Exercise 4

Problem 3

4) Iteration in plot

$$x_{i+1} = \mu x_i (1 - x_i)$$
$$= \mu (x_i - x_i^2) \qquad (1)$$

Newton iteration:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \qquad (2)$$

Mark (1) and (2) as equal:

$$\mu (x_i - x_i^2) = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\Leftrightarrow \frac{f(x_i)}{f'(x_i)} - \mu x_i^2 + (\mu - 1) x_i = 0 \quad \Big| \cdot f'(x_i)$$

$$\Leftrightarrow f(x_i) + (-\mu x_i^2 + (\mu - 1) x_i) f'(x_i) = 0 \quad \Big| \cdot \frac{1}{-\mu x_i^2 + (\mu - 1) x_i}$$

$$\Leftrightarrow f'(x_i) + \frac{1}{-\mu x_i^2 + (\mu - 1) x_i} f(x_i) = 0 \qquad \boxed{y = f(x_i)}$$

$$\rightarrow \frac{dy}{dx_i} + \frac{y}{-\mu x_i^2 + (\mu - 1) x_i} = 0 \quad \Big| \cdot \frac{dx_i}{y}$$

Homogenic equation

$$\Leftrightarrow \frac{dy}{y} = - \frac{dx_i}{-\mu x_i^2 + (\mu - 1) x_i} \quad \Big| \int (\cdot)$$

$$\Leftrightarrow \ln |y| = - \int \frac{dx_i}{-\mu x_i^2 + (\mu - 1) x_i} + A$$

$$\Leftrightarrow y = C \cdot e^{\left[- \int \frac{dx_i}{-\mu x_i^2 + (\mu - 1) x_i} \right]}$$

$$\rightarrow \underline{\underline{f(x_i) = C \cdot e^{\left[\frac{\ln(\mu (x_i - 1) + 1) - \ln(x_i)}{\mu - 1} \right]}}}$$

(Wolfram Alpha)

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This formula has undefined values with $\ln(0)$ terms.

Modify:

$$f = e\left[\frac{\ln\{\mu(x-1)+1\} - \ln(x)}{\mu-1}\right]$$

$$= e\left[\frac{\ln\left\{\frac{\mu(x-1)+1}{x}\right\}}{\mu-1}\right] \quad \text{-> No improvement!}$$

$$= \sqrt[\mu-1]{e^{\ln\left\{\frac{\mu(x-1)+1}{x}\right\}}}$$

$$= \sqrt[\mu-1]{\frac{\mu(x-1)+1}{x}}$$

$$f = \left(\frac{\mu(x-1)+1}{x}\right)^{\frac{1}{\mu-1}}$$ → this is computable but not continuous

$$f' = (a(x-1)+1)^{\frac{1}{a-1}-1} \quad x^{-\frac{a}{a-1}}$$

$$f(\overset{x}{0{,}5}, \overset{\mu}{2{,}05}) = \left(\frac{2{,}05(0{,}5-1)+1}{0{,}5}\right)^{\frac{1}{2{,}05-1}}$$

$$= \left(-\frac{1}{20}\right)^{\frac{20}{21}}$$

Function calculator:

$$= (-0{,}05)^{0{,}95238} = 0{,}0576\ldots$$

$$f(\overset{x}{0{,}5}, \overset{\mu}{2{,}1}) = \left(\frac{2{,}1(0{,}5-1)+1}{0{,}5}\right)^{\frac{1}{2{,}1-1}}$$

$$= \left(-\frac{1}{10}\right)^{\frac{10}{11}}$$

$$= 0{,}1232\ldots$$

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