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The nurse scheduling problem: a goal programming and nonlinear optimization approaches

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Abstract. Nurses scheduling is an activity of allocating nurses to conduct a set of tasks at certain room at a hospital or health centre within a certain period. One of obstacles in the nurse scheduling is the lack of resources in order to fulfil the needs of the hospital. Nurse scheduling which is undertaken manually will be at risk of not fulfilling some nursing rules set by the hospital. Therefore, this study aimed to perform scheduling models that satisfy all the specific rules set by the management of Bogor State Hospital. We have developed three models to overcome the scheduling needs. Model 1 is designed to schedule nurses who are solely assigned to a certain inpatient unit and Model 2 is constructed to manage nurses who are assigned to an inpatient room as well as at Polyclinic room as conjunct nurses. As the assignment of nurses on each shift is uneven, then we propose Model 3 to minimize the variance of the workload in order to achieve equitable assignment on every shift. The first two models are formulated in goal programming framework, while the last model is in nonlinear optimization form.

1. Introduction

Staff scheduling is a universal problem that can be encountered in many organizations, such as call centers, educational institution, industry, hospital, and any other public services. It is one of the most important aspects of workforce management strategy and the one that is most prone to errors or issues as there are many entities should be considered, such as the staff turnover, employee availability, time between rotations, unusual periods of activity, and even the last minute shift changes.

The nurse scheduling problem (NSP) is a variant of staff scheduling problems which appoints nurses to shifts as well as rooms per day taking both hard constraints, i.e., hospital requirements, and soft constraints, i.e., nurse's preferences, into account. Thus, the timetable is commonly designed to maximize the preferences of the nurses and to minimize the total penalty cost from violations of the soft constraints [3]. Various models and techniques have been proposed to address the NSP, ranging from simple to advanced approaches.

It is proposed in [4] a simple heuristic approach, flexible and easy enough to be implemented on spreadsheets, and requiring almost no investment. In [5], it is considered an exact branch-and-price algorithm for solving the nurse scheduling problem incorporating multiple objectives and discuss different branching and pruning strategies. While, a search technique for nurse scheduling, which deals with it as a multi-objective problem, based on Pareto search methodology is presented by [2].

In this present work we formulate a specific NSP in a hospital within the framework of goal programming. We aimed to minimize the total penalty cost of violations of the number of days off as well as that of assigned shifts. As an alternate we also proposed a nonlinear optimization problem whose objective is to evenly distribute the workload among nurses by minimizing the variance.

2. Problem description

We considered an NSP at Bogor State Hospital which has 76 nurses, including several chief nurses, to be allocated into 4 inpatient rooms, namely Flamboyant, Vanda, Dahlia, and Pafio. The number of nurses assigned to each room is provided by Table 1. It can be seen from the number of chiefs that nurses in Dahlia room is divided into three teams, while those of Flamboyant and Vanda rooms into two teams, and there is only one team for Pafio room. The number of nurses required in each room and each shift is given by Table 2.

Table 1. The number of nurses available in each room

	Flamboyant	Dahlia	Vanda	Pafio
Nurse	23	20	13	12
Chief nurse	2	3	2	1
Total	25	23	15	13

Table 2. The number of nurses required in each room

Shift	Flamboyant	Dahlia	Vanda	Pafio
Morning	8	7	4	3
Afternoon	6	6	4	3
Night	6	6	4	3

The following regulations are imposed by hospital management and considered as a number of constraints, either hard or soft, in the scheduling models:

1. A day consists of three shifts of seven to ten hours, namely morning shift (7 am to 2 pm), afternoon shift (2 pm to 9 pm), and night shift (9 pm to 7 am).
2. A nurse has a total of 22 to 24 shifts within one scheduling period. In this case study, one scheduling period is equivalent to 28 days.
3. A nurse may have at most 3 consecutive night shifts.
4. A nurse should not be assigned morning shift immediately after night shift.
5. A nurse should have one day-off immediately after 2 consecutive night shifts and 2 consecutive days-off immediately after 3 consecutive night shifts.
6. There is a possibility that a nurse may be on duty for at most 6 consecutive days.
7. Chief nurse should always be assigned in morning shift and have day-off in Sunday.
8. Each nurse should have about 6 days-off within one scheduling period.
9. Each nurse should have about 6 morning shifts, 8 afternoon shifts, and 8 night shifts within one scheduling period.
10. All nurses are assigned in the same room as indicated in Table 1.
11. Beyond the typical constraints commonly imposed by a scheduling problem, we also consider a specific requirement, namely the nurses assigned in Pafio room, excluding the chief nurse, are also designated to have a limited number of shifts in Polyclinic room. This additional work is considered as an overtime.
12. Overtime shifts in Polyclinic room are conducted in Friday and Saturday during the morning and afternoon shifts, involving 4 nurses in each shifts. Each nurse should have 4 to 6 overtime shifts within period.

3. Mathematical models

Since all nurses are assigned in the same room, then the NSP for Flamboyant, Dahlia, and Vanda rooms can be formulated in a single model, i.e., Model 1. While that of Pafio room we introduced

Model 2. Both Models 1 and 2 are expressed in goal programming setting. A nonlinear optimization model is formulated in Model 3, aiming to evenly distribute the workload among nurses. In this development we mainly follow [1].

3.1. Model 1

To facilitate our models, we define the following sets: $\mathbb{D} = \{1,2,\dots,n\}$ the set of all days within one scheduling period, $\mathbb{L} \subset \mathbb{D}$ is the set of all Sundays, $\mathbb{R} = \{1,2,\dots,m\}$ is the set of all nurses, including the chief nurses, and $\mathbb{T} \subset \mathbb{N}$ is the set of all chief nurses. For Flamboyant room we have $m = 25$ and $\mathbb{T} = \{1,14\}$, for Dahlia room we have $m = 23$ and $\mathbb{T} = \{1,9,17\}$, and for Vanda room we have $m = 15$ and $\mathbb{T} = \{1,9\}$. We also define the following decision variables:

$$P_{ij} = \begin{cases} 1, & \text{if nurse } i \text{ is assigned a morning shift at day } j \\ 0, & \text{otherwise,} \end{cases}$$

$$S_{ij} = \begin{cases} 1, & \text{if nurse } i \text{ is assigned an afternoon shift at day } j \\ 0, & \text{otherwise,} \end{cases}$$

$$M_{ij} = \begin{cases} 1, & \text{if nurse } i \text{ is assigned a night shift at day } j \\ 0, & \text{otherwise,} \end{cases}$$

$$H_{ij} = \begin{cases} 1, & \text{if nurse } i \text{ has a day-off at day } j \\ 0, & \text{otherwise.} \end{cases}$$

The abovementioned regulations asked by the hospital management are then expressed in a set of hard and soft constraints alongside with other constraints commonly required in scheduling formulations. The following are the hard constraints:

1. The minimum number of nurses required in each shift every day should be fulfilled:
   $\sum_{i\in\mathbb{R}} P_{ij} \geq b_p, \sum_{i\in\mathbb{R}} S_{ij} \geq b_s, \sum_{i\in\mathbb{R}} M_{ij} \geq b_m, \forall j \in \mathbb{D}$.
   Here we set b_p, b_s, b_m according to Table 2.
2. In a day, a nurse should be assigned only one shift. Otherwise, he/she gets a day-off:
   $P_{ij} + S_{ij} + M_{ij} + H_{ij} = 1, \forall i \in \mathbb{R}, \forall j \in \mathbb{D}$.
3. A nurse should not be assigned morning shift immediately after night shift:
   $M_{ij} + P_{i(j+1)} \leq 1, \forall i \in \mathbb{R}, \forall j \in \mathbb{D}$.
4. A nurse is not assigned more than α consecutive night shifts:
   $M_{ij} + M_{i(j+1)} + \cdots + M_{i(j+\alpha-1)} \leq \alpha, \forall i \in \mathbb{R} - \mathbb{T}, \forall j = 1,2,\dots n - \alpha + 1$.
   In this case we set $\alpha = 3$.
5. There is no possibility that a nurse is on duty more than β consecutive days:
   $H_{ij} + H_{i(j+1)} + \cdots + H_{i(j+\beta)} \geq 1, \forall i \in \mathbb{R} - \mathbb{T}, \forall j = 1,2,\dots,n - \beta$,
   where $\beta = 6$.
6. Each nurse should have one day-off immediately after 2 consecutive night shifts and 2 consecutive days-off immediately after 3 consecutive night shifts:
   $M_{ij} + M_{i(j+1)} + M_{i(j+2)} - \left(H_{i(j+1)} + H_{i(j+2)} + H_{i(j+3)} + H_{i(j+4)}\right) \leq 1, \forall i \in \mathbb{R} - \mathbb{T}, \forall j = 1,2,\dots,n - 4$,
   $M_{ij} + M_{i(j+1)} + S_{i(j+2)} \leq 2, \forall i \in \mathbb{R} - \mathbb{T}, \forall j = 1,2,\dots,n - 2$.
7. Chief nurse should always be assigned in morning shift and should have day-off in Sunday:
   $\sum_{j\in\mathbb{D}} P_{ij} = n - \gamma, \sum_{j\in\mathbb{L}} H_{i,j} = \gamma, \forall i \in \mathbb{T}$,
   where γ is the number of Sundays within period of scheduling.

