Exercise 10 Problem 1

First find out mean and variance of raw data:

$$\bar{x} = 29.20$$

$$\text{Var}(x) = 23.33$$

Scale the data to the distribution of the Gaussian process

Given parameters

$$\mu = 3,7 \quad , \quad \sigma^2 = 76.6$$

Scaling $x \to x'$

$$x' = \frac{(x - \bar{x})}{\sqrt{\text{var}(x)}} \cdot \sigma + \mu$$

Scaled data points

$$x' = \{ -9.4370, \; 1.8630, \; 2.8829, \; 7.3101, \; 18.4989, \; -11.9478, \; 7.4232, \; 8.2021, \; 18.6618, \; -1.9206, \; 5.0511, \; 17.1444, \; 12.6668, \; -2.3480, \; 10.9805, \; 4.0849, \; -4.2336, \; -6.1346, \; -3.5464, \; -1.2017 \}$$

$$\bar{x}' = 3.6999$$

$$\text{Var}(x') = 76.6$$

Calculate $\chi^2 = \sum_i \frac{(x_i' - \bar{x}')^2}{\mathrm{var}(x')}$

$= 20.0$

Degrees of freedom

$\nu = N - M$

$= 20 - 2$ (parameters μ, σ)

$= 18$

Since $\chi^2 \approx \nu$, the model assuming μ and σ scaled by $\bar{x}$ and $\mathrm{var}(x)$, fit well with the data.