Ngay

Bài Tập Lần 20

Câu 10, d,

$$u_{tt}(x,y,z,t) = 3^2 \Delta u(x,y,z,t) \quad (x,y,z) \in \mathbb{R}^3,\ t>0.$$
$$u(x,y,z,0) = 0$$
$$u_t(x,y,z,0) = \chi_D(x,y,z)$$
$$D = B_1(0,2,0) \cup B_1(2,0,0) \cup B_1(-2,0,0)$$

$$R = \vee(0,0,+\infty) \quad P = (0,100,0)$$

Áp dụng CT Kirchoff:

$$u(x,y,z,t) = \partial_t\left[\frac{1}{36\pi t}\iint_{\partial B_{3t}(x,y,z)} \varphi(x,y,z)\,dS\right] + \frac{1}{36\pi t}\iint_{\partial B_{3t}(x,y,z)} \psi(x,y,z)\,dS$$

$$= \frac{1}{36\pi t}\iint_{\partial B_{3t}(x,y,z)} \chi_D(x,y,z)\,dS \quad \left(\text{Do } \varphi(x,y,z) = u(x,y,z,0) = 0\right.$$
$$\left.\psi(x,y,z) = u_t(x,y,z,0) = \chi_D(x,y,z)\right)$$

$$= \frac{1}{36\pi t}\iint_{\partial B_{3t}(x,y,z)\cap D} dS = \frac{1}{36\pi t}\left|\partial B_{3t}(x,y,z) \cap D\right|$$

$$= \frac{1}{36\pi t}\left|\partial B_{3t}(x,y,z) \cap B_1(0,2,0)\right| + \frac{1}{36\pi t}\left|\partial B_{3t}(x,y,z) \cap B_1(2,0,0)\right|$$
$$+ \frac{1}{36\pi t}\left|\partial B_{3t}(x,y,z) \cap B_1(-2,0,0)\right|$$

(Vì $B_1(0,2,0) \cap B_1(2,0,0) \cap B_1(-2,0,0) = \emptyset$)

Thứ
Ngày . . No.

$\Rightarrow U(P,t) = u(0,100,0,t)$

$= \frac{1}{36\pi t}\left[\left| \partial B_{3t}(0,100,0) \cap B_1(0,2,0) \right| + \left| \partial B_{3t}(0,100,0) \cap B_1(2,0,0) \right| \right.$

$\left. + \left| \partial B_{3t}(0,100,0) \cap B_1(-2,0,0) \right| \right]$

+) Tính $S_1 = \left| \partial B_{3t}(0,100,0) \cap B_1(0,2,0) \right|$

$r = \sqrt{0^2 + (100-2)^2 + 0^2} = 98$

TH1) Nếu $0 < 3t < 98 - 1 = 97$

$\leftrightarrow 0 < t < \frac{98}{3}$ $\frac{97}{3}$

$\Rightarrow \partial B_{3t}(0,100,0) \cap B_1(0,2,0) = \emptyset$

$\rightarrow S_1 = 0$

TH2) Nếu $3t > 98 + 1 \quad (\Leftrightarrow t > 33)$

$\rightarrow \partial B_{3t}(0,100,0) \cap B_1(0,2,0) = \emptyset$

$\Rightarrow S_1 = 0$

TH3) Nếu $\frac{97}{3} < t < 33$

$\rightarrow S_1 = 2\pi t.h = 2\pi t . \frac{1 - (3t - r)^2}{2r}$

$= 3\pi t\left[1 - \frac{(3t - 98)^2}{98}\right]$

+) Tính $S_2 = \left| \partial B_{3t}(0,100,0) \cap B_1(2,0,0) \right|$

$r = \sqrt{(2-0)^2 + (0-100)^2 + 0^2} = \sqrt{10004}$

TH1) Nếu $0 < 3t < \sqrt{10004} - 1$ hoặc $3t > \sqrt{10004} + 1$.

$\leftrightarrow \left[\begin{array}{l} 0 < t < \frac{\sqrt{10004} - 1}{3} \\ t > \frac{\sqrt{10004} + 1}{3} \end{array} \right.$

$\rightarrow \partial B_{3t}(0,100,0) \cap B_1(2,0,0) = \emptyset$

$\rightarrow S_2 = 0$

+) Nếu $\frac{\sqrt{10004}-1}{3} < t < \frac{\sqrt{10004}+1}{3}$

$\rightarrow S_2 = 2\pi \cdot 3t \cdot h$

$= 2\pi \cdot 3t \cdot \frac{1-(3t-r)^2}{2r}$

$= 3\pi t \cdot \frac{[1-(3t-\sqrt{10004})^2]}{\sqrt{10004}}$

+) Tính $S_3 = |\partial B_{3t}(0,100,0) \cap B_1(-2,0,0)|$

$r = \sqrt{(0+2)^2+(100-0)^2+0^2} = \sqrt{10004}$

+) Nếu $\left[\begin{array}{l} 0 < t < \frac{\sqrt{10004}-1}{3} \\ t > \frac{\sqrt{10004}+1}{3} \end{array}\right.$

$\rightarrow \partial B_{3t}(0,100,0) \cap B_1(-2,0,0) = \emptyset \Rightarrow S_3 = 0$

+) Nếu $\frac{\sqrt{10004}-1}{3} < t < \frac{\sqrt{10004}+1}{3}$

$\rightarrow S_3 = 2\pi 3t h = 2\pi 3t \frac{1-(3t-r)^2}{2r}$

$= 3\pi t \frac{[1-(3t-\sqrt{10004})^2]}{\sqrt{10004}}$

$$\rightarrow U(P,t) = \frac{1}{36\pi t} \cdot [S_1 + S_2 + S_3]$$

Vậy $U(P,t) =$

$$\begin{cases} 0 & \text{nếu } 0 < t < \frac{97}{3} \\ \frac{1-(3t-98)^2}{1176} & \text{nếu } \frac{97}{3} < t < 33 \\ 0 & \text{nếu } 33 < t < \frac{\sqrt{10004}-1}{3} \\ \frac{1-(3t-\sqrt{10004})^2}{6\sqrt{10004}} & \text{nếu } \frac{\sqrt{10004}-1}{3} < t < \frac{\sqrt{10004}+1}{3} \\ 0 & \text{nếu } t > \frac{\sqrt{10004}+1}{3}. \end{cases}$$