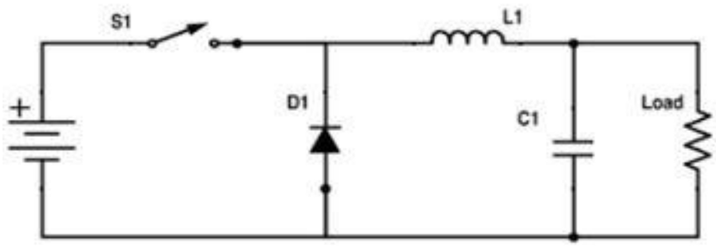
Project 1 – Buck Converter

Ideal Buck Converter Circuit:



Initial Specifications:

Average output voltage: $\langle V_{out} \rangle = 400V = \langle V_{load} \rangle$,

Output ripple voltage: $\Delta V_{load} = 10V$,

Input voltage: $V_{in} = 800V$,

Switch frequency: $f_{sw} = 10kHz$,

Power range: $P_{min} = 50kW$, $P_{max} = 250kW$.

Parameters Calculations:

We know $V_{out} = DV_{in}$ for buck converter, with the known values of output and input voltages, we achieve duty cycle D:

$$D = \frac{1}{2}$$

And we can calculate period T_{sw} and fundamental frequency ω using $f_{sw} = 10kHz$, respectively:

$$T_{sw} = 10^{-4}s$$

$$\omega = 20\pi kHz$$

In steady-state, voltage output stays constant. Using this, we can have the values below:

$$R_{light} = \frac{V_{load}^2}{P_{min}} = 3.2\Omega$$

$$R_{heavy} = \frac{V_{load}^2}{P_{max}} = 0.64\Omega$$

We know, for buck converter, $L_{min} = L_{crit} = \frac{R_{light}(1-D)T_{sw}}{2}$. Therefore, we got our inductance:

$$L = 1.1L_{crit} = 1.1\frac{R_{light}(1-D)T_{sw}}{2} = 8.8 \times 10^{-5}H$$

The minimum value C for capacitor needed is:

$$C = \frac{T_{sw}^2(1-D)V_{load}}{8L\Delta V_{load}} = 28.409 \times 10^{-5}C$$

Steady-state Analysis:

From the values we have above, we can calculate the expected currents and voltages in steady-state of each components in the buck converter.

We have the equations:

$$I_{L2} = \frac{V_{load}}{R_{load}} + \frac{(1-D)T_{sw}V_{load}}{2L}$$

$$I_{L1} = \frac{V_{load}}{R_{load}} - \frac{(1-D)T_{sw}V_{load}}{2L}$$

$$I_{SW2} = I_{L2} = I_{D2}$$

$$I_{SW1} = I_{L1} = I_{D2}$$

$$I_{C2} = I_{L2} - I_{LOAD}$$

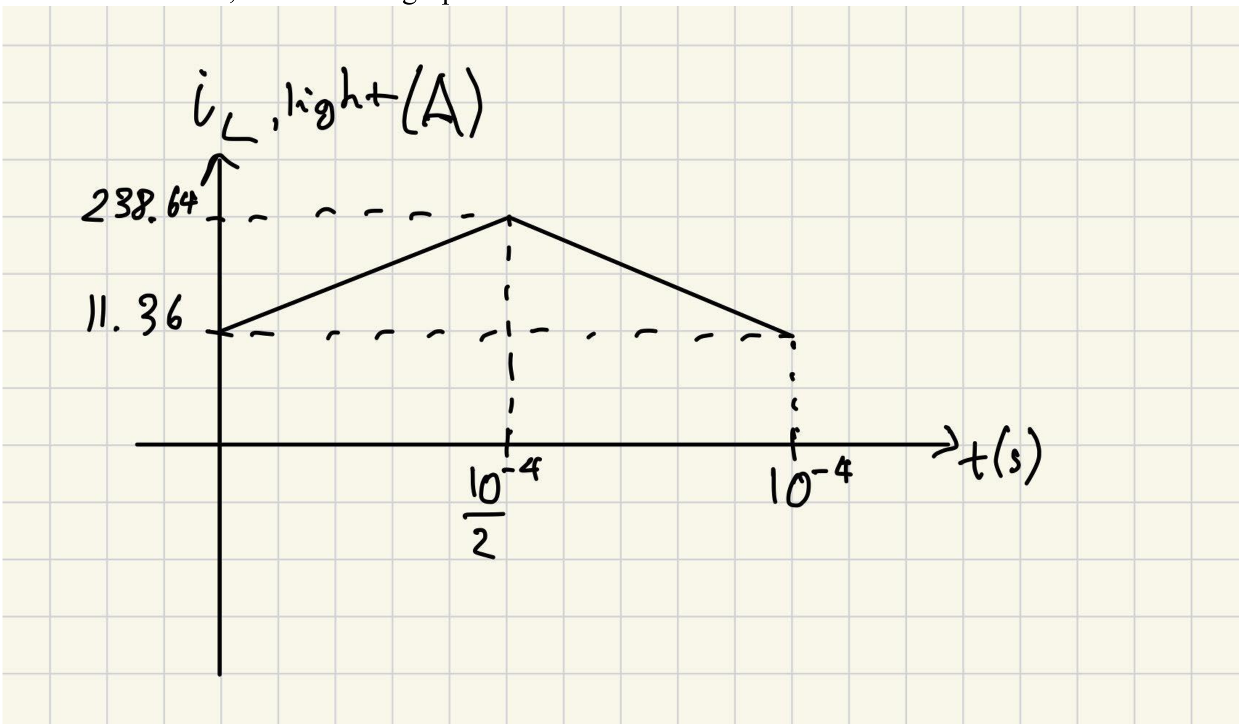
$$I_{C1} = I_{L1} - I_{LOAD}$$

Using these facts and equations, we compose a table below to calculate the current and voltage values we'll observe in the graphs, for both light and heavy resistance load.

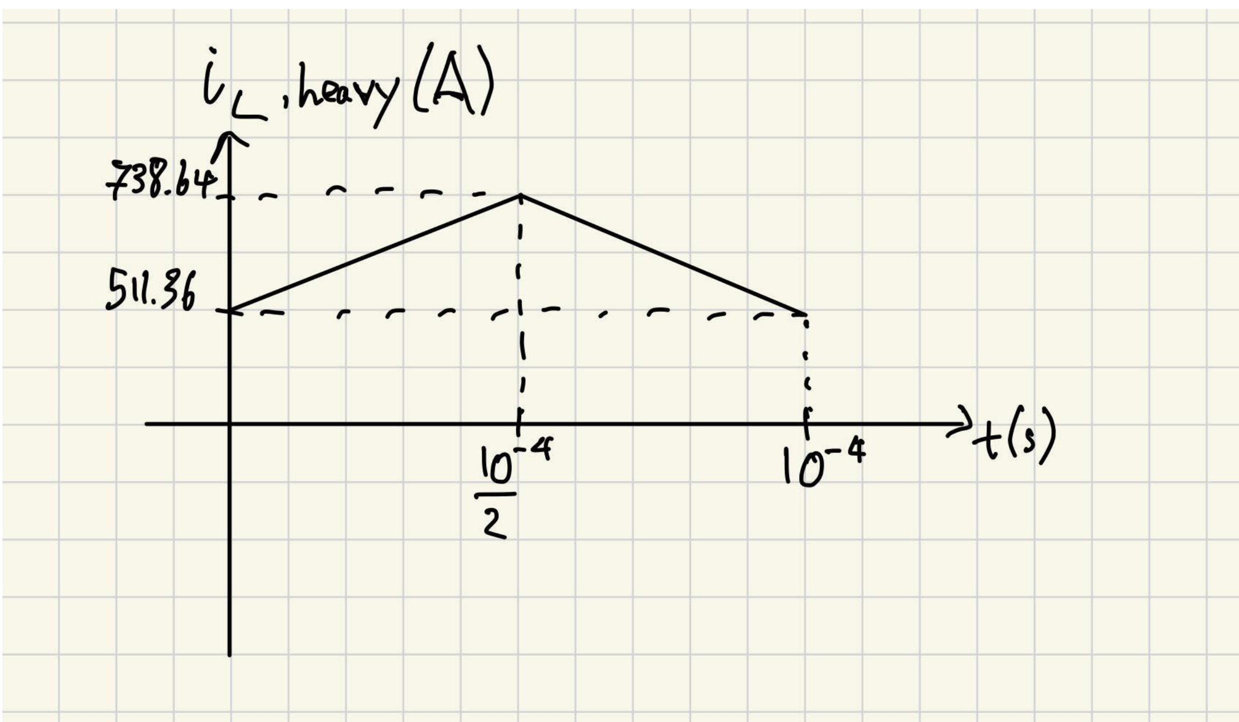
Table 1. Currents and Voltages Values for Light and Heavy Loads

	R_{light}	R_{heavy}
I_{L2} (A)	238.64	738.64
I_{L1} (A)	11.36	511.36
I_{SW2} (A)	238.64	738.64
I_{SW1} (A)	11.36	511.36
I_{C2} (A)	113.64	113.64
I_{C1} (A)	-113.64	-113.64
I_{D2} (A)	238.64	738.64
I_{D1} (A)	11.36	511.36
V_{L2} (V)	400	400
V_{L1} (V)	-400	-400

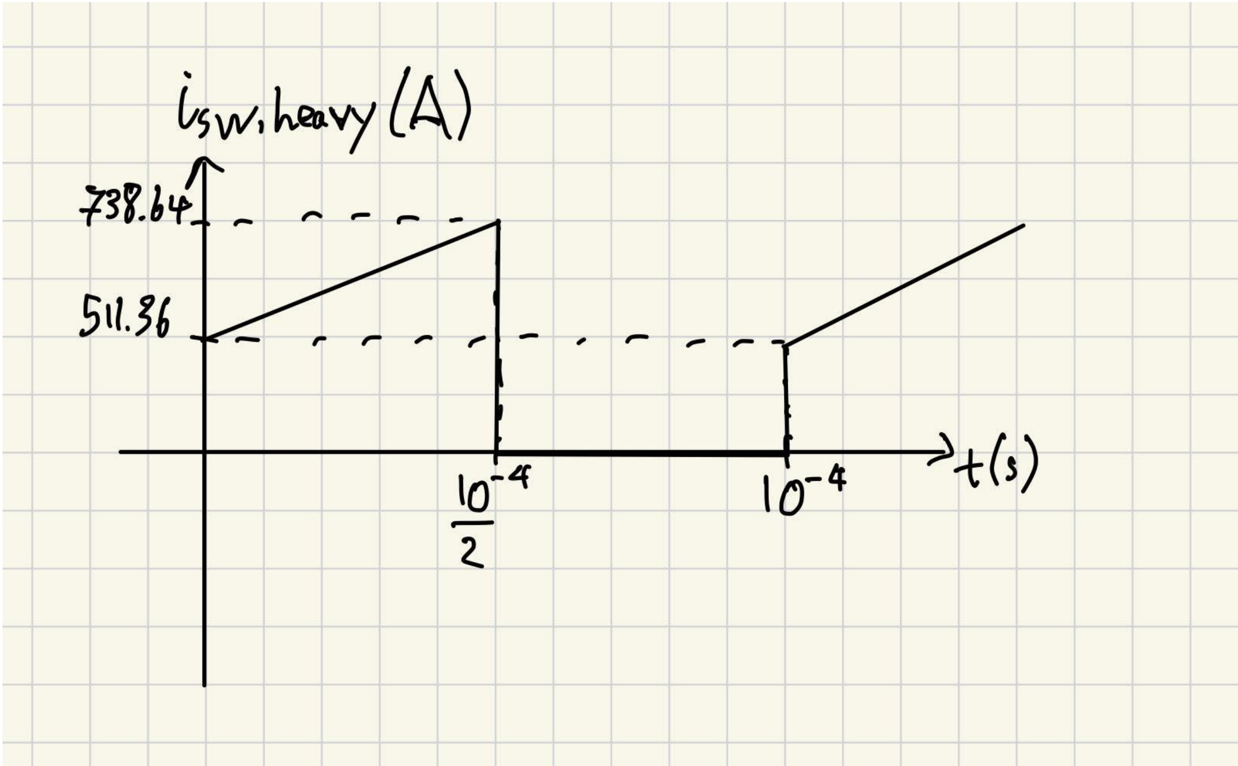
From these values, we form the graphs below:



