OPTIMIZING THE THREE-SHIFT NURSE SCHEDULE USING REVISED SIMPLEX METHOD (Nurse Scheduling Problem)

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OPTIMIZING THE THREE-SHIFT NURSE SCHEDULE USING REVISED SIMPLEX METHOD

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DECLARATION

We hereby declare that this dissertation is our own work under the able supervision of Dr. Peter Amoako-Yirenkyi, towards the award of the Bachelor of Science(BSc.) in Mathematics and that, to the best of our knowledge, it contains no material previously published by another person nor that which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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DEDICATION

This research paper is dedicated to our supervisor Dr. Peter Amoako-Yirenkyi, our families and friends who have supported us throughout this journey.

ABSTRACT

Staff scheduling is a major problem that can be encountered in many organizations, such as educational institutions, industries, hospitals, and many other places. It is one of the most crucial aspects of the workforce management strategy but yet the one which is prone to many issues as there are several factors to consider before drawing a schedule. The Nurse Scheduling Problem is a variant of Staff scheduling problems which assigns nurses to shifts as well as rooms per day taking both hard constraints (hospital policies) and soft constraints (nurses' preference) into account. Nurse scheduling is an essential problem in hospitals. In this project, we developed a model based on the organizational policies of a Hospital and the various nurses' preferences. We then solved this model using the Revised Simplex Method which was implemented on a parallel computer using the Pulp library and the GLPK solver to generate an optimal schedule for the nurses. Our case study was the University Hospital, KNUST.

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CONTENTS

LIST OF ABBREVIATION

KNUST .... Kwame Nkrumah University of Science and Technology

NIMS .... National Institute for Mathematcial Sciences

NSP .... Nurse Scheduling Problem

GLPK .... GNU Linear Programming Kit

GNU .... GNU's Not Unix

LR .... Lagrangian Relaxation

LPP .... Linear Programming Problem(s)

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CHAPTER 1

INTRODUCTION

1.1 Background of Study

The problem of scheduling nurses affects most healthcare systems. Scheduling typically consists of two parts: a "shift plan" determining how many nurses are required to work each shift, then a "shift assignment" which assigns individual nurses to those shifts in the shift plan. Scheduling the right nursing staff will improve the performance and quality of the nursing unit. Proper scheduling will help recruitment, nurse preferences, and maintain an overtime budget for a logical nurse.

The most important thing in scheduling is increasing the nurses' spirit and ensuring patient safety. Scheduling of nurse or commonly referred to as Nurse Scheduling Problems (NSP) are challenging because the number of nurses is relatively limited compared to the number of patients in the hospital, so the scheduling of nurses is required to get a schedule with a fair workload for each nurse and fulfill existing scheduling restrictions.

Nurse scheduling problem(NSP) is one of the complex problems in assignment problems. It has been assigned a complexity class of NP-hard. NSP requires schedules in which employees are assigned shifts and working times throughout some planning period. Solving the scheduling problem, i.e. making sure all shifts are covered with sufficient staff while respecting numerous employee preferences and complying with work place regulations, is no trivial task. The scheduling problem is simply too complex for any good schedule to be identified easily. Nevertheless, the majority of hospitals still create schedules through manual methods, although these have proven both time-consuming and expensive (Kelloggand Walczak, 2007)

Good scheduling can contribute to increase the nurse welfare and distribute workload evenly. This is something that manually generated schedules cannot guarantee. Unfortunately, most hospitals still generate their nurses' schedules manually including the University Hospital, KNUST.

This project seeks to develop a model and an algorithm that can be used to generate an optimal nurses' roster for the nurses at the University Hospital, KNUST to maximize the preferences of the nurses.

1.2 Problem Statement

There are many problems associated with manually generated nurse schedules since they are not optimal. Some of these problems include; the time required to generate a schedule, each nurse's satisfaction with his or her shift, and the low level of productivity in the work of the nurses.

This project aims to solve these problems by developing a model to generate an optimal nurses' schedule for the University Hospital, KNUST, ensuring nurses are satisfied with their shift assignments while still ensuring organizational constraints are satisfied.

1.3 Objectives

The goal of this research is to:

i. Formulate the Nurse Scheduling Problem as a linear programming model

ii. Solve the mathematical model using the Revised Simplex Method which will be implemented on a parallel computer using the Pulp library.

iii. To generate an optimal nurse schedule for some number of nurses.

iv. To automate the schedule to make it easy to affect changes.

1.4 Methodology

We outlined the policies governing the schedule, made known some assumptions, and developed a model for the problem.

The model was formulated as a linear programming model so, we used the Revised Simplex Method to solve it. The method was implemented on a parallel computer by using an external library called pulp with a solver called GLPK (GNU(GNU's Not UNIX) Linear Programming Kit).

1.5 Justification

Generating a schedule using an optimization technique as compared to manually generated nurses' schedule by head nurses provide better rosters void of any human biasness.

1.6 Report Organization

The first chapter discusses the background of the study, statement of the problem as well as the objectives of this study. It also discusses the methodology and the justification of the study.

The second chapter reviews various methods used by other researchers related to this study.

The third chapter discusses the methodology. It also explains the mathematical formulation of the problem in this study.

The fourth chapter shows the results and interpretations.

The fifth chapter concludes the study with some recommendations. It also acknowledges the reference used in the study.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The nurse rostering problem has been solved by many researchers using mathematical methods that give exact solutions as well as heuristics that approximate near-optimal solutions.

Articles on nurse rostering problems differ by objectives, constraints to satisfy as well as methodology. Over the last decade, the overall objective of works published fall under some categories, namely, Financial Cost, Scheduling, Nurses Number Optimization and Nurses Satisfaction. In this chapter, we review some articles under these categories.

