Numerical methods in scientific computing III 2023

Exercise 7
Return by Tuesday 13.3.2023 23:59 to Moodle
Exercise session: Thursday 17.3.2023

Note the exceptional dates (exam week)!

Problem 1. *(computer)* (6 points)
Use the cubic B-splines ($k=3$) of exercise 6, problem 4 to plot a curve in xy plane using the parametric presentation of the curve:

$$\begin{cases} x(t)=\sum_{i=0}^{n-1} x_i N_i^3(t) \\ y(t)=\sum_{i=0}^{n-1} y_i N_i^3(t) \end{cases},$$

where $\{x_i, y_i\}$, $i=0,\dots,n$, are the control points of the curve[1]. You may choose these points freely. A suitable number is $n\sim 14$ and the parameter t runs e.g. from 0 to 1.

Problem 2. *(computer)* (6 points)
Estimate the value of

$$\int_0^\infty e^{-x}\cos^2(x)\,dx$$

(a) by adaptive Simpson's method and by simply truncating the integral to a finite upper limit.
(b) by adaptive Simpson‘s method and doing the transformation $x=t/(1-t)$.
(c) by Gauss-Laguerre quadrature with 2,4, and 8 points.

Problem 3. *(pen and paper, computer)* (6 points)
(a) Show that

$$\int_0^1 \sin(\sqrt{x})\,dx=\int_0^1 [\sin(\sqrt{x})-\sqrt{x}]\,dx+\frac{2}{3}.$$

Calculate the integral numerically with the Romberg method and using the forms

(b) $\int_0^1 \sin(\sqrt{x})\,dx$

(c) $\int_0^1 [\sin(\sqrt{x})-\sqrt{x}]\,dx+\frac{2}{3}$.

Comment your results.

1 Consult the lecture notes, chapter 6, page 94.

Problem 4. *(pen and paper)* (6 points)
Show that the element $R(1,1)$ of the Romberg method is equal to Simpson's rule.