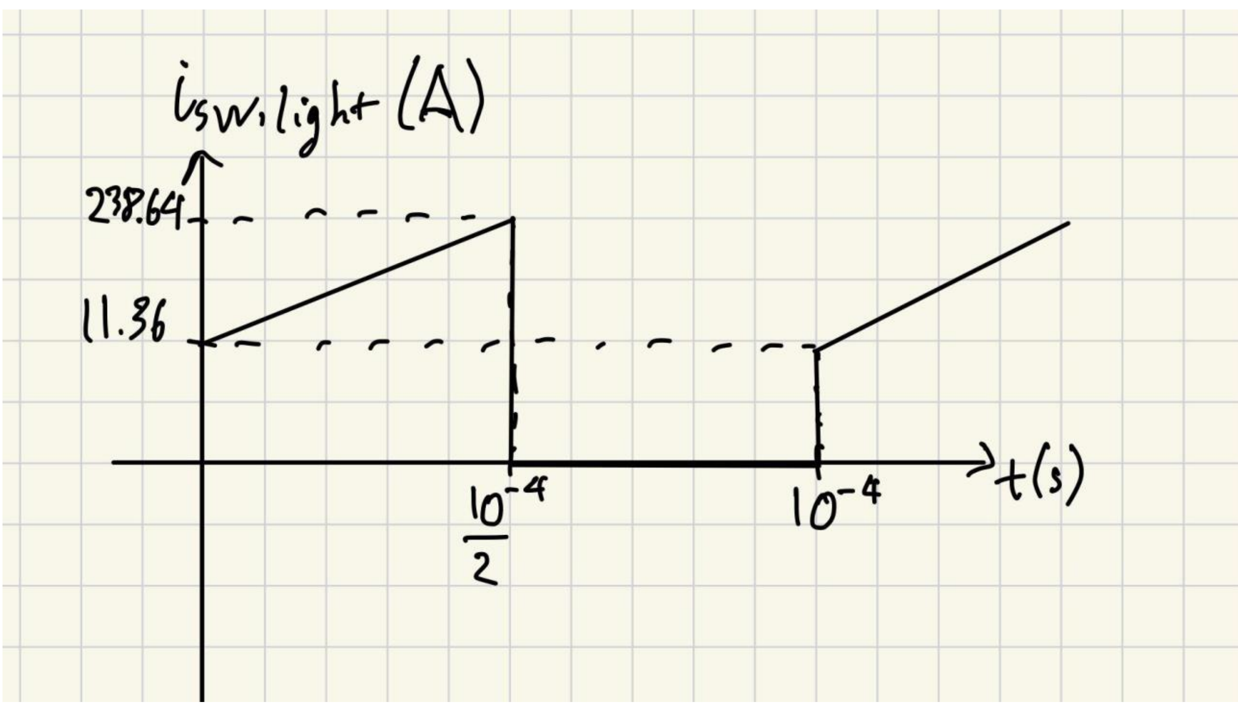
Graph 1. Inductor Current with Light Load vs Time



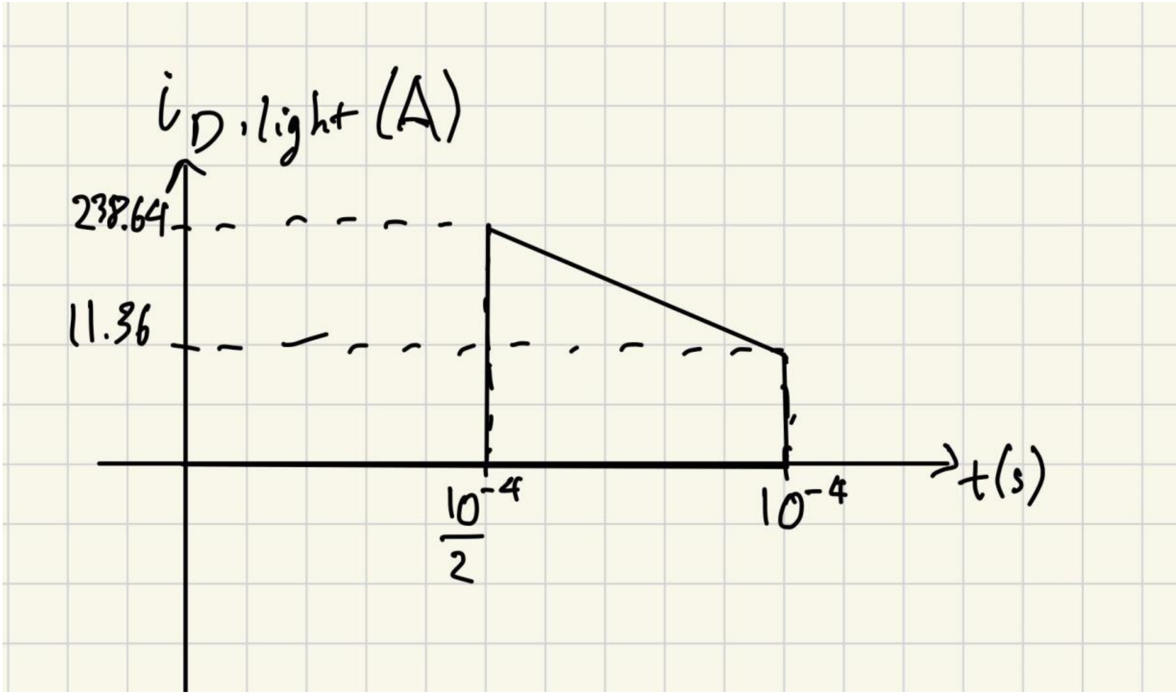
Graph 2. Inductor Current with Heavy Load vs Time



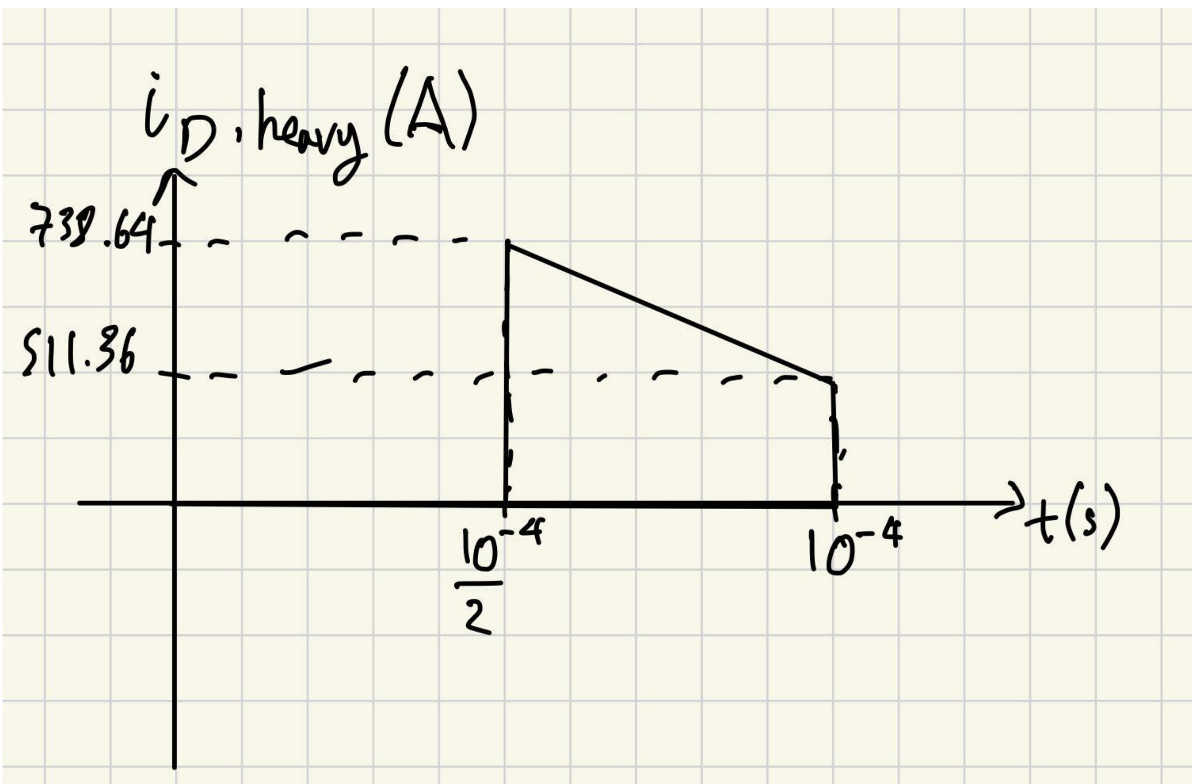
Graph 3. Switch Current with Heavy Load vs Time



