1. In the Scheduling on Unrelated Parallel Machines problem, we are given n jobs and m machines. We want to assign every job to some machine. The time of running job j on machine i is given by $p_{i,j}$. The goal is to minimize the *maximum total processing time* required by a machine.

   Show that this problem cannot be $(3/2 - \varepsilon)$-approximated.

   Hint. Reduce from 3-Dimensional Matching, which is an NP-hard problem. In this problem, we are given sets $A, B, C \subseteq [n]$ and a family of sets $\mathcal{F} \subseteq A \times B \times C$. The goal is to find a subset $\mathcal{F}' \subseteq \mathcal{F}$ such that every element of A, B, and C is contained in *exactly* one of the sets of $\mathcal{F}'$.

2. Show that Independent Set cannot be $(7/8 + \varepsilon)$ approximated.

   Hint. Standard reduction from 3SAT suffices. What kind of L-reduction is this?

3. The Cook-Levin theorem states that 3SAT is NP-hard. We want to show that its standard proof does not show inapproximability of MAX3SAT.

   The proof defines for every NP language a reduction f such that

$$x \in L \Leftrightarrow f(x) \in \text{3SAT}.$$

   Prove that there exists $x \notin L$ such that $f(x)$ is a formula with m clauses and a assignment satisfying more than $(1 - o(1))m$ of them.

   Hint. What does the reduction *encode*? Do not worry about specific numbers, this is enough to get the idea.

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