NMSC
exercise 5

Problem 4.

function $P_N(x) = \sum_{j=0}^{N} c_j e^{jx} = c_0 + \sum_{j=1}^{N} c_j e^{jx}$ (1)

exponential can be expressed as polynomial series:

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \rightarrow e^{jx} = 1 + \sum_{k=1}^{\infty} \frac{(jx)^k}{k!} \quad (2)$$

Combining (1) and (2)

$$P_N(x) = c_0 + \sum_{j=1}^{N} c_j \left(1 + \sum_{k=1}^{\infty} \frac{(jx)^k}{k!}\right)$$

$$= c_0 + \sum_{j=1}^{N} c_j + \sum_{j=1}^{N} c_j \sum_{k=1}^{\infty} \frac{(jx)^k}{k!}$$

$$= \sum_{j} c_j + \sum_{k=1}^{\infty} \sum_{j=1}^{N} \left(\frac{c_j j^k}{k!}\right) x^k$$

which is clearly a polynomial of the form

$$P_N(x) = a_0 + \sum_{k=1}^{\infty} a_k x^k \quad (3)$$

where $\begin{cases} a_0 = \sum_{j=1}^{N} c_j \\ a_k = \frac{1}{k!} \sum_{j=1}^{N} c_j j^k \end{cases}$

According to lesson 6 page 5
There is a unique polynomial
$P_N(x)$

$$\begin{cases} a_0 = c_0 + c_1 + \ldots + c_N \\ a_k = \frac{1}{k!}\left(c_1 + c_2 2^k + c_3 3^k + \ldots + c_N N^k\right) \end{cases}$$

the coefficients a_k with increasing k:

$k=0$: $a_0 = c_1 + c_2 + \ldots + c_N$

$k=1$: $a_1 = c_1 + 2c_2 + 3c_3 + \ldots + Nc_n$

$k=2$: $a_2 = c_1 + 4c_2 + 9c_2 + \ldots + N^2 c_n$

From the format it is clear that for a unique selection of a_k there is a unique selection of c_j.

BRUNNEN