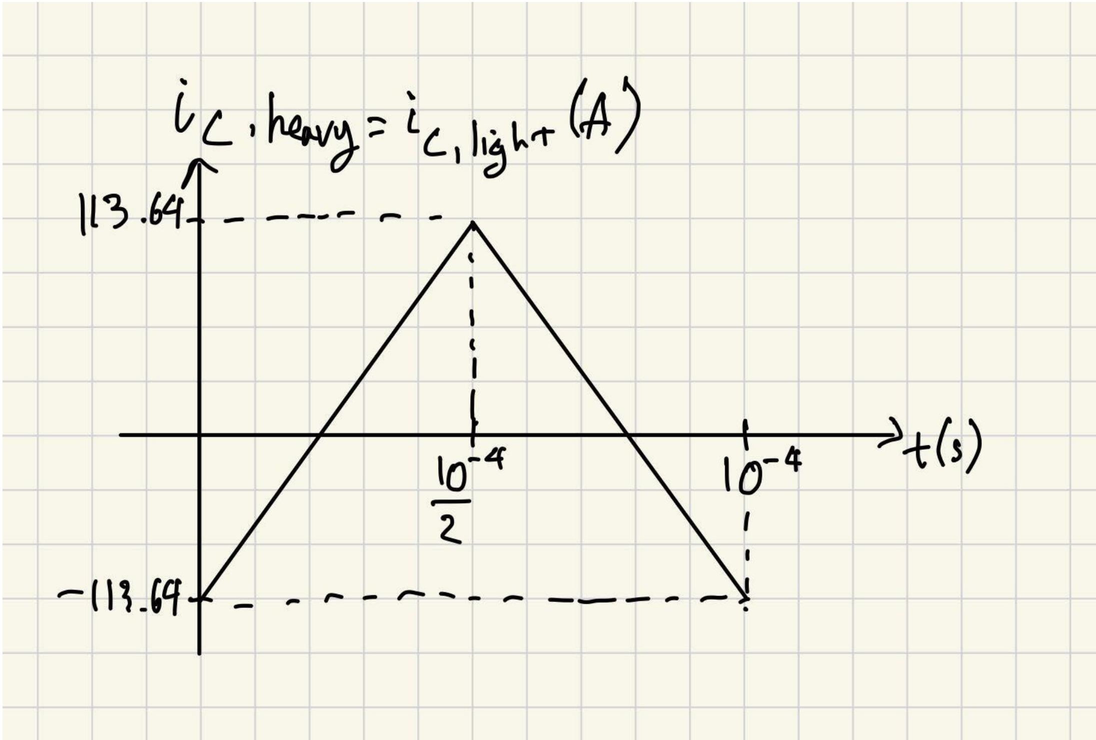
Graph 4. Switch Current with Light Load vs Time



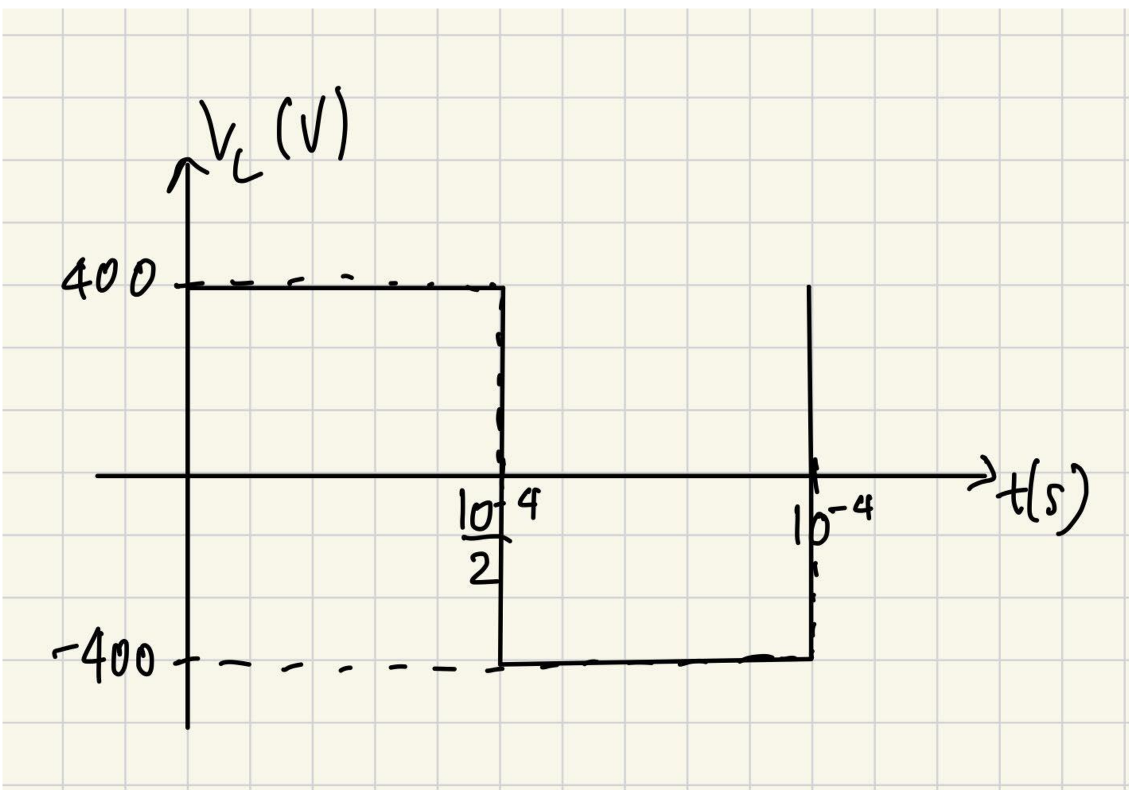
Graph 5. Diode Current with Light Load vs Time



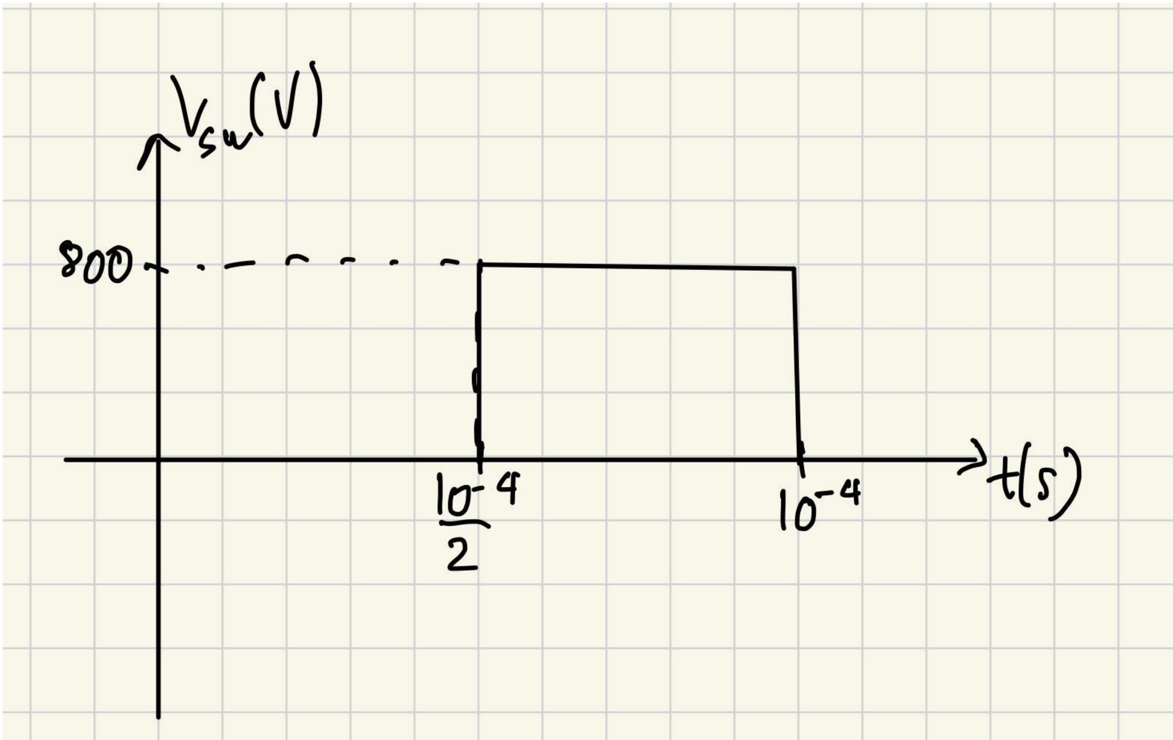
Graph 6. Diode Current with Heavy Load vs Time



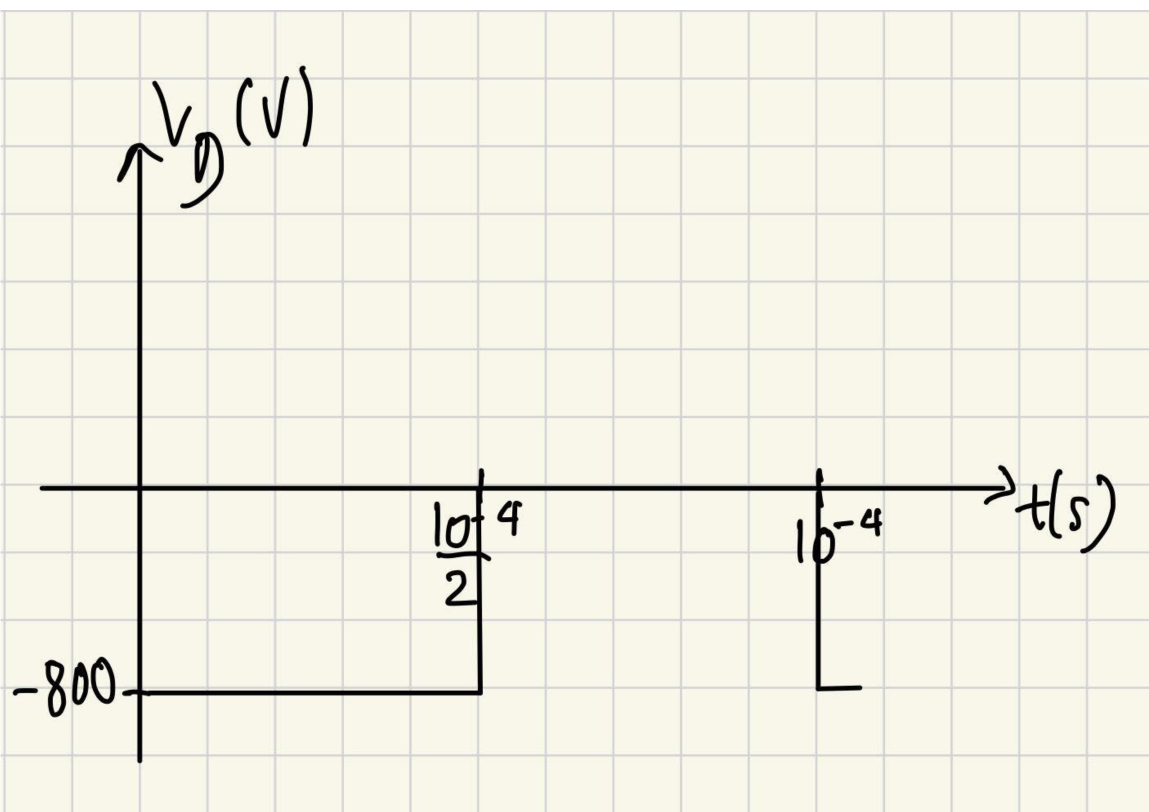
Graph 7. Capacitor Current with Light and Heavy Load vs Time



Graph 8. Inductor Voltage vs Time



Graph 9. Switch Voltage vs Time



Graph 10. Diode Voltage vs Time

Steady-state Matlab Code:

The Matlab code and functions below will be used for both light and heavy load resistance.

