4. The coefficient of variation (the ratio of the standard deviation and mean) is constant across target size.

Our results are consistent with the notion that adult humans and animals have comparable nonverbal representations of number magnitude. However, this conclusion depends on our assumption that the performance we measured depended on nonverbal counting.

Were subjects verbally counting?

Of particular concern is the possibility that subjects were verbally counting either overtly or covertly to produce the appropriate number of presses. Anecdotal reports from subjects suggest they did not verbally count (“I could stop myself from counting”; “If I tried to count, I consistently lost track anyway because I pressed [the button] faster than I could count”). These reports were evaluated by comparing key-press latencies and verbal-counting latencies.

Subvocal-counting latencies were measured in the first and last experimental sessions. Subjects were presented with Arabic numerals in the same range as in the key-press paradigm and pressed a button when they had silently counted as fast as possible to the presented number. Given the number of trials in the key-press experiment (400), we expected practice benefits between the first and last sessions if verbal counting was used to perform the key-press task, but no difference in verbal-counting latencies was found, $t(6) = 0.04$. In Figure 4, we contrast the total time Subject 1 took to produce key-press responses and her subvocal-counting times. A total of 20 trials per item was used to calculate mean silent-counting latencies. As can be readily observed, her verbal-counting latencies were typically much greater than the total response time for her key presses for the same target