The remaining requirements regarding the ideal level for the number of morning, afternoon, and night shifts as well as the number of days-off are then formulated as soft constraints, i.e., goals. Thus, we attempt to achieve the ideal level as close as possible. We define deviation variables d_{ki}^- and d_{ki}^+, which are nonnegative integers, to measure the shortage and excess from the goal, respectively.

1. Each nurse should have about δ morning shifts, ε afternoon shifts, and θ night shifts within one scheduling period:

$\sum_{j\in\mathbb{D}} P_{ij} + d_{1i}^- - d_{1i}^+ = \delta, \sum_{j\in\mathbb{D}} S_{ij} + d_{2i}^- - d_{2i}^+ = \varepsilon, \sum_{j\in\mathbb{D}} M_{ij} + d_{3i}^- - d_{3i}^+ = \theta, \forall i \in \mathbb{R} - \mathbb{T}$.
For our case we have $\delta = 6$, $\varepsilon = 8$, and $\theta = 8$.

2. Each nurse should have about γ days-off within one scheduling period:
$\sum_{j\in\mathbb{D}} H_{ij} + d_{4i}^- - d_{4i}^+ = \gamma$,
where $\gamma = 6$.

By this goal programming framework, we then aim to minimize the shortage of the number of morning shifts and days-off from their ideal levels, as well as the excess of the number of afternoon and night shifts from their ideal levels, measured for all nurses, not including chief nurses. Thus, we consider the following objective function:

$$\min Z \coloneqq \sum_{i\in\mathbb{R}-\mathbb{T}} (d_{1i}^- + d_{2i}^+ + d_{3i}^+ + d_{4i}^-).$$

3.2. Model 2

Model 2 is devoted to undertake the NSP of Pafio room, where nurses of this room are required to have a number of additional shifts at Polyclinic room, counting as overtime shifts. This problem is obviously an extension of that formulated previously. Thus, it is only needed to add several new constraints to Model 1. For this, we define by $\mathbb{K}$ the set of all days for overtime shift, i.e., Friday and Saturday in the scheduling period. We also introduce the following new decision variables.

$$LP_{ij} = \begin{cases} 1, & \text{if nurse } i \text{ is assigned a morning overtime shift at day } j \\ 0, & \text{otherwise,} \end{cases}$$
$$LS_{ij} = \begin{cases} 1, & \text{if nurse } i \text{ is assigned an afternoon overtime shift at day } j \\ 0, & \text{otherwise.} \end{cases}$$

With the same objective function, we impose the following constraints in addition to those of Model 1.