The switch function (sw.m) takes inputs of duty cycle D and time instance t and outputs the state of the switch as a numerical 1 or 0. It is to calculate a Fourier series triangle wave and compare with duty cycle D to determine if the transistor is on or off:

```
%sw.m
function [st] = sw(D, t)
aksum = 1 / 2; % a0 value
for k = 1:30
    aksum = aksum + (2 * cos(pi * k) - cos(2 * pi * k) - 1) / ((pi ^ 2) ...
            * (k ^ 2)) * cos(20000 * pi * k * t);
end
if aksum >= D
    st = 1;
else
    st = 0;
end
end
```

The average calculation function (aver.m) is to calculate the average of any waveform produce by the other files, utilizing the concept of Riemann sum. It takes the waveform, period, and time between samples as inputs, and output the average of the waveform:

```
%aver.m
function [av] = aver(x, T, dt)
sum = 0;
t = 0:dt:T;
t_len = length(t);
dx = T / dt;
for i = t_len:-1:(t_len - dx)
    sum = sum + dt * x(i);
end
av = sum / T;
end
```

The buck.m file is a script file, in which all the circuit voltages and currents are to be calculated. It makes use of the switch function and is called via file buckproc.m:

```
%buck.m
k = 1;
while (t(k) < tend)
    state = sw(D, t(k));
    I_L(k + 1) = (Vin * state - Vload(k)) * dt / L + I_L(k);
    I_C(k + 1) = I_L(k) - Vload(k) / Rload;
    Vload(k + 1) = Vload(k) + dt * (I_L(k) - Vload(k) / Rload) / C;
    I_sw(k + 1) = I_L(k) * state;
    I_D(k + 1) = I_L(k) * (1 - state);
    t(k + 1) = t(k) + dt;
    k = k + 1;
end
```

Lastly, the file buckproc.m is where the parameter values of inductor, capacitor, etc. are initialized. It also initialized the initial values of some voltages and currents in the circuit. It uses bucks to calculate the waveforms and plot the waveforms, while calculate average values, etc. via aver.m:

```
%buckproc.m
T = 1 / f;
dt = T / 100;
tend = 100 * T;
t = 0:dt:tend;
t_len = length(t);
D = 0.5;

Vin = 800;
R_light = 3.2;
R_heavy = 0.64;
Vload = zeros(1,t_len);
I_L = zeros(1,t_len);
I_C = zeros(1,t_len);
I_sw = zeros(1,t_len);
I_D = zeros(1,t_len);

Vload(1) = 0;
I_L(1) = 0;
%Rload = R_light; uncommon for light load conditions
Rload = R_heavy;
buck;

aVout = aver(Vload, tend, dt);
aIL = aver(I_L, tend, dt);
eff = (aVout ^ 2 / Rload) / (Vin * D * aIL);
disp(aVout);
disp(aIL);
disp(eff);

figure
plot(t, Vload)
title('Vload vs time')
xlabel('time(s)')
ylabel('Vload(V)')
```

The buckproc.m file is to change into the file below to accommodate the steady-state analysis:

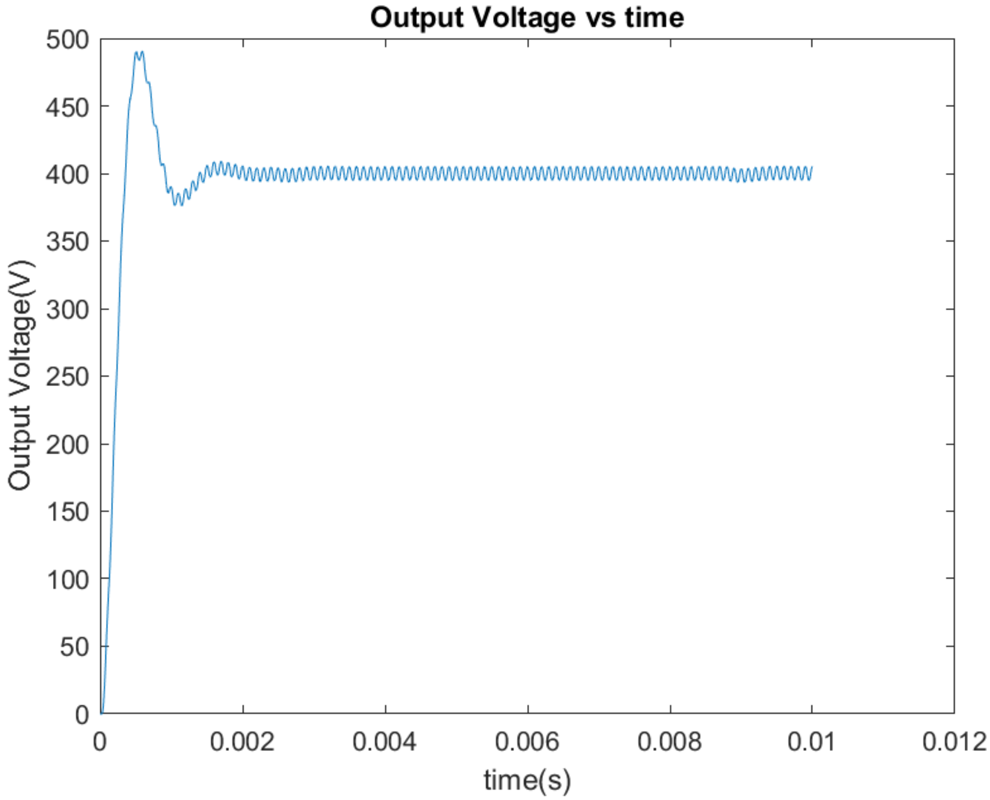
```
%buckproc.m
L = 8.8 * 10 ^ -5;
C = 28.409 * 10 ^ -5;
f = 10000;
T = 1 / f;
dt = T / 100;
tend = 100 * T;
t = 0:dt:tend;
t_len = length(t);
t_2 = t_len:-1:(t_len - 2 * T / dt);

D = 0.5;
```

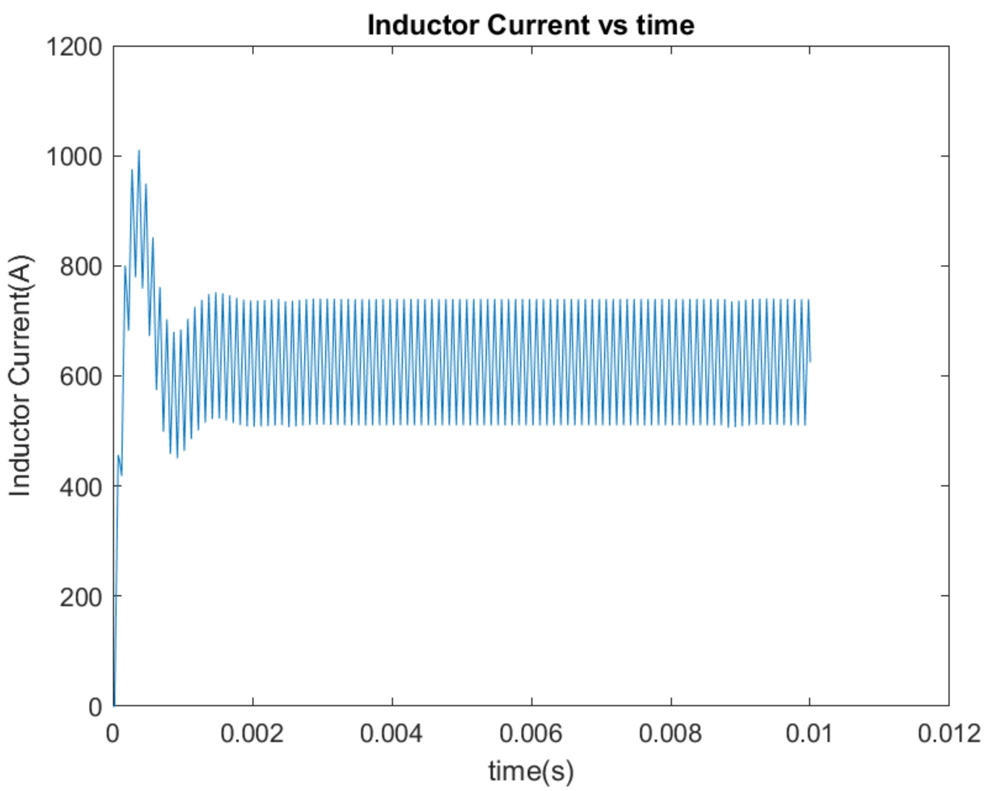
```
Vin = 800;
R_light = 3.2;
R_heavy = 0.64;
Vload = zeros(1,t_len);
I_L = zeros(1,t_len);
I_C = zeros(1,t_len);
I_sw = zeros(1,t_len);
I_D = zeros(1,t_len);

Vload(1) = 400;
I_L(1) = 511.36;
%Rload = R_light;
Rload = R_heavy;
buck;

aVout = aver(Vload, tend, dt);
aIL = aver(I_L, tend, dt);
eff = (aVout ^ 2 / Rload) / (Vin * D * aIL);
disp(aVout);
disp(aIL);
disp(eff);

figure
plot(t_2, I_sw(t_2))
title('Switch Current vs time')
xlabel('time(s)')
ylabel('Switch Current(A)')

figure
plot(t_2, I_D(t_2))
title('Diode Current vs time')
xlabel('time(s)')
ylabel('Diode Current(A)')

figure
plot(t_2, I_L(t_2))
title('Inductor Current vs time')
xlabel('time(s)')
ylabel('Inductor Current(A)')

figure
plot(t_2, Vload(t_2))
title('Output Voltage vs time')
xlabel('time(s)')
ylabel('Output Voltage(V)')
```

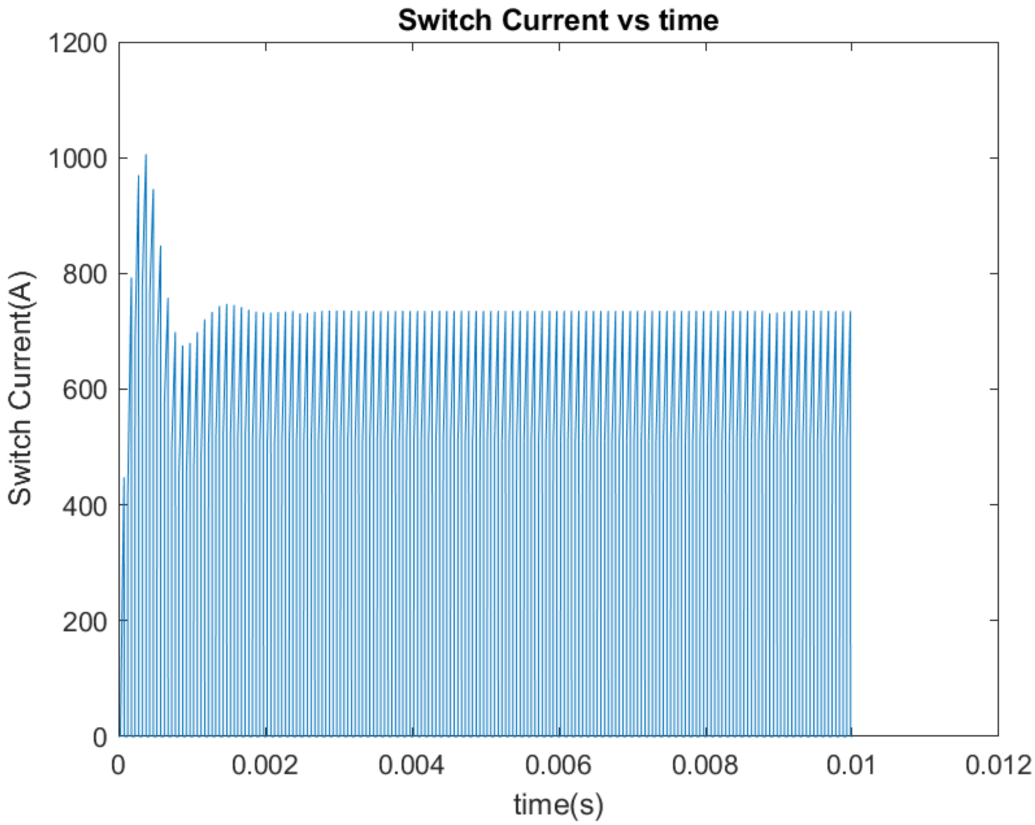
Steady-state Matlab Analysis:

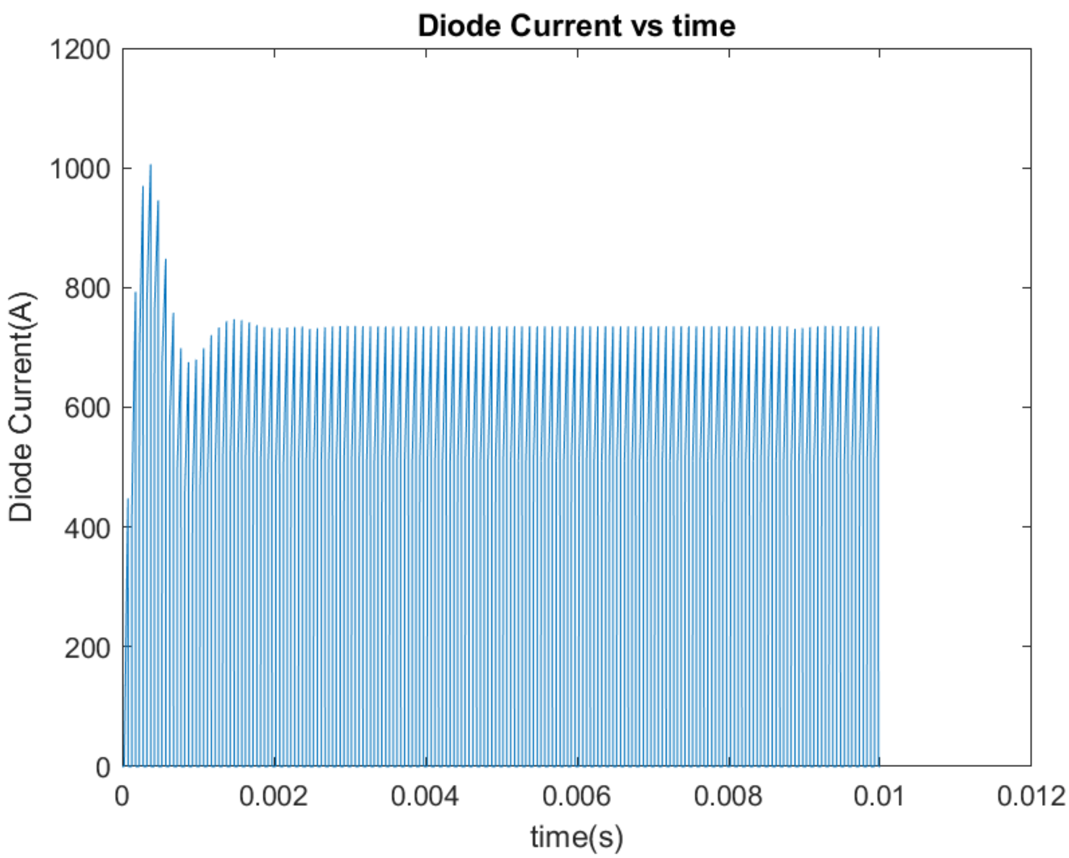
Running the Matlab files has produced these graphs and plots below:

Heavy Load Condition:

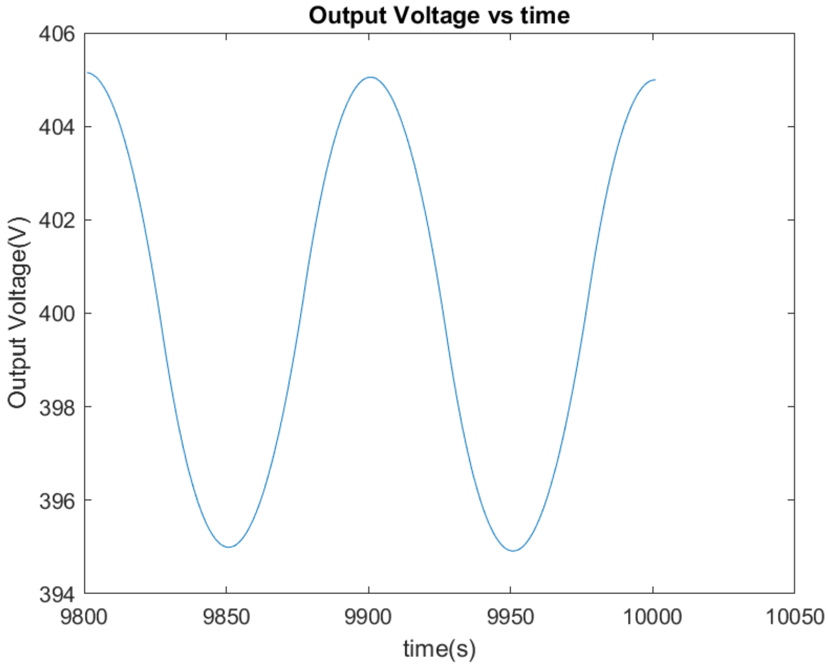


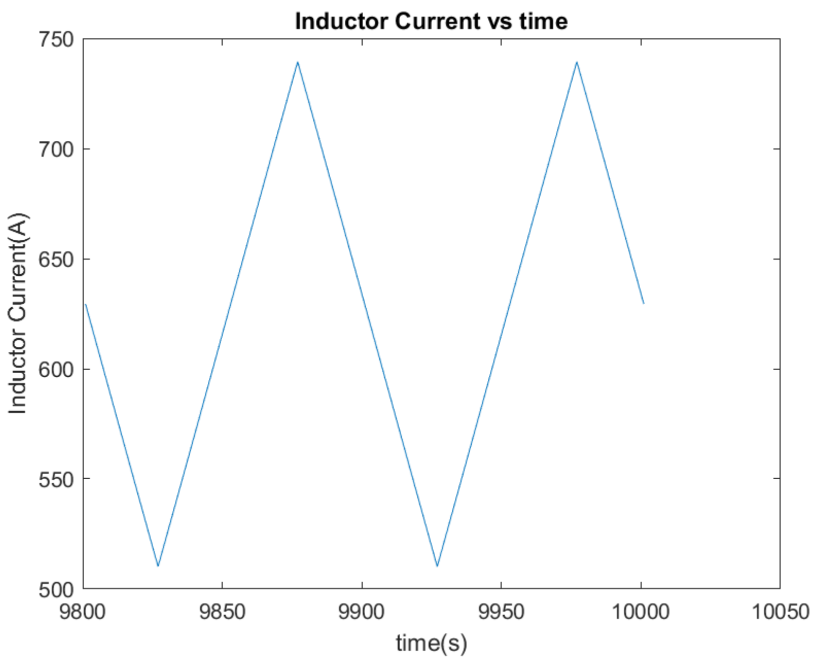


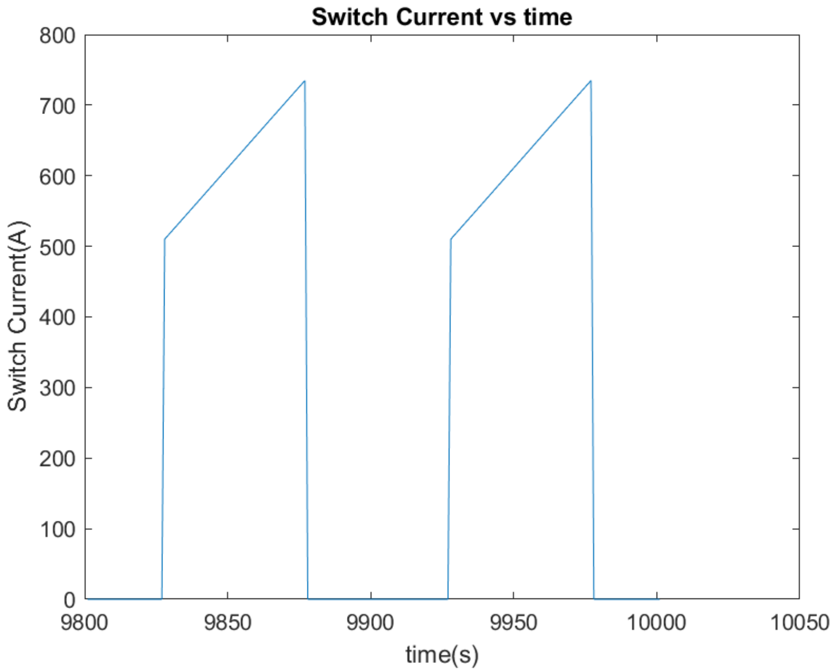

Switch Current vs time
Switch Current(A)
time(s)
0
200
400
600
800
1000
1200
0.002
0.004
0.006
0.008
0.01
0.012

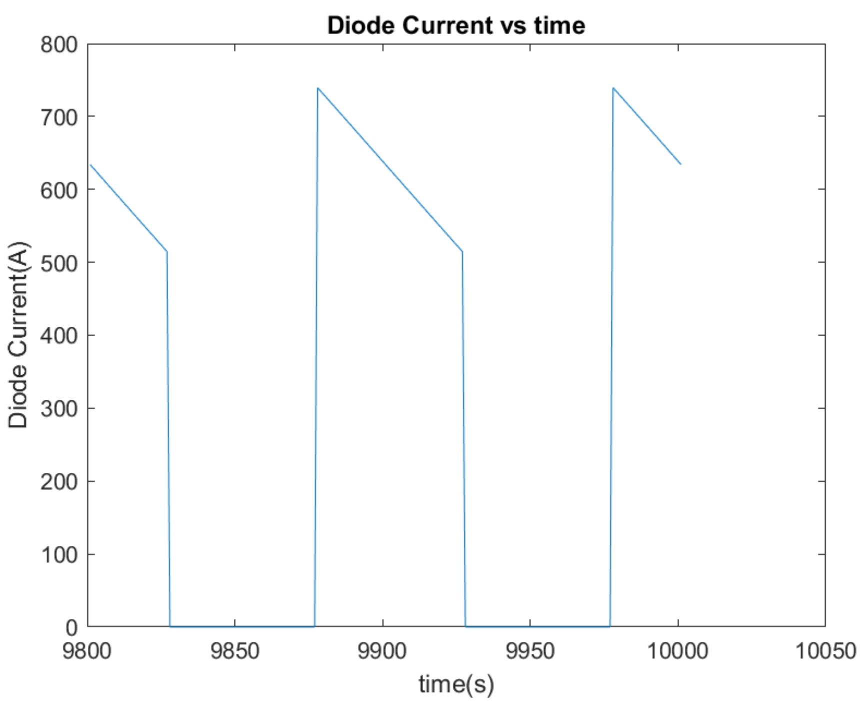

Diode Current vs time
Diode Current(A)
time(s)
0
200
400
600
800
1000
1200
0.002
0.004
0.006
0.008
0.01
0.012

