Exercise 3

Problem 1

Let us denote matrix Q_{pq} as a combination of column vectors

$$Q_{pq} = [q_1, q_2, \dots q_N]$$

Matrix Q_{pq} is orthogonal if all col vectors $q_1 \dots q_N$ are orthogonal with each other, i.e.

$$q_m^T q_n = 0 \quad \forall\, m,n \in \{1, \dots, N\}$$

A) For vectors $m,n \notin \{p,q\}$ this is trivial, since these col vectors are unit vectors $\hat{\imath}_m, \hat{\imath}_n \in I$

B) Non-zero elements in rows p and q are only present in col vectors q_p and q_q and for q_p and q_q the row elements with indexes p and q are the only non-zero elements. Therefore $q_m^T q_p, q_m^T q_q = 0$ $\forall m \in \{1, \dots, N\}$

C) For the inner product

$$q_p^T q_q = q_q^T q_p = c \cdot (-s) + s \cdot c = 0$$

$\Rightarrow$ <u>all vectors q_m, q_n $\forall\, m,n \in \{1, \dots, N\}$</u>
<u>are orthogonal and thus</u>
<u>matrix Q_{pq} is orthogonal</u> $\therefore$