Quantum walk

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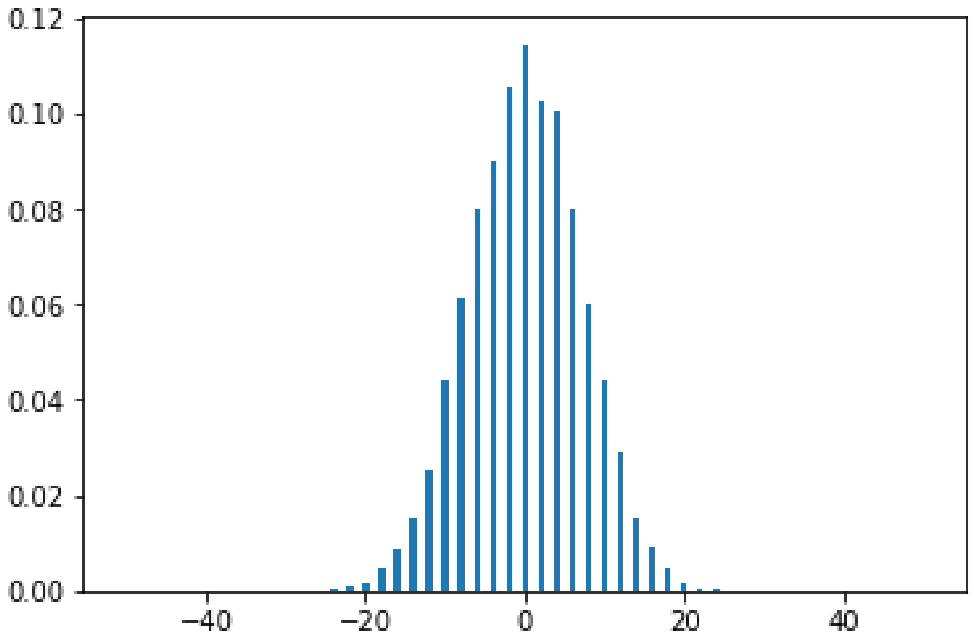
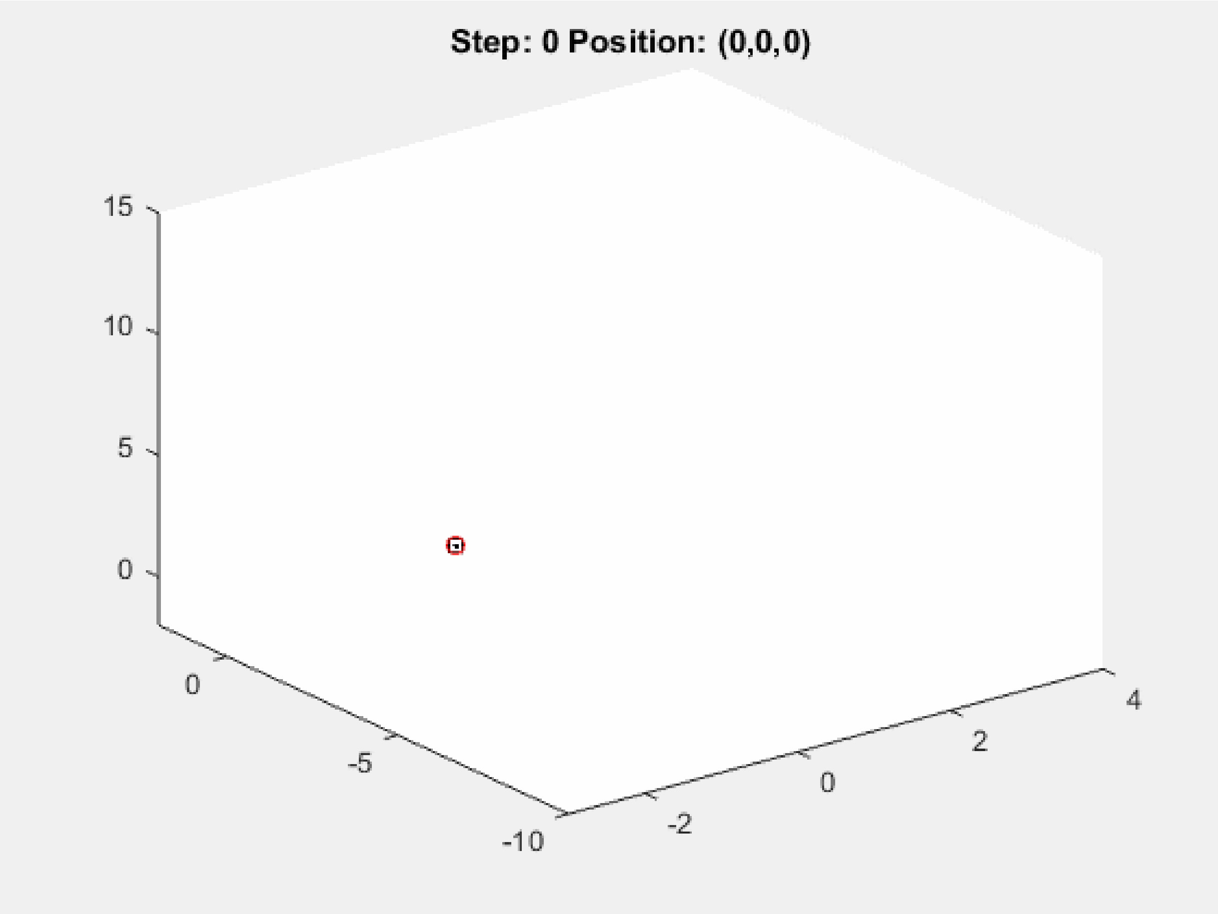
Vu Tuan Hai

Random walk

Take the random variable $Z \in \{-1, 1\}$

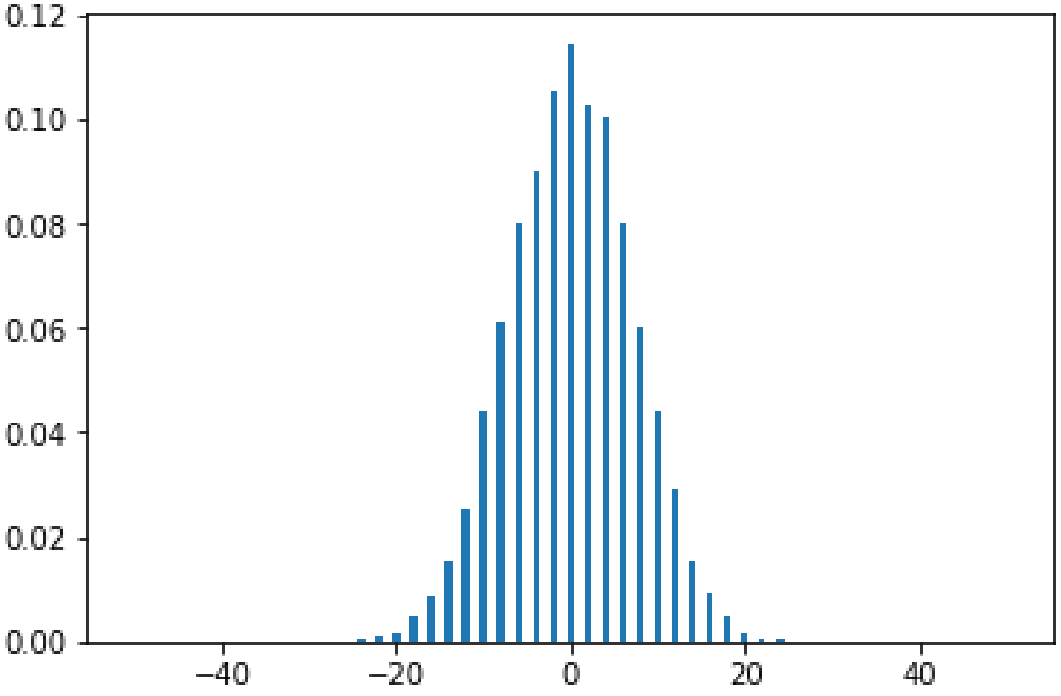
The series $\{S_n\}$ is a 1-D random walk with $S_0 = 0$ and $S_n = \sum_{i=1}^{n} Z_i$



