# Explicit Model Predictive Control via Piecewise Nonlinear Approximations

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# Explicit MPC As Identification Problem

## Concepts

Implicit MPC: control law is computed via online programming

Explicit MPC: control law is computed offline

## Main idea

Assume that an implicit MPC is available, we try to identify each control input as piecewise nonlinear function of a regressor

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## Main idea

Assume that an implicit MPC is available, we try to identify each control input as piecewise nonlinear function of a regressor

### Problem statement

Given data $\mathcal{D} = \{q(k), Z(k)\}_{k=1}^{N}$ generated from implicit MPC

- A scalar quantity $q(k) \in \mathbb{R}$ which is a control input
- A regressor $Z(k) \in \mathbb{R}^{n_z}$ which is problem-dependent (trivial choice is plant states)

Find a multi-input single-output nonlinear map $F$ such that

$$q \approx F(Z)$$

# A Continuous Nonlinear Approximator

Search $F$ of the form

$$q \approx F(Z) = \Gamma^{-1}(L^T Z)$$

- $L \in \mathbb{R}^{n_z}$ is linear parameter
- $\Gamma$ is strictly increasing

# A Continuous Nonlinear Approximator

Search $F$ of the form

$$q \approx F(Z) = \Gamma^{-1}(L^T Z)$$

$$\Downarrow$$

$$\Gamma(q) \approx L^T Z$$

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Search $F$ of the form

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$$B(\xi(q))\mu = \Gamma(q) \approx L^T Z$$

- $L$ is linear parameter
- $\mu$ is nonlinear parameter
- $B(\cdot)$ is a basis function
- $\xi(q)$ is a normalization map

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## Optimization problem

Finding $(\mu, L)$ such that

$$B(\xi(q))\mu \approx Z^T L$$

and $\Gamma$ is strictly increasing

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## Optimization problem

Finding $(\mu, L)$ such that

$$\left\| \begin{bmatrix} B(\xi(q)) & -Z^T \end{bmatrix} \begin{bmatrix} \mu \\ L \end{bmatrix} \right\| \approx 0$$

and $[\frac{dB}{d\eta}(\eta)]\mu \geq \epsilon$ for all $\eta \in [0,\ 1]$

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and $[\frac{dB}{d\eta}(\eta_i)]\mu \geq \epsilon$ for some $0 = \eta_1 < \eta_2 \cdots < \eta_{n_{\text{grid}}} = 1$

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Search $F$ of the form

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$$B(\xi(q))\mu = \Gamma(q) \approx L^T Z$$

- $L$ is linear parameter
- $\mu$ is nonlinear parameter
- $B(\cdot)$ is a basis function
- $\xi(q)$ is a normalization map
- $\omega(q, Z)$ is positive weight

## Optimization problem

$$\begin{aligned} \min_{\mu, L} \quad & \omega(q, Z) \left\| \begin{bmatrix} B(\xi(q)) & -Z^T \end{bmatrix} \begin{bmatrix} \mu \\ L \end{bmatrix} \right\| \\ \text{s.t.} \quad & [\frac{dB}{d\eta}(\boldsymbol{\eta})]\mu \geq \epsilon \cdot \mathbf{1} \end{aligned}$$

# Extension to Piecewise Nonlinear Approximations

Search the partitions $\{\mathcal{R}_{(i)}\}_{i=1}^{s}$ of the regression domain such that

$$q \approx F(Z) = \begin{cases} \Gamma_{(1)}{}^{-1}(L_{(1)}{}^{T} Z) & \text{if } Z \in \mathcal{R}_{(1)} \\ \vdots & \\ \Gamma_{(s)}{}^{-1}(L_{(s)}{}^{T} Z) & \text{if } Z \in \mathcal{R}_{(s)} \end{cases}$$

