The Nurse Scheduling Problem in Real-Life

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Abstract The aim of this paper is to study the scheduling process for two types of nursing teams, regular teams from care units and the float team that covers for shortages in the hospital. When managers address this problem, they either use a manual approach or have to invest in expensive commercial tool. We propose a simple heuristic approach, flexible and easy enough to be implemented on spreadsheets, and requiring almost no investment. The approach leads to streamlined process and higher-quality schedules for nurses. The multi-objective model and heuristics are presented, and additional analysis is performed to compare the performance of the approach. We show that our approach compares very well with an optimization software (CPLEX solver) and may be implemented at no cost. It addresses the lack of choice between either manual solution method or a commercial package at a high cost.

Keywords Health service · scheduling · manpower planning · optimization

1 Introduction

The province of Quebec is experiencing an ever-increasing demand for healthcare because of universal access and the aging of the population. However, budget cuts are unavoidable and human resources are increasingly scarce. Human resources and nursing resources in particular are the main focus of this paper. Nurses are responsible for many medical activities, and "account for approximately 25% of the total hospital operating budget and 44% of direct care costs" [20]. Offering flexible schedules is a primary objective, it encourages the stability of the workforce and makes the profession more attractive in a context where there are chronic staff shortages. Providing nurses with good working conditions is also important, particularly in the Quebec context where unions are powerful and threats of strikes are often in the news.

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Nurse scheduling is a complex monthly exercise with multiple and contradictory objectives such as minimizing the total costs while maximizing the satisfaction of the nurses' preferences or equally distributing the workload. The literature confirms that nurses' satisfaction is related to their workload and work conditions. In addition, the constraints imposed by collective agreements, unions, and contracts must be respected.

Constraints typically relate to:

- the quota requirement or the minimum number of nurses required to cover the units' needs for each shift;
- the load to be assigned to each nurse in terms of the number of shifts or the category of shifts (usually defined by contract);
- the skill requirements on units;
- the maximum number of consecutive days of work;
- the minimum break between two shifts;
- the ergonomic rules related to the number of isolated days of work or days-off for example.

Preferences are usually expressed as wished days-off or working days. In the context of nursing shortages, quota requirements should not be a "hard" constraint. This quota requirement is also referred to in the literature as a covering constraint. This constraint is the most important and challenging one. For regular units, this quota is usually defined in advance by budgets and patient-to-nurse ratio requirements. In the context of Quebec's shortages, the challenge is to be as close as possible to this quota. There is an obvious link between the regular units and the float team, since the latter absorbs shortages. The float team has available (regular) resources and also uses overtime hours, replacements, and outside on-call or agency nurses. Figure 1 illustrates the relationship between the two teams.

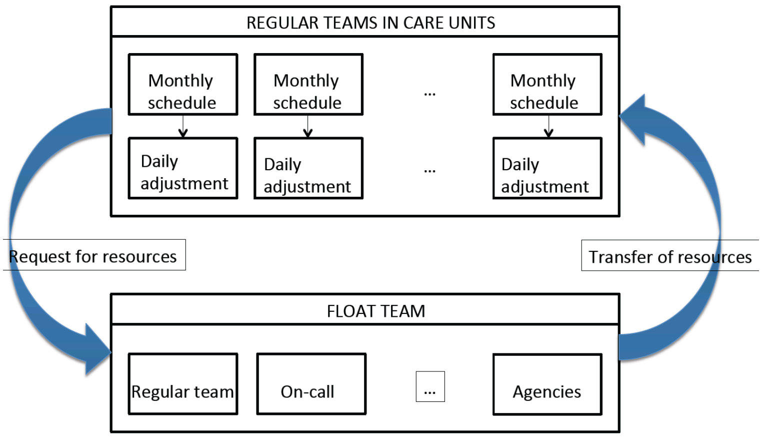


Fig. 1 Interaction between float team and regular care units

The adjustment of nurse schedules and determining the use of resources such as floaters and on-call nurses is discussed in [3, 4, 10] and is not the focus of this paper. We are interested in the schedule for the regular teams and the float team.

To understand how these constraints, preferences, and workload-distribution requirements affect the scheduling process, it is essential to obtain first-hand information from a hospital unit.

Two hospitals in Montreal were contacted, Notre-Dame and Sainte-Justine. These are large public and university hospitals. The nursing structures of these hospitals are standard. There are regular nurse teams in each care unit of the hospital, supported by a centralized float team for the entire hospital, whose purpose is to absorb demand variations. The nurse scheduling process in both hospitals is decentralized, and each unit delegates this task to a scheduler responsible for manually setting up a 28-day schedule for each nurse. The workforce size in a typical unit is about 30.

Scheduling in healthcare is more often than not, manual and time-consuming and does not always provide the best results. On the other hand, commercial softwares specialized on automated scheduling may be used, but usually at a high price.

