Bài tập lần 2,

Đề 1 - K64TT

Câu 1 $xU_x + (1+y^2)U_y + xu = x^2$ $(x>0, y>0)$

a)
$\Leftrightarrow xU_x + (1+y^2)U_y = x^2 - xu$

$\to a(x,y) = x, \; b(x,y) = 1+y^2, \; c(x,y,u) = x^2 - xu.$

Phương trình đặc trưng

$$\frac{dx}{x} = \frac{dy}{1+y^2} = \frac{du}{x^2-xu}$$

$\Rightarrow$ $\boxed{\ln x = \arctan y + C}$

$\Leftrightarrow \ln x - \arctan y = C$

Đặt $\begin{cases} s = \ln x - \arctan y \\ t = x \end{cases}$ $\to u(x,y) = v(s,t)$

$\Rightarrow \begin{cases} U_x = v_s \, s_x + v_t \, t_x = \dfrac{v_s}{x} + v_t \\[4mm] U_y = v_s \, s_y + v_t \, t_y = \dfrac{-v_s}{1+y^2} \end{cases}$

$\Rightarrow \quad xU_x + (1+y^2)U_y = v_s + xv_t - v_s = xv_t = x^2 - xu$

$\Leftrightarrow \quad v_t = x - u$

$\Leftrightarrow v_t = t - v$

$\Leftrightarrow v_t + v = t$

$\Leftrightarrow e^t v_t + e^t v = t e^t$

$\Leftrightarrow e^t v = t e^t - e^t + f(s)$

$\Leftrightarrow v = t - 1 + e^{-t} f(s)$

$\Rightarrow N^o TQ \; u(x,y) = x - 1 + e^{-x} f(\ln x - \arctan y)$

Thử lại $\begin{cases} U_x = 1 - e^{-x} f + \dfrac{e^{-x} f'}{x} \\[4mm] U_y = \dfrac{-e^{-x} f'}{1+y^2} \end{cases}$ $\to xU_x + (1+y^2)U_y = x(1 - e^{-x} f)$
$= x(x-u) = x^2 - xu \; (TM)$

b) Ta có $u(x,y) = x - 1 + e^{-x} f(\ln x - \arctan y)$

Để $u(x,y) = ax + b + 2e^{-x}$ thì $\ln x - \arctan y = 0$.

$\Rightarrow \begin{cases} a = 1 \\ b = -1 \\ f(0) = 2 \end{cases}$

+) Chọn $f(t) = t + 2 \Rightarrow f(\ln x - \arctan y) = \ln x - \arctan y + 2$

$\Rightarrow u(x,y) = x - 1 + e^{-x}(\ln x - \arctan y + 2)$

Kiểm tra lại $u_x = 1 - e^{-x}(\ln x - \arctan y + 2) + \dfrac{e^{-x}}{x}$

$u_y = -\dfrac{e^{-x}}{1+y^2}$

$\Rightarrow x u_x + (1+y^2) u_y = x - x e^{-x}(\ln x - \arctan y + 2)$
$$= x^2 - ux.$$

+) Chọn $f(t) = \cos(t) + 1$

$\Rightarrow f(\ln x - \arctan y) = \cos(\ln x - \arctan y) + 1$

$\Rightarrow u(x,y) = x - 1 + e^{-x}[\cos(\ln x - \arctan y) + 1]$

Kiểm tra

$u_x = 1 - e^{-x}[\cos(\ln x - \arctan y) + 1] + \dfrac{e^{-x}\sin(\ln x - \arctan y)}{x}$

$u_y = \dfrac{e^{-x}\sin(\ln x - \arctan y)}{1+y^2}$

$\Rightarrow x u_x + (1+y^2) u_y = x - x e^{-x}[\cos(\ln x - \arctan y) + 1]$
$$= x^2 - ux$$