MA513: Formal Languages and Automata Theory
Topic: Turing Machine
Lecture Number 36   Date: November 12, 2011

Example A: Figure 1(a) illustrates a machine consisting of two copies of R. The machine represented by this diagram moves its head right one square; then, if that square contains an a, or a b, or a $\rhd$, or a $\sqcup$, it moves its head one square further to the right.

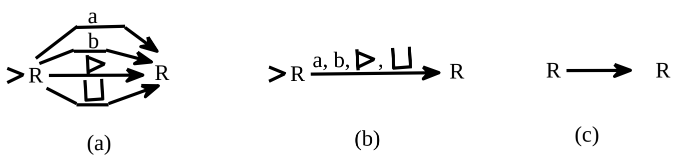

Figure 1: Demonstration of example A

It will be convenient to represent this machine as in Figure 1(b). That is, an arrow labeled with several symbols is the same as several parallel arrows, one for each symbol. If an arrow is labeled by all symbols in the alphabet Σ of the machine, then the labels can be omitted. Thus, if we know that $\Sigma = \{a, b, \rhd, \sqcup\}$, then we can display the machine as shown in Figure 1(c), where by convention, the leftmost machine is always the initial one. Sometimes an unlabeled arrow connecting two machines can be omitted entirely, by juxtaposing the representations of the two machines. Under this convention, the machine in Figure 1(c) becomes simply RR, or even R^2.

Example B: If $a