When referring to the literature on nurse rostering and scheduling, one can see that the problem is extensively studied. Van den Bergh et al [19] present indepth review. They describe the methodology, models, and algorithms. A wide variety of methods have been used for nurse scheduling: mathematical programming, constraint programming, heuristics and meta-heuristics, hybrid methods, and simulation.

Different objectives are considered in the literature:

- to decrease manual scheduling;
- to increase demand coverage in terms of workforce size and also according to required skills;
- to maximize nurse preferences;
- to obtain equity between the schedules.

Researchers agree that although nurse scheduling is a well-studied problem, its practical solution and the implementation at the institution are still problematic. They emphasize that better solutions are obtained when the specific features of each application are included [7]. Maenhout and Vanhoucke [16, 17] are among the few researchers who focus on developing generic models and algorithms for the nurse scheduling problem.

Most studies are application-focused and use approaches such as tabu search [8], genetic algorithms [1], learning methodologies [2], scatter search [9] ,combinations [11], or even mathematical programming [22]. They deal with the constraints by penalizing their violation in the objective function. It is difficult to find feasible solutions, and in numerous applications, the quota requirement constraint cannot be satisfied; see e.g., [12]. Therefore, some researchers introduce an acceptable shortage or surplus that allows flexibility in the quota requirement. In [5], the demand constraint and respect of preferences are relaxed and a Lagrangian-based heuristic is used.

Three studies are particularly pertinent. Ferland et al [12] introduce an assignment-type model for the scheduling problem. They consider a set of objectives consisting of the formal objectives of the problem as well as a set of constraints. They use a tabu search [13] where at each iteration, two solutions are compared by considering their objectives in a lexical order. This prioritization of objectives is central in scheduling. In [6], a multi-objective approach is introduced that differentiates between hard and soft constraints. Valouxis and Housos [18] formalize the nurse scheduling problem using directly the rosters (alternating between work days and rest days) in the model. A non-optimal solution is generated by solving the mathematical model and a post-optimization phase using tabu search is performed. Wong et al [21] solve the nurse scheduling problem in a Hong Kong emergency department with a two-phase heuristic implemented in Excel. They build a feasible planning which is then improved by a local search taking into account soft constraints. However, the nurse scheduling problem for an emergency department is a particular case as the work environment is very dynamic.

We propose solving the nurse scheduling problem for both regular and float team using a scientific method based on operations research tools, simple and easy to implement at no extra cost. Indeed, one objective is to implement our method in a spreadsheet; nursing units already use Excel. Furthermore, as Kellogg and Walczak [15] note, one of the reasons that even approaches based on practical studies are not implemented is the use of complicated technology. Solutions based on free softwares such as COIN-OR are therefore not suitable. We address directly this issue with our approach. Because application-based approaches are more suitable for implementation in hospitals, we focus on a specific application in the constrained context of Quebec. To summarize, our objectives are threefold: we first conduct a practical study of the process of nurse scheduling in two different large size teaching hospitals, we then introduce a procedure based on local search that can be easily implemented at no extra cost. In addition of being user-friendly, it aims for standardization and efficiency. We then conduct an analysis on the performance of the tool. We primarily focus on the practical implementation of the proposed approach rather than the optimality of the solution.

This paper is organized as follows. The next section introduces new heuristics as well a description of the transferable prototypes. The results and discussions section shows the benefits of the proposed approaches in terms of process and scheduling method and we close with concluding remarks.

2 Problem statement and methods

To better address the nurse scheduling problem, we first review the scheduling process and analyze non valued added tasks, and finally introduce the mathematical model and the heuristics used to solve it.

2.1 Analyzing the current process

In both of the hospitals that we studied, the scheduler uses three inputs to build the planning: the constraints related to work agreements, the quota requirement, and the preferences of the nurses.

Work rules We first reviewed the collective agreements to collect work rules. These rules concern the shift (or set of shifts) assigned to each person, the number of shifts per week (usually five), per two weeks, and per four weeks, the definition of fixed days if any (in some applications weekend assignments may be fixed in advance), the length of the work sequence (the number of consecutive worked days, usually limited to five), and the minimum break between two shifts (typically 16 hours). In addition to these constraints, members of the float team are, in our case, given a *typical schedule* when signing their contract. In this case, the schedule specifies when they will work (shifts and sequences) if no modifications (such as hard preferences) are requested. Since the schedule is modified every month to match the nurses' preferences, the objective is to find a compromise among the preferences without moving too far from the typical schedule. The equivalent for a regular team may be viewed as their long term (recurrent) preferences.

Quota requirement The quota requirement for a regular unit is usually set in advance and referred to as a quota. Quotas differ from day to day and from shift to shift and are fixed by considering the budget and the units' needs. In the case of regular units, at least one head nurse during the day shift is requested. Generally, quotas vary (increase or decrease) following the units' workload. An informal priority is associated with the demand for each day, with Mondays and Fridays having a higher priority because these two days appear to be critical. Shortages are prohibited on these two days.