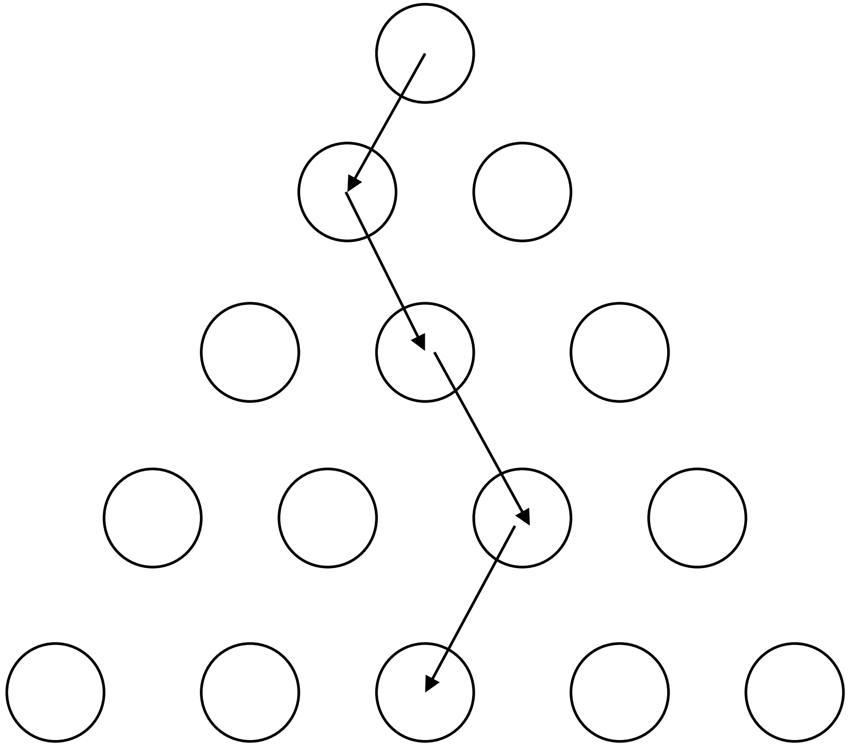


Random walk

Normal distribution of S_N



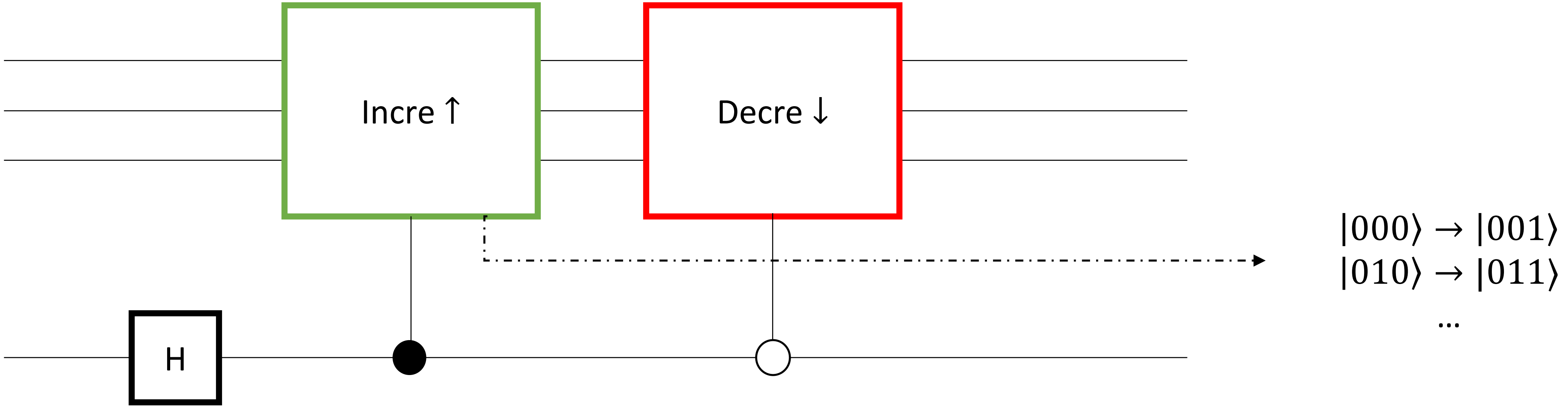
Values of $\{S_N\}$



More likely

Unlikely

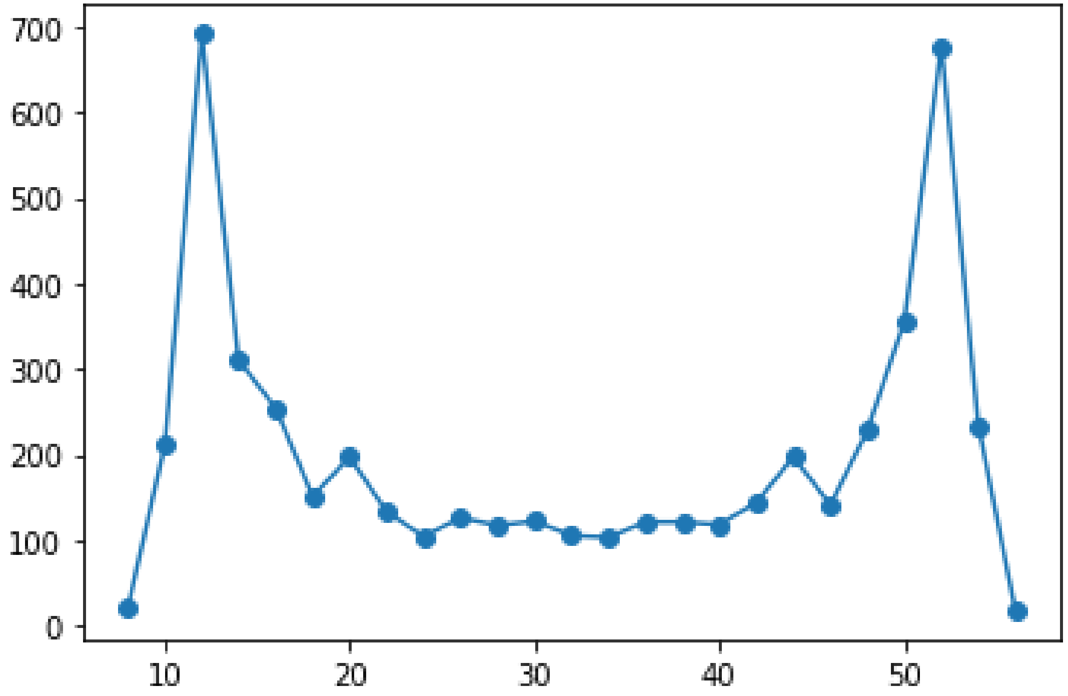
Quantum walk (using superposition)



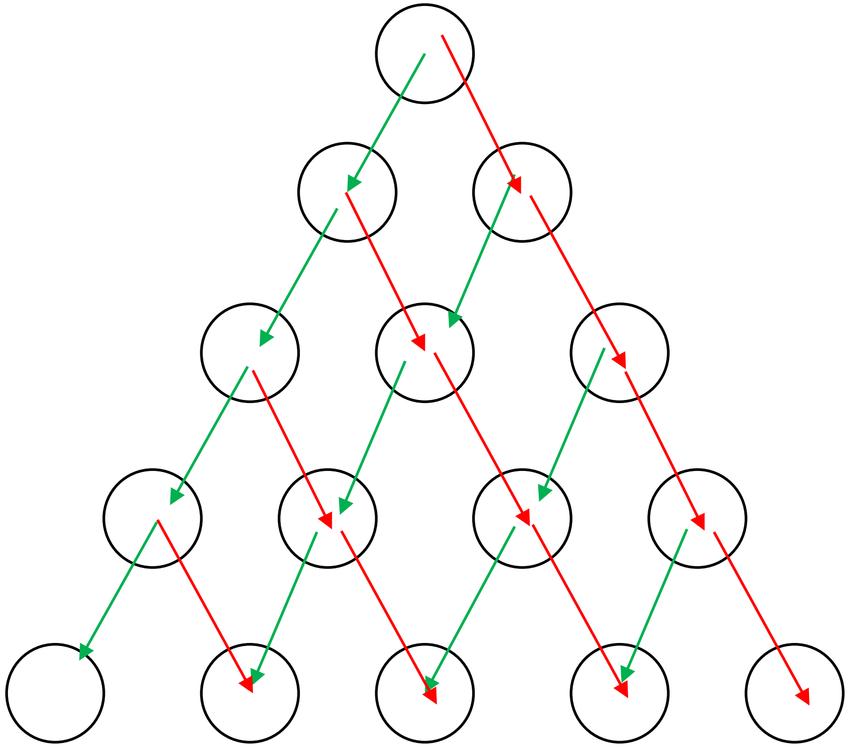
$$|\uparrow\rangle \otimes |0\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \otimes |0\rangle \xrightarrow{S} \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |1\rangle + |\downarrow\rangle \otimes |-1\rangle)$$

Quantum walk

Paranormal distribution of S_N



Values of $\{S_N\}$



Unlikely

More likely

Quantum walk

Quantum walk is described as product of 2 unitary operator: coin flip operator & shift operator

1. Spatial search (discrete time) – face with large graph

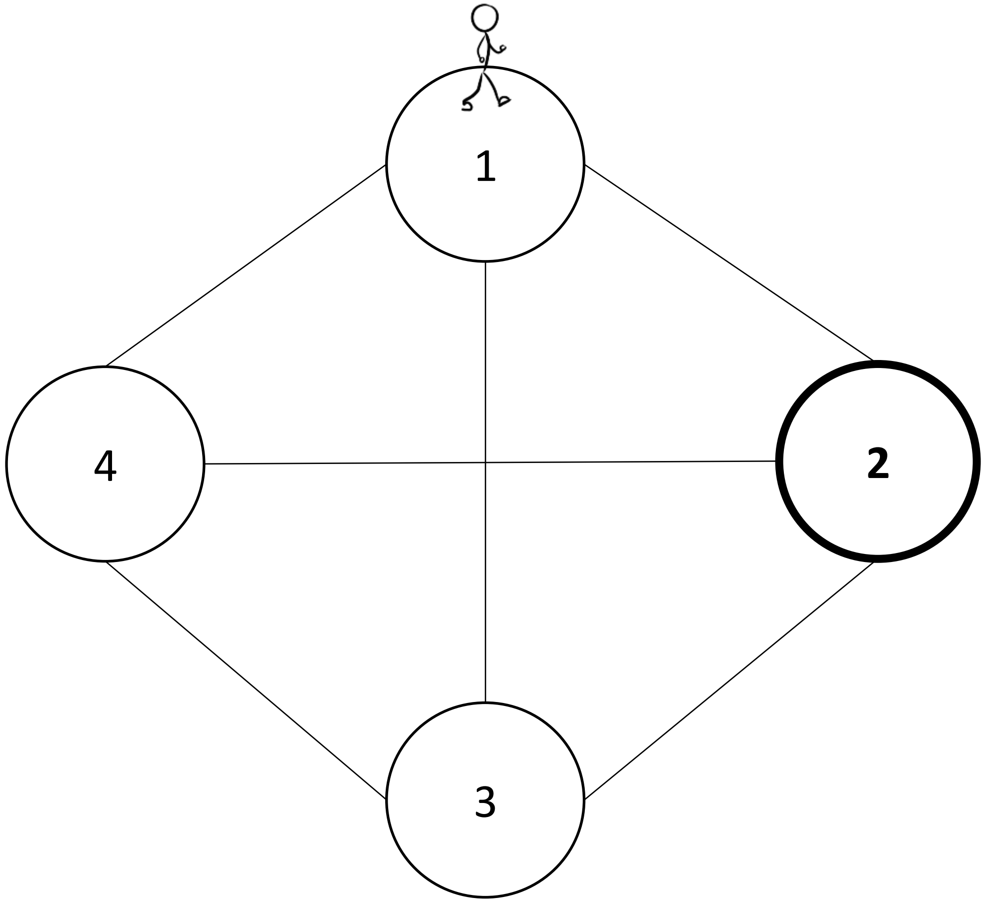
Assume that we have a complete graph with 4 nodes.

