Exercise 2

Problem 1

The ∞-norm of a vector

$$\|x\|_\infty = \lim_{p\to\infty}\left[\sum_{i=1}^{n}|x_i^p|\right]^{1/p}$$

Let x_j denote the vector element with the largest absolute value i.e. $|x_j| = \max_{1\le j\le n}(|x_n|)$.

Dividing all terms of the series by $|x_j^p|$ we get

$$\|x\|_p = \left(|x_j^p|\sum_{i=1}^{n}\frac{|x_i^p|}{|x_j^p|}\right)^{1/p}$$

$$= |x_j|\left(\sum_{i=1}^{n}\left|\frac{x_i}{x_j}\right|^p\right)^{1/p}$$

Now for the sum terms:

$$\begin{cases} i\neq j \;\to\; \left|\frac{x_i}{x_j}\right| < 1 \\ i=j \;\to\; \left|\frac{x_i}{x_j}\right| = \left|\frac{x_j}{x_j}\right| = 1 \end{cases} \Rightarrow \begin{cases} \lim_{p\to\infty}\left|\frac{x_i}{x_j}\right|^p = 0 \\ \lim_{p\to\infty}\left|\frac{x_j}{x_j}\right|^p = 1 \end{cases}$$

The only significant term remains

$$\|x\|_\infty = \lim_{p\to\infty}|x_j|\left(\sum_{i=1}^{n}\left|\frac{x_i}{x_j}\right|^p\right)^{1/p}$$

$$= |x_j|(1)^{1/p} = |x_j| = \max_{1\le i\le n}|x_n|$$