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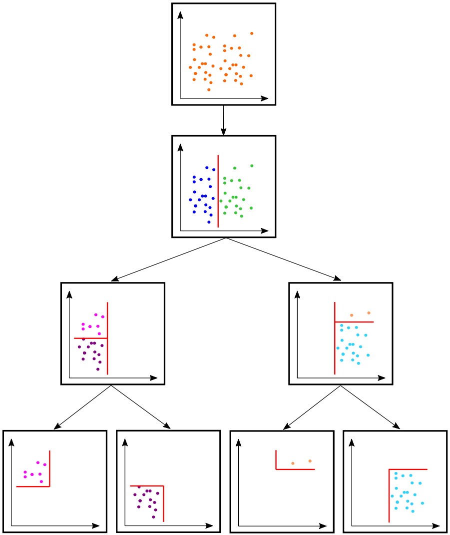
## A common strategy to find the satisfactory partitions

Iterative partitioning the regression domain into regions (by some heuristic rules) until a sufficiently small approximation error is obtained

# Extension to ... ons

Search the part ... that

## Hyper-rectangular domain partitioning



### A common st...

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## A common strategy to find the satisfactory partitions

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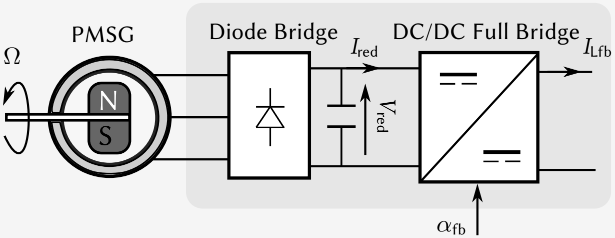
## Post-processing: complexity reduction

The number of regions $s$ can be reduced by forcing the model parameters of some regions to be identical $\Longrightarrow$ see paper for details

# The Power Electronic Module in A Power Plant

## Control Concept, Challenge & Solution



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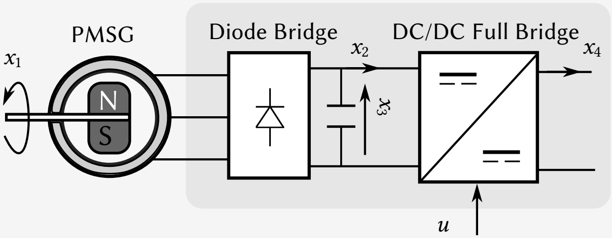

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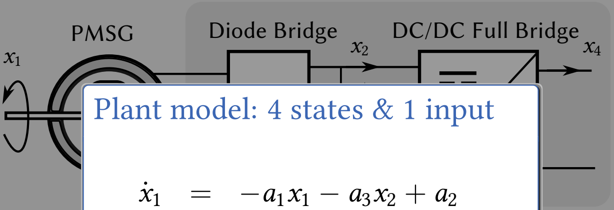


### Plant model: 4 states & 1 input

$$
\begin{aligned}
\dot{x}_1 &= -a_1 x_1 - a_3 x_2 + a_2 \\
\dot{x}_2 &= -a_4 x_2 + a_6 x_1 - a_7 x_3 \\
\dot{x}_3 &= a_8 (x_2 - k x_4 u) \\
\dot{x}_4 &= a_9 (-x_5^{\mathrm{st}} + k x_3 u)
\end{aligned}
$$

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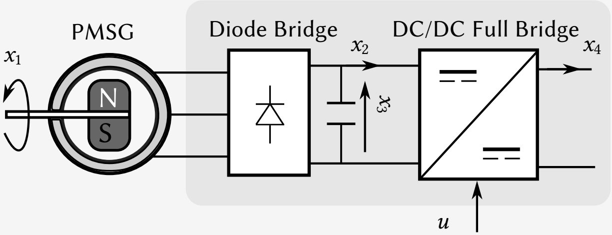