Changing the initial values to our analysis values above inside the buckproc.m file, we obtain the graphs of the same circuit components under steady-state:





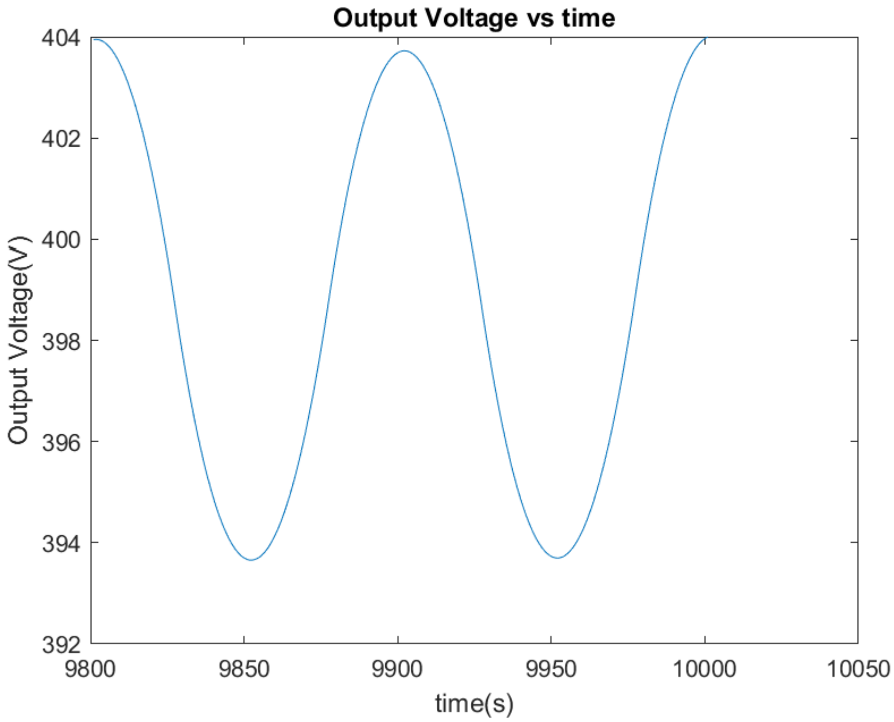


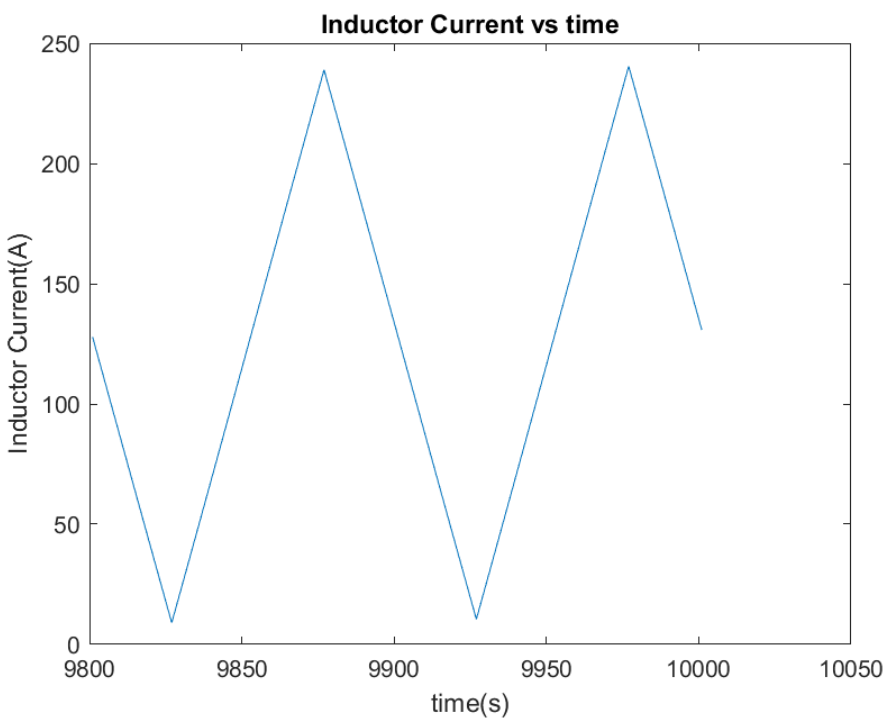


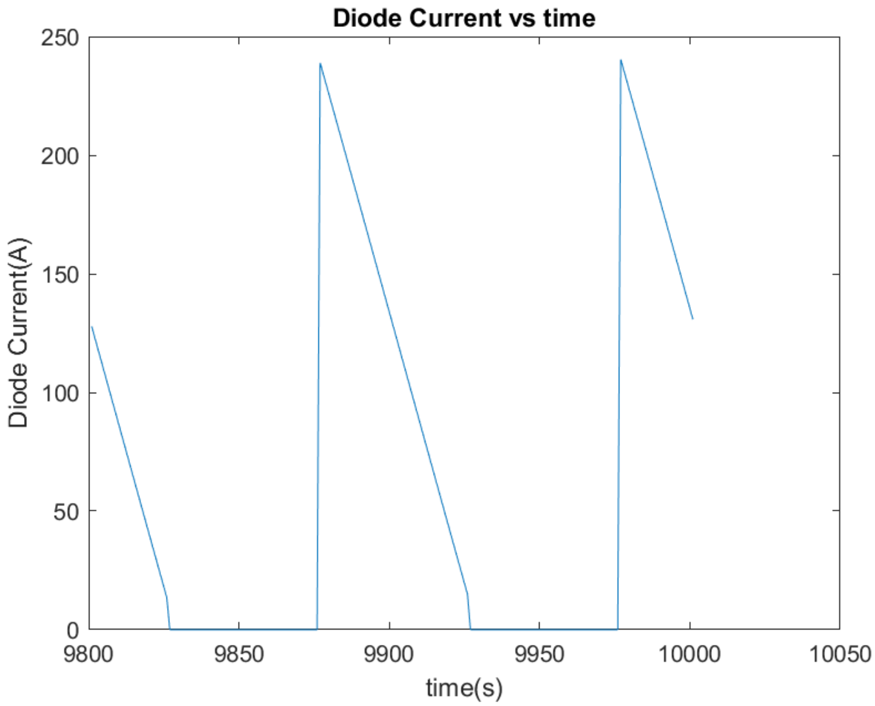
The average output voltage calculated is 398.9219, and the efficiency of the circuit is 0.9971.

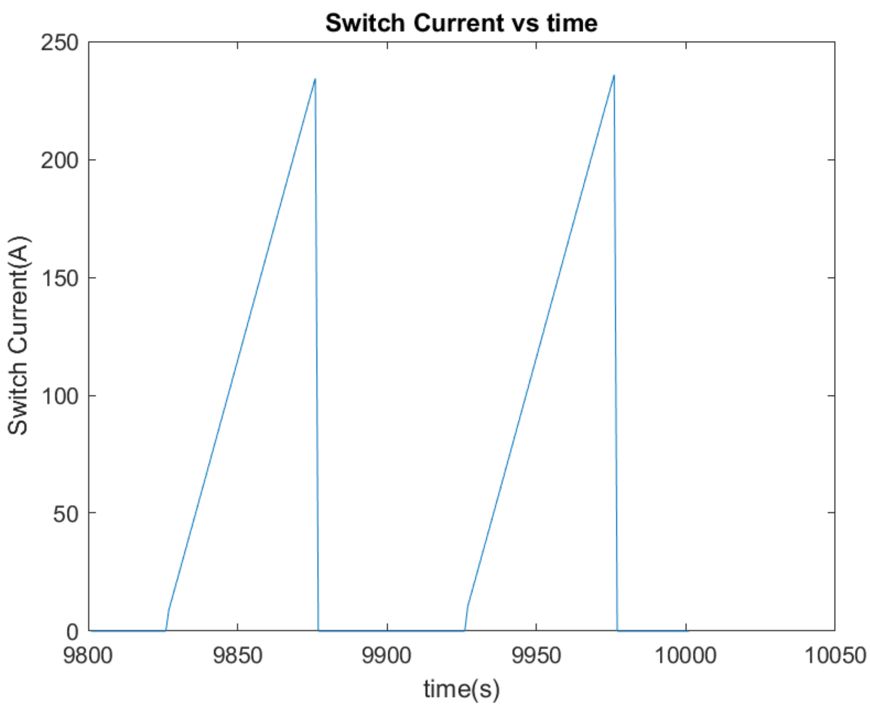
Light Load Conditions:

Using the same setup code, we can get the graphs of the components' voltages and currents in steady-state as below:


Output Voltage vs time
404
402
400
398
396
394
392
Output Voltage(V)
9800
9850
9900
9950
10000
10050
time(s)


Inductor Current vs time
250
200
150
100
50
0
Inductor Current(A)
9800
9850
9900
9950
10000
10050
time(s)





The average output voltage calculated is 403.3098, and the efficiency of the circuit is 1.0074.

Steady-state Results and Conclusion:

As we can observe from the graphs and plots from both the Matlab and the manual analysis, we can see that they agree with each other on how the waveforms for the circuit voltages and currents should look like, with the expected values for the peaks. This statement is true for both heavy and light load conditions.

Non-ideal Switches Analysis:

Expected and Initial Specifications:

Expected output voltage: $V_{out} = 400V$,

On-stage Diode and Switch resistance: $R_{D,on} = R_{sw,on} = 0.01\Omega$,

On-stage Diode and Switch voltage drop: $V_{sw,on} = V_{D,on} = 1V$.