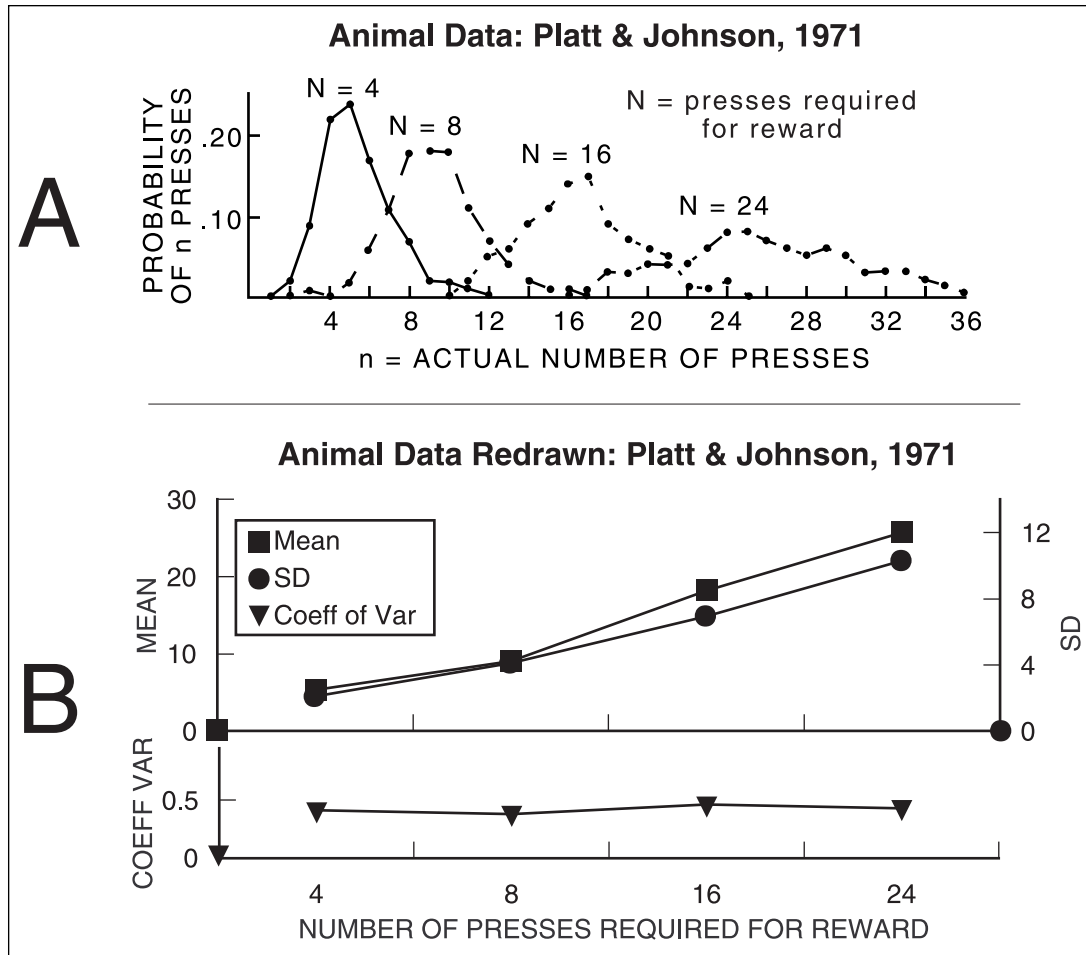


Fig. 2. Evidence of scalar variability in the rat's representation of number. The graph in (a), which is reprinted from Platt and Johnson (1971), shows probability distributions for conditions in which different numbers of presses were required for reward. In each case, the rat's most probable response was the same number of presses that was required for reward, although the variability in responses increased as the magnitude of the target number increased. Panel (b) graphs the means, standard deviations, and coefficients of variation ("Coeff Var") from the same response data as a function of the number of presses required for reward. The means and standard deviations are directly related, resulting in a constant coefficient of variation.

number. Further, counting times significantly increased in the teens and 20s (e.g., "thirteen," "fourteen," "fifteen"; "twenty-one," "twenty-two") because the terms contain more phonemes and words. Key-press times showed no such effect, suggesting subvocal verbal counting was not used to estimate number.

Another possible counting strategy might involve chunking presses into groups and counting the number of groups (e.g., fives, tens). Were this the case, we would expect systematic variation in the key-press-to-key-press intervals at boundaries between chunks. For example, if the subjects was counting by fives, we might anticipate slower responses for every 5th press. However, press-to-press intervals were virtually identical for the 2nd through 25th presses for each individual subject, with means of 115 to 127 ms and standard deviations of approximately 30 ms. No systematic deviations (e.g., every 5th press slower or faster) were observed, suggesting that chunking strategies were not employed to judge the number of key presses. Given the increases in production time for the number words with more syllables

(e.g., teens, 20s), it is hard to see how this unchanging interpress interval could obtain if verbal counting were involved.

Were subjects using time durations?

The consistency of the intervals between key presses resulted in a strong correlation between the total response time and the number of key presses. Therefore, subjects might have used total response time rather than number of key presses as a stopping criterion in the key-press experiment. To evaluate this hypothesis, we presented subjects with a tone and asked them to replicate the tone's duration by controlling the start and stop of a second tone with button presses. The tone durations were in the range of the total key-press response times, varying between 1,700 ms and 2,300 ms and differing by 120 ms. Each tone was presented 10 times over the course of the two final testing sessions. The results were used to calculate a coefficient of variation for timing ability as a measure of timing precision. The coefficient of

