(g) if $f(t)$ is imaginary and even, then $\tilde{f}(\omega)$ is imaginary and even

imaginary: $f(t)^* = -f(t) \rightarrow \tilde{f}(-\omega) = -\tilde{f}(\omega)^*$ (b)(1)

even: $f(-t) = f(t) \rightarrow \tilde{f}(-\omega) = \tilde{f}(\omega)$ (c)(2)

$\rightarrow$ for $\tilde{f}(\omega)$ to be imaginary and even,

$$\tilde{f}(\omega)^* = -\tilde{f}(\omega) \qquad (1)$$

Using (c)(2) $\quad \tilde{f}(\omega) = \tilde{f}(-\omega)$

$\leftrightarrow \quad -\tilde{f}(\omega) = -\tilde{f}(-\omega)$

$\rightarrow \quad \tilde{f}(\omega)^* = -\tilde{f}(-\omega)$

Using (c)(1) $\quad = -\int_{-\infty}^{\infty} f(-t) e^{2\pi i \omega t} dt$

$f(-t) = f(t) \quad = -\int_{-\infty}^{\infty} f(t) e^{2\pi i \omega t} dt$

$= -\tilde{f}(\omega)$

which is same as (1) $\therefore$

(h) if $f(t)$ is imaginary and odd, then $\tilde{f}(\omega)$ is real and odd

imaginary: $f(t)^* = -f(t) \rightarrow \tilde{f}(-\omega) = -\tilde{f}(\omega)^*$ (b)(1)

odd: $f(-t) = -f(t) \rightarrow \tilde{f}(-\omega) = -\tilde{f}(\omega)$ (d)(1)

$\rightarrow$ for $\tilde{f}(\omega)$ to be real and odd,

$$\tilde{f}(\omega)^* = \tilde{f}(\omega) \qquad (1)$$

$$\tilde{f}(\omega)^* = \int_{-\infty}^{\infty} f(t)^* e^{-2\pi i \omega t} \, dt$$

$f(t)^* = -f(t)$

$$= \int_{-\infty}^{\infty} \{-f(t)\} e^{-2\pi i \omega t} \, dt$$

$$= -\int_{-\infty}^{\infty} f(t) e^{-2\pi i \omega t} \, dt$$

$$= -\tilde{f}(-\omega)$$

$$= \tilde{f}(\omega) \text{ for odd } f(t) \quad \therefore$$