In contrast, we typically do not plan for a quota requirement for the float team. Still a size for the core team of floaters has to be determined for each shift. The exercise of well forecasting the demand is thus essential. This demand is not known in advance since it is related to:

- the variation in the workload of the units, and
- the variation in the workforce size (for example, because of absenteeism).

In order to use a general and flexible model, we studied the historical data based on one year information to evaluate the average demand for each shift and each day (Average Demand) and the variance (Variance) for the float team. Because the quota may not realistically be met (most of the time, there is up to a 50% workforce shortage in some cases), the average demand is modified for each day of the week using the following formula:

$$(\text{Average Demand}) * (\frac{\text{Available shifts}}{\text{Total Demand}}) * (\frac{\text{Variance}}{\text{Average Variance}}),$$

where (Available shifts) is the number of shifts available in the planning period and (Total demand) is the average demand in the same period. (Average Variance) is the aggregated Variance per shift. The ratio (Available shifts / Total Demand) ensures that the total modified demand can be covered by the total available shifts over a planning period. The ratio (Variance / Average Variance) will take into account the demand variability of resources in order to schedule more nurses on high variability shifts. One should note that this ratio is easily adjustable and is used as a parameter only.

In the remaining, we will refer to this modified demand as a quota requirement for the float team.

Preferences Surprisingly, the collection of the data on preferences was not as simple as it might seem. Preferences typically relate to whether or not nurses are willing to work on a particular date. Since the schedule planning is performed every four weeks, the scheduler usually allows the nurses a few days to indicate their

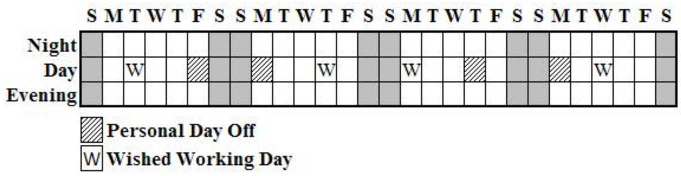


Fig. 2 Annotated schedule with preferences

preferences. Each nurse will annotate the draft schedule illustrated in Figure 2 to indicate her preferences.

All recurrent preferences are not mentioned because the scheduler is already aware of them. Clearly, this situation becomes very quickly problematic when the scheduler is absent or replaced.

The gathering of the preferences is interesting. The scheduler for the regular team is well organized; she asks for preferences, and the nurses have two weeks to respond. The preferred schedule satisfies all the nurses' preferences. The scheduler for the regular floaters has no defined process: she accepts changes to the schedule throughout the planning process until the final schedule is posted. This process is time-consuming, inefficient, and certainly not optimal. We analyzed different final schedules to determine the preferences for October and November 2010 based on the number of shift combinations per nurse, the seniority, the skills, the type of rotation or shift, and the weekends.

After collecting the preferences, schedulers follow independently a three-step process starting with the sketch schedule step to create an official schedule each month. Figure 3 illustrates the whole process. As stated previously, all constraints relative to work rules are hard constraints and must be respected when building the schedules. Since there are shortages of the human resources, the quota requirement may be seen as a goal to reach rather than mandatory to fulfill, and particular attention is paid to Monday and Friday shifts. The weekend assignments are fixed and cannot be modified. Both teams use software, based on Excel, that is designed to manage the human resources and is linked to the payroll. This does not optimize the schedule but is used as a visual tool.

We have identified two procedures to construct the nurse schedules in the studied hospitals: constructive or improving. The constructive procedures consists of using the draft schedule and adding shifts to reach the quota requirement. When all the mandatory shifts have been added, the following step is to try to improve the schedule one week at a time using a flip movement. Figure 4 illustrates this move. In the example, status of days 5 and 11, and days 18 and 21 are switched. If one was a working day, it becomes an off-day and vice versa. Allowed moves consist of moving a shift assignment from one day to another in the general case, and from one shift to another for the eligible subset of nurses.

The scheduler for the core team of the float team has an improving procedure: she moves from a completed schedule to another that better satisfies her requirements using the same flip movement. The personal judgment and experience of the scheduler are the key of choosing which schedule is best, and this is clearly difficult to model since no objective criteria are available. However, it may be summarized as reaching the quota requirement (because of the lack of resources) while

