

BT 3:

1. $T(n) = 2T(n-1) + n - 1$

$$T(n-1) = 2T(n-2) + (n-1) - 1 = 2T(n-2) + n - 2$$

$$T(n) = 2[2T(n-2) + n - 2] + n - 1 = 4T(n-2) + 2n - 3$$

$$\Rightarrow T(n) \sim 2^n. \quad \Rightarrow T(n) = O(2^n).$$

2. $T(n) = 2T(n-1) + 3n + 1 = 2T(n-1) + O(n).$

$$T(1) = 2T(0) + O(1)$$

$$T(2) = 2T(1) + O(1) = 2^2 T(0) + O(n)$$

...

$$\Rightarrow T(n) = 2^n T(0) + O(n) = O(2^n).$$

3. $T(n) = 2T\left(\frac{n}{2}\right) + 3n + 2.$

$$\begin{cases} a=2 \\ b=2 \\ f(n) = O(n) \end{cases}$$

$$\Rightarrow n^{\log_2 2} = n^1$$

$$\Rightarrow T(n) = O(n \log n)$$

4. $T(n) = 6T\left(\frac{n}{6}\right) + 2n + 3$

$$\begin{cases} a=6 \\ b=6 \\ f(n) = O(n) \end{cases}$$

$$\Rightarrow T(n) = O(n \log n).$$

5. $T(n) = 6T\left(\frac{n}{6}\right) + 3n - 1.$

$$\begin{cases} a=6 \\ b=6 \\ f(n) = O(n) \end{cases}$$

$$\Rightarrow \text{Applying Master Theorem} \Rightarrow O(n \log n)$$