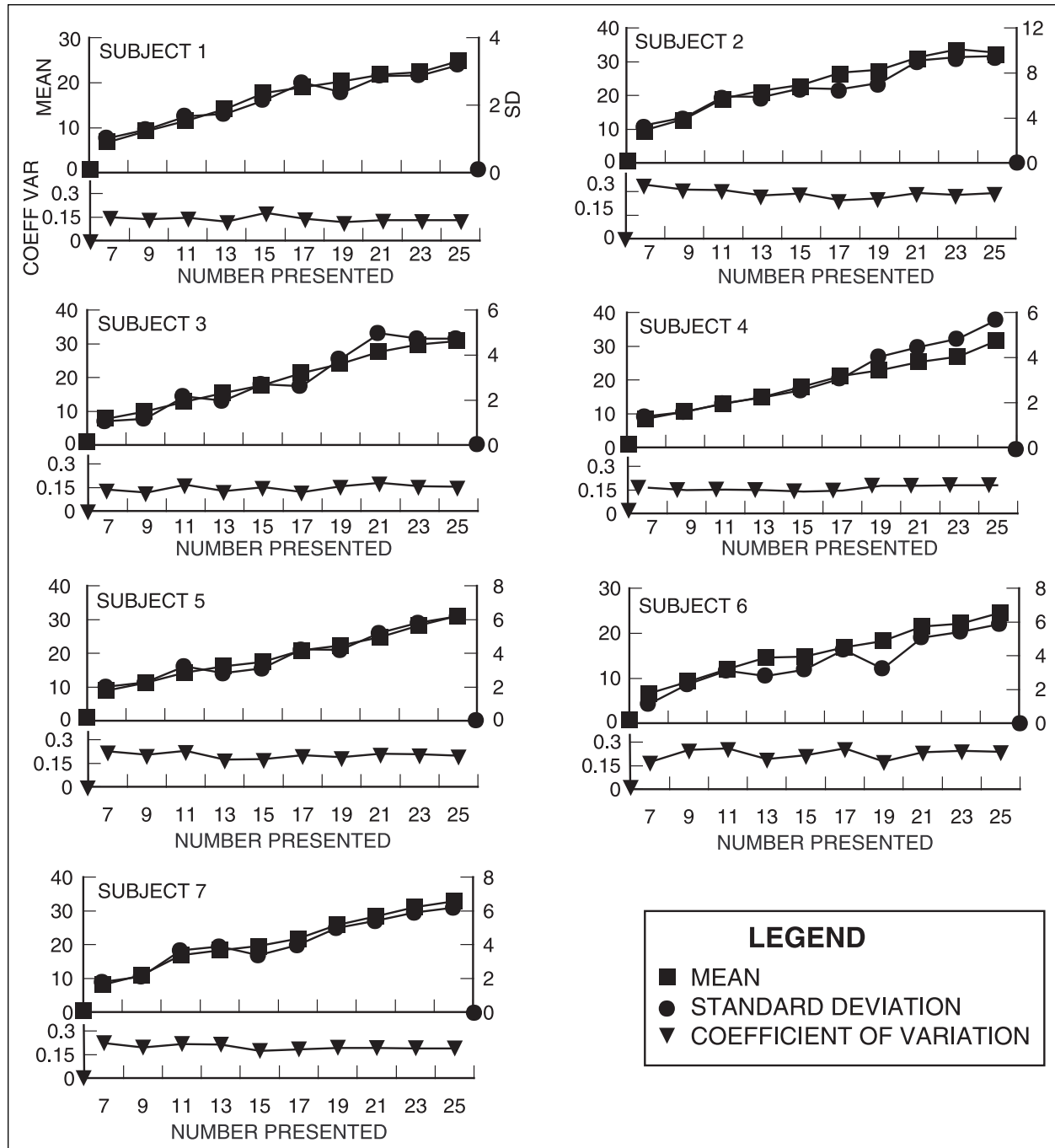


Fig. 3. Key-press experiment results revealing mean number of presses, standard deviation, and the coefficient of variation across different set sizes. Note that the scales for mean and standard deviation vary for each subject to emphasize that in each case the increase in the standard deviation is proportional to the increase in the mean. The coefficient of variation is the constant of proportionality. The appearance of equality between the means and standard deviations results from using different scales on the left and right ordinates to emphasize that the increase in the standard deviation is proportional to the increase in the mean.

variation for timing judgments (ranging from 0.25 to 0.35) was much greater than that found for the variation in the times taken to complete a run of presses for any one target in the key-press task (0.081 to 0.19). If subjects were targeting the time required to give a certain number of

presses in the key-press task, then when they were explicitly targeting comparable duration in a timing task, they should have shown no greater variability. In fact, however, their temporal variability was substantially greater in the timing task than in the key-press task. Our