One approach is using random walk.

Note that:

Walker must be in somewhere

$$\Rightarrow \sum prob = 1$$

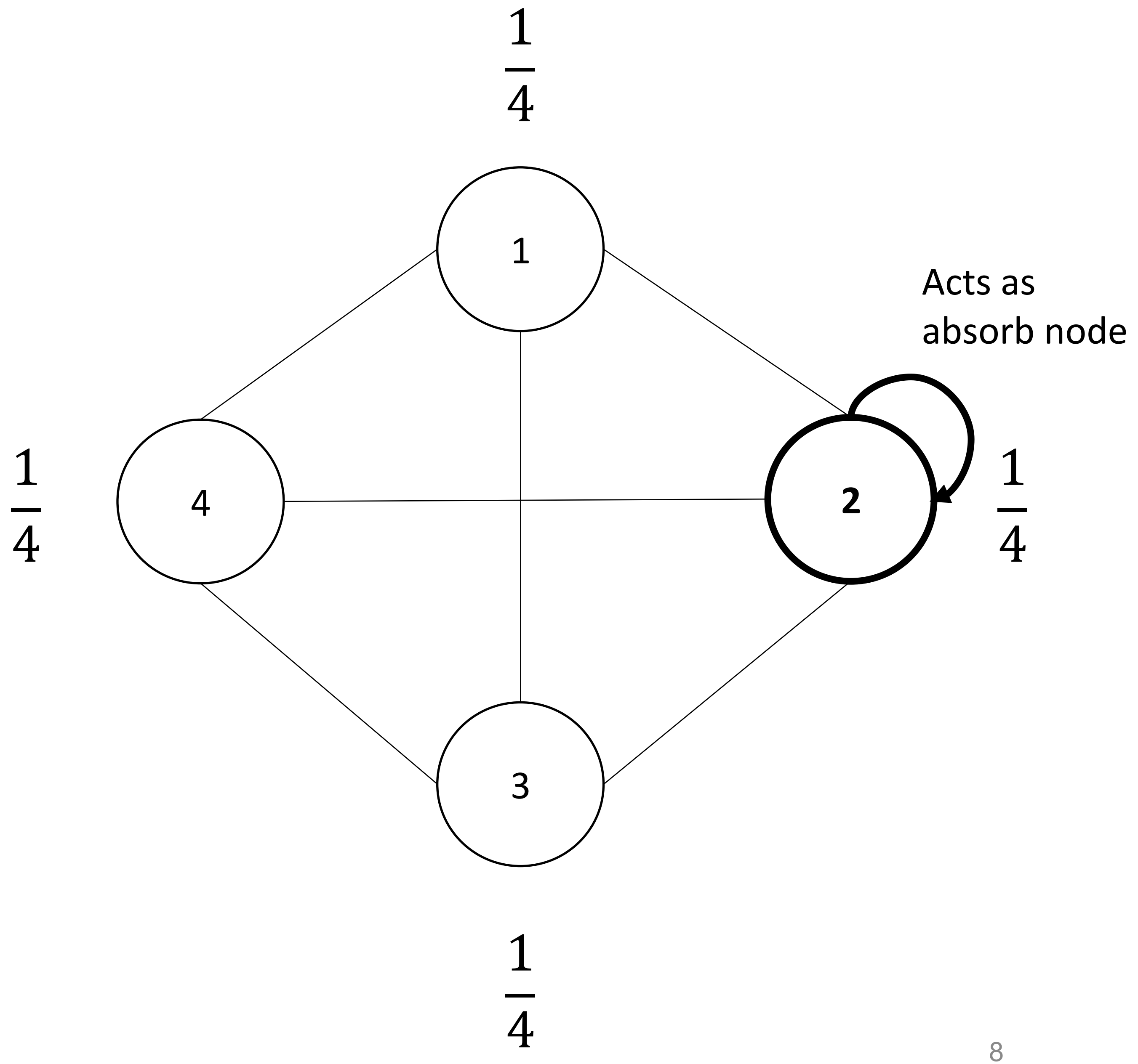


2. Random walk

Initial state: ¼ at all node

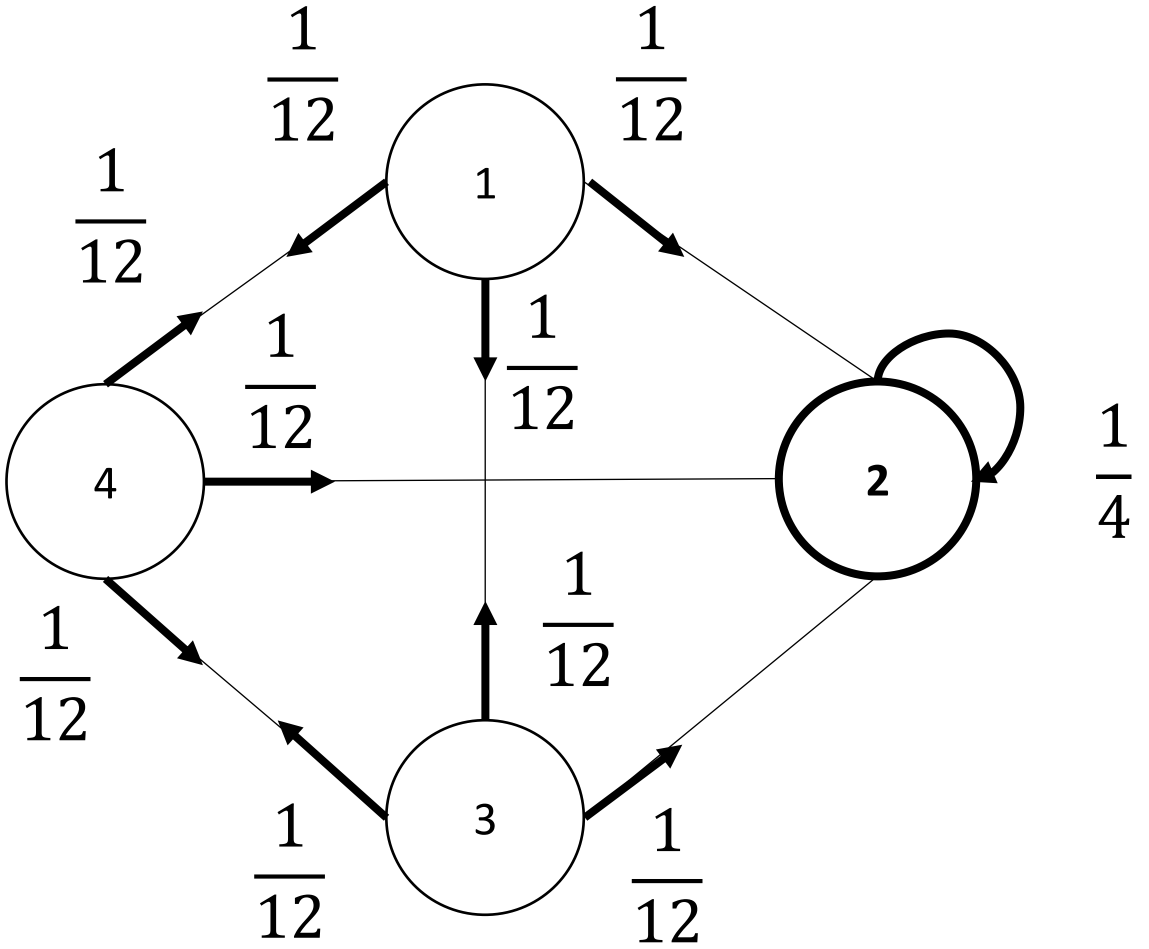
At step k: jump to other random nodes

Prob(1) = Prob(2) =
Prob(3) = Prob(4) = $\frac{1}{4}$



2. Random walk

At step 1:

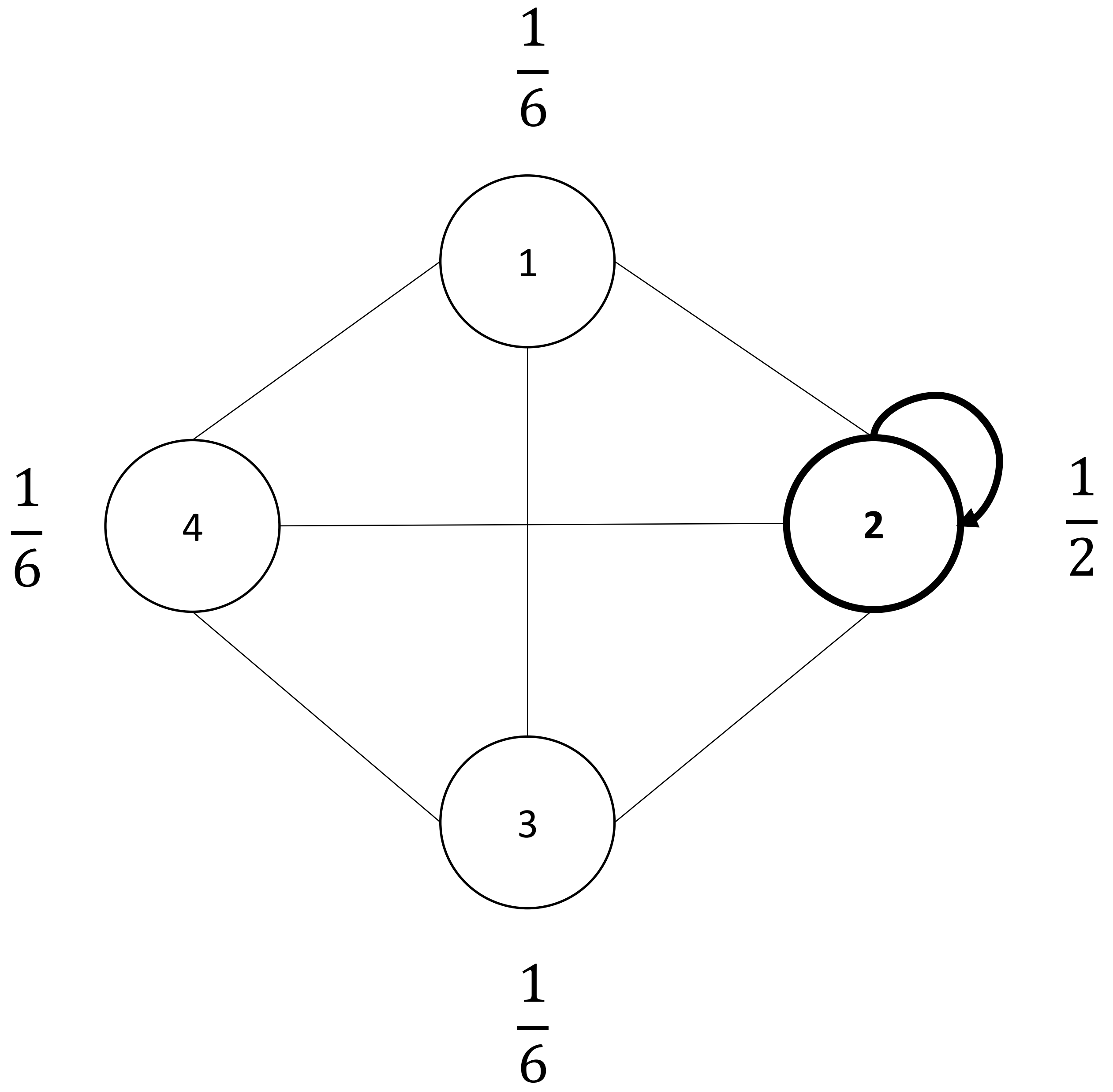


2. Random walk

When done step 1:

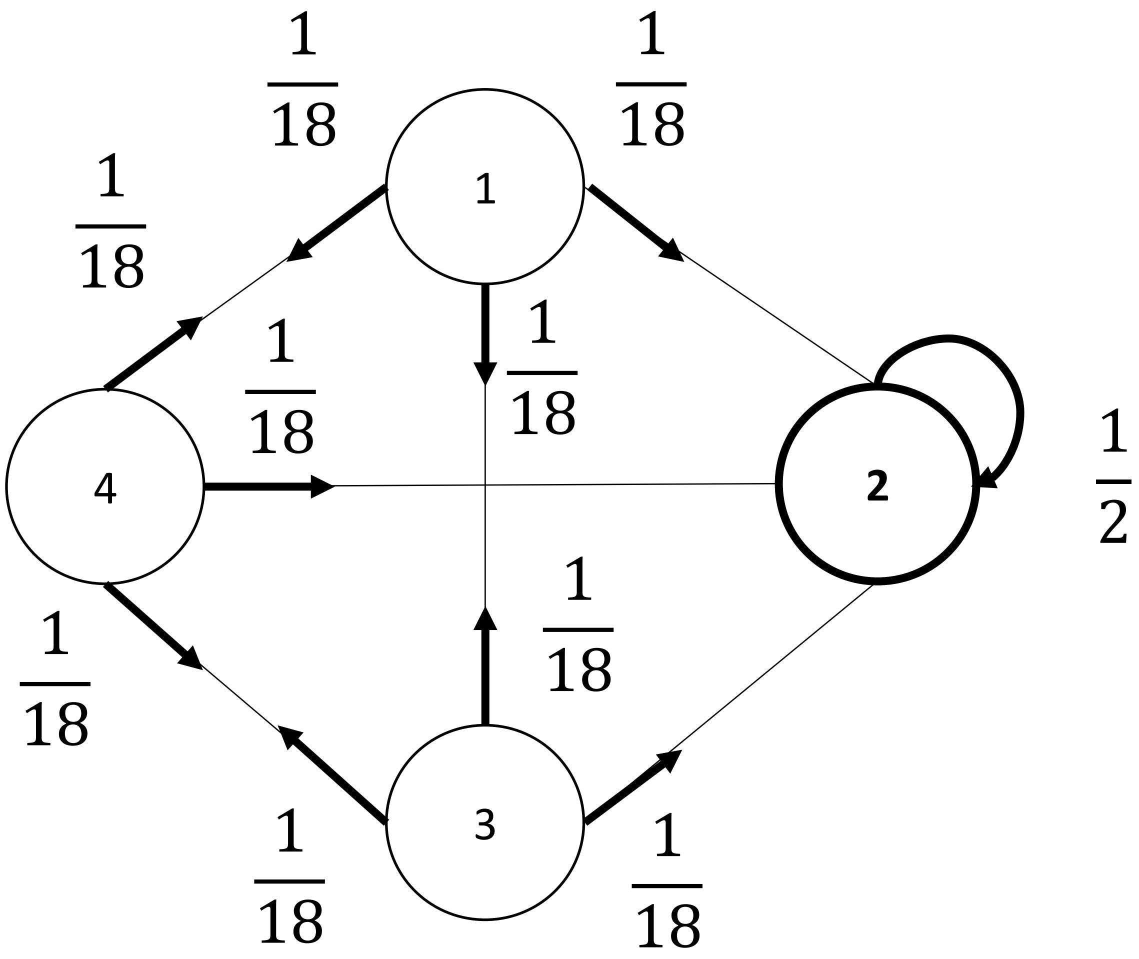
Prob(1) = Prob(3) = Prob(4) = 1/12*2 = 1/6

Prob(2) = 1/4+1/12*3 = 1/2



2. Random walk

At step 2:

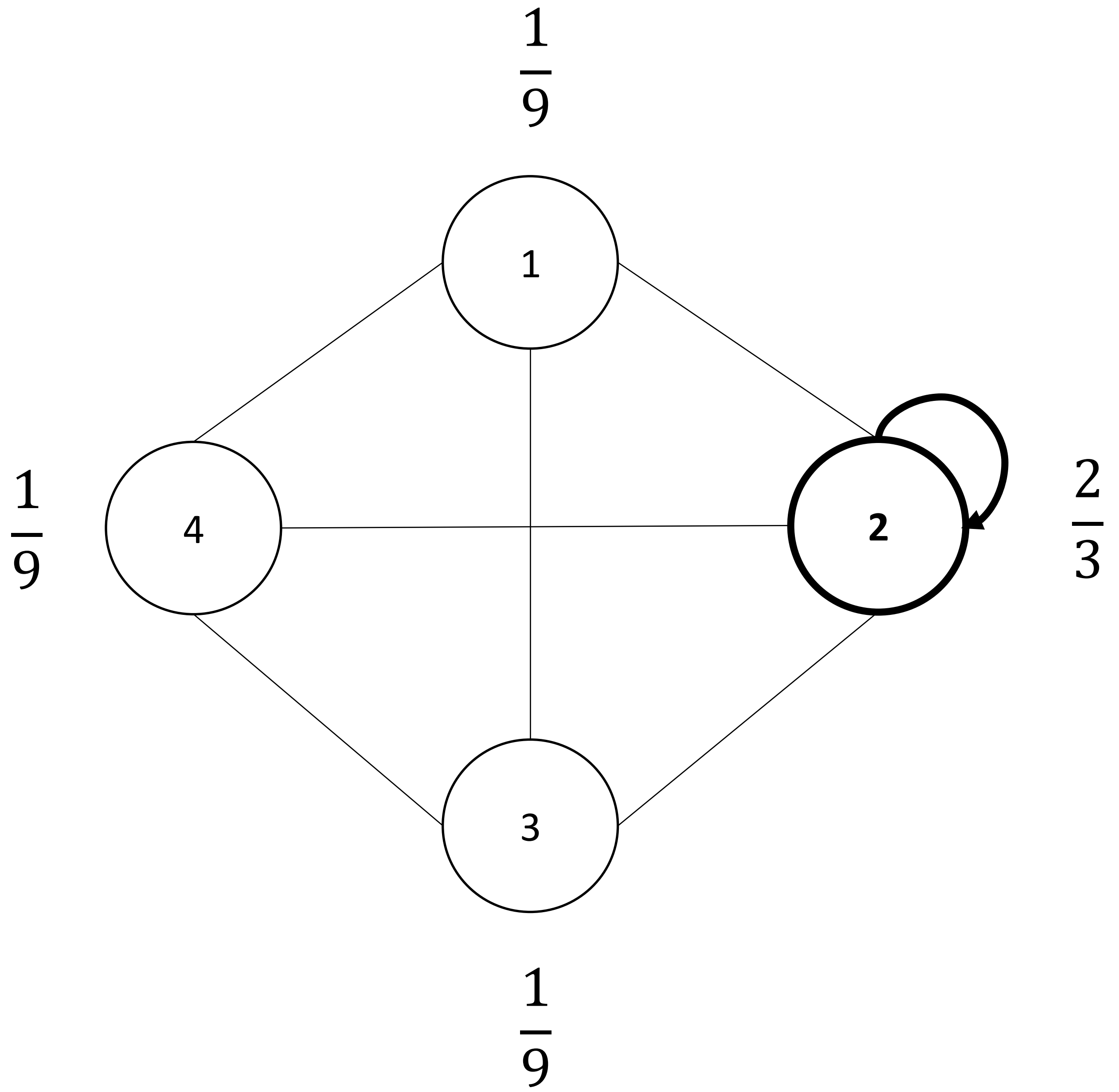


2. Random walk

When done step 2:

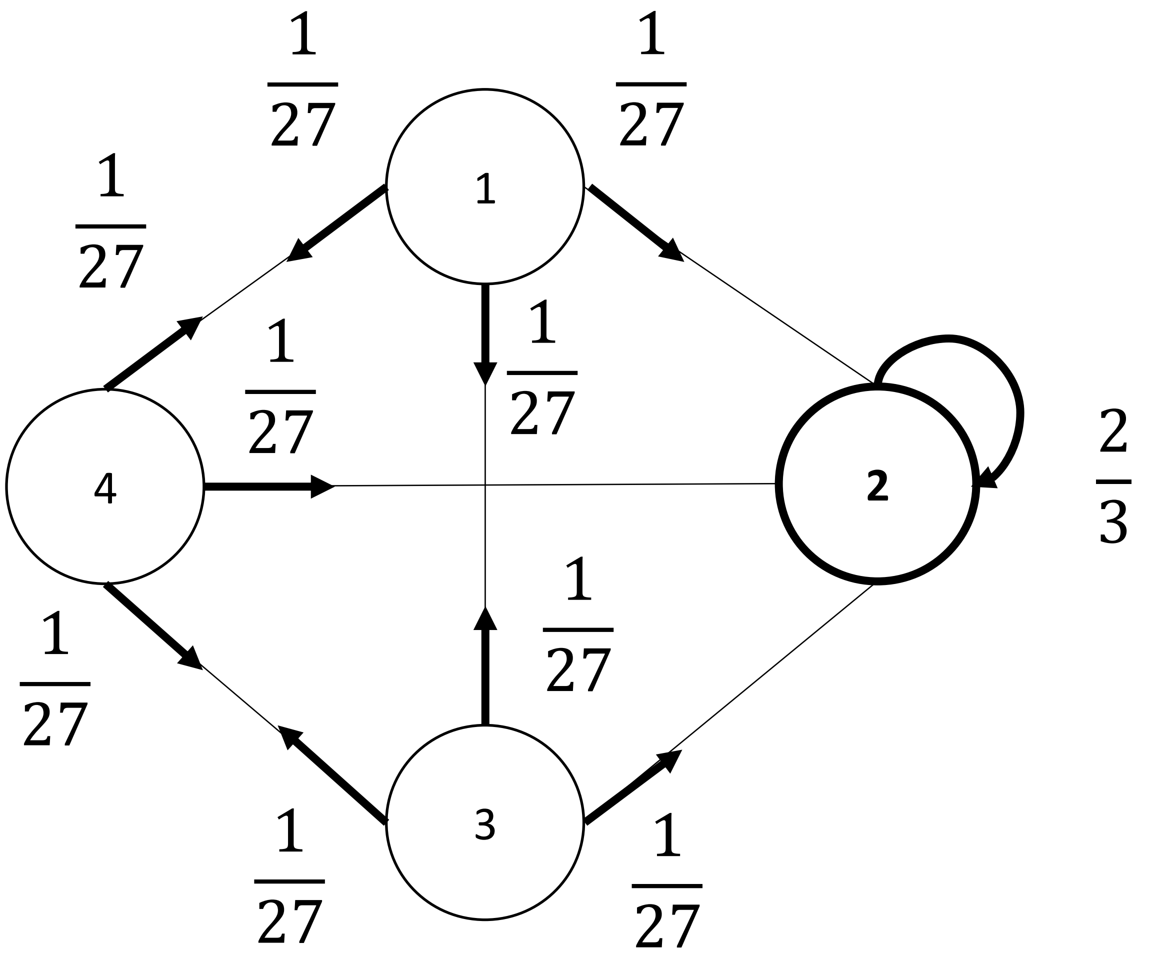
Prob(1) = Prob(3)= Prob(4)
1/18*2=1/9

Prob(2) = 1/2+1/18*3=2/3



2. Random walk

At step 3:

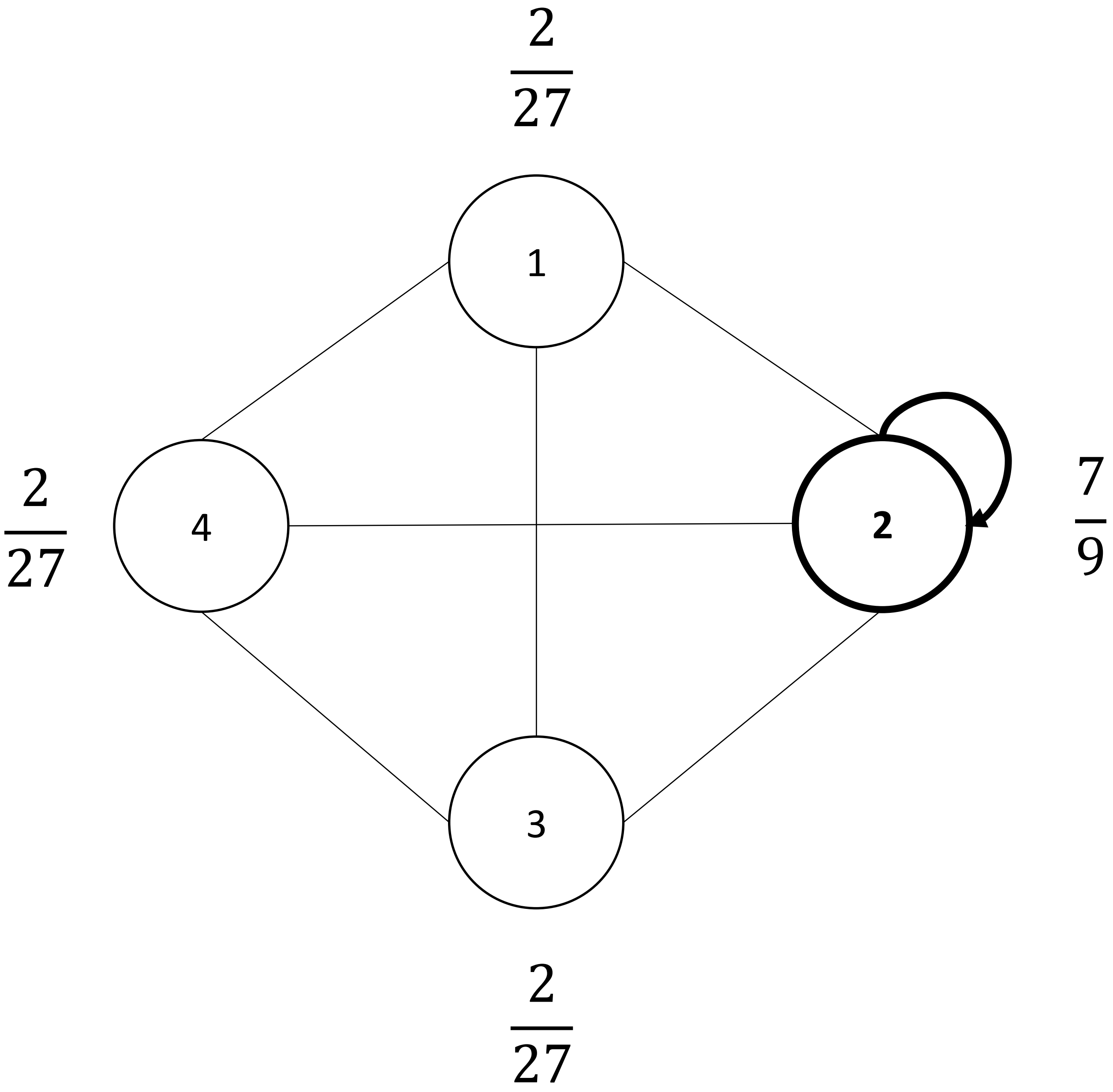


2. Random walk

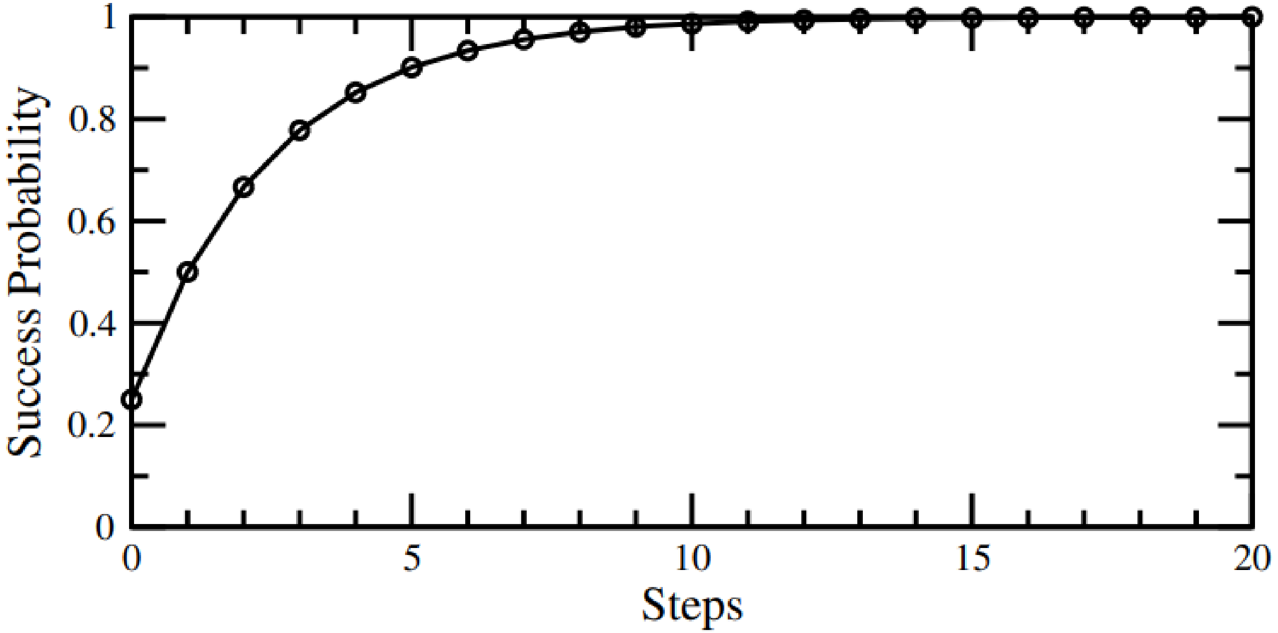
When done step 3:

Prob(1) = Prob(3)= Prob(4)
1/27*2=2/27

Prob(2) = 2/3+1/27*3=7/9



2. Random walk



Prob(Success) at time t =

$$1 - \frac{N-1}{N}\left(\frac{N-2}{N-1}\right)^{t}$$

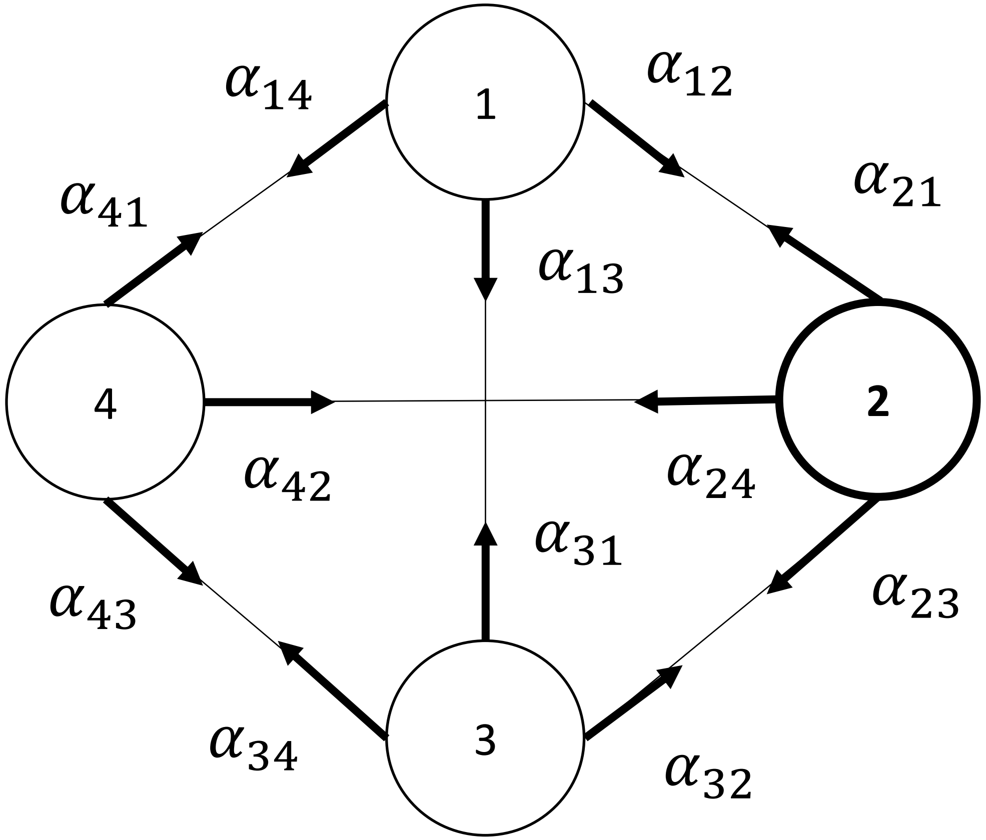
Runtime: $O(N)$

3. Quantum walk

Change 4 probs to 12 amplitudes of a wave function $|\psi\rangle \coloneqq \{\alpha_i\}$

Init: $|\psi(0)\rangle$

Note that at intermediate steps we cannot observe the system. Then we do not know the current node is the marked node or not.



At node 2: A = $\frac{-1}{\sqrt{12}} * \frac{3}{3}$

3. Quantum walk

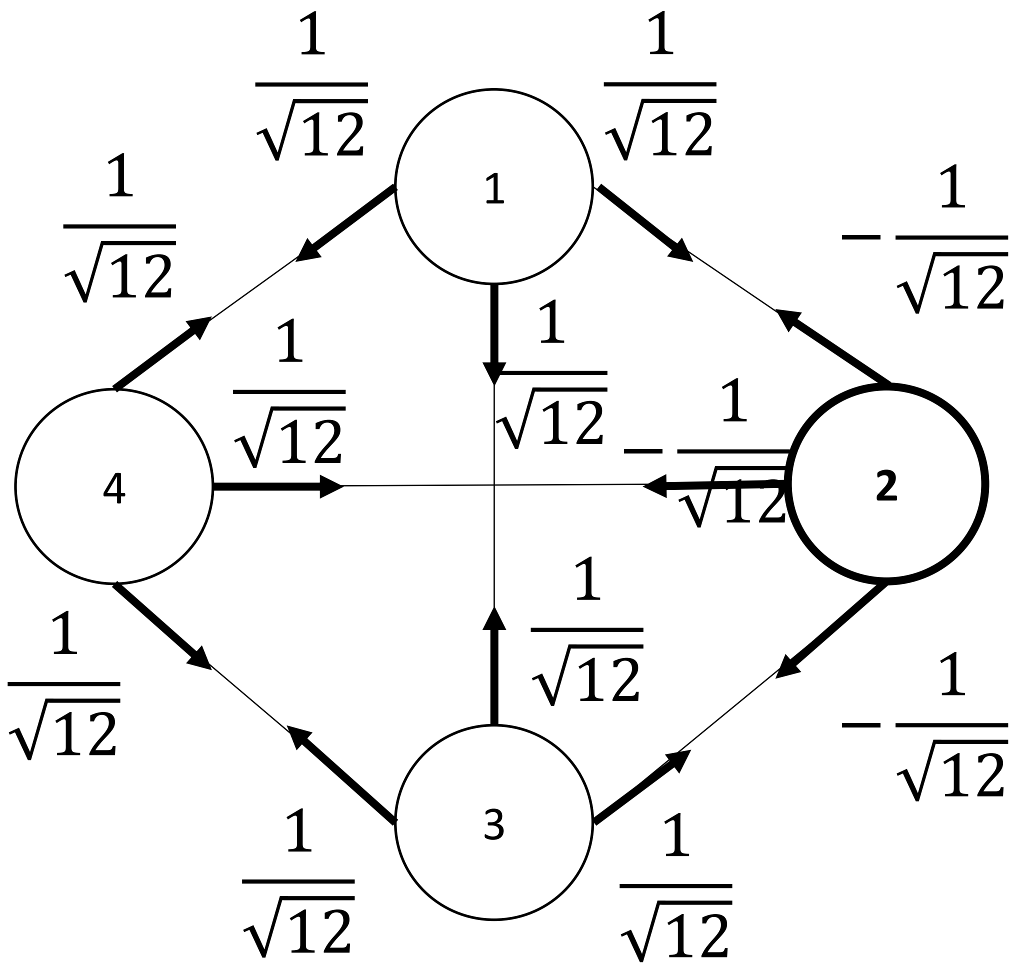
At step 1

1a. Query oracle:

$$Q|\psi(0)\rangle$$

1b. Apply a quantum coin flip (-I+2A) that inverts each amplitude about the average amplitude at its vertex. (Nothing happen)