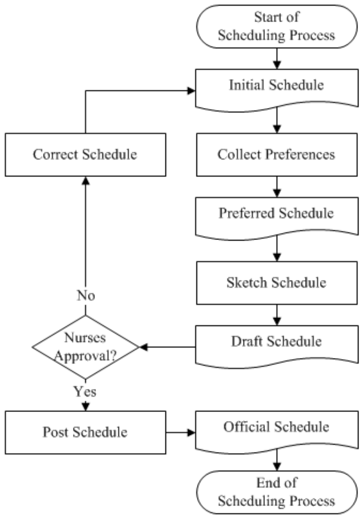


Fig. 3 Scheduling process map

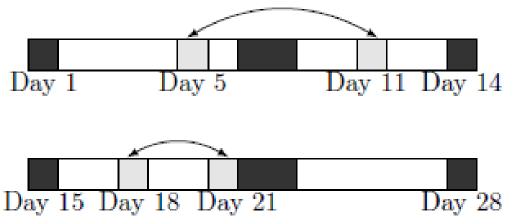


Fig. 4 Illustration of flip movements

satisfying the nurses' preferences. We interviewed the scheduler to recreate real situations and understand the trade-offs needed to satisfy both preferences and quota requirements. In summary, there are no firm rules: the win-win relationship between the scheduler and the nurses often leads to informal and subjective rules that are difficult to track. The schedulers claimed that they consider the equity of different schedules, but they did not indicate how they evaluate this equity.

Once the schedule is constructed, both schedules start the "correct schedule" phase. It consists of asking for the nurses' feedback on the fulfillment of their preferences. A negotiating phase follows to convince them to accept the schedule. The schedulers then try to implement changes where possible. Finally, two weeks before the beginning of the new period, the schedule is posted in the unit. It is now final and no major changes should be made besides unpredictable daily changes (step 5 - Adjust schedule). One can see the link between regular care units and the float team who will coverage for shortages.

We carefully reviewed each step and analyzed the value added of the activities. Results and recommendations on the process are reported in the results section.

2.2 Solving the nurse scheduling problem

We have previously described the nurse scheduling problem and we have formally stated the mathematical model in the appendix A. It considers both the regular team and the core members of the float team. Our objective is to minimize the penalties associated with ergonomics (changing shifts), quota requirement, preferences and differences from the typical schedule while ensuring a small change. This last objective is introduced to reduce resistance to change by introducing at least one difference in each period. One should note that for the float team, part of the objective function and some constraints do not apply and hence, the problem is easier to solve. Both problems are solved to optimality using the most efficient commercial optimization software package: CPLEX [14].

As stated earlier, CPLEX is very costly and our objective is to develop reasonable heuristics, one for each of the two teams, that are standard and simple to use, have no additional cost and resource requirements, give better results, and are less time-consuming than the manual approach. In this standardized context, the steps and rules are structured, organized, and clearly stated. Standardization leads to improvements and eases knowledge transfer in the process. It also brings objectivity on the quality of the schedule which reduces potential conflicts among staff. Furthermore, the steps are standard and simple enough to be used by a beginner. Finally, they are manual and require only pen and paper.

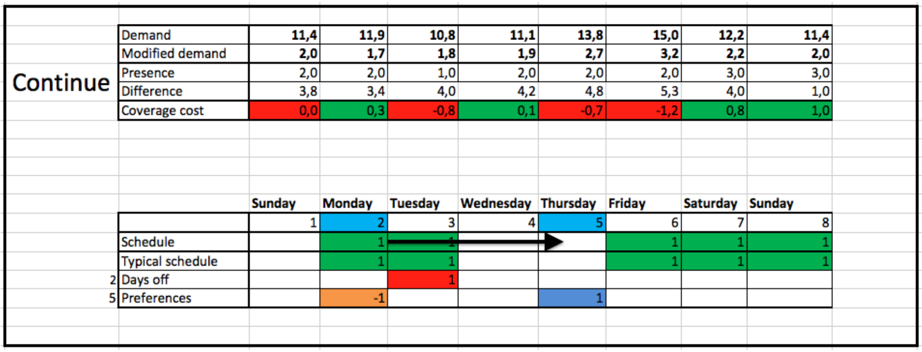
To facilitate their adoption, the algorithms are designed to be similar to the current working methods of the schedulers. We are using two different heuristics with different algorithms mainly to address the fact that weights of the objective function differ between the regular team scheduling problem and the float team scheduling problem. They both use the same type of movements currently used by the schedule, but in a structured manner. These algorithms are described in the appendix B.

Both algorithms are implemented using Excel and if desired can be used manually. We have chosen Excel since it is simple and already used in the hospitals. Figures 5 and 6 show how the permutations are performed with Excel. A permutation is implemented by changing a value from 1 to 0 (or vice versa): moving from an assignment to a particular shift on a particular day to no assignment (or the reverse). The schedule of each nurse is represented on a table as in Figure 5. Because the coverage cost on column 2 is equal to 0.3 and is negative on column 5 (respectively Monday and Thursday), the algorithm will choose a shift for a nurse whose preference is to work on Thursday rather than Monday as illustrated in the bottom table. Movements continue as long as the CONTINUE status on the left side of the table is on, as soon as it switches to STOP, the algorithm ends. The implicit stopping criteria is a local minimum related to the objective function.