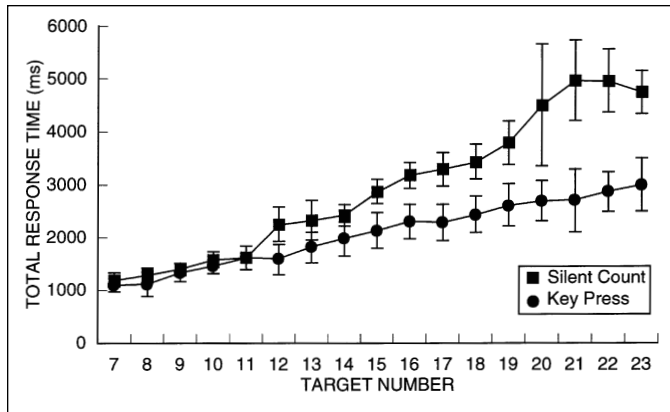


Fig. 4. Total silent-counting durations and total key-press durations across set sizes for Subject 1. Vertical ranges represent ± 1 SD.

findings, consistent with recent reports of human timing precision (Wearden, Denovan, Fakhri, & Haworth, 1997), reveal that timing accuracy is too variable to have generated the low-variability response durations in the numerical key-press task.

Subjects also estimated the amount of time their 7 to 25 key presses took on average. Each subject was presented with the odd numerals from 7 through 25 and asked to produce a time duration equal to the total duration of their key-press response in Experiment 1. Subjects produced the duration by hitting a button twice, once to start and once to stop a timer. They were instructed not to press the button or tap their finger (simulating button pressing) during this interval. Almost all subjects greatly overestimated the duration of their responses in the numerical task. For example, Subject 1's estimate of the time taken to press the key 21 times (6.1 s) was almost 4 s, or 200%, larger than the actual duration (2.1 s).

In short, subjects did not know with any accuracy the time it took them to produce a specific number of key presses, and the variability in their attempts to reproduce a just-heard duration was much greater than the variability in the duration of their nonverbal counts. Both findings indicate rather strongly that subjects did not rely on estimated duration rather than on estimated number.

Discussion

In summary, the evidence presented thus far supports the notion that human adults represent number nonverbally in terms of a linear magnitude with scalar variability, a representation that is qualitatively and quantitatively similar to that found in animals. The additional analyses suggest performance in the key-press task depended on a nonverbally generated representation of the number of key presses. Rapid subvocal counting is unlikely as the basis of how the subjects determined the number of presses in light of both the rapidity of the counts (125 ms per item) and the fact that the intercount interval showed no effect of the number of syllables in the count words.

However, one possibility we have not yet considered is that judgments of number from one's own behavior might employ a mechanism very different from the one used to passively obtain number from an external source in the world. Another possibility is that the repeated motoric movements may have influenced the response variability reported in the key-press experiment. The next experiment addressed these issues by asking subjects to judge the number of flashes in a rapid sequence.

FLASH-COUNT EXPERIMENT

In another well-studied numerical paradigm involving animals, the experimenter presents a series of tones or flashes and requires the animal to make a decision about the numerosity of that series (Meck & Church, 1983; Wearden et al., 1997). Our second experiment was designed to replicate approximately this animal paradigm with humans, provide a measure of nonverbal counting from a passively obtained numerical stimulus, and reduce the correlation between time and number.

Method

Subjects were presented with a dot that repeatedly flashed on and off in one location, and were asked to say approximately how many times the dot flashed, without verbally counting.

Stimuli and trial description

Each trial began 750 ms after the subject pressed a button to remove a "Ready?" message. All stimulus dots had identical visual angle (5°), circular shape, black color, and central location. The number of dot flashes varied between the odd numbers from 7 through 25. To ensure that the time when a dot would occur could not be predicted, and to reduce the correlation between total presentation time and the number of flashes, we varied the duration of each individual dot presentation and the interstimulus interval according to a geometric approximation to a Poisson (random-rate) process with an expected interevent interval of 83 ms. There was a constant probability of .167 that a flashed dot would terminate after each successive 16-ms (60-Hz) screen refresh cycle. This gave a discrete distribution of flash durations, with 16-ms increments between successive possible durations, and with the frequency of successive values in the distribution given by the geometric series p , $p(1-p)$, $p(1-p)^2$, and so on ($p = .167$). The intervals between flashes had the same distribution. Thus, the expected interval from the onset of one dot flash to the onset of the next was 166 ms, considerably shorter than the count-to-count interval in rapid subvocal counting, as estimated from our own key-press experiment and the subitizing literature (Kaufman, Lord, Reese, & Volkman, 1949; Mandler & Shebo, 1982).