**Objective** Force $x_4$ to track a reference signal $x_4^r$

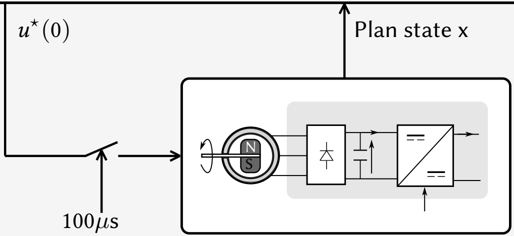
**Challenge** Fast sampling period $100\mu s$
Input saturation $u \in [0,\ 1]$
Positivity constraint $x_i \geq 0$, $i = 1, \ldots, 4$

**Solution** Fast nonlinear model predictive control

# Implicit Nonlinear Model Predictive Control

Reference $\begin{bmatrix} x^r \\ u^r \end{bmatrix}$

$$
\begin{aligned}
&\min_{u^\star} \textstyle\sum_{i=1}^{N_p-1} (||x(i+1) - x^r||_Q^2 + ||u(i) - u^r||_R^2) + \rho\delta^2 \\
&\text{s.t.} \quad x(0) = x && \text{measurement} \\
&\qquad\; x(i+1) = f(x(i), u(i)) && \text{prediction model} \\
&\qquad\; g(x(i), u(i), \delta) \leq 0 && \text{constraints}
\end{aligned}
$$



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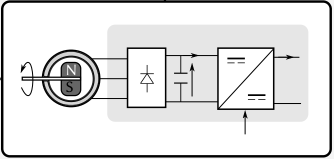
& g(x(i), u(i), \delta) \leq 0 & \text{constraints}
\end{aligned}$$

$$\begin{aligned}
& 0 \leq u(k) \leq 1 \\
& 0 - \delta \leq x_1(k) \leq 40 + \delta \\
& 4.5 - 0.1\delta \leq x_2(k) \leq 5.5 + 0.1\delta \\
& 55 - \delta \leq x_3(k) \leq 200 + \delta \\
& 1 - \delta \leq x_4(k) \leq 25 + \delta \\
& 0 \leq \delta
\end{aligned}$$

$u^\star(0)$

Plan state x

100$\mu$s



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Reference $\begin{bmatrix} x^r \\ u^r \end{bmatrix}$

$$\min_{u^\star} \sum_{i=1}^{N_p-1} (||x(i+1) - x^r||_Q^2 + ||u(i) - u^r||_R^2) + \rho\delta^2$$

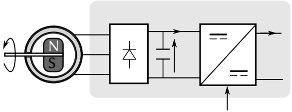
| | | |
|---|---|---|
| s.t. | $x(0) = x$ | measurement |
| | $x(i+1) = f(x(i), u(i))$ | prediction model |
| | $g(x(i), u(i), \delta) \leq 0$ | constraints |

$\exists$ NMPC control law $u(x, x^r, u^r)$

$u^\star(0)$

Plan state x

$100\mu s$



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$$\min_{u^\star} \sum_{i=1}^{N_p-1} (||x(i+1) - x^r||_Q^2 + ||u(i) - u^r||_R^2) + \rho\delta^2$$