1. In Friday or Saturday, a nurse should have either morning overtime shift, afternoon overtime shift, night shift, or day-off:
$LP_{ij} + LS_{ij} + M_{ij} + H_{ij} \le 1, \forall i \in \mathbb{R} - \mathbb{T}, \forall j \in \mathbb{K}$.
2. In Friday or Saturday, morning shift and morning overtime shift cannot be assigned in the same time. So do afternoon shift and afternoon overtime shift:
$P_{ij} + LP_{ij} \le 1, S_{ij} + LS_{ij} \le 1, \forall i \in \mathbb{R} - \mathbb{T}, \forall j \in \mathbb{K}$.
3. The number of nurses required for overtime should be fulfilled:
$\sum_{i\in\mathbb{R}-\mathbb{T}} LP_{ij} = b_l, \sum_{i\in\mathbb{R}-\mathbb{T}} LS_{ij} = b_l, \forall j \in \mathbb{K}$,
where b_l is the required number of nurses for overtime. Here we have $b_l = 4$.
4. As before, a nurse should not be assigned morning overtime shift immediately after night shift:
$M_{i(j-1)} + LP_{ij} \le 1, \forall i \in \mathbb{R} - \mathbb{T}, \forall j \in \mathbb{K}$.
5. Each nurse will only have a limited number of overtimes:
$\tau \le \sum_{j\in\mathbb{K}} (LP_{ij} + LS_{ij}) \le \varphi, \forall i \in \mathbb{R} - \mathbb{T}$,
where $\tau = 4$ and $\varphi = 6$.

3.3. Model 3

Model 3 is intended to evenly distribute the workload and day-off among nurses by minimizing the variance. As the formula of variance is nonlinear, this leads to a nonlinear optimization problem. In fact, we only need to design a new model by keeping all hard constraints prescribed in Model 1 and change its objective function into the following variance formulae:

$$\min Z \coloneqq \sum_{k=1}^{4} V_k,$$

where

$$V_1 = \frac{\sum_{i\in\mathbb{R}-\mathbb{T}} G_i^2}{m-|\mathbb{T}|} - \left(\frac{\sum_{i\in\mathbb{R}-\mathbb{T}}\sum_{j\in\mathbb{D}} P_{ij}}{m-|\mathbb{T}|}\right)^2, V_2 = \frac{\sum_{i\in\mathbb{R}-\mathbb{T}} N_i^2}{m-|\mathbb{T}|} - \left(\frac{\sum_{i\in\mathbb{R}-\mathbb{T}}\sum_{j\in\mathbb{D}} S_{ij}}{m-|\mathbb{T}|}\right)^2,$$

$$V_3 = \frac{\sum_{i\in\mathbb{R}-\mathbb{T}} B_i^2}{m-|\mathbb{T}|} - \left(\frac{\sum_{i\in\mathbb{R}-\mathbb{T}}\sum_{j\in\mathbb{D}} M_{ij}}{m-|\mathbb{T}|}\right)^2, V_4 = \frac{\sum_{i\in\mathbb{R}-\mathbb{T}} Y_i^2}{m-|\mathbb{T}|} - \left(\frac{\sum_{i\in\mathbb{R}-\mathbb{T}}\sum_{j\in\mathbb{D}} H_{ij}}{m-|\mathbb{T}|}\right)^2.$$

In this formulation, we define by G_i, N_i, B_i, and Y_i represent the number of morning shifts, afternoon shifts, night shifts, and days-off assigned to nurse i during the period, respectively. In this model, those of chief nurses are excluded from the variance calculation as $|\mathbb{T}|$ denotes the cardinality of the set $\mathbb{T}$. Thus, V_i denotes the variance of the workload among nurses within shift i, where i can be morning, afternoon, night, or day-off.

4. Result and discussion

Table 3 provides the difference between the value of objective function Z obtained by the OR models, i.e., Models 1 and 2, and that discovered in manual rooster applied by the hospital.

Table 3. The value of objective function by Models 1 and 2

Method	Inpatient Room			
	Flamboyant	Dahlia	Vanda	Pafio
Manual	29	42	41	20
OR Model	6	38	32	0

Since the objective function Z accounts the shortage of the number of morning shifts and days-off from their ideal levels, as well as the excess of the number of afternoon and night shifts from their ideal levels, it can be shown that in general by using OR modeling we are able to minimize the shortage of the number of morning shifts and days-off as it is preferable to have more and reduce the excess of the number of afternoon and night shifts it is preferable to have less.