At the end of each flash sequence, the question "How many?" appeared on the screen, and the subject stated the number of dots he or she had seen. Subjects were specifically instructed to approximate, or go "by feel," and not to verbally count the flashes.

Subjects and apparatus

The same 7 student volunteers who participated in the key-press experiment also participated in this experiment. Each subject provided 40 estimates for each number of flashes over the course of eight hourly sessions (which also included related tasks). Presentation and recording apparatus were the same as in the previous experiment.

Results

The similarity in the pattern of performance between experiments was remarkable. Presented in Figure 5 are the results from all subjects. Mean estimates of number of dot presentations were approximately correct, and the response means and standard deviations increased in direct proportion to the target, resulting in a constant coefficient of variation across set size.

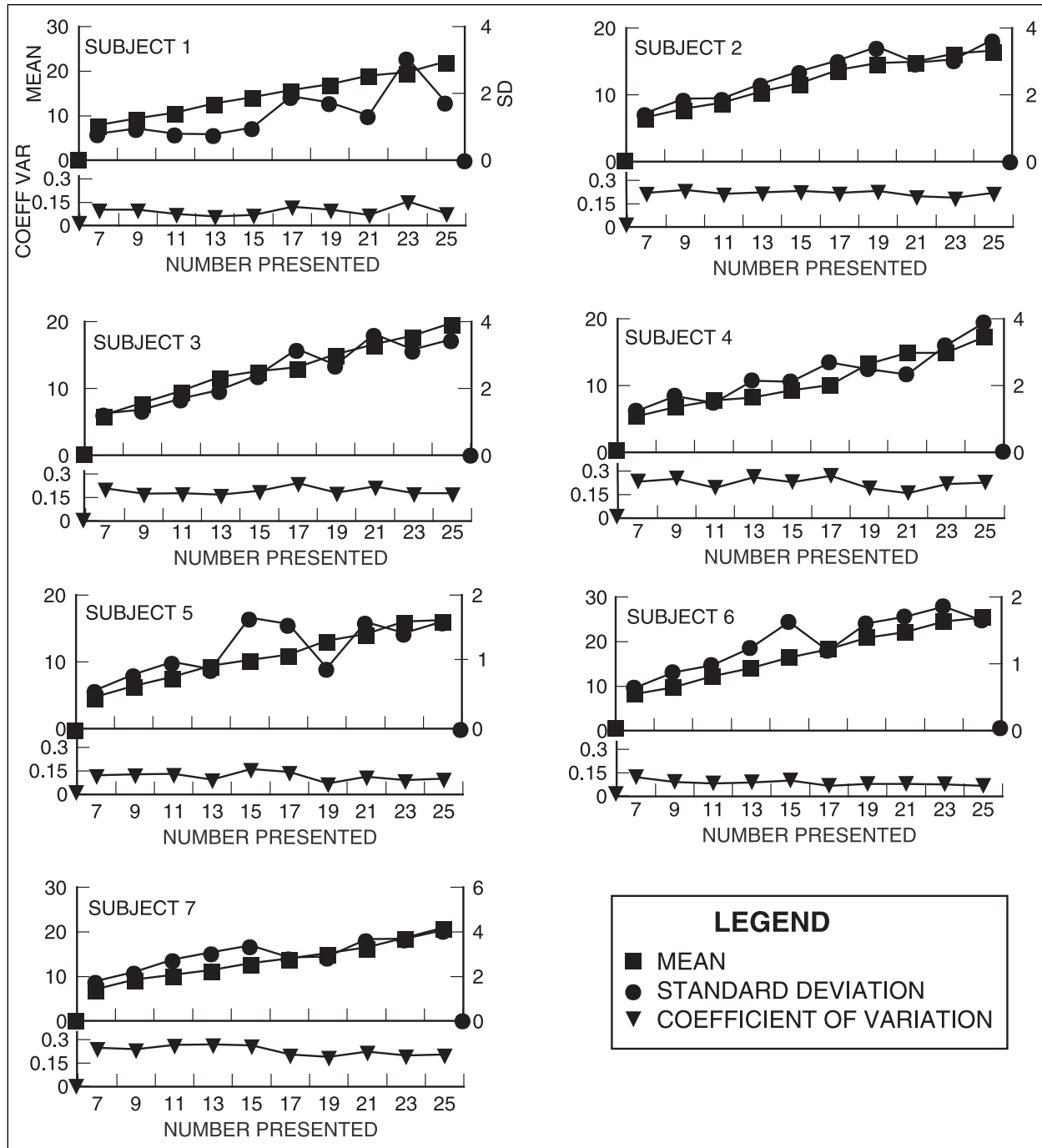


Fig. 5. Flash-count experiment results revealing mean reported number of flashes, standard deviation, and the coefficient of variation across different numbers of flashes. Note that the scales for mean and standard deviation vary for each subject to emphasize the proportional increase in the standard deviation. The appearance of equality between the means and standard deviations results from using different scales on the left and right ordinates to emphasize that the increase in the standard deviation is proportional to the increase in the mean.

Were subjects subvocally counting?

Again, we considered the possibility that subjects verbally counted the number of dots presented. A contrast of total presentation time with covert counting times revealed that dot presentation rates were