It is very interesting to combine these criteria for evaluating schedules since they address more than computable standards. From a general hospital scheduling point of view, it makes sense to take such a broad interpretation of cost (to generate the schedule) into account. However, it would also make sense to add other criteria (like personnel cost, for example) to the list. Nearly all the criteria are very hard to measure. Warner compares three scheduling approaches against these 3 criteria:

- In the Traditional Approach, the schedules are generated by hand. This policy is flexible, which is the only advantage with respect to the criteria.

- Cyclical Scheduling generally provides good schedules but it cannot easily address personal requests.

- Computer Aided Traditional Scheduling enables a fast and more complete search for good schedules. The advantages of this approach are high with respect to all the criteria considered.

2.2 Scheduling

Warner (3), also deals with staffing in some detail. He defines the staffing problem as an annual decision in which seasonal variation can be considered. It consists of determining an appropriate number of full-time equivalent nurses for each skill category. A methodology for the staffing decision is proposed by Warner and many hospitals accept it (subject to small adaptations). After the scheduling phase comes the third step: the reallocation of nurses. This phase is a fine-tuning of staffing and scheduling. It involves determining how float nurses are assigned to units based on unforecastable changes or absenteeism. Among hospital schedulers, the potential benefits of this reallocation step in the process are uncertain. However, Warner is convinced that the combination of the three stages in the end leads to a better scheduling policy. Warner's paper represents a key contribution to nurse staffing.

Smith-Daniels, Schweikhart, and Smith-Daniels (1988) present a literature overview on capacity planning in healthcare. They distinguish between capacity decisions on facility resources and on workforce resources. In these categories, two decision levels are selected: acquisition decisions and allocation decisions. The acquisition decisions for workforce resources match the meaning of 'staffing' as it is defined in this section. Two other decisions in the group are the assignment of workers to units and to tasks. A useful contribution of the paper is the collection of many different strategies and approaches for the decision-maker. Smith-Daniels et al. predict that the strict staffing and timetabling of people and other resources will all be combined in an objective for new large-scale health organizations. The allocation decisions for work-force resources, namely the assignment of workers to days and shifts, is not deeply studied in the paper. It may therefore be problematic to believe that the integration of all the planning categories into one objective will be applicable to complex real world problems.

De Vries (1987), developed a ‘management control framework’ to balance the supply and the demand for nursing care. There seems to be an acceptable range of balance between supply and demand instead of a strict equilibrium. De Vries calculates the actual capacity utilization by dividing the workload per hour by the available staff per hour. Theoretically, uniform criteria could hold for all wards in the hospital. However, differences in workload between wards can be registered and result in a mechanism for coordination between wards. The performance of the framework has been tested for real examples and the stability could be improved in a few cases. We believe that this is mainly due to the flexibility of setting parameters separately per ward, and to the expert knowledge on the floor that is used for forecasting the workload.
Bard et al.(2009) formulated a mixed-integer program for physicians shift scheduling and solved it with the CPLEX optimization software package. Even though physician scheduling and nurse scheduling are similar problems, physician scheduling is more complex than nurse scheduling. In 2010, Asgeirsson (2010) introduced the same problem using a local-search-based algorithm to find a solution.

2.3 Minimizing Cost

Sitompul and Randhawa (1990), concentrate on financial cost. The goal is to reduce the personnel cost. Characteristics of manpower scheduling in hospitals are fluctuating demand, human effort (which cannot be inventoried), and critical customer convenience, while the schedules are subject to different kinds of constraints. They define four stages in nurse scheduling:

- Determine a set of feasible schedules that satisfy the constraints.
- Select the best schedule in terms of cost, coverage, and/or other criteria.
- Fine tune to accommodate changes.
- Make specific shift assignments.

It is interesting to compare schedules with respect to the perceived quality by the personnel members instead of using the violation of constraints as a criterion. In practice, it often makes no sense to separate specific shift assignments from the schedule design because assignment to different people influences the quality of the schedule. Sitompul and Randhawa advocate the approach of tackling staffing and rostering at the same time. They argue that separating the rostering from management decisions leads to sub-optimal schedules. From a theoretical point of view, this is true.

M.Moz et al in 2004 treated the problem of the cyclic schedule for nurses while minimizing the external nurses' coverage cost. The problem is formulated as an integer linear programming and then decomposed by using the Lagrangian relaxation (LR).
In 2006, J.F Bard discussed an integrated methodology for the nurses' allocation based on the service size, the nurse characteristics and the specific profiles. The general objective is also to minimize the workloads costs related to regular, extra and temporary hours assigned to nurses. The second objective is to minimize the cases number where the service is facing a staff shortage and the cases number where too many nurses are assigned. The problem is modeled with linear programming.
(Tsai and Lee, 2010) presented a mathematical programming model in two stages to solve an NSP. The first stage is designed to optimize the holiday periods in the nurse schedules, and it is solved using the LINGO software. The second stage is modeled as a mixed-integer programming model to obtain the most appropriate shift-table for the nurses and was solved using the Genetic Algorithm in MATLAB software.

2.4 Nurses Satisfaction

(Jafari and Salmasi, 2015) implemented the simulated annealing algorithm to solve an NSP in Iran. Their objective was to maximize the nurses' preferences for working shifts and weekends. They focused on the Milad Hospital in Iran. Their outcome showed that the SA algorithm provided significantly better schedules than the schedules provided by head nurses.

Lin, Wang, Li Hauang 2017), developed an integer programming model to solve a nurse scheduling problem. Their objective was to maximize the satisfaction of all nurses of different ranks about the shift schedule. The performance of their model was evaluated on the obstetrics and gynecology department of a hospital for a planning period of 14 days with a total of 20 nurses, where nurses 1 to 15 are senior nurses, and the remaining are junior nurses. The result showed that preferred shifts and days-of the nursing staff are assigned fairly.

