(c) If $f(t) = f(-t)$ then $\hat{f}(\omega) = \hat{f}(-\omega)$

$$\hat{f}(-\omega) = \int_{-\infty}^{\infty} f(t) e^{-2\pi i \omega t} dt$$

Substitute $f(t) \Rightarrow f(-t)$, $t \to -t$, $dt \to (-dt)$

$$\hat{f}(-\omega) = \int_{\infty}^{-\infty} f(-t) e^{2\pi i \omega t} (-dt)$$

$$= \int_{-\infty}^{\infty} f(-t) e^{2\pi i \omega t} dt \qquad (1)$$

Since $f(-t) = f(t)$

$$\to \tilde{f}(-\omega) = \int_{-\infty}^{\infty} f(t) e^{2\pi i \omega t} dt$$

$$= \tilde{f}(\omega) \qquad \therefore \qquad (2)$$

(d) If $f(-t) = -f(t)$, then $\tilde{f}(-\omega) = -\tilde{f}(\omega)$

Starting from (c)(1)

Since $f(-t) = -f(t)$

$$\tilde{f}(-\omega) = \int_{-\infty}^{\infty} \{-f(t)\} e^{2\pi i \omega t} dt$$

$$= -\int_{-\infty}^{\infty} f(t) e^{2\pi i \omega t} dt$$

$$= -\tilde{f}(\omega) \qquad \therefore \qquad (1)$$