typically significantly faster than corresponding covert verbal-counting rates, making verbal counting an unlikely strategy. For example, Figure 6 contrasts Subject 1's mean silent-counting latencies with corresponding durations for the dot-flash sequences.

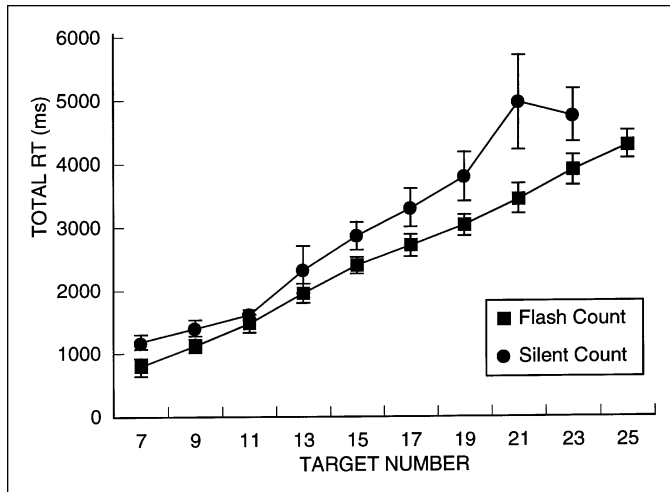


Fig. 6. Total silent-counting durations and total duration of flash sequences across set sizes for Subject 1. Vertical ranges represent ± 1 SD. RT = response time.

Were subjects using time duration?

We must also consider if subjects were using estimates of the duration of dot-flash sequences to estimate numerosity, rather than using nonverbal counts. To evaluate this possibility, we performed a multiple regression in which the actual number of dots and total duration of the dot-flash sequence were used to predict the reported number of dots. For each individual subject, when both number and time duration were entered into the regression, the actual number of dots was a significant predictor of the reported number, whereas total time duration was not. Further, when variation attributed to total time duration was partialled out, there was still a significant correlation between actual and reported number of dots, $r^2 = .21$, $p < .01$.

Discussion

In summary, we found no evidence to support the notion that the estimates of the number of flashes were generated using either verbal counting or timing strategies. Rather, the results are best interpreted as representing the precision with which people can map from a mental magnitude generated by a nonverbal counting process to the corresponding verbal symbol for numerosity. The similarity in the patterns of performance in this experiment (in which subjects were passively obtaining the number) and the previous experiment (in which subjects produced the quantities) implies that a common representation of number was involved.