CHAPTER 3

METHODOLOGY

3.1 Introduction

In this chapter, we will focus on the formulation of the mathematical model for scheduling the nurses for shift periods.
We would also discuss and explain the Revised Simplex Method.

3.2 Problem Description

The Nurse Scheduling Problem is the task of distributing shifts over nurse personnel to meet work regulations as well as hospital requirements.
In our research, the number of nurses required for each shift on each day is given, and we are concerned with assigning shifts to the nurses to satisfy the demands of the hospitals and to also maximize the preferences of the nurses.
Some terms used in this research are explained below;

3.2.1 Some Terms

- *Planning horizon*: It is the number of days considered to generate a schedule for the nurse personnel.
- *Scheduling Period*: Each day is divided into separate time slots called scheduling periods or shift periods and the number of nurses required for each shift period is specified.
- *Day-off*: A nurse is off on a particular day if no shift period is assigned to the

nurse on that particular day.

- Constraints: They are instances that every schedule generated must satisfy. They can be categorized under hard and soft constraints.

- *Hard Constraints*: They are constraints that every generated schedule must satisfy for it to be valid or acceptable.
  It comprises the hospital's policies, number of nurses needed on each shift on each day.

- *Soft Constraints*: These constraints need not necessarily be satisfied, however, satisfying as much as possible improves the quality of the schedule.
  In essence, they determine the relative quality of the schedule(1). The soft constraint comprises the nurses' preferences for working shifts and their preferences for days-off during the planning horizon.

3.2.2 Assumptions

1. The planning horizon considered for this study will be 7 days. In essence, a new schedule is generated to assign shifts to nurses at the beginning of each 7-day period.

2. Monday is considered the first day of the week.

3. Each day is divided into three 8-hour shift periods that the number of nurses in demand is given for each of them.
   Morning period (M) 6:00 AM to 14:00 PM (8 h).
   Afternoon period (A) 14:00 PM to 22:00 PM (8 h).
   Evening period (E) 22:00 PM to 6:00 AM (8 h).

4. No difference in the skill level of nurses.

3.2.3 Policies

These are the policies that are going to help us come out with our model.

These policies have been grouped into organizational policies which correspond to hard constraints and nurses' policies which correspond to soft constraints (nurses' preferences).

Organizational Policies (Hard Constraints)

1. Each nurse must be scheduled for at most one shift each day.

2. No nurse may be scheduled to work a night shift followed immediately by a morning shift.

3. Each nurse must have at least one day off in the planning horizon.

4. Nurses assigned shifts each morning should be equal to the number of nurses needed for morning shifts each day(which is constant).

5. Nurses assigned shifts each afternoon should be equal to the number of nurses needed for afternoon shifts each day(which is constant).

6. Nurses assigned shifts each night should be equal to the number of nurses needed for night shifts each day(which is constant).

7. Each nurse is not assigned more than two consecutive morning shifts.

8. Each nurse is not assigned more than two consecutive afternoon shifts.

9. Each nurse is not assigned more than two consecutive night shifts.

Nurses' Policies (Soft Constraints)

1. Maximum of three-night shifts for each nurse for each planning horizon.

2. Each nurse gets the same number of shifts.

3.2.4 Parameters

- **n** : the total number of nurses.
- **i** : index of nurses $\{1, 2, ..., n\}$.
- **j** : index of shifts $\{1, 2, 3\}$ such that, the indices 1,2,3 refer to shifts Morning, Afternoon and Evening respectively.
- **k** : index of days $\{1, 2, 3, ..., 7\}$.
- **a1** : number of nurses required for morning shift period each day.
- **a2** : number of nurses required for afternoon shift period each day.
- **a3** : number of nurses required for evening shift period each day
- p_{ijk} : 9,5 and 1, if the preference of nurse i is high, medium or low at shift j on day k, respectively.
- **c** : be the number of shifts for each nurse in a week.

3.2.5 Decision Variables

$$x_{i,j,k} = \begin{cases} 1, & \text{if nurse i is assigned to shift j on day k} \\ 0, & \text{if otherwise} \end{cases}$$

3.2.6 Mathematical Expression for the Policies

$$\sum_{j=1}^{3} x_{i,j,k} \leq 1, \forall i, \forall k \tag{3.1}$$

(3.1) ensures that each nurse works at most one shift each day.

$$x_{i,3,k} + x_{i,1,k+1} \leq 1, \forall i, k = 1, 2, ..., k-1 \tag{3.2}$$

(3.2) ensures that a night shift is not followed immediately by a morning shift

$$\sum_{k=1}^{7} x_{i,j,k} \leq 6, \forall i, \forall j \tag{3.3}$$

(3.3) ensures that each nurse has at least one day off in the planning horizon.

$$\sum_{k=1}^{7} x_{i,1,k} = a1, \forall i, \forall k \tag{3.4}$$

(3.4) ensures that the demand of nurses for morning shifts each day is met.

$$\sum_{k=1}^{7} x_{i,2,k} = a2, \forall i, \forall k \tag{3.5}$$

(3.5) ensures that the demand of nurses for afternoon shifts each day is met.

$$\sum_{k=1}^{7} x_{i,3,k} = a3, \forall i, \forall k \tag{3.6}$$

(3.6) ensures that the demand of nurses for evening shifts each day is met.

$$x_{i,1,k} + x_{i,1,k+1} + x_{i,1,k+2} \leq 2, \forall i, k = 1, 2, ..., k-2 \tag{3.7}$$

(3.7) ensures that each nurse is not assigned more than two consecutive morning shifts.

$$x_{i,2,k} + x_{i,2,k+1} + x_{i,2,k+2} \leq 2, \forall i, k = 1, 2, ..., k-2 \tag{3.8}$$

(3.8) ensures that each nurse is not assigned more than two consecutive afternoon shifts.

$$x_{i,3,k} + x_{i,3,k+1} + x_{i,3,k+2} \leq 2, \forall i, k = 1, 2, ..., k-2 \tag{3.9}$$

(3.9) ensures that each nurse is not assigned more than two consecutive evening shifts.