3 Results and Discussions

3.1 Removing the non-added values activities

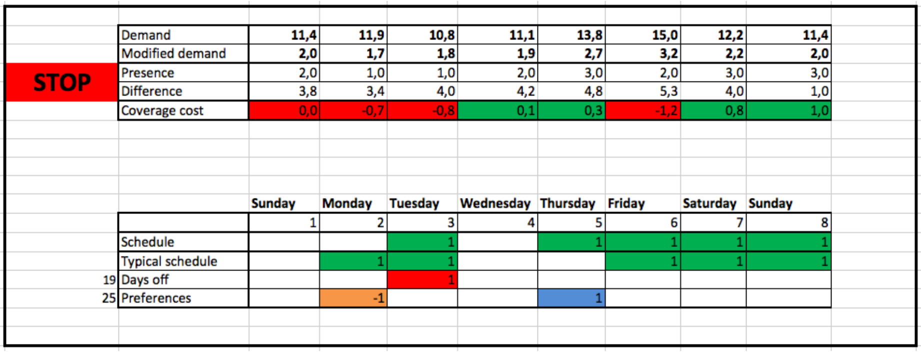
One can easily see that in practice many steps of the current process are non-value added. Two improvements are proposed, at different phases of the process, and both save time and energy.



Demand	11,4	11,9	10,8	11,1	13,8	15,0	12,2	11,4
Modified demand	2,0	1,7	1,8	1,9	2,7	3,2	2,2	2,0
Presence	2,0	2,0	1,0	2,0	2,0	2,0	3,0	3,0
Difference	3,8	3,4	4,0	4,2	4,8	5,3	4,0	1,0
Coverage cost	0,0	0,3	-0,8	0,1	-0,7	-1,2	0,8	1,0

		Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
		1	2	3	4	5	6	7	8
	Schedule		1	1			1	1	1
	Typical schedule		1	1			1	1	1
2	Days off			1					
5	Preferences		-1			1			

Fig. 5 Intermediate step of algorithm



Demand	11,4	11,9	10,8	11,1	13,8	15,0	12,2	11,4
Modified demand	2,0	1,7	1,8	1,9	2,7	3,2	2,2	2,0
Presence	2,0	1,0	1,0	2,0	3,0	2,0	3,0	3,0
Difference	3,8	3,4	4,0	4,2	4,8	5,3	4,0	1,0
Coverage cost	0,0	-0,7	-0,8	0,1	0,3	-1,2	0,8	1,0

		Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
		1	2	3	4	5	6	7	8
	Schedule			1		1	1	1	1
	Typical schedule		1	1			1	1	1
19	Days off			1					
25	Preferences		-1			1			

Fig. 6 Stopping criteria of algorithm

In the sketch-schedule step, when constructing the schedule by adding shifts, the scheduler should use one spreadsheet only to avoid switching between two different, one showing the assignments and the other the quota requirements. This constant switching could be eliminated by adding three lines to count the number of nurses for each shift and day.

In the collect preferences step, the scheduler should establish a clear rule for when the list of preferences is expected and a deadline after which no more changes are accepted. Currently, the scheduler spends much time cycling between the first and third steps because she accepts a new preference, sketches the schedule, corrects it, then adds another preference, and so on. At least 50% of her time could be saved by this simple change; the current process is time-consuming, inefficient, and certainly not optimal.

3.2 Comparisons of schedules quality

All the methods are compared using two main criteria: cost and time. Tables 1 and 2 summarize the results based on the following criteria. The gap from quota requirement is the under (over) coverage. The preferences are the total number of preferences respected. The equity measures the respecting of preferences across nurses; these should be balanced. The resolution time is for the scheduler and the heuristic solution the time to manually follow all the steps; the model column gives the CPU time consumed by the solver CPLEX. For the regular team we also compare the number of times that nurses switch shifts in a month. Finally, ergonomics indicates the quality of the schedule in terms of alternating work days and days-off. One should note that the tables do not list the objectives on the same order as the priorities for each objective depend on the scheduler or the environment.

Table 1 Results for regular team

Rank	Criterion	Clerk	Heuristic	Model
1	Quota requirement	12	8	0
2	Preferences	104	112	83
3	Equity	Very high	High	High
5	Number of alternating shifts	12	15	25
6	Ergonomics	273	291	311
	Resolution time	2 to 4 days	0.5 days	0.02 s
	Cost	Low	Very high	High

Table 1 presents the results for the schedule for the regular team. The quota requirement and the preferences are contradictory objectives that are difficult to satisfy simultaneously for the scheduler, and the heuristic. The mathematical model clearly provides the best solution. The equity between the nurses is difficult to measure since the scheduler considers their long-term satisfaction. Seniority is considered only by the heuristic and the model. The scheduler pays more attention to the alternating shifts and the ergonomics; the heuristic and the model focus more on the higher-ranking criteria. Finally, the value of the objective function is as expected, minimal with the model (-45) followed by the heuristic (160) and finally the scheduler (283). For the scheduler the construction of the schedule is time-consuming and therefore costly. The heuristic can be executed manually with no extra resources and at a low cost. In contrast, the model is solved by a commercial software CPLEX. Furthermore, even if a free solver was used, it would remain the implementation of a graphical user interface. The time reduction for providing the schedule is substantial, only half-day is necessary when using the heuristic and the solution is of better quality. Four additional quota requirements are met and preferences are better respected. The ergonomics have less weight in the objective and the heuristic, due to the preference of the scheduler. However, this heuristic is flexible enough to switch priorities.