GENERAL DISCUSSION

This series of experiments provides evidence that humans possess a nonverbal representation of natural numbers that is qualitatively and quantitatively similar to the representation found in nonverbal animals. Our findings suggest that number is represented by a mental magnitude that is approximately proportional to the number and has scalar variability when retrieved from memory—hence a coefficient of variation that is constant across set size. In other words, the semantics of number (the psychological meaning) is given by the process that maps from numerosity to magnitudes,

and from symbols to magnitude, and by the processes that operate on those magnitudes (e.g., the processes that mediate making decisions about the relative magnitude of numbers represented by verbal or written symbols).

Scalar variability is inconsistent with a verbal or nonverbal count using discrete mental number symbols rather than noisy magnitudes. In such a model, miscounts (skips and double-counts) are the dominant source of variability. The count on any one trial is $n - n_s + n_d$, where n is the number of items and n_s and n_d are the number of skips and double-counts (counting errors). The variance in the counting errors will equal $np(1-p)$, where p is the probability of an error in the counting of any one of the n items. Thus, the standard deviation of the error distribution will increase in proportion to the square root of n , which means that the coefficient of variation should decline by a factor of almost 2 between $n = 7$ and $n = 25$.

Several theories of number processing now include a representation of the magnitude the numerals represent (Campbell, 1994; Dehaene & Cohen, 1995; McCloskey, Macaruso, & Whetstone, 1992). However, few provide specific details about the nature of this representation. For example, McCloskey (1992) proposed that numerical quantities are represented by a semantic, amodal representation that includes a representation of each individual numeral (0–9) and its base-10 place value. However, McCloskey did not detail how the magnitudes 0 through 9 are represented abstractly within this theory. Our findings suggest that representation of numerical magnitude might best be characterized with an accumulator-like representation.

These results may also serve to unify several experimental findings within numerical cognition. For example, an underlying representation of magnitude that obeys Weber's law can account for the findings from magnitude comparison and same/different judgments (Dehaene & Akhavein, 1995; Moyer & Landauer, 1967). Nonverbal magnitude representations with scalar variability can also account for counting abilities both within and well beyond the so-called subitizing range of 1 to 6 items. Subitizing itself may be rapid nonverbal counting followed by a mapping from the resulting magnitude to the corresponding number word, with scalar variability in the magnitudes limiting the reliability of the procedure to numerosities of 4 or fewer (Gallistel & Gelman, 1992).

Finally, the nonverbal accumulator theory can account for the multiple reports that numerical magnitude can be represented very early in life, prior to learning verbal counting (Wynn, 1992; Xu & Spelke, 1997). Although babies have typically been found to discriminate 1 and 2, 2 and 3, and sometimes 3 and 4, recent evidence suggests that babies can also discriminate much larger numbers (i.e., 8 and 16; Xu & Spelke, 1997). This is consistent with the use of an underlying representation by magnitudes with scalar variability. Even if adjacent magnitudes (e.g., 3 and 4) are indistinguishable from one another because of the variability in the magnitude representations, larger quantities should be discriminable if sufficiently dissimilar.

If it is the case that accumulated magnitude representations are used to represent number prior to the acquisition of verbal counting and arithmetic, it would suggest that a fundamental challenge for children is to learn to map back and forth between a nonverbally generated sequence of magnitudes, with scalar variability, and a linguistically generated sequence of discrete verbal and graphemic number symbols (Gelman, 1993).

These findings pose several questions, such as How are verbal numerals mapped onto numerical magnitudes? and What role do representations of numerical magnitude play in the acquisition and

retrieval of simple arithmetic facts, complex mathematical skills, and everyday arithmetic (e.g., evaluating sale prices)? Also, numerical meaning might arise from and depend on the nonverbal representation of discrete numerosities by continuous and noisy mental magnitudes, together with the mental machinery for operating arithmetically with these magnitudes, a machinery clearly present in nonverbal animals (Gallistel, 1990). If so, this would challenge the view that the inherently arbitrary linguistic symbols for different numerosities acquire meaning by virtue of their use in language and the inferences and associations arising from that use (Fuson, 1988).

