

(c) If $f(t) = f(-t)$ then $\tilde{f}(\omega) = \tilde{f}(-\omega)$

$$\tilde{f}(-\omega) = \int_{-\infty}^{\infty} f(t) e^{-2\pi i \omega t} dt$$

Substitute $f(t) \rightarrow f(-t)$, $t \rightarrow -t$, $dt \rightarrow (-dt)$

$$\begin{aligned}\tilde{f}(-\omega) &= \int_{\infty}^{-\infty} f(-t) e^{2\pi i \omega t} (-dt) \\ &= \int_{-\infty}^{\infty} f(-t) e^{2\pi i \omega t} dt \quad (1)\end{aligned}$$

Since $f(-t) = f(t)$

$$\begin{aligned}\rightarrow \tilde{f}(-\omega) &= \int_{-\infty}^{\infty} f(t) e^{2\pi i \omega t} dt \\ &= \tilde{f}(\omega) \quad \circ \circ \quad (2)\end{aligned}$$

(d) If $f(-t) = -f(t)$, then $\tilde{f}(-\omega) = -\tilde{f}(\omega)$

Starting from (c)(1)

Since $f(-t) = -f(t)$

$$\begin{aligned}\tilde{f}(-\omega) &= \int_{-\infty}^{\infty} \{-f(t)\} e^{2\pi i \omega t} dt \\ &= - \int_{-\infty}^{\infty} f(t) e^{2\pi i \omega t} dt \\ &= - \tilde{f}(\omega) \quad \circ \circ \quad (1)\end{aligned}$$