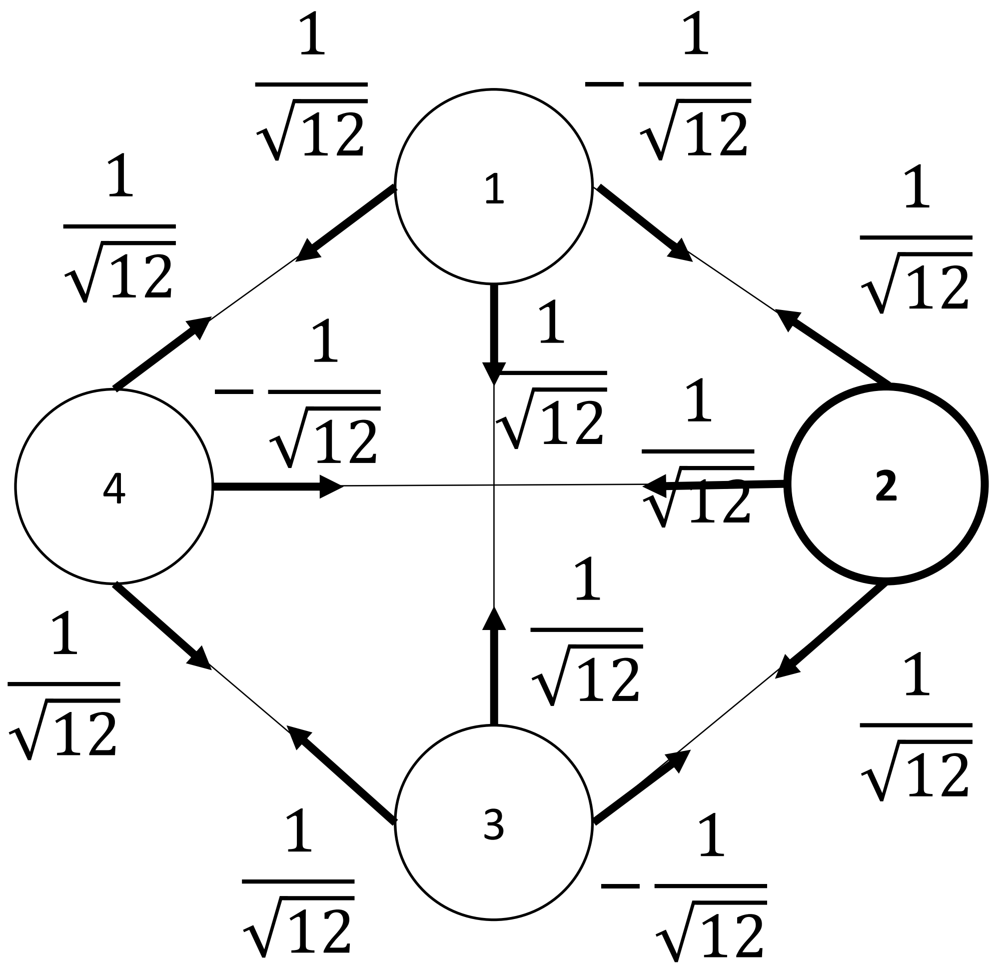
$$C(Q|\psi(0)\rangle)$$



3. Quantum walk

1c. Shift operator (walk): walker at vertex i pointing to vertex j to hop to vertex j and point back to vertex i ⇒ Swap α_{ij} with α_{ji}.

$$SCQ|\psi(0)\rangle = |\psi(1)\rangle$$

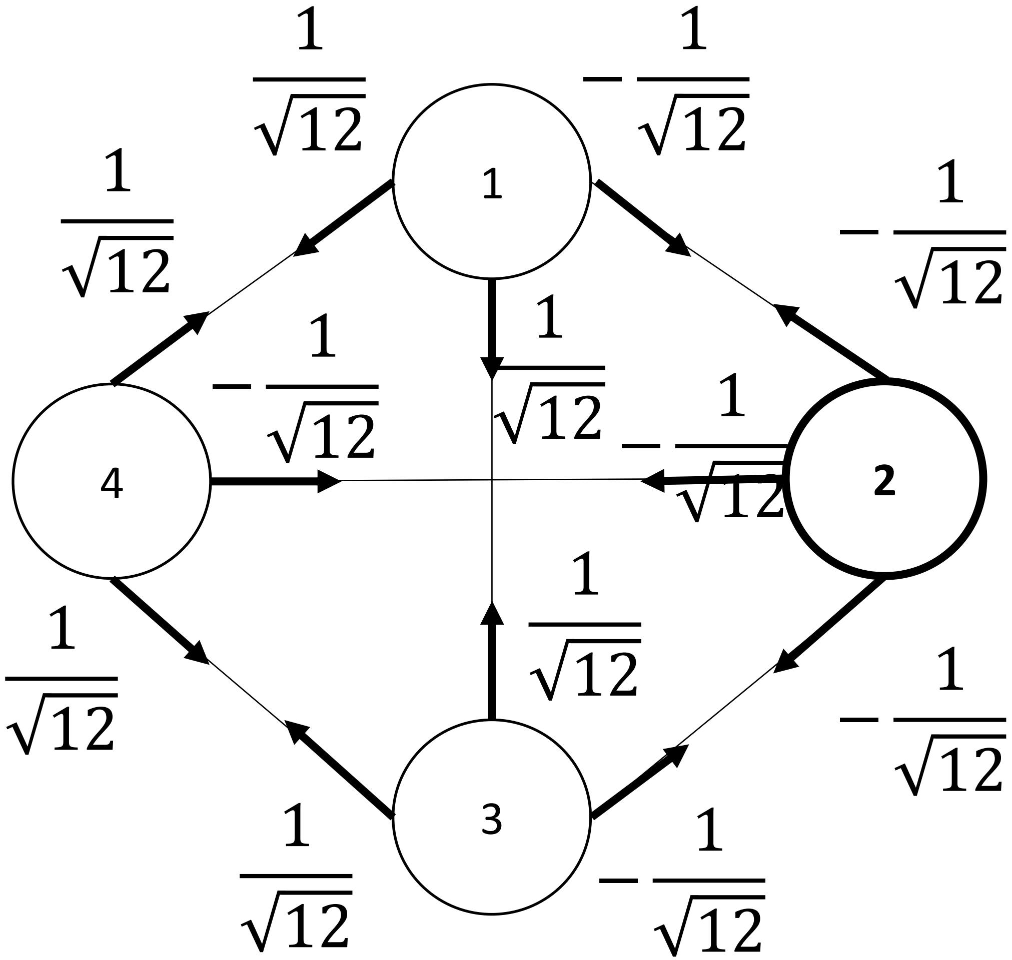


3. Quantum walk

At step 2

2a. Query oracle:

$$Q|\psi(1)\rangle$$



3. Quantum walk

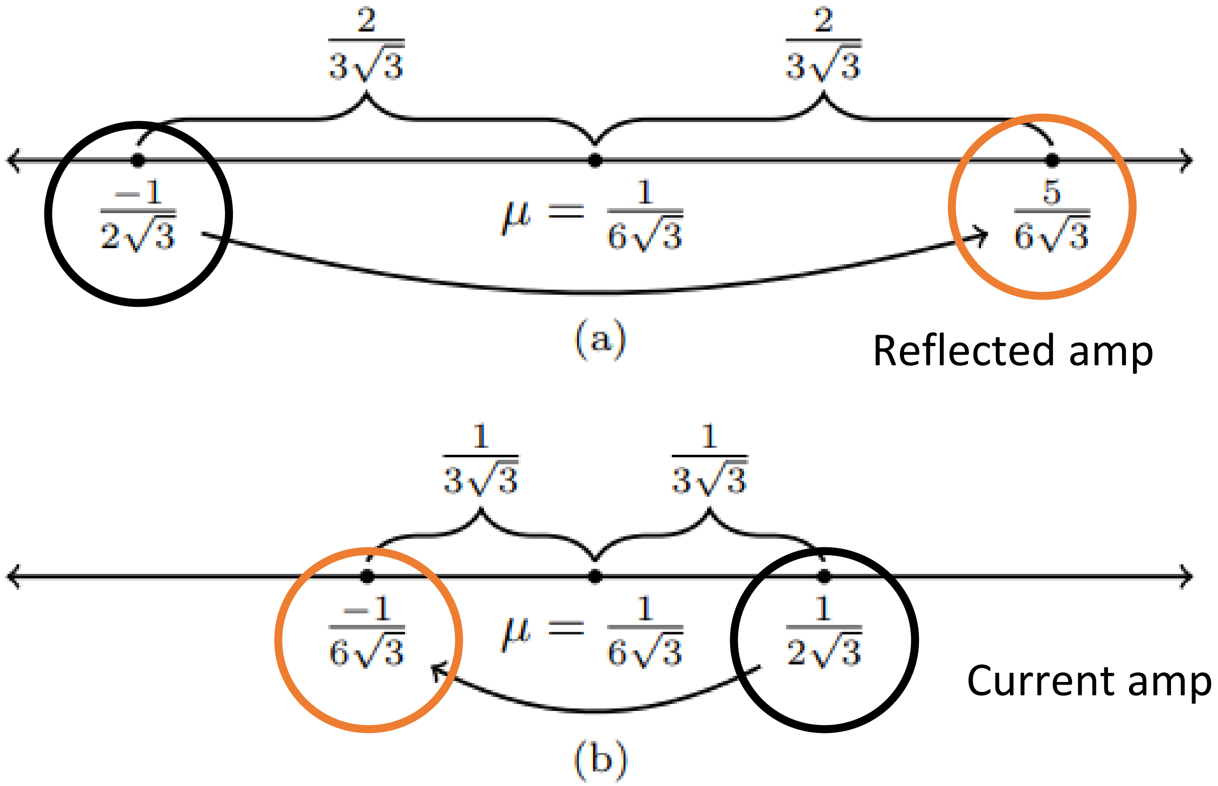
2b. The coin inverts or reflects each amplitude about their average.

