Numerical methods in scientific computing 2023

Exercise 2
Submit your solution to Moodle no later than Monday 30.1.2023 23:59
Exercise session: Friday 3.2.2023

Problem 1. *(pencil and paper)* (6 points)
Show that for a vector $\mathbf{x}$ of length n

$$\lim_{p\to\infty}\left[\sum_{i=1}^{n}\left|x_i{}^{p}\right|\right]^{1/p}=\max_{1\le i\le n}(|x_i|)\,,$$

which justifies the notation $\|\mathbf{x}\|_\infty$ for this norm.

Problem 2. *(computer)* (6 points)
In the Kahan summation algorithm the error due to the finite precision (`e`) is calculated in every iteration of the summation loop in addition to the sum itself (`s`). The algorithm can be written in C as below:

```
float s,x,y,t,e;
. . .
s= ...the first term in the sum...;
e=0.0;
for (i=2;i<=imax;i++) {
      x= ...whatever we have to sum...;
      y=x-e;
      t=s+y;
      e=(t-s)-y;
      s=t;
}
```

A) Write a function `harmonic_kahan(N)` that uses the above algorithm to calculate and return the sum of first *N* terms of the harmonic sum of Exercise 1, problem 2. Put your function in a file named `harmonic_kahan`. Use single precision.
B) By using your function, check if the value of the sum saturates to a finite value as in Exercise 1. Explain your findings.

Problem 3. *(computer; C, C++ or Fortran)* (6 points)
As mentioned in the lectures, LAPACK library is the most common software used in numerical linear algebra. Precompiled packages can be found for most Linux distros. For example, in Ubuntu you have to install packages `liblapack3` and `liblapack-dev`. Package `liblapack-doc` contains the `man` pages for all routines. This problem demonstrates the direct use of LAPACK library from Fortran and C.

A) Download the attached package `ex2_p3.zip`. The package contains a small program that reads in the a system and solves it using the LAPACK library routine `dgesv`[1]. Compile (either the Fortran or C version) and solve the systems given in the files `matrix6` and `matrix100`. In your answer give and

1 Documentation for the routine can be found at shorturl.at/aILOU

explain the compilation and run commands you used and the corresponding solution vectors $\mathbf{x}$.

B) Edit `matrix6` in such a way that it become singular. Return the matrix itself and the program output. Explain your findings.

Problem 4. *(computer; C, C++ or Fortran)* (6 points)

A) Write a function `residual(N,A,x,b,m)` that calculates and returns the norm-*m* of the residual $(\|\mathbf{A}\,\mathbf{x}-\mathbf{b}\|_m)$ of the solution of an $N\times N$ linear system $\mathbf{A}\,\mathbf{x}=\mathbf{b}$. Use the convention that for `m=0`, the norm-∞ is calculated. Put your function in a source file named `residual`. You can use library implementations for matrix multiplication but not for norm calculation.

B) Use your program to calculate the residual for the systems of Problem 3 for $m=1,2,\infty$.