s.t. $x(0) = x$ measurement

$x(i+1) = f(x(i), u(i))$ prediction model

$g(x(i), u(i), \delta) \leq 0$ constraints

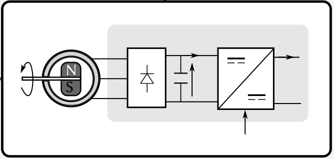
Choose $q = u$ and

$Z = [x, x^r, u^r]$

$u^\star(0)$

Plan state x

$100\mu$s

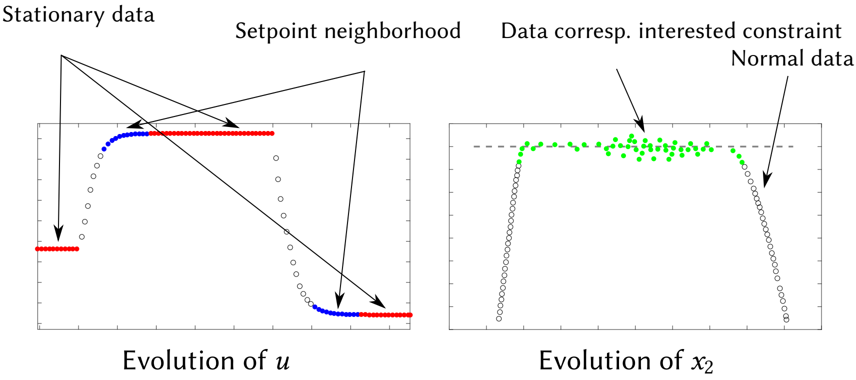


# Explicit Nonlinear Model Predictive Control

## Data & Weight Tuning

Data cardinality $> 24000$ generated from a 10s simulation scenario

### Weight tuning



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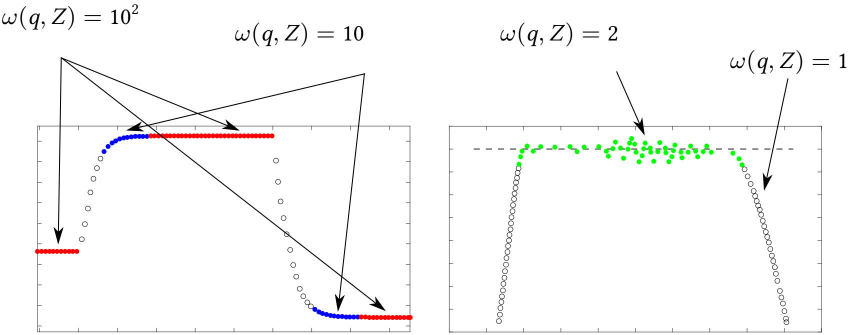


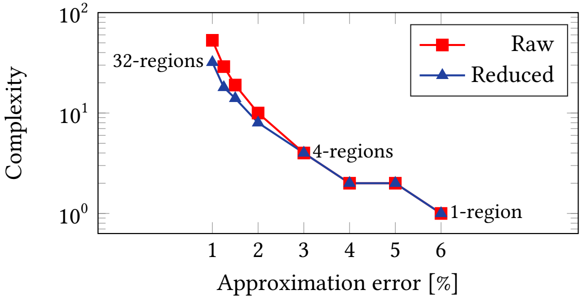

Evolution of $u$

Evolution of $x_2$

# Explicit Nonlinear Model Predictive Control

## Identification Results

### Trade-off curve



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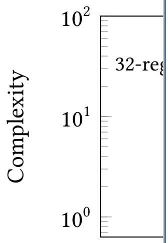
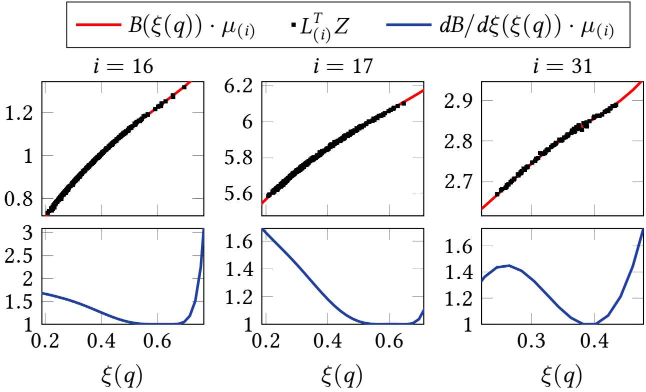
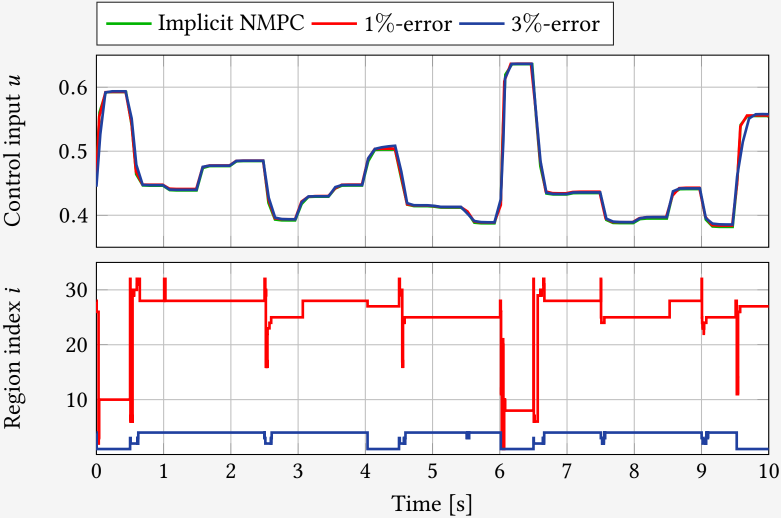