(-I+2A)

At node 1: $A = \frac{\frac{1}{\sqrt{12}}+\frac{1}{\sqrt{12}}-\frac{1}{\sqrt{12}}}{3} = \frac{1}{6\sqrt{3}}$

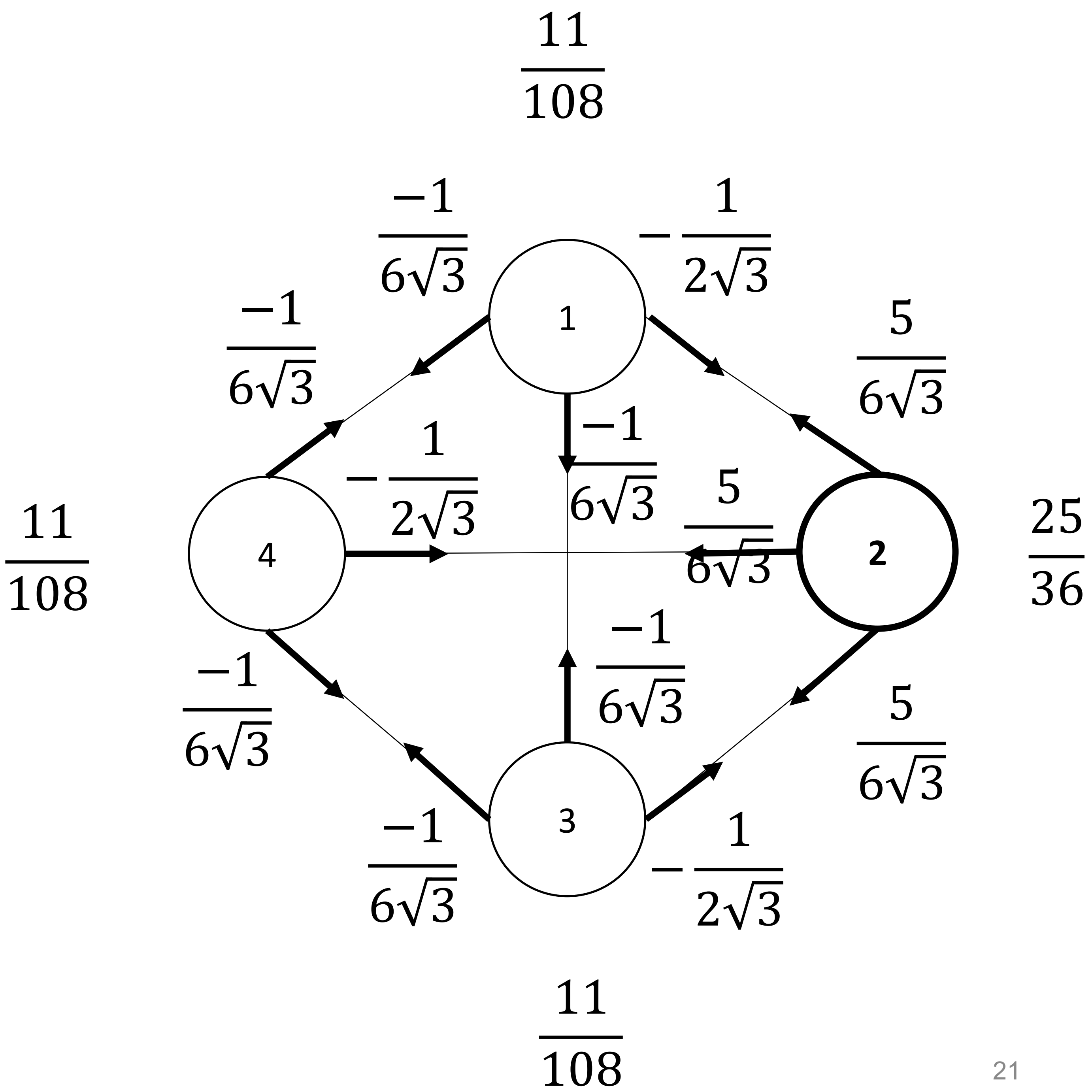
$$\Rightarrow \alpha_{12} = -\alpha_{12} + 2A = \frac{5}{6\sqrt{3}}$$

$$\Rightarrow \alpha_{13} = -\alpha_{13} + 2A = \frac{-1}{6\sqrt{3}}$$

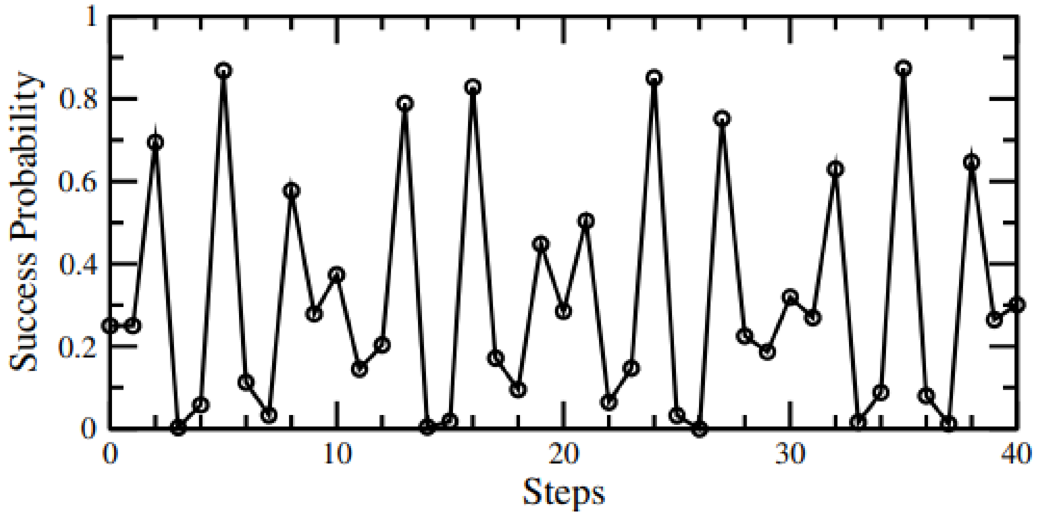


3. Quantum walk

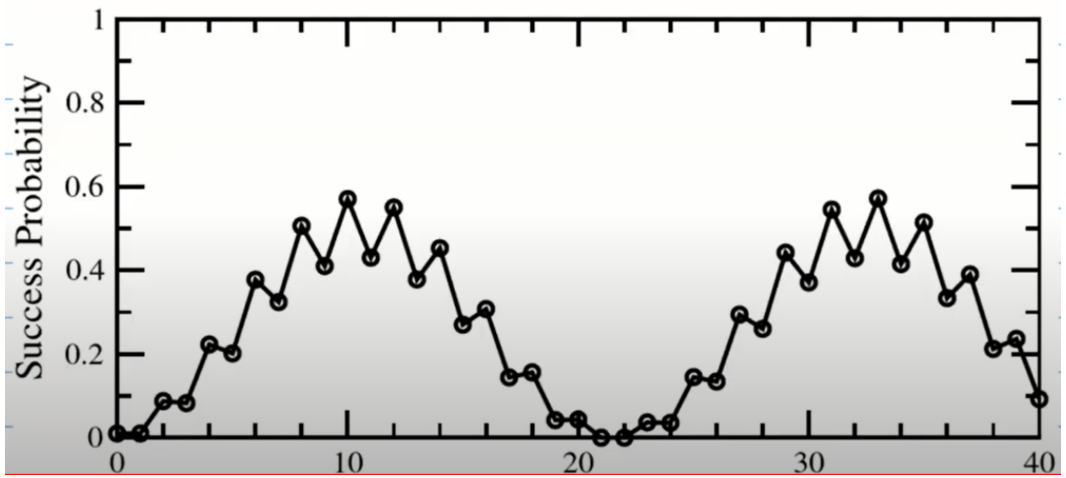
2c. Shift operator (walk): swap α_{ij} with α_{ji}.



3. Quantum walk



N=4



N=100

Prob(Success) at time t =

$$\Big\{(N-1)\Big[\cos(\phi t)+\sqrt{2N-3}\,\sin(\phi t)\Big]^2+(-1)^t(N-2)\Big\}^2/\left[(2N-3)^2N\right],$$
$$\text{where } \phi=\sin^{-1}\left(\frac{\sqrt{2N-3}}{N-1}\right)$$

Runtime: $O(\sqrt{N})$

References

Thomas Wong,
Creighton University

TITLE	CITED BY	YEAR
Global symmetry is unnecessary for fast quantum search J Janmark, DA Meyer, TG Wong Physical Review Letters 112 (21), 210502	66	2014
Connectivity is a Poor Indicator of Fast Quantum Search DA Meyer, TG Wong Physical Review Letters 114 (11), 110503	63	2015
Grover Search with Lackadaisical Quantum Walks TG Wong Journal of Physics A: Mathematical and Theoretical 48 (43), 435304	62	2015
Laplacian versus adjacency matrix in quantum walk search TG Wong, L Tarrataca, N Nahimov Quantum Information Processing 15 (10), 4029-4048	37	2016
Faster search by lackadaisical quantum walk TG Wong Quantum Information Processing 17 (3), 1-9	29	2018
Spatial Search by Continuous-Time Quantum Walk with Multiple Marked Vertices TG Wong Quantum Information Processing 15 (4), 1411-1443	26 *	2016
Nonlinear quantum search using the Gross–Pitaevskii equation DA Meyer, TG Wong New Journal of Physics 15 (6), 063014	26	2013
Quantum search with general nonlinearities DA Meyer, TG Wong Physical Review A 89 (1), 012312	25	2014