Table 4. The percentage level of constraints accomplishment

No	Component	Flamboyant		Dahlia		Vanda	
		Manual	Model 1	Manual	Model 1	Manual	Model 1
1	Fulfillment of the required nurses in						
	morning shift	82.14	100	89.29	100	100	100
	afternoon shift	82.14	100	25	100	82.14	100
	night shift	100	100	100	100	100	100
2	Total of 22-24 shifts in one period	96	100	73.91	100	93.33	100
3	Day-off in Sunday for chief nurses	100	100	100	100	100	100
4	Overtime	0	0	100	0	38.46	0
5	Duties more than 6 consecutive days	13.04	0	20	0	7.69	0
6	Duties more than 3 consecutive night shifts	0	0	5	0	0	0
7	One day-off after 2 consecutive night shifts	100	100	100	100	100	100
8	Two consecutive days-off after 3 consecutive night shifts	91.67	100	90	100	53.85	100
9	No morning shift immediately after night shift	100	100	100	100	100	100

Table 4 provides the percentage level of constraints accomplishment by using manual rooster and by performing OR model. Obviously utilizing OR models give a better achievement. By OR model we can assure that there will be sufficient number of nurses in each shift, while manual rooster it is shown that it was an inadequate number of nurses in Flamboyant, Dahlia, and Vanda rooms. It is also guaranteed by the model that all nurses will have 22-24 shifts within one scheduling period, while manual scheduling of Dahlia room shows a 74 percent of nurses with the preferred number of shifts. Table 5 presents the shift, overtime, and day-off distribution among nurses in Pafio room by running Model 2.

Table 5. The shift, overtime, and day-off distribution in Pafio room

Nurse	Shift			Overtime		Day-off
	Morning	Afternoon	Night	Morning	Afternoon	
1*	24	0	0	0	0	4
2	6	8	7	2	3	7
3	6	7	8	2	2	7
4	6	8	8	2	2	6
5	7	8	7	3	3	6
6	10	8	4	4	2	6
7	6	8	8	3	3	6
8	6	8	7	4	2	7
9	8	8	6	2	4	6
10	6	8	8	3	1	6
11	8	8	6	2	4	6
12	6	8	8	2	3	6
13	6	7	8	3	3	7

*: chief nurse, not assigned to have overtime.

Table 6. The variance of workload and day-off

Inpatient Room	Method	Shift			
		Morning	Afternoon	Night	Day-off
Vanda	Manual	3.08	3.69	0.59	0.17
	Model 1	0.76	0.42	0.59	0.59
	Model 3	0	0	0	0
Pafio	Manual	4.16	5.25	0	0.47
	Model 2	1.66	0.15	1.5	0.24
	Model 3	0	0	0	0

Even though nurse scheduling by using OR model provides better solutions than manual approach and satisfies all requirements addressed by the hospital, there still exists some less preferable facts regarding the distribution of shifts and days-off among nurses. For example, nurse scheduling in Vanda room by using Model 1 provides variance of 0.59 on the number of night shifts and days-off as depicted by Table 6. It means that nurses responsible in this room possess different number of night shifts and days-off, where the former is ranging from 8 to 10 times within period and the later is ranging from 4 to 6 times. Of course, having more night shifts but less days-off is less preferable for most nurses.

Model 3 is developed to reduce the difference in shift workload and day-off among nurses by minimizing the variance. Table 6 shows that, in the case of Vanda and Pafio rooms, the variance can be minimized to zero, indicating that all nurses, not including the chief nurses, have the same number of morning, afternoon, night, and day-off shifts. However, this fair distribution of workload and day-off is compensated by the increase of computation time. Regarding this drawback, Model 3 can still be considered as an alternate model in preparing nurses timetable.

5. Conclusion

In this paper we have developed three mathematical models to address the nurse scheduling problem arising at Bogor State Hospital. The first two models are formulated in term of goal programming and the last model is a nonlinear programming of variance minimization. It is shown that all conditions required by the hospital can be fulfilled by Models 1 and 2 and the variance due to shift workload and day-off among nurses can be minimized by Model 3.

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