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REFERENCES

- Campbell, J.I.D. (1994). Architectures for numerical cognition. *Cognition*, 53, 1–44.
- Church, R.M. (1984). Properties of the internal clock. *Annals of the New York Academy of Sciences*, 423, 566–582.
- Cohen, J.D., MacWhinney, B., Flatt, M., & Provost, J. (1993). Psyscope: A new graphic interactive environment for designing psychology experiments. *Behavior Research Methods, Instruments, & Computers*, 25, 257–271.
- Dehaene, S., & Akhavein, R. (1995). Attention, automaticity and levels of representation in number processing. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 21, 314–326.
- Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition*, 1, 83–120.
- Fetterman, J.G. (1993). Numerosity discrimination: Both time and number matter. *Journal of Experimental Psychology: Animal Behavior Processes*, 19, 149–164.
- Fuson, K.C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag.
- Gallistel, C.R. (1990). *The organization of learning*. Cambridge, MA: MIT Press.
- Gallistel, C.R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, 44, 43–74.
- Gelman, R. (1993). A rational-constructivist account of early learning about numbers and objects. In D. Medin (Ed.), *Learning and motivation* (pp. 61–96). New York: Academic Press.
- Gibbon, J. (1977). Scalar expectancy theory and Weber's law in animal timing. *Psychological Review*, 84, 279–325.
- Gibbon, J. (1992). Ubiquity of scalar timing with a Poisson clock. *Journal of Mathematical Psychology*, 36, 283–293.
- Gibbon, J., Church, R.M., & Meck, W.H. (1984). Scalar timing in memory. *Annals of the New York Academy of Sciences*, 423, 52–77.
- Kaufman, E.L., Lord, M.W., Reese, T.W., & Volkman, J. (1949). The discrimination of visual number. *American Journal of Psychology*, 62, 498–525.
- Klahr, D. (1973). A production system for counting, subitizing and adding. In F. Huck & S. Park (Eds.), *Visual information processing* (pp. 47–91). New York: Academic Press.
- Landauer, T. (1962). Rate of implicit speech. *Perceptual and Motor Skills*, 15, 646.
- Mandler, G., & Shebo, B.J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General*, 11, 1–22.
- McCloskey, M. (1992). Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. *Cognition*, 44, 107–157.
- McCloskey, M., Macaruso, P., & Whetstone, T. (1992). The functional architecture of numerical processing mechanisms: Defending the modular model. In I.D.C. Jamie (Ed.), *The nature and origins of mathematical skills: Advances in psychology*, 91 (pp. 493–537). Amsterdam: North-Holland.
- Mechner, F. (1958). Probability relations within response sequences under ratio reinforcement. *Journal of the Experimental Analysis of Behavior*, 5, 463–466.
- Meck, W.H., & Church, R.M. (1983). A mode control model of counting and timing processes. *Journal of Experimental Psychology: Animal Behavior Processes*, 9, 320–334.
- Moyer, R.S., & Landauer, T.K. (1967). Time required for judgments of numerical inequality. *Nature*, 215, 1519–1520.
- Platt, J.R., & Johnson, D.M. (1971). Localization of position within a homogeneous behavior chain: Effects of error contingencies. *Learning and Motivation*, 2, 386–414.
- Rilling, M., & McDermid, C. (1965). Signal detection in fixed ratio schedules. *Science*, 148(3669), 526–527.
- Wearden, J.H., Denovan, L., Fakhri, M., & Haworth, R. (1997). Scalar timing in temporal generalization in humans with longer stimulus durations. *Journal of Experimental Psychology: Animal Behavior Processes*, 23, 502–511.
- Wilkie, D.M., Webster, J.B., & Leader, L.G. (1979). Unconfounding time and number discrimination in a Mechner counting schedule. *Bulletin of the Psychonomic Society*, 13, 390–392.
- Wynn, K. (1992). Addition and subtraction by human infants. *Nature*, 358, 749–750.
- Xu, F., & Spelke, E. (1997). Large number discrimination in 6-month-old infants. In M. Shafto & P. Langley (Eds.), *Nineteenth Annual Conference of the Cognitive Science Society* (pp. 1093). London: Erlbaum.

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