SOFT CONSTRAINTS

$$\sum_{k=1}^{7} x_{i,3,k} \leq 3, \forall i \tag{3.10}$$

(3.10) ensures that each nurse works at most three night shifts in the planning horizon.

$$\sum_{j=1}^{3}\sum_{k=1}^{7} x_{i,j,k} = c, \forall i \tag{3.11}$$

(3.11) ensures that the nurses have the same number of shifts.

OTHER CONSTRAINTS

$$\sum_{i=1}^{3} a_i < n \tag{3.12}$$

(3.12) guarantees that the number of nurses assigned to work on a particular day is not more than or equal to the number of nurses

$$x_{i,j,k} \in \{0,1\} \tag{3.13}$$

(3.13) indicates that the decision variable $x_{i,j,k}$ is a binary variable

$$a_1 \geq 0, a_2 \geq 0, a_3 \geq 0 \tag{3.14}$$

(3.14) indicates that a_1 , a_2 and a_3 take non-negative values

OBJECTIVE FUNCTION

Maximize

$$\sum_{i=1}^{n}\sum_{j=1}^{3}\sum_{k=1}^{7} (p_{ijk}.x_{ijk}) \tag{3.15}$$

(3.15) means that we are maximizing the nurses' preferences to be scheduled for a shift j on day k

3.2.7 Model

The model for the problem that we seek to solve then becomes

Maximize

$$\sum_{i=1}^{n}\sum_{j=1}^{3}\sum_{k=1}^{7}(p_{ijk}.x_{ijk}) \tag{3.16}$$

subject to

$$\sum_{j=1}^{3} x_{i,j,k} \leq 1, \forall i, \forall k \tag{3.17}$$

$$x_{i,3,k} + x_{i,1,k+1} \leq 1, \forall i, \forall k, \forall i, k = 1, 2, ..., k-1 \tag{3.18}$$

$$\sum_{k=1}^{7} x_{i,j,k} \leq 6, \forall i, \forall j \tag{3.19}$$

$$\sum_{i=1}^{n} x_{i,1,k} = a1, \forall k \tag{3.20}$$

$$\sum_{i=1}^{n} x_{i,2,k} = a2, \forall k \tag{3.21}$$

$$\sum_{i=1}^{n} x_{i,3,k} = a3, \forall k \tag{3.22}$$

$$x_{i,1,k} + x_{i,1,k+1} + x_{i,1,k+2} \leq 2, \forall i, k = 1, 2, ..., k-2 \tag{3.23}$$

$$x_{i,2,k} + x_{i,2,k+1} + x_{i,2,k+2} \leq 2, \forall i, k = 1, 2, ..., k-2 \tag{3.24}$$

$$x_{i,3,k} + x_{i,3,k+1} + x_{i,3,k+2} \leq 2, \forall i, k = 1, 2, ..., k-2 \tag{3.25}$$

$$\sum_{k=1}^{7} x_{i,3,k} \leq 3, \forall i \tag{3.26}$$

$$\sum_{j=1}^{3} \sum_{k=1}^{7} x_{i,j,k} = c, \forall i \tag{3.27}$$

$$\sum_{i=1}^{3} a_i < n \tag{3.28}$$

$$x_{i,j,k} \in \{0, 1\} \tag{3.29}$$

$$a_1 \geq 0, a_2 \geq 0, a_3 \geq 0 \tag{3.30}$$

This model is a linear programming model.

3.3 Linear Programming

A Linear Programming model seeks to maximize or minimize a linear function, subject to a set of linear constraints.

The linear model consists of the following components;

1. *A set of decision variables:* Variables, x_1 x_2 x_3, and so on, which are the inputs - things you can control. They are abbreviated x_n to refer to individuals or x to refer to them as a group

2. *An objective function:* The output you are trying to maximize or minimize.

3. *A set of constraints:* Which are equations that place limits on how big or small some variables can get. We can have equality constraints or inequality constraints.

3.3.1 Importance of Linear Programming

Many real world problems lend themselves to linear programming modeling and can be approximated by linear models.

There are well-known successful applications in:

1. Manufacturing

2. Marketing

3. Finance (Investment)

4. Advertising

5. Agriculture

6. Scheduling Problems

7. Transportation Problems

3.3.2 Methods for Solving Linear Programming Models

There are several known methods for solving Linear Programming Problems (LPP) or models and, some of them are listed below;

1. *Graphical Method:* This method is used when we only have two decision variables. Hence it is not mostly adopted to solve complex linear programming models.

2. *Simplex Method:* This method uses an iterative procedure to solve LPP. It can be used on LPP with more than two decision variables unlike, the case of the Graphical Method.

3. *Revised Simplex Method:* It is a variant of the Simplex Method. It is mathematically equivalent to the Simplex Method but differs in its implementation.

In this paper, we would solve our problem by using the Python programming language. The solver we intend to use is the **glpk** solver and the **Revised Simplex Method** is the algorithm it uses so we would briefly talk about the Revised Simplex Algorithm, its Applications, the Algorithm and illustrate it with an example.

