Exercise 4

Problem 3

A) Iteration in plot

$$x_{i+1} = \mu x_i (1 - x_i)$$

$$= \mu (x_i - x_i^2) \quad (1)$$

Newton iteration:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad (2)$$

Mark (1) and (2) as equal:

$$\mu (x_i - x_i^2) = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\leftrightarrow \frac{f(x_i)}{f'(x_i)} - \mu x_i^2 + (\mu - 1) x_i = 0 \quad | \cdot f'(x_i)$$

$$\leftrightarrow f(x_i) + (-\mu x_i^2 + (\mu - 1) x_i) f'(x_i) = 0 \quad | \cdot \frac{1}{-\mu x_i^2 + (\mu - 1) x_i}$$

$$\leftrightarrow f'(x_i) + \frac{1}{-\mu x_i^2 + (\mu - 1) x_i} f(x_i) = 0 \qquad \boxed{y = f(x_i)}$$

$$\rightarrow \frac{dy}{dx_i} + \frac{y}{-\mu x_i^2 + (\mu - 1) x_i} = 0 \quad | \cdot \frac{dx_i}{y}$$

$$\leftrightarrow \frac{dy}{y} = - \frac{dx_i}{-\mu x_i^2 + (\mu - 1) x_i} \quad | \int (\cdot)$$

$$\leftrightarrow \ln |y| = - \int \frac{dx_i}{-\mu x_i^2 + (\mu - 1) x_i} + A$$

$$\leftrightarrow y = C \cdot e^{\left[- \int \frac{dx_i}{-\mu x_i^2 + (\mu - 1) x_i} \right]}$$

BRUNNEN

$$\rightarrow f(x_i) = C \cdot e^{\left[\frac{\ln(\mu(x_i - 1) + 1) - \ln(x_i)}{\mu - 1} \right]}$$ (Wolfram Alpha)