Table 2 presents the results for the float team. As mentioned earlier, since the process is not streamlined for this float team, no documentation on preferences with the associated final planning of a same period is available. It is referred to as "None" in the table. To provide a comparison between the different methods, we have asked for the scheduler insight. The first line is the number of respected preferences. The second line is the number of differences between the solution and the typical schedule. The quota requirement is calculated here as being the over-coverage. The equity and the cost have not been measured. Because the seniority is taken into account in the parameters, the equity is good for every solution.

Table 2 Results for float team

Rank	Criterion	Typical Schedule	Clerk	Heuristic	Model
1	Preferences	0	None	7	12
2	Gap with the typical schedule	0	None	7	15
3	Quota requirement	7	None	5.5	5.7
4	Equity	Very high	Very high	High	High
	Resolution time	0 s	0 .5 day	30 min	0 .08 s
	Cost	Very high	Low	Very high	High

However, the scheduler can also check that the preferences are equitably accepted; this second objective is not modeled in our method. This explains why the scheduler's solution is better in terms of equity. The cost of the implementation and the time consumed allow us to measure the cost. As the scheduler requires much time to build the schedule, the heuristic and the model are clearly better. The resolution time is also very important. As the global scheduling process is not completely changed, the clerk run the problem each time she receives requirements from nurses. The 0.5 days in the table shows the time spent to collect nurses' preferences. As the scheduler performs this step repeatedly, providing her team with a schedule takes up to two weeks. Finally, our solutions would be much better if we were allowed more flexibility in the planning. A smaller emphasis on the typical schedule would introduce such flexibility. In conclusion, time reduction in planning the schedules is substantial (from half a day to 30 minutes) and the solution obtained with the heuristic is better than the manual one.

A work reorganization, improvement methods, and a standardized approach to the scheduling reduce the time required and allow both hospitals to make more efficient use of their resources. The optimization software CPLEX gives the best results, but both heuristics, considering that they are implemented in Excel, give very good results.

Additional benefits of the proposed methods should be highlighted. In our context one scheduler is assigned to each unit, and performs additional tasks to building the monthly schedule. Direct labor saving can not be observed in this context since no position can be closed. However, in other contexts where schedulers may be merged, reducing the amount of time taken to build the schedules from half a day to 30 minutes will have a direct impact in savings.

4 Conclusion

We have presented a practical study of nurse scheduling in medical units and for the float team in two large hospitals. The planning process is decentralized, so each scheduler develops a new schedule every month. We analyzed this process and presented a mathematical model based on multi-objective optimization for the schedule construction. We have developed simple procedures based on local search to solve the problem, which are more efficient than the manual method at no extra cost. These procedures are standard, easy to use and quick, and are based on simple permutations that can be implemented in Excel. Preferences of nurses are better respected than in the manual approach, quota requirement for each shift is closer to be achieved and generally the quality of the solutions is improved.

We are confident that our heuristic approach addresses the lack of choice between either manual solution method or a commercial package at a high cost.

5 Conflict of Interest

The authors declare that they have no conflict of interest.

A Mathematical model

This section presents in detail the mathematical model.
We define the following sets.
N: set of nurses considered;
$N_R \subset N$: set of head nurses;
J: set of days in the period (28 days with j = 1 referring to a Sunday);
K: set of 8-hour shifts for each day: Night, Day and Evening.

We use the following parameters.
f_{ij}: typical schedule of nurse i for each day j: 0 (no work) and +1 (work) ;
A_i: matrix of days-off for nurse i ($a_{ij} = 0$ for work or +1 for a day-off on day j);
$\bar{A} = \mathbb{1} - A$, (complementary matrix of A);
p_{ij}: preferences of nurse i expressed as -1 (no work), 0 (no preference), and +1 (work) for each day j (preferences are related to days, not shifts);
D_{jk}: quota requirement for nurses for day j and shift k;
Q_{ik}: available shift k for nurse i (=1 if the shift is available, 0 otherwise);
T: maximum number of days worked by a nurse in one week;
m_i: length of final work sequence in previous period for nurse i.

We introduce the following parameter costs.
c^+: cost of over-covering;
c^-: cost of under-covering;
β_i: cost of not satisfying a preference for nurse i;
γ_{ik}: cost of switching assignment;
r_i: benefit from satisfying typical schedule for nurse i (aggregation of seniority, experience, skills, etc.).