### Nonlinearity of a 32-regions 1%-error EMPC



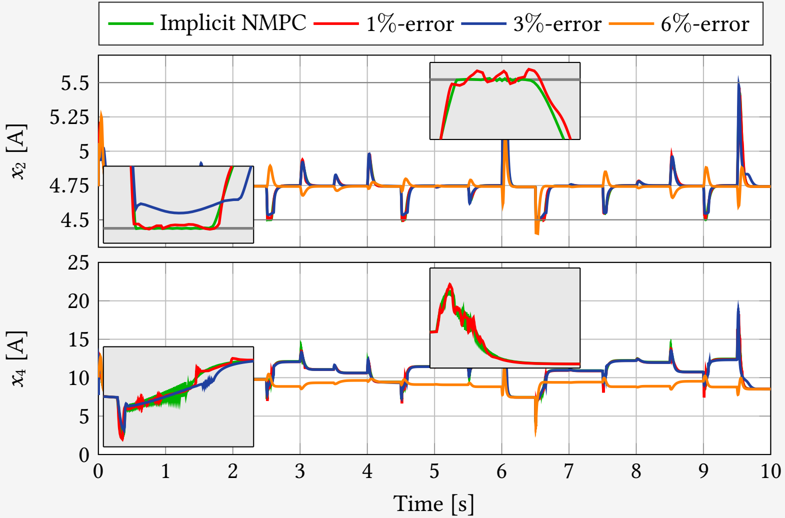
# Closed-Loop Validation

## Control Input & Region Index

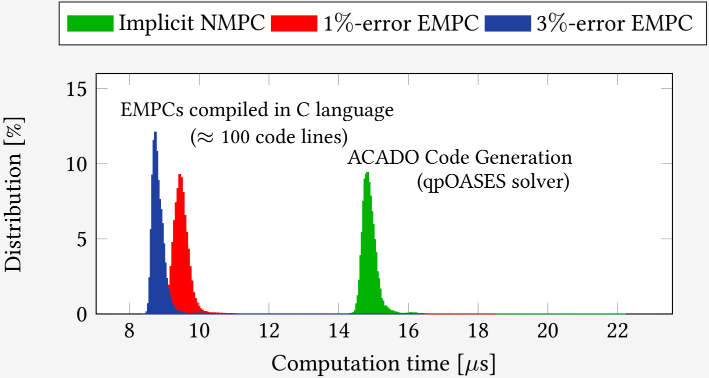


# Closed-Loop Validation

## Constraint Satisfaction, Setpoint Stabilization & Tracking Error



# Real-Time Comparison




| | Implicit NMPC | 1%-error EMPC | 3%-error EMPC |
|---|---|---|---|
| Worst | 22.22 | 18.51 | 16.43 |
| *Mean* | 14.89 | 9.41 | 8.83 |

Platform: 2.6 GHz Intel(R) Core(TM) i7 and 16GB of RAM.

# Conclusion, Future Work & Acknowledgement

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- A practical computational methodology for explicit piecewise nonlinear representation of MPC control laws derived from learning data
- Illustrative example with a fast constrained nonlinear application

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## Acknowledgement

This work is financially supported by the project CRYOGREEN from French National Research Agency (ANR)