3.4 The Revised Simplex Method

The original Simplex method is a straight forward algebraic procedure. However, this way of executing the algorithm (in either algebraic or tabular form) is not the most efficient computational procedure for computers because it computes and stores many numbers that are not needed at the current iteration and that may not even become relevant for decision making at subsequent iterations. The only pieces of information relevant at each iteration are:

1. The coefficients of the non-basic variables.

2. The coefficients of the entering basic variable in the other equations.

3. The right-hand sides of the equations.

It would be very useful to have a procedure that could obtain this information efficiently without computing and storing the other coefficients. These considerations motivated the development of the revised simplex method. This method was designed to accomplish exactly the same things as the original simplex method, but in a way that is more efficient for execution on computer. Thus, it is a streamlined version of the original procedure. It computes and stores only the information that is currently needed, and it carries along the essential data in a more compact form.

In Mathematical Optimization, the Revised Simplex Algorithm is a variant of George Dantzig's Simplex Method for Linear Programming.
The Revised Simplex Method is mathematically equivalent to the Simplex Algorithm but differs in its implementation.
In stead of maintaining a tableau which explicitly represents the constraints adjusted to a set of variables, it maintains a representation of a basis of the matrix representing the constraints.
The matrix-oriented approach allows for greater computational efficiency by enabling sparse matrix operations.
The basic premise of the Revised Simplex Method is to avoid useless computations in the Simplex Tableau: only compute the entries we need.
Revised simplex method is computationally more efficient and accurate. As a result, it is used in all commercial available package. (e. g. IBM MPSX, CDC APEX III)

3.4.1 Applications of the Revised Simplex Method

1. Used for solving linear programming models.

2. Because of it computational efficiency and accuracy, it is used in almost all commercial solvers.

3. It can be used to perform sensitivity analysis and solve the dual problem.

3.4.2 Algorithm or Steps for the Revised Simplex Method

We are going to illustrate the steps of the Method with an example.

Note that for the Revised Simplex Method, we need the following for testing and/ or improving the current solutions

- The net evaluation row Δ_j to determine the non-basic variable that enters the basis.
- The pivot column.
- The current basis variables and their values (X_B column) to determine the minimum positive ratio and then identify the basis variable to leave the basis.

The above information is directly obtained from the original equations by making use of the inverse of the current basis matrix at any iteration.

There are two standard forms for revised simplex method;

1. *Standard Form-I:* In this form, it is assumed that an identity matrix is obtained after introducing slack variables only.
2. *Standard Form-II:* If artificial variables are needed for an identity matrix, then two-phase method of ordinary simplex method is used in a slightly different way to handle artificial variables.

Example to Show the Steps

Solve by Revised simplex method

Max

$$Z = 2x_1 + x_2$$

Subject to

$$3x_1 + 4x_2 \leq 6$$

$$6x_1 + x_2 \leq 3$$

and

$$x_1, x_2 \geq 0$$

First thing before we begin the steps is to convert the problem to a standard form

Max

$$Z = 2x_1 + x_2 + 0s_1 + 0s_2$$

Subject to

$$\begin{array}{lllll} 3x_1 & +4x_2 & +s_1 & & = 6 \\ 6x_1 & +x_2 & & +s_2 & = 3 \end{array}$$

and

$$x_1, x_2, s_1, s_2 \geq 0$$

STEP 1

Express the given problem in standard form -I

- Ensure all $b_i \geq 0$
- The objective function should be of maximization
- Use of non-negative slack variables to convert inequalities to equations

The objective function is also treated as first constraint equation

$$Z - 2x_1 - x_2 + 0s_1 + 0s_2 = 0$$

$$3x_1 + 4x_2 + s_1 + 0s_2 = 6$$

$$6x1 + x2 + 0s1 + s2 = 3$$

and

$$x_1, x_2, s_1, s_2 \geq 0$$

STEP 2

Construct the starting table in the revised simplex form

Express the equations in the matrix form with suitable notation

$$\begin{bmatrix} 1 & -2 & -1 & 0 & 0 \\ 0 & 3 & 4 & 1 & 0 \\ 0 & 6 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$$

Column vector corresponding to Z is usually denoted by e_1 which is usually denoted as $B_1 = [, \beta_0^{(1)}, \beta_1^{(1)}, ..., \beta_n^{(1)}]$

Hence the column $\beta_0^{(1)}, \beta_1^{(1)}, \beta_2^{(1)}$ constitute the basis matrix B_1 (whose inverse B_1^{-1} is also B_1)

Basic Variables	$e_1/\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$	X_B	X_k	$\frac{X_B}{X_k}$	Non-Basic Variables	$a_1^{(1)}$	$a_2^{(1)}$
Z	1	0	0	0				-2	-1
s_1	0	1	0	6				3	4
s_2	0	0	1	3				6	1

STEP 3

Computation for Δ_j for $a_1^{(1)}$ and $a_2^{(1)}$

$\Delta 1 =$ first row of $B_1^{-1} \times a_1^{(1)} = 1(-2) + 0(3) + 0(6) = -2$

$\Delta 2 =$ first row of $B_1^{-1} \times a_2^{(1)} = 1(-1) + 0(4) + 0(1) = -1$

STEP 4

Apply the test of optimality

Both Δ_1 and Δ_2 are negative. So find the most negative value and determine the incoming vector.

Therefore most negative value is $\Delta_1 = -2$. This indicates $a_1^{(1)}(x_1)$ is incoming vector.

STEP 5

Compute the column vector X_k

$X_k = B_1^{-1} \times a_1^{(1)}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix}$$

STEP 6

Determine the outgoing vector. We are not supposed to calculate for Z row.

