(e) if $f(t)$ is real and even, $\tilde{f}(\omega)$ is real and even

reality: $f(t)^* = f(t)$

even: $f(t) = f(-t)$

$\rightarrow\ f(t)^* = f(-t) \ \wedge\ \tilde{f}(\omega)^* = \tilde{f}(-\omega)$

Using (a)(1):

$$\tilde{f}(\omega)^* = \int_{-\infty}^{\infty} f(t)^* e^{-2\pi i\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} f(-t)\, e^{-2\pi i\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{-2\pi i\omega t}\, dt$$

$$= \tilde{f}(-\omega) \ \therefore$$

(f) if $f(t)$ is real and odd, then $\tilde{f}(\omega)$ is imaginary and odd

reality: $f^*(t) = f(t) \rightarrow \tilde{f}(-\omega) = \tilde{f}(\omega)^*$ (a)(1)
odd: $f(-t) = -f(t) \rightarrow \tilde{f}(-\omega) = -\tilde{f}(\omega)$ (d)(1)

$\rightarrow$ for $\tilde{f}(\omega)$ to be imaginary and odd
$\tilde{f}(\omega)^* = -\tilde{f}(\omega)$ (1)

Using (a)(1) $\tilde{f}(\omega)^* = \int_{-\infty}^{\infty} f(t)^* e^{-2\pi i\omega t}\, dt$

$$= \int_{-\infty}^{\infty} f(t)\, e^{-2\pi i\omega t}\, dt$$

$$= \tilde{f}(-\omega) \quad (2)$$

Combining this with (d)(1)

$\rightarrow\ \tilde{f}(\omega)^* = -\tilde{f}(\omega)$

Which is same as (1) $\because$