Heavy power conditions:

- Heavy load: $R_{heavy} = \frac{V_{load}^2}{P_{max}} = 0.64\Omega$

Analysis:

For nonideal switches buck converter, when switch is on, we have the voltage equation using Kirchoff's Voltage Law, and Kirchoff's Current Law to determine the value of the average current for inductor:

$$\langle I_L \rangle = \langle I_C \rangle + \langle I_{load} \rangle = I_{load} = \frac{V_{load}}{R_{heavy}} = 625A$$

$$V_{L1} = V_{in} - V_{sw,on} - R_{sw,on} I_L - V_{load} = V_{in} - V_{sw,on} - R_{sw,on} I_{L1} - V_{load}$$

$$V_{L1}{}' = V_{in} - V_{sw,on} - R_{sw,on} I_L - V_{load} = V_{in} - V_{sw,on} - R_{sw,on} I_{L2} - V_{load}$$

And for when switch is off, we have:

$$V_{L2}' = -V_{D,on} - R_{D,on} I_L - V_{load} = -V_{D,on} - R_{D,on} I_{L1} - V_{load}$$

$$V_{L2} = -V_{D,on} - R_{D,on} I_L - V_{load} = -V_{D,on} - R_{D,on} I_{L2} - V_{load}$$

Knowing that the average inductance voltage $\langle V_{out} \rangle = 0$, we can derive the equation below, regarding duty cycle D and V_{L2} and V_{L1}:

$$0 = (1 - D) V_{L2} + D V_{L1}$$

To calculate the current of the inductor, a more complex process needs to be taking place, therefore, I'd done it by hand:

$$L\frac{di}{dt} = V_L$$

(1): $L\frac{di}{dt} = V_{in} - V_{sw,on} - R_{sw,on}I_L - V_{load}$ → switch on

We have: $L\frac{di}{dt} + R_{sw,on}I_L = V_{in} - V_{load} - V_{sw,on} = 399$

$$\Leftrightarrow I_L(t) = \frac{399}{0.01}\left(1 - e^{-\frac{0.01}{L}t}\right) + i_{L1}e^{-\frac{0.01}{L}t} \quad (*)$$

(2): $L\frac{di}{dt} = -V_{D,on} - R_{D,on}I_L - V_{load}$ → switch off

$$\Rightarrow L\frac{di}{dt} = -V_{D,on} - R_{D,on}\frac{V_{load}}{R_{load}} - V_{load}$$

Same concept ⇒ $I_L(t) = \left(\frac{-401}{0.01}\right)\left(1 - e^{-\frac{0.01}{L}t}\right) + i_{L2}e^{-\frac{0.01}{L}t} \quad (**)$

Switch on → $t_1 = DT_{sw} = 5.09 \times 10^{-5}$ s

$(*) \rightarrow I_L(t_1) = 230.119 + i_{L1} \times 0.994 = i_{L2}$

Switch off → $t_2 = (1-D)T_{sw} = 4.91 \times 10^{-5}$ s

$(**) \rightarrow I_L(t_2) = -223.117 + i_{L2} \times 0.994 = i_{L1}$

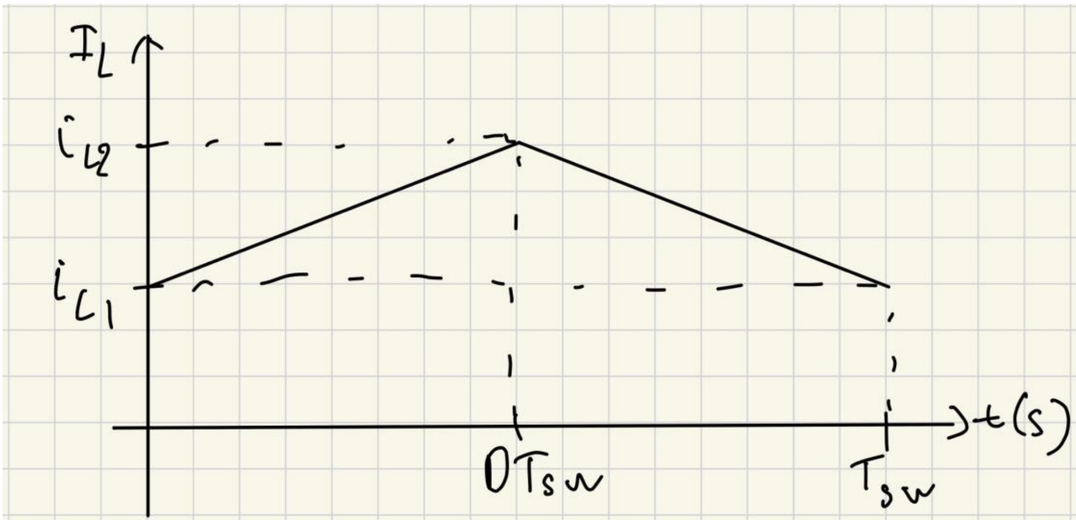
$$\Rightarrow \begin{cases} i_{L1} = 506.4 \text{ A} \\ i_{L2} = 733.6 \text{ A} \end{cases}$$

From here, we can have the values of the voltages above as:

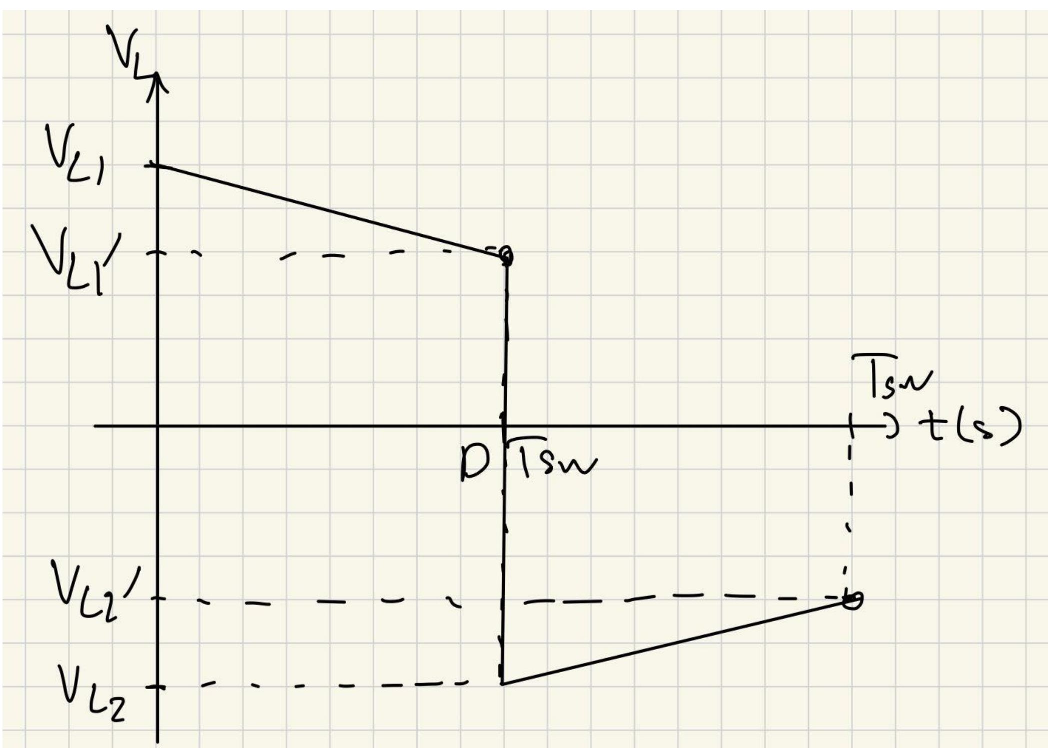
$V_{L1} = 393.936V$, $V_{L1}' = 391.664V$, $V_{L2}' = -406.064V$, $V_{L2} = -408.336A$.

Hence, we can calculate the value of duty cycle D as 0.509.

The graphs for inductor voltage and current are as shown below:



Graph 11. Inductor Current vs Time



Graph 12. Inductor Voltage vs Time

The efficiency of the circuit is calculated as below:

Efficiency: $\eta = \frac{\langle P_{out} \rangle}{\langle P_{in} \rangle} = \frac{\frac{V_{load}^2}{R_{heavy}}}{V_{in} D \langle I_L \rangle} = \frac{250000}{800(0.509)\frac{i_{L2}+i_{L1}}{2}}$

$\Rightarrow \eta = 0.99$

Non-ideal Switches Matlab Code:

The switch function (sw.m) stays the same as it serves the same purpose:

```
%sw.m
function [st] = sw(D, t)
aksum = 1 / 2; % a0 value
for k = 1:30
    aksum = aksum + (2 * cos(pi * k) - cos(2 * pi * k) - 1) / ((pi ^ 2) ...
            * (k ^ 2)) * cos(20000 * pi * k * t);
end
if aksum >= D
    st = 1;
else
    st = 0;
end
end
```

The average calculation function (aver.m)) stays the same as it serves the same purpose:

```
%aver.m
function [av] = aver(x, T, dt)
sum = 0;
t = 0:dt:T;
t_len = length(t);
dx = T / dt;
for i = t_len:-1:(t_len - dx)
    sum = sum + dt * x(i);
end
av = sum / T;
end
```

The buck.m file is a script file, though almost the same as the ideal circuit's, there are some changes. There are equations to calculate power loss in diode (P_D) and transistor (P_T), and the different calculation of inductor current I_L:

```
%buck.m
k = 1;
while (t(k) < tend)
    state = sw(D, t(k));
    I_L(k + 1) = (Vin * state - Vload(k) - ...
                 VT - RT * I_L(k)) * (dt / L) + I_L(k);
    I_C(k + 1) = I_L(k) - Vload(k) / Rload;
    Vload(k + 1) = Vload(k) + (dt / C) * (I_L(k) - Vload(k) / Rload);
```

```
    I_sw(k) = I_L(k) * state;
    I_D(k) = I_L(k) * (1 - state);
    P_D(k) = state * (1 + I_D(k) * RD) + ...
                (1 - state) * (VD - Vin - I_sw(k) * RD) * (I_D(k));
    P_T(k) = state * (1 + I_sw(k) * RT) + ...
                (1 - state) * (Vin - VT - I_D(k) * RT) * (I_sw(k));
    t(k + 1) = t(k) + dt;
    k = k + 1;
end
```

