Intelligence Program 2017

Math Test (HCMC)

1> Suppose that the 2004 state of land use in a city of 60 mi^2 of built-up area is

C: Commercially Used 25%
I: Industrially Used 20%
R: Residentially Used 55%.

Find the states in 2009, 2014, and 2019, assuming that the transition probabilities for 5-year intervals are given by the matrix **A** and remain practically the same over the time considered.

$$\mathbf{A} = \begin{array}{c} \begin{matrix} \text{From C} & \text{From I} & \text{From R} \end{matrix} \\ \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{bmatrix} \end{array} \begin{array}{l} \text{To C} \\ \text{To I} \\ \text{To R} \end{array}$$

2> An elastic membrane in the x_1x_2 plane with boundary circle $x_1^2 + x_2^2 = 1$ as Figure 1 is stretched so that a point P(x_1, x_2) goes over into the point Q(x_1, x_2) given by

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = Ax = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Find the **principal directions**, that is, the directions of the position vector **x** of *P* for which the direction of the position vector **y** of *Q* is the same or exactly opposite **(Fig.1)**. What shape does the boundary circle take under this deformation?

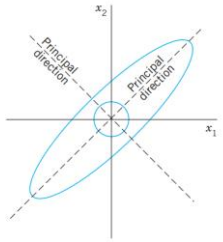
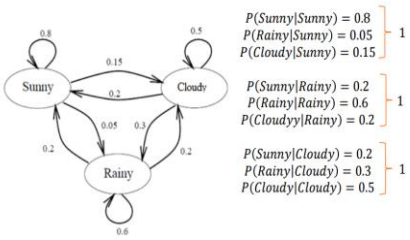


Figure 1: Principal direction

3> Assume the data produces the following transition probabilities:



Given that today is Sunny, what's the probability that tomorrow is Sunny and the next day Rainy?

4> Given an amount of a radioactive substance, say, 0.5 g (gram), find the amount present at any later time.

Physical Information. Experiments show that at each instant a radioactive substance decomposes—and is thus decaying in time—proportional to the amount of substance present.

5> AB Steel, Inc., produces two kinds of iron by using three kinds of raw material (scrap iron and two kinds of ore) as shown. Make an operation plan to maximize the daily profit.

Raw Material	Raw Material Needed per Ton		Raw Material Available per Day (tons)
	Iron I_1	Iron I_2	
R_1	2	1	16
R_2	1	1	8
R_3	0	1	3.5
Net profit per ton	\$150	\$300	