NMSC

Exercise 5

Problem 2

(a) $f(x) = e^{x}$ (1)

Use the identity

$e^{x} = e^{x_{map}} e^{x_{sc}}$, where

$\begin{cases} x_{map} = x \text{ mapped to } \left[-\frac{1}{2}, \frac{1}{2}\right] \\ x_{sc} = \text{scaler exponent} \end{cases}$

It is convenient to keep the scaler term $e^{x_{sc}}$ simple to calculate. By selecting

$x_{sc} = n \cdot \log(2)$ we have

$e^{x_{sc}} = 2^{n}$

The original function (1) then has an identity

$$\boxed{f(x) = f(x_{map}, n) = e^{x_{map}} \cdot 2^{n}} \quad (2)$$

Algorithm for selecting n and x_{map}

1: $n = \text{round}\left(\frac{x}{\ln(2)}\right)$

2: $x_{map} = x - n \cdot \ln(2)$, $x_{map} \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Now (2) can be used to evaluate $f(x)$

BRUNNEN

Is $x_{map} \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ true?

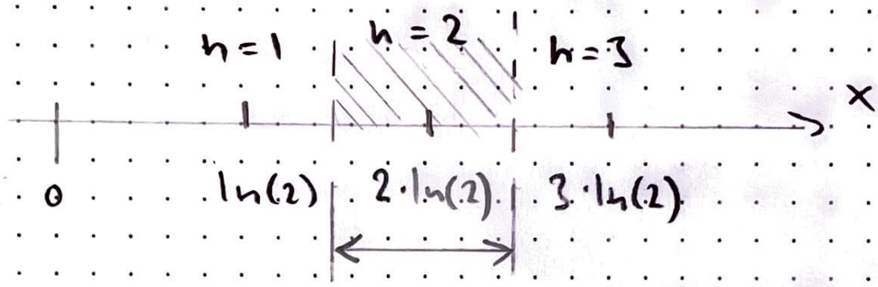
Step 2 means that

$$x_{map} = \text{round}\left[x - \{\ln(2), 2\cdot\ln(2), \dots, n\cdot\ln(2)\}\right]$$

whichever n minimizes x_{map}

$$\rightarrow x_{map} \in \left[-\frac{\ln(2)}{2}, \frac{\ln(2)}{2}\right]$$

$$\approx [-0{,}34, 0{,}34]$$



$$\begin{cases} x = 2\cdot\ln(2) \pm \varepsilon, & \varepsilon \approx 0{,}34 \\ x_{map} = \pm\varepsilon \end{cases}$$