To determine the most accurate value for these parameters, we have used the scheduler's insight and his personal judgment when developing the schedules to evaluate the priorities between the objectives.

The decision variables are as follows.
x_{ijk}: 1 if nurse i is assigned shift k on day j and 0 otherwise;
z^+_{jk}: Over-coverage of day j in shift k;
z^-_{jk}: Under-coverage of day j in shift k;
y^+_{ijk}: auxiliary variable, positive part of $((\bar{a}_{ij} x_{ijk} - \bar{a}_{ij+1} x_{ij+1,k})$;
y^-_{ijk}: auxiliary variable, negative part of $(\bar{a}_{ij} x_{ijk} - \bar{a}_{ij+1} x_{ij+1,k})$.

We present the model using the three types of constraints: quota requirement, work rules and alternating shifts.

Quota requirement constraints

$$\sum_{i \in N_R} x_{ijDay} \geq 1, \qquad \forall j \in J \tag{1}$$

Constraints (1) ensure that at least one head nurse is present during day shifts.

$$z^+_{jk} \geq \sum_{i \in N} \bar{a}_{ij} x_{ijk} - D_{jk} \qquad \forall j \in J, \forall k \in K \tag{2}$$

$$z^-_{jk} \geq D_{jk} - \sum_{i \in N} \bar{a}_{ij} x_{ijk} \qquad \forall j \in J, \forall k \in K \tag{3}$$

Constraints (2) and (3) measure the over-coverage and the under-coverage (the gap between the quota requirement and the actual workforce); the days off are not taken into account ($\bar{a}_{ij} = 0$).

Work rules constraints

$$x_{ijk} \leq Q_{ik}, \qquad \forall i \in N, \forall j \in J, \forall k \in K \tag{4}$$

Constraints (4) ensure that nurses are assigned only to shifts that they are allowed to work.

$$\sum_{k \in K} x_{ijk} = 1, \qquad \forall i \in N, \forall j \in J \tag{5}$$

$$x_{ijDay} + x_{ij+1Night} \leq 1, \qquad \forall i \in N, \forall j = 1, ..., 27 \tag{6}$$

$$x_{ijEvening} + x_{ij+1Night} + x_{ij+1Day} \leq 1, \qquad \forall i \in N, \forall j = 1, ..., 27 \tag{7}$$

Constraints (5) ensure that each nurse works at most one shift per day; constraints (6) and (7) impose a minimum break between two shifts.

$$\sum_{j=b}^{b+6} \sum_{k \in K} x_{ijk} \leq T, \qquad \forall i \in N, b = 1, 8, 15, 22 \tag{8}$$

$$\sum_{j=b}^{b+5} \sum_{k \in K} x_{ijk} \leq T, \qquad \forall i \in N, b = 6, ..., 23 \tag{9}$$

$$\sum_{j=b}^{b+5-m_i} \sum_{k \in K} x_{ijk} \leq T - m_i, \qquad \forall i \in N, \forall i \in N, b = 1, ..., 5 \tag{10}$$

Constraints (8) ensure the maximum number of work days in one week. Constraints (9) set the maximum number of work days to T during six consecutive days while constraints (10) ensure the same rules for the beginning of the month (we take into account the number of work days at the end of the last month).

$$\sum_{j=b}^{b+13} \sum_{k \in K} x_{ijk} \leq \sum_{j=b}^{b+13} f_{ij}, \qquad \forall i \in N, b = 1, 15 \tag{11}$$

Constraints (11) ensure nurses to work in two weeks exactly the same number of shifts than in their typical schedule.

$$\sum_{k \in K} x_{ijk} = f_{ij}, \qquad \forall i \in N, j = 1, 7, 8, 14, 15, 21, 22, 28 \tag{12}$$

Constraints (12) ensure they work only the weekend assigned in their typical schedule. Finally to consider days-off, constraints (13) set $x_{ijk} = 1$ for one shift for a day off. Even if these constraints seem to assign a shift to the nurse, we consider it to be a dummy shift to keep using the same model. This shift will can therefore be counted in the previous constraints such as the collective agreements that assign T shifts per nurse per week. One can refer to constraints (2) and (3) to understand these dummy shifts are not considered in the covering constraints.

$$\sum_{k \in K} x_{ijk} = 1, \qquad \forall (i,j) \in \{(i,j) | i \in N, j \in J, a_{ij} = 1\} \tag{13}$$

Alternating shifts constraints

$$y^+_{ijk} - y^-_{ijk} = \bar{a}_{ij} x_{ijk} - \bar{a}_{ij+1} x_{ij+1,k}, \qquad \forall i \in N, j = 1, ..., 27, \forall k \in K \tag{14}$$

Constraints (14) define the variables y^+_{ijk} and y^-_{ijk}. $y^+_{ijk} - y^-_{ijk}$ is equal to 1 or -1 if the nurse i does not work on the same shift the day j and $j+1$. We have added this variable since we need to minimize the number of alternating shifts.

