b) Exponential distribution

$$P(x) = \frac{1}{2} e^{-|x|}$$

Moment generating function MGF

$$M(t) = \int e^{tx} \frac{1}{2} e^{-|x|} dx$$

$$= \frac{1}{2} \left[\int_{-\infty}^{0} e^{tx} e^{x} dx + \int_{0}^{\infty} e^{tx} e^{-x} \right]$$

$$= \frac{1}{2} \left[\int_{-\infty}^{0} e^{(t+1)x} dx + \int_{0}^{\infty} e^{(t-1)x} dx \right]$$

$$= \frac{1}{2} \left[\Big/_{-\infty}^{0} \frac{e^{(t+1)x}}{(t+1)} + \Big/_{0}^{\infty} \frac{e^{(t-1)x}}{(t-1)} \right]$$

$\boxed{x = 0}$ $\quad \frac{e^{(t+1)x}}{t+1} = \frac{1}{t+1} \quad , \quad \frac{e^{(t-1)x}}{t-1} = \frac{1}{t-1}$

$\boxed{x \to -\infty}$ $\quad \frac{e^{(t+1)x}}{t+1} \to 0 \quad , \quad t \geq -1$

$\boxed{x \to \infty}$ $\quad \frac{e^{(t-1)x}}{t-1} \to 0 \quad , \quad t \leq 1$

Convergence requires $t \in [-1, 1]$

$$M(t) = \frac{1}{2} \left[\frac{1}{t+1} - \frac{1}{t-1} \right]$$

$$= \frac{(t-1) - (t+1)}{2(t^2 - 1)}$$

$$= \frac{1}{1 - t^2}$$

Cumulant generating function CGF:

$$K(t) = \ln [M(t)]$$

$$= \ln\left[\frac{1}{1-t^2}\right]$$

$$= \ln(1) - \ln(1-t^2)$$

$$= -\ln(1-t^2)$$

$$K'(t) = \frac{d}{dt} - \ln(1-t^2)$$

$$= -\frac{1}{1-t^2} \cdot \frac{d}{dt}(1-t^2)$$

$$= -\frac{1}{1-t^2}(-2t)$$

$$= \frac{2t}{1-t^2} \quad , \; t^2 \neq 1$$

$$K''(t) = \frac{2\cdot(1-t^2) - 2t(-2t)}{(1-t^2)^2}$$

$$= \frac{2 - 2t^2 + 4t^2}{(1-t^2)^2}$$

$$= \frac{2(1-t^2)}{(1-t^2)^2}$$

$$= \frac{2}{1-t^2}$$

$$K'''(t) = 2(-1)(1-t^2)^{-2}$$

$$= \frac{-2}{(1-t^2)^2}$$

Cumulants from derivatives:

$$c_1 = K'(0) = \frac{2 \cdot 0}{1 - 0^2} = \underline{\underline{0}}$$

$$c_2 = K''(0) = \frac{2}{1 - 0^2} = \underline{\underline{2}}$$

$$c_3 = K'''(0) = \frac{-2}{(1 - 0^2)^2} = \underline{\underline{-2}}$$