Thứ

Ngày . . No.

Bài tập lần 12

Bài 68

$$\begin{cases} u_t = 9u_{xx} \\ u(x,0) = \sin 2x \cos 4x = \varphi(x) \quad x \in \mathbb{R} \end{cases}$$

Nghiệm của bài toán có dạng

$$u(x,t) = \frac{1}{6\sqrt{\pi t}} \int_{-\infty}^{+\infty} \sin 2y \cos 4y \, e^{-\frac{(x-y)^2}{36t}} dy$$

$$= \frac{1}{12\sqrt{\pi t}} \int_{-\infty}^{+\infty} (\sin 6y - \sin 2y) e^{-\frac{(x-y)^2}{36t}} dy$$

=> Đặt $x - y = k \to dy = -dk$

$y = -\infty \to k = +\infty$, $y = +\infty \to k = -\infty$

$$\Rightarrow u(x,t) = \frac{1}{12\sqrt{\pi t}} \int_{-\infty}^{+\infty} [\sin 6(x-k) - \sin 2(x-k)] e^{-\frac{k^2}{36t}} dk$$

$$= \frac{1}{12\sqrt{\pi t}} \int_{-\infty}^{+\infty} (\sin 6x \cos 6k - \cos 6x \sin 6k - \sin 2x \cos 2k + \sin 2k \cos 2x) e^{-\frac{k^2}{36t}} dk$$

Ta có các hàm $f_1(k) = \sin 6x \cos 6k \, e^{-\frac{k^2}{36t}}$, $f_2(k) = -\sin 2x \cos 2k \, e^{-\frac{k^2}{36t}}$ } là các hàm chẵn theo k

$g_1(k) = \cos 6x \sin 6k \, e^{-\frac{k^2}{36t}}$, $g_2(k) = \sin 2k \cos 2x \, e^{-\frac{k^2}{36t}}$ } là các hàm lẻ theo k

dk))

~~$\Rightarrow u(x,t) =$~~ Mà $\Rightarrow \int_{-\infty}^{+\infty} (f_1(k) + f_2(k)) dk = 2\int_0^{+\infty} (f_1(k) + f_2(k)) dk$

$$\int_{-\infty}^{+\infty} (g_1(k) + g_2(k)) dk = 0$$

$$\Rightarrow u(x,t) = \frac{1}{6\sqrt{\pi t}} \int_0^{+\infty} (\sin 6x \cos 6k - \sin 2x \cos 2k) e^{-\frac{k^2}{36t}} dk$$

$$= \frac{1}{6\sqrt{\pi t}} \left(\sin 6x \int_0^{+\infty} \cos 6k \, e^{-\frac{k^2}{36t}} dk - \sin 2x \int_0^{+\infty} \cos 2k \, e^{-\frac{k^2}{36t}} dk \right)$$

$$= \frac{1}{6\sqrt{\pi t}} \left(\frac{\sin 6x \sqrt{\pi}}{2 \cdot \frac{1}{6\sqrt{t}}} e^{-\frac{36}{4 \cdot \frac{1}{36t}}} - \frac{\sin 2x \sqrt{\pi}}{2 \cdot \frac{1}{6\sqrt{t}}} e^{-\frac{4}{4\frac{1}{36t}}} \right)$$

KOKUYO

$\rightarrow u(x,t) = \frac{1}{2}\left(\sin 6x\, e^{-324t} - \sin 2x\, e^{-36t}\right)$

Bài 70

$$\begin{cases} u_t = 4u_{xx}, & t > 0 \\ u(x,0) = e^{-x^2}, & x \in \mathbb{R} \end{cases}$$

Nghiệm của bài toán có dạng

$$u(x,t) = \frac{1}{4\sqrt{\pi t}} \int_{-\infty}^{+\infty} e^{-y^2} e^{\frac{-(x-y)^2}{16t}}\, dy$$

$$= \frac{1}{4\sqrt{\pi t}} \int_{-\infty}^{+\infty} e^{\frac{-x^2+2xy-y^2(16t+1)}{16t}}\, dy$$

$$= \frac{1}{4\sqrt{\pi t}} \int_{-\infty}^{+\infty} e^{\frac{-x^2}{16t} + \frac{2xy}{16t} - y^2\frac{(16t+1)}{16t}}$$

$$= \frac{1}{4\sqrt{\pi t}} \int_{-\infty}^{+\infty} e^{\frac{-x^2}{16t} + \frac{x^2}{16t(16t+1)} - \frac{x^2}{16t(16t+1)} + \frac{2x}{\sqrt{16t}\sqrt{16t+1}} \cdot \frac{y\sqrt{16t+1}}{\sqrt{16t}} - \frac{y^2(16t+1)}{16t}}\, dy$$

$$= \frac{1}{4\sqrt{\pi t}} \int_{-\infty}^{+\infty} e^{\frac{-x^2}{16t+1} - \left(\frac{x}{\sqrt{16t}\sqrt{16t+1}} - y\frac{\sqrt{16t+1}}{\sqrt{16t}}\right)^2}\, dy$$

$$= \frac{1}{4\sqrt{\pi t}} e^{\frac{-x^2}{16t+1}} \int_{-\infty}^{+\infty} e^{-\left(\frac{x}{\sqrt{16t}\sqrt{16t+1}} - \frac{y\sqrt{16t+1}}{\sqrt{16t}}\right)^2}\, dy$$

$$\cancel{= \frac{1}{4\sqrt{\pi t}} e^{\frac{-x^2}{16t+1}}}$$

Đặt $\frac{x}{\sqrt{16t}\sqrt{16t+1}} - \frac{y\sqrt{16t+1}}{\sqrt{16t}} = k$

$\rightarrow dy = \frac{-\sqrt{16t}}{\sqrt{16t+1}}\, dk$

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$$\rightarrow u(x,t) = \frac{1}{4\sqrt{\pi t}} e^{\frac{-x^2}{16t+1}} \frac{4\sqrt{t}}{\sqrt{16t+1}} \int_{-\infty}^{+\infty} e^{-k^2} dk$$

$$= \frac{2}{\sqrt{\pi}\sqrt{16t+1}} e^{\frac{-x^2}{16t+1}} \int_{\cancel{-\infty}\,0}^{+\infty} e^{-k^2} dk$$

$$= \frac{2}{\sqrt{\pi}} \frac{1}{\sqrt{16t+1}} e^{\frac{-x^2}{16t+1}} \frac{\pi}{\sqrt{2}} = \frac{1}{\sqrt{16t+1}} e^{\frac{-x^2}{16t+1}}$$

$$\rightarrow u(x,t) = \frac{1}{\sqrt{16t+1}} e^{\frac{-x^2}{16t+1}}$$

Bài 73

$$\begin{cases} u_t = 4u_{xx} & t \geq 0 \\ u(x,0) = x^2 - 2x + 3 & x \in \mathbb{R} \end{cases}$$

Nghiệm của bài toán có dạng

$$u(x,t) = \frac{1}{4\sqrt{\pi t}} \int_{-\infty}^{+\infty} (y^2 - 2y + 3) e^{\frac{-(x-y)^2}{16t}} dy.$$