Model

$$\begin{aligned} \min \sum_{j=1}^{27} \sum_{i \in N} \sum_{k \in K} \gamma_{ik} * (y^+_{ijk} + y^-_{ijk}) + \\ \sum_{j \in J} \sum_{k \in K} [c^+ (z^+_{jk})^2 + c^- (z^-_{jk})^2 - \sum_{i \in N} (\beta_i p_{ij} x_{ijk} - r_i f_{ij} x_{ijk})] \end{aligned} \tag{15}$$

subject to:

$$\begin{aligned} &\text{Constraints} && (1) - (14) \\ &x_{ijk} \in \{0, 1\} && \forall i \in N, \forall j \in J, \forall k \in K && (16) \\ &z^+_{jk}, z^-_{jk} \in \mathrm{R}^+ && \forall j \in J, \forall k \in K && (17) \\ &y^+_{ijk}, y^-_{ijk} \in \mathrm{R}^+ && \forall i \in N, \forall j = 1, ..., 27, \forall k \in K && (18) \end{aligned}$$

The objective function (15) has three terms. The first specifies that rotation from one shift to another is minimized . The second is a quadratic term that imposes a rapidly increasing penalty as the solution deviates from the quota requirements; it will try to avoid situations where, for example, there is one over-coverage of two nurses instead of two over-coverages of one nurse. The third ensures that the preferences are maximized and equity is respected. Finally constraints (16), (17), and (18) ensure that the relevant variables are binary or nonnegative.

This model reflects exactly the problem for the regular team. For the float team, $\gamma_{ik} = 0$ since nurses do not rotate over shifts and constraints (1) is removed since no head nurse is mandatory. As there is no rotation, the model of the float team can be solved separately for each shift.

B Heuristic

This section presents heuristics for the regular team and the float team problems. Algorithm 1 presents the heuristic for the regular team. The initial schedule should contain the preferences and days-off. To balance the deficit between shifts, we calculate a score using the following formula:

$$SCORE = |G_{Night} - G_{Day}| + |G_{Night} - G_{Evening}| + |G_{Day} - G_{Evening}|$$

where G_{shift} represents the monthly gap between the quota and the number of nurses assigned to a shift. Using this score, flip movements as illustrated in Figure 4 are performed first for the nurses eligible to shift rotation and then the set of nurses in general. Compared to the current approach used by the scheduler, the algorithm reproduces the same movements in a larger neighborhood. The flip movements are performed not in a one-week period but with the whole horizon, for all nurses with a fixed schedule (such as head nurses), and are restrained to two weeks due to a hard constraint on number of days worked every 14 days. These movements are very performant and are used in most heuristic methods when applied to large scale problems.

Algorithm 2 presents the heuristic developed for the float team. It uses permutations to minimize the objective function. A permutation $\pi^i_{j_1,j_2}$ permutes days j_1 and j_2 for nurse i. π_i represents the set of authorized permutations.

A permutation $\pi^i_{j_1,j_2}$ is authorized if it respects $x^i_{j1} + x^i_{j2} = 1$ as well as the load to be assigned (8)-(11), and is similar to the flip movement illustrated in Figure 4. Once again, shifts

Algorithm 1 Heuristic: Regular team

```
INITIALIZE with the typical schedule
for each nurse do
   INSERT days off
   if ergonomics rules respected then
      INSERT preferences
   end if
end for
CALCULATE SCORE
while SCORE decreases do
   FIND shift s⁻ highest shortage AND shift s⁺ highest surplus
   FIND week t of shift s⁻ highest shortage
   FIND nurse i in rotation [s⁺; s⁻] working week t in shift s⁺
   for each day in W do
      SWITCH from shift s⁺ to shift s⁻ for nurse i
   end for
   CALCULATE SCORE
end while
for each shift, each week do
   REDUCE SHORTAGE with flip movements as illustrated in Figure 4
end for
```

Algorithm 2 Heuristic: Float team

$x_j^i = F_j^i, \forall i \in N, \forall j = 1, ..., 28$

$\Pi cost \Leftarrow \min_{\pi^i_{j1,j2} \in \Pi} \pi cost^i_{j1,j2}$

while $\Pi cost < 0$ **do**

  FIND $(i, j1, j2)$ via $\pi cost^i_{j1,j2} = \Pi cost$

  PERMUTE x^i_{j1} and x^i_{j2}

  $\Pi cost \Leftarrow \min_{\pi^i_{j1,j2} \in \Pi} \pi cost^i_{j1,j2}$

end while

can be moved only within a two-week period, from a non-working shift to a working shift. The idea behind heuristic is to keep the same movement that the clerk is currently making. We just evaluate each movement quantitatively; that is why our heuristic performs well on measurable objectives.

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