Exercise 6

Problem 1

The Richardson iteration triangular form using $D(n,m)$

$D(1,1)$

$D(2,1)$ $D(2,2)$

$D(3,1)$ $D(3,2)$ $D(3,3)$

$\ldots$

$D(N,1)$ $D(N,2)$ $D(N,3)$ $\cdots$ $D(N,N)$

is equal to $L +$

USE THESE TERMS:

$\sum_{k=1}^{\infty} -a_{2k}\left(\frac{h}{2}\right)^{2k}$			
$\sum_{k=1}^{\infty} -a_{2k}\left(\frac{h}{4}\right)^{2k}$	$\sum_{k=2}^{\infty} -a_{2k}\left(\frac{h}{4}\right)^{2k}$		
$\sum_{k=1}^{\infty} -a_{2k}\left(\frac{h}{2^{N-1}}\right)^{2k}$	$\sum_{k=2}^{\infty} -a_{2k}\left(\frac{h}{2^{N-1}}\right)^{2k}$	$\sum_{k=N-1}^{\infty} -a_{2k}\left(\frac{h}{2^{N-1}}\right)^{2k}$	
$\sum_{k=1}^{\infty} -a_{2k}\left(\frac{h}{2^{N}}\right)^{2k}$	$\sum_{k=2}^{\infty} -a_{2k}\left(\frac{h}{2^{N}}\right)^{2k}$	$\sum_{k=N-1}^{\infty} -a_{2k}\left(\frac{h}{2^{N}}\right)^{2k}$	$\sum_{k=N}^{\infty} -a_{2k}\left(\frac{h}{2^{N}}\right)^{2k}$

For the "base" term L:

$$L = \frac{4^N}{4^N-1}L - \frac{1}{4^N-1}L$$

$$= \frac{4^N-1}{4^N-1}L$$

$$= L$$

For the error term:

$$\sum_{k=N}^{\infty} -a_{2k}\left(\frac{h}{2^N}\right)^{2k}$$

$$= \frac{4^N}{4^N-1}\sum_{k=N-1}^{\infty} -a_{2k}\left(\frac{h}{2^N}\right)^{2k} - \frac{1}{4^N-1}\sum_{k=N-1}^{\infty} -a_{2k}\left(\frac{h}{2^{N-1}}\right)^{2k}$$

$$= \frac{4^N}{4^N-1}\left(-a_{2(N-1)}\left(\frac{h}{2^N}\right)^{2(N-1)} + \sum_{k=N}^{\infty} -a_{2k}\left(\frac{h}{2^N}\right)^{2k}\right)$$

$$- \frac{1}{4^N-1}\left(-a_{2(N-1)}\left(\frac{h}{2^{N-1}}\right)^{2(N-1)} + \sum_{k=N}^{\infty} -a_{2k}\left(\frac{h}{2^{N-1}}\right)^{2k}\right)$$

$$= \frac{4^N}{4^N-1}\left(-a_{2(N-1)}\left(\frac{h}{2^N}\right)^{2(N-1)} + \sum_{k=N}^{\infty} -a_{2k}\left(\frac{h}{2^N}\right)^{2k}\right)$$

$$- \frac{1}{4^N-1}\left(-a_{2(N-1)}\left(\frac{h}{2^N}\right)^{2(N-1)} 2^{2(N-1)} + \sum_{k=N}^{\infty} -a_{2k}\left(\frac{h}{2^N}\right)^{2k}\left(2^{2k}\right)\right)$$

$$= \frac{4^N}{4^N-1}\sum_{k=N}^{\infty} -a_{2k}\left(\frac{h}{2^N}\right)^{2k} - \frac{1}{4^N-1}\sum_{k=N}^{\infty} -a_{2k}\left(\frac{h}{2^{N-1}}\right)^{2k}$$

$$-a_{2(N-1)}\left(\frac{h}{2^N}\right)^{2(N-1)}\left(\frac{4^N}{4^N-1} - \frac{1}{4^N-1}2^{2(N-1)}\right)$$

$$\boxed{\text{When } N \to \infty : \quad 2^{N-1} \to 2^N}$$

$$\to \sum_{k=N}^{\infty} -a_{2k}\left(\frac{h}{2^N}\right)^{2k} = \sum_{k=N} -a_{2k}\left(\frac{h}{2^N}\right)^{2k}\left(\frac{4^N}{4^N-1} - \frac{1}{4^N-1}\right)^{=1}$$

$$-a_{2(N-1)}\left(\frac{h}{2^N}\right)^{2(N-1)}\left(\frac{4^N}{4^N-1} - \frac{4^N}{4^N-1}\right)^{=0}$$

<u>which is true</u> $\circ_{\circ}^{\circ}$