Basic Variables	$e_1/\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$	X_B	X_k	$\frac{X_B}{X_k}$
Z	1	0	0	0	-2	-
s_1	0	1	0	6	3	2
s_2	0	0	1	3	$6 \uparrow$	$0.5 \longrightarrow$

STEP 7

Determination of improved Solution

Column e_1 will never change, x_1 is incoming so place it outside the rectangular boundary

$$\begin{array}{c} \left[\begin{array}{ccc|} \beta_1^1 & \beta_2^1 & X_B \\ 0 & 0 & 0 \\ 1 & 0 & 6 \\ 0 & 1 & 3 \end{array}\right] \begin{array}{c} X_1 \\ -2 \\ 3 \\ 6 \end{array} \end{array}$$

Make the pivot element as 1 and the respective column elements to zero.

$$\begin{array}{c} \left[\begin{array}{ccc|} \beta_1^1 & \beta_2^1 & X_B \\ 0 & \frac{1}{3} & 1 \\ 1 & -\frac{1}{2} & \frac{9}{2} \\ 0 & \frac{1}{6} & \frac{1}{2} \end{array}\right] \begin{array}{c} X_1 \\ 0 \\ 0 \\ 1 \end{array} \end{array}$$

Construct the table to start with second iteration

Basic Variables	$e_1/\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$	X_B	X_k	$\frac{X_B}{X_k}$	Non-Basic Variables	$a_4^{(1)}$	$a_2^{(1)}$
Z	1	0	$\frac{1}{3}$	1				0	-1
s_1	0	1	-$\frac{1}{2}$	$\frac{9}{2}$				0	4
s_2	0	0	$\frac{1}{6}$	$\frac{1}{2}$				1	1

$$\Delta_4 = 1(0) + 0(0) + \frac{1}{3}(1) = \frac{1}{3}$$

$$\Delta_2 = 1(-1) + 4(0) + 1(\frac{1}{3}) = -\frac{2}{3}$$

Δ_2 is most negative. Hence $a_2^{(1)}$ is the incoming vector

Compute the column vector

$$\begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{6} \end{bmatrix} \times \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} \\ \frac{7}{2} \\ \frac{1}{6} \end{bmatrix}$$

Determine the outgoing vector

Basic Variables	$e_1/\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$	X_B	X_k	$\frac{X_B}{X_k}$
Z	1	0	$\frac{1}{3}$	1	-$\frac{2}{3}$	-
s_1	0	1	-$\frac{1}{2}$	$\frac{9}{2}$	$\frac{7}{2}$	$\frac{9}{7} \longrightarrow$
s_2	0	0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6} \uparrow$	3

Determination of Improved Solution

$$\begin{array}{c} \begin{bmatrix} \beta_1^{(1)} & \beta_2^{(1)} & X_B \\ 0 & \frac{1}{3} & 1 \\ 1 & -\frac{1}{2} & \frac{9}{2} \\ 0 & \frac{1}{6} & \frac{1}{2} \end{bmatrix} \begin{matrix} X_1 \\ -\frac{2}{3} \\ \frac{7}{2} \\ \frac{1}{6} \end{matrix} \end{array}$$

Making the pivot element 1 and elements above or beneath it 0

$$\begin{array}{c} \left[\begin{array}{ccc|} \beta_1^{(1)} & \beta_2^{(1)} & X_B \\ \frac{4}{21} & \frac{5}{21} & \frac{13}{7} \\ \frac{2}{7} & -\frac{1}{7} & \frac{9}{7} \\ -\frac{1}{21} & \frac{8}{42} & \frac{2}{7} \end{array}\right] \begin{array}{c} X_1 \\ 0 \\ 1 \\ 0 \end{array} \end{array}$$

Construct the table to start with third iteration

Basic Variables	$e_1/\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$	X_B	X_k	$\frac{X_B}{X_k}$	Non-Basic Variables	$a_4^{(1)}$	$a_2^{(1)}$
Z	1	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{13}{7}$				0	0
s_1	0	$\frac{2}{7}$	-$\frac{1}{7}$	$\frac{9}{7}$				0	1
s_2	0	-$\frac{1}{21}$	$\frac{8}{42}$	$\frac{2}{7}$				1	0

$$\Delta_4 = 1(0) + \frac{4}{21}(0) + \frac{5}{21}(1) = \frac{5}{21}$$

$$\Delta_3 = 1(0) + \frac{4}{21}(1) + \frac{5}{21}(0) = \frac{4}{21}$$

Δ_4 and Δ_3 are positive .

Therefore the optimal solution is Max $Z = \frac{13}{7}$, $x_1 = \frac{2}{7}$ and $x_2 = \frac{9}{7}$

CHAPTER 4

ANALYSIS

4.1 Introduction

We shall discuss in this chapter the problem instances used in this research, the implementation of our model on these problem instances and the interpretation of the results obtained thereof.

4.2 Test Specification

The number of required nurses for each shift on each day in the planning horizon is specified.

Here, we are going to generate a schedule using two different instances.
The instances are as follows:

1. **Instance 1:** The number of nurses used here is **9** and the number of nurses required for each shift on each day is shown in Table 4.1;

Days	No. of Nurses for Morning Shifts	Afternoon	Evening
Monday	2	3	2
Tuesday	2	3	2
Wednesday	2	3	2
Thursday	2	3	2
Friday	2	3	2
Saturday	2	3	2
Sunday	2	3	2

Table 4.1: Demand of nurses for each shift on each day (Instance 1)

2. **Instance 2:** The number of nurses used here is **21** and the number of nurses required for each shift on each day is shown in Table 4.2;

Days	No. of Nurses for Morning Shifts	Afternoon	Evening
Monday	6	7	5
Tuesday	6	7	5
Wednesday	6	7	5
Thursday	6	7	5
Friday	6	7	5
Saturday	6	7	5
Sunday	6	7	5

Table 4.2: Demand of nurses for each shift on each day (Instance 2)

4.3 Results

1. **Instance 1:** The schedule generated for instance 1 is shown in Table 4.3;

Nurses	Mon	Tue	Wed	Thur	Fri	Sat	Sun
A	M	A	M	E	A	M	0
B	E	A	M	0	A	M	M
C	0	M	E	0	E	A	M
D	A	E	0	A	M	E	A
E	A	0	E	A	M	E	E
F	A	M	A	M	E	A	0
G	E	0	A	M	A	0	A
H	0	A	0	A	0	A	A
I	M	E	A	E	0	0	E

Table 4.3: Schedule Generated for 9 nurses (Instance 1)

The M represents morning shifts, the A represents Afternoon shifts, the E represents Evening shifts and the 0 represents a day-off.

For instance, the schedule for **Nurse A** implies that he works Morning Shifts on Monday, Wednesday and Saturday, works Afternoon shifts on Tuesday and Friday, works an evening shift on Thursday and has a day-off on Sunday

Also, the schedule for Monday means that Nurses A and I cover morning shifts, Nurses D,E and F cover afternoon shifts, Nurses B and G cover evening shifts and Nurses C and H have Monday off.

It is clear from this schedule generated that all the hard constraints are satisfied. For instance, one can observe the following from the schedule generated;

1. The demand of nurses for each shift on each day is satisfied: That is, on all the days, 2 nurses are working morning shifts, 3 are working afternoon shifts and 2 are working evening shifts as required.

2. Each nurse has at least one day-off in the planning horizon

3. No nurse works a morning shift immediately after an evening shift.

In summary, all the organizational policies are satisfied and the soft constraints are also satisfied as much as possible

2. **Instance 2:** The schedule generated for instance 2 is shown in Table 4.4;

We can observe from the table that all hard constraints are satisfied and the soft constraints are satisfied as much as possible.

Nurses	Mon	Tue	Wed	Thur	Fri	Sat	Sun
A	A	M	0	M	A	E	A
B	0	A	M	E	A	M	E
C	M	A	E	A	E	0	E
D	E	0	M	A	M	A	E
E	A	E	A	0	M	E	A
F	A	M	A	E	A	0	E
G	E	A	E	A	E	0	A
H	M	A	E	0	E	A	A
I	A	E	A	E	0	E	A
J	E	A	M	A	M	E	0
K	A	M	0	M	E	A	A
L	A	M	E	A	M	A	0
M	0	M	A	M	A	M	M
N	A	E	0	A	M	A	M
O	M	E	A	M	0	A	M
P	E	0	M	A	M	E	E
Q	E	A	M	E	0	M	M
R	0	M	A	M	A	M	M
S	M	0	M	E	A	M	M
T	M	E	A	M	A	M	0
U	M	A	E	0	E	A	M

Table 4.4: Schedule Generated for 21 nurses (Instance 2)

4.4 Summary of the Results

We generated optimal rosters for some number of nurses to check the convergence and speed of our algorithm and method. Below is a summary of our results.

Total Nurses	Running time(secs)	Feasible Solution Cost	Constraint Violation
6	1.25	235	0
9	2.05	329	0
21	4.41	846	0
100	16.97	3525	0
200	38.25	6580	0

Table 4.5: Summary of the Results

We can see in Table 4.5 that as the number of nurses is increased, the time taken to generate a schedule also increases including the cost involved.

No constraint was violated in all instances.
Below is a graph to describe the relationship between the number of nurses and the running time for an optimal schedule to be generated.

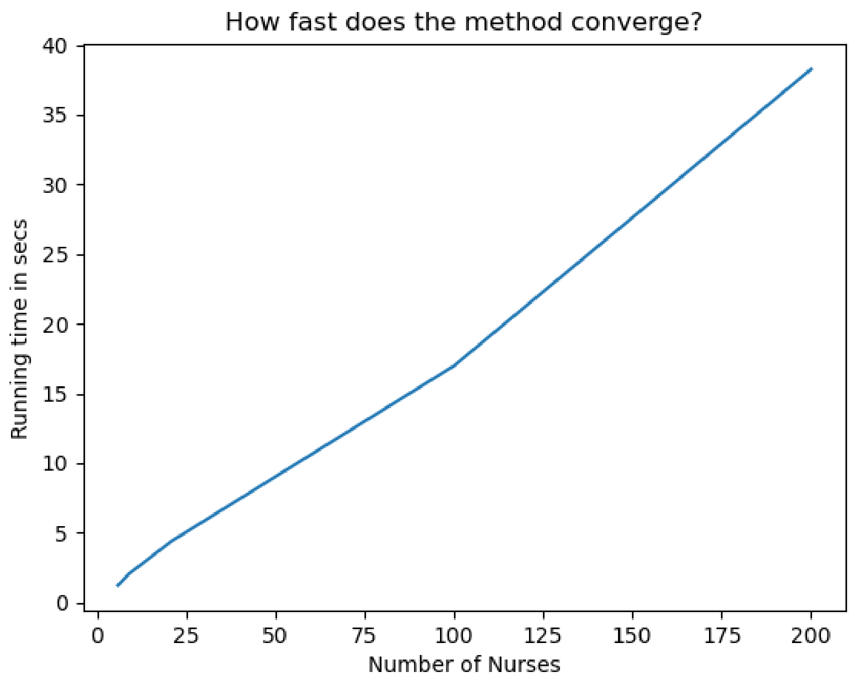

Figure 4.1: Running time to generate an optimal nurses' roster for some number of nurses

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

Creating impartial nurses' schedule helps to maintain an efficient and pleasant working environment in hospitals.
In this project report,

1. We formulated the three-shift nurse schedule problem as a linear programming model.

2. We then solved the model using the Revised Simplex Method which was implemented on a parallel computer using the Pulp library and a solver called GLPK.

3. We also generated two optimal schedules that fairly satisfied the preferences of the nurses and the organizational policies of the hospitals.

4. The code we wrote in solving the problem was able to automate the schedule whoch makes it very easy to affect changes.

5.2 Recommendation

Optimization techniques are very helpful in apportioning limited resources to maximize output. Our implemented method was a good one and solved the nurses' scheduling problem. So, we recommend that hospitals should employ the use of optimization-based techniques in generating their nurse schedules since;

1. It saves time: it took few seconds for optimal schedules to be generated for a

specified number of nurses but in the case of the manually generated schedules, it takes hours and even days to generate an appropriate schedule for the nurses.

2. It increases productivity among the nurses: as a result of the optimality of the schedule, the schedule generated takes into consideration all the policies regarding the scheduling process and that means that nurses are satisfied with their shifts whilst organizational policies are also met. This increases the level of productivity among the nurses.

3. It automates the schedule which makes it easy to affect changes: The code used in generating the schedule is automated and this makes it very easy to generate new schedules when there are changes for instance in the total number of nurses, the demand of nurses for a particular shift period etc.

4. It can be used to generate a nurse roster for any number of nurses: Because the code used in generating the optimal schedules was run on a parallel computer with parallel processing, we can generate optimal nurses' schedules for any number of nurses. This means that this method can be implemented in hospitals with any number of nurses.

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APPENDIX

```
# -*- coding: utf-8 -*-
"""
Created on Sun Jul 25 16:18:28 2021

@author: Reindolf Ocran Boadu
"""

#Solving the Nurse Scheduling Problem
#Number of nurses
n=6 #This number can be changed
#A list for number of shifts
jj=[1,2,3] #Index of shifts
kk=[1,2,3,4,5,6,7] #index of days
ii=[nurses for nurses in range(n)]
a1=2 #Number of nurses required for morning shift each day
a2=2#Number of nurses required for afternoon shift each day
a3=1 # Number of nurses required for evening shift each day


#Matrix for nurse i preferences for a shift j on day k
f=[1,5,9,9,1,5,9,9]

#Importing the pulp library
import pulp as p
#Creating the Lp Problem with title; "Nurse Scheduling Problem"
model = p.LpProblem("Nurse Scheduling Problem", sense = p.LpMaximize)

#Defining variables
#Which is 1 when nurse i is assigned to shift j on day k, 0 otherwise
x = p.LpVariable.dicts("x", [ (i, j, k) for i in ii
                                            for j in jj
```

```
                                          for k in kk], 0, 1,
                                          'Binary')

#objective function
#Our objective is to maximize the nurse's preferences for a shift j on
                                    a day k

model += p.lpSum(f[k] * x[(i, j, k)]
                 for k in kk
                 for j in jj
                 for i in ii
                 )

#Constraint 1
#Each nurse must be scheduled for at most one shift each day
for i in ii:
    for k in kk:

        model += p.lpSum(x[(i, j, k)] for j in jj
                      ) <= 1



#Constraint 2
#No nurse may be scheduled to work a night shift followed immediately
                                   by a morning shift
for k in range(1,len(kk)):
    for i in ii:
        model += x[(i, 3, k)] + x[(i, 1, k+1)] <= 1

#Constraint 3
#Each nurse must have at least one day-off in the planning horizon
for i in ii:
    model += p.lpSum(x[(i, j, k)]
                     for j in jj
                     for k in kk) <= 6
```

```
#...........................................
#Constraint 4
#Nurses assigned each morning should be equal to the number of nurses
#needed for morning
#shifts each day which we assume to be constant for all the days
for k in kk:

    model += p.lpSum(x[(i, 1, k)] for i in ii) == a1

#Constraint 5
#Nurses assigned each afternoon should be equal to the number of
#nurses needed for a afternoon
#shift each day which we assume to be constant for all the days
for k in kk:

    model += p.lpSum(x[(i, 2, k)] for i in ii) == a2

#Constraint 6
#Nurses assigned each morning should be equal to the number of nurses
#needed for an evening
#shift each day which we assume to be constant for all the days
for k in kk:

    model += p.lpSum(x[(i, 3, k)] for i in ii) == a3

#.............................................
#More constraints

#Constraint 7
#Each nurse is not assigned more than two consecutive morning shifts
for k in range(1,len(kk)-1):
    for i in ii:
        model += x[(i, 1, k)] + x[(i, 1, k+1)] + x[(i, 1, k+1)] <= 2
```

```
#Constraint 8
#Each nurse is not assigned more than two consecutive afternoon shifts
for k in range(1,len(kk)-1):
    for i in ii:
        model += x[(i, 2, k)] + x[(i, 2, k+1)] + x[(i, 2, k+1)] <= 2

#Constraint 9
#Each nurse is not assigned more than two consecutive evening shifts
for k in range(1,len(kk)-1):
    for i in ii:
        model += x[(i, 3, k)] + x[(i, 3, k+1)] + x[(i, 3, k+1)] <= 2



#Constraint 10
#Maximum of three night shifts for a nurse for each planning horizon

for i in ii:
    model += p.lpSum(x[(i, 3, k)]
                          for k in kk) <= 3






#Solution
soln=model.solve()

#Generating the status of the solution
print("The solution has a status of", soln)
print("This implies that it is", p.LpStatus[model.status])
print('')
f=open("C:/Users/USER/Desktop/Schedule2.txt","a+")
#Printing the values for each decision variable
print("The values of the decision variables are: ")
for var in x:
    var_value=x[var].varValue
```

```
    print(x[var],"=" ,var_value)
    hello=str(x[var]) + "=" +str(var_value)
    f.write(str(hello)+"\n")

#Printing the vaule of the objective
obj=model.objective.value()
print('')
print('The cost of the scheduling is', obj)
f.close()
```