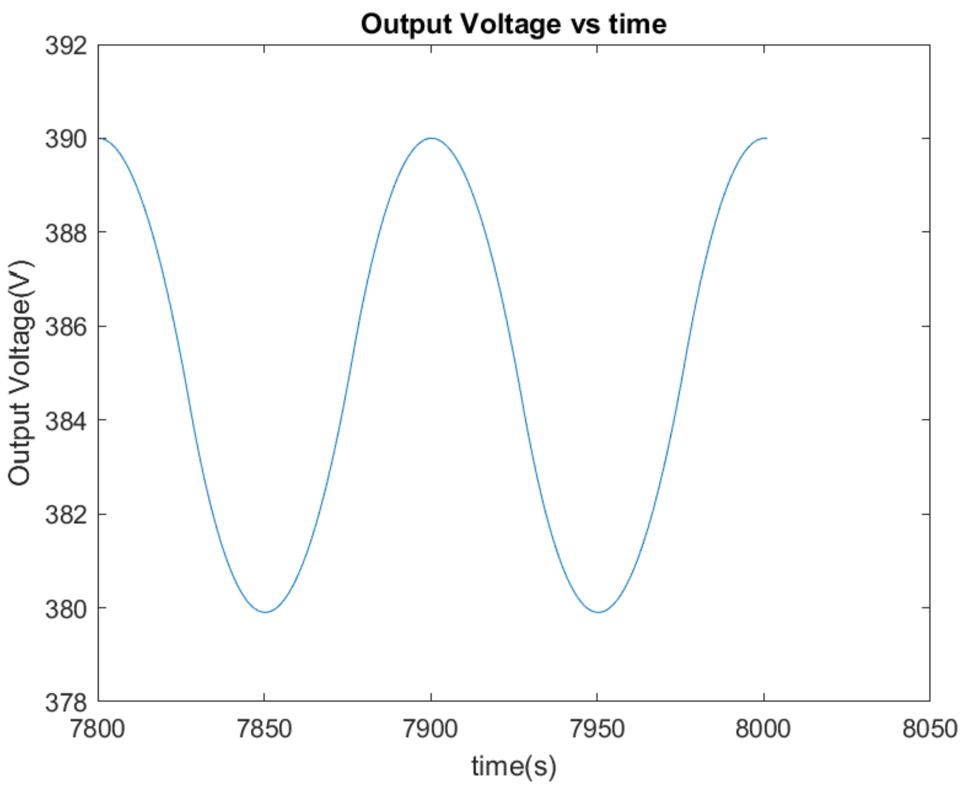
Lastly, the file buckproc.m adds the initial values of diode and transistor resistances, along with their respective voltage drops:

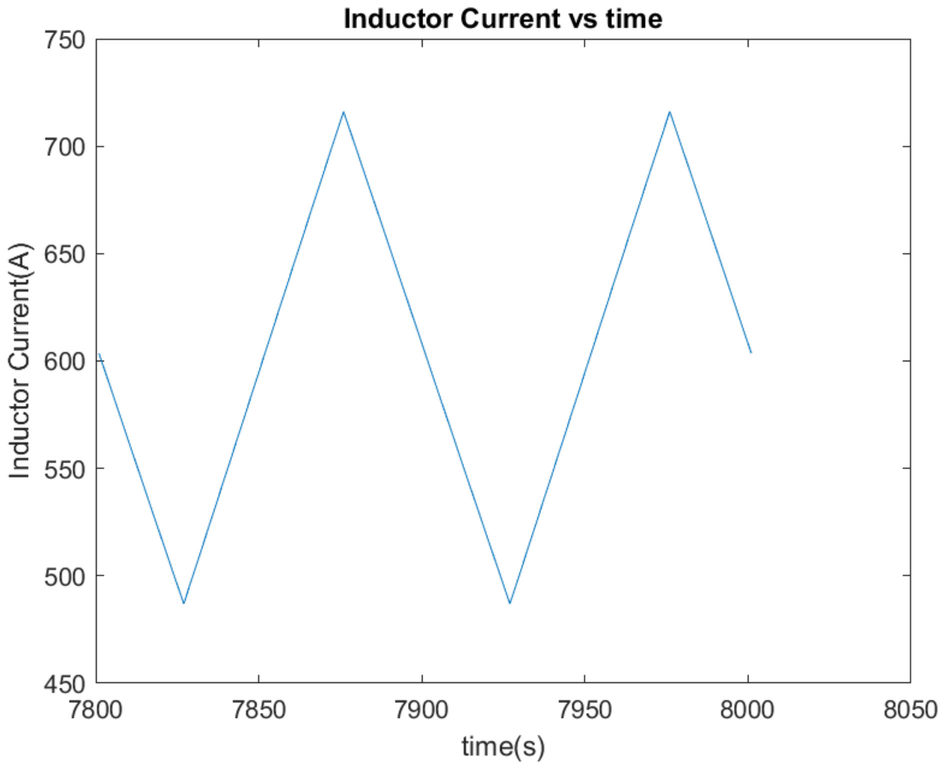
```
%buckproc.m
L = 8.8 * 10 ^ -5;
C = 28.409 * 10 ^ -5;
f = 10000;
T = 1 / f;
dt = T / 100;
tend = 80 * T;
t = 0:dt:tend;
t_len = length(t);
t_2 = t_len:-1:(t_len - 2 * T / dt);

D = 0.509;

Vin = 800;
R_heavy = 0.64;

Vload = zeros(1,t_len);
I_L = zeros(1,t_len);
I_C = zeros(1,t_len);
I_sw = zeros(1,t_len);
I_D = zeros(1,t_len);
P_T = zeros(1,t_len);
P_D = zeros(1,t_len);

Vload(1) = 400;
I_L(1) = 506.4;
RT = 0.01;
RD = 0.01;
VT = 1;
VD = 1;

Rload = R_heavy;
buck;

aVout = aver(Vload, tend, dt);
aIL = aver(I_L, tend, dt);
eff = (aVout ^ 2 / Rload) / (Vin * D * aIL);
disp(aVout);
disp(aIL);
disp(eff);

figure
plot(t_2, P_T(t_2))
```

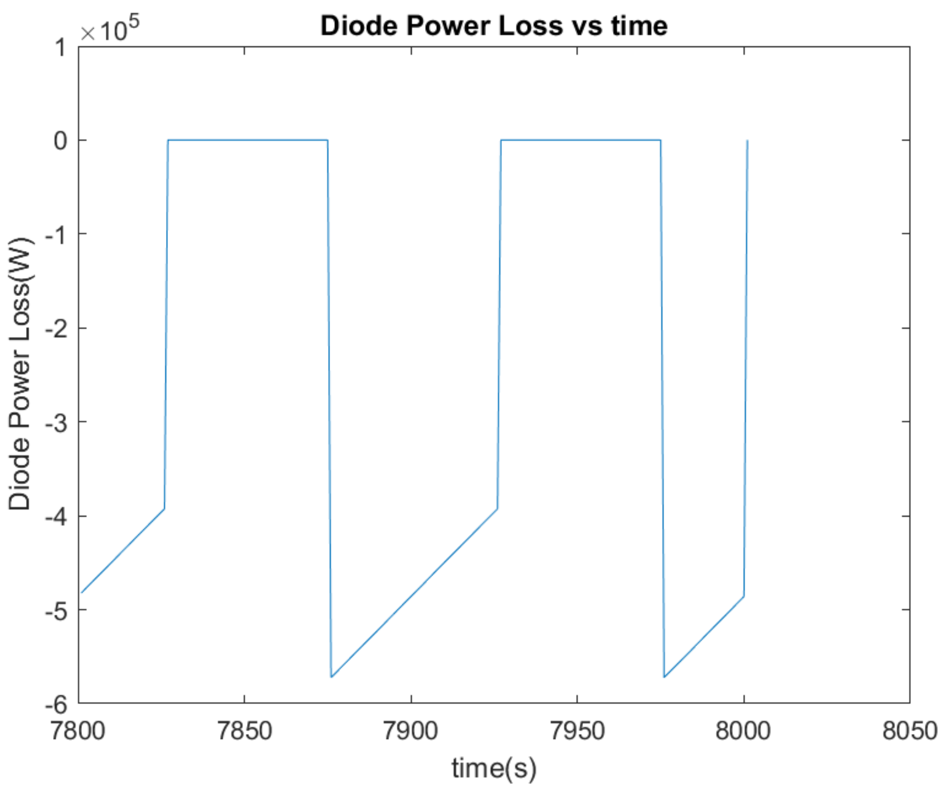
```
title('Transistor Power Loss vs time')
xlabel('time(s)')
ylabel('Transistor Power Loss(W)')

figure
plot(t_2, P_D(t_2))
title('Diode Power Loss vs time')
xlabel('time(s)')
ylabel('Diode Power Loss(W)')

figure
plot(t_2, I_L(t_2))
title('Inductor Current vs time')
xlabel('time(s)')
ylabel('Inductor Current(A)')

figure
plot(t_2, Vload(t_2))
title('Output Voltage vs time')
xlabel('time(s)')
ylabel('Output Voltage(V)')
```

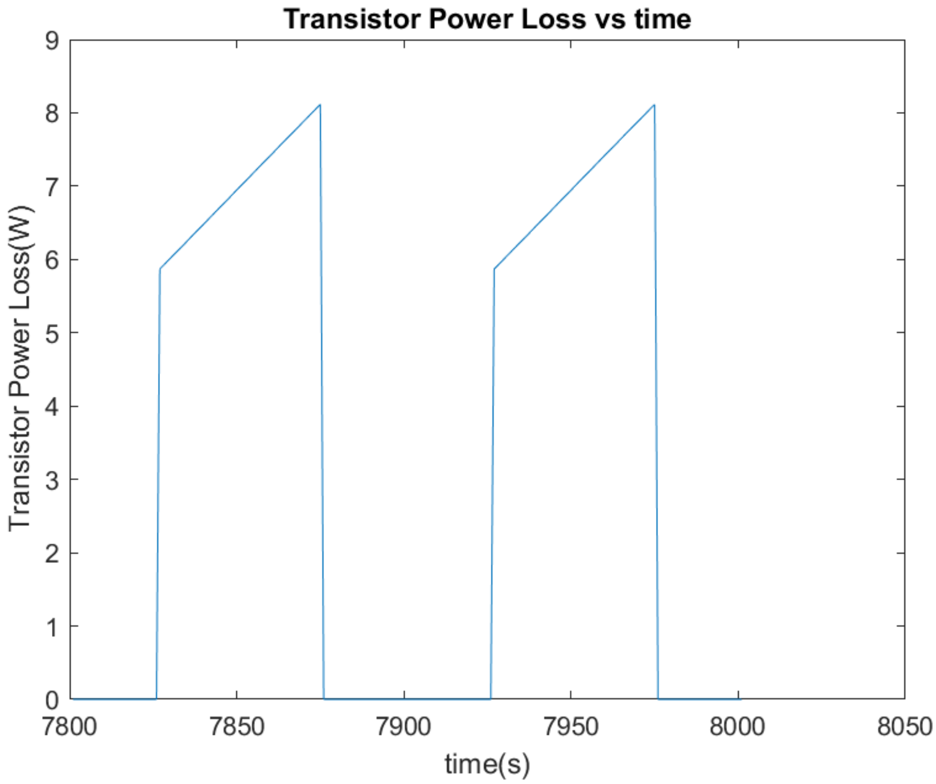
Non- ideal Switches Matlab Analysis:

We retrieved the plots below after running the Matlab code:




Inductor Current vs time
Inductor Current(A)
time(s)
750
700
650
600
550
500
450
7800
7850
7900
7950
8000
8050


Diode Power Loss vs time
$\times 10^5$
Diode Power Loss(W)
time(s)
1
0
-1
-2
-3
-4
-5
-6
7800
7850
7900
7950
8000
8050



The average output voltage was calculated to be 383.9841V and the efficiency is calculated to be 0.9435.

Non-ideal Switches Results and Conclusion:

As we can observe from the graphs and plots from both the Matlab and the manual analysis, we can see that they agree with each other on how the waveforms for the circuit voltages and currents should look like. One discrepancy shown from both analyses is that the efficiency of the circuit calculated by hand and that by Matlab is about 0.5 in unit different. I believe this stems from the fact that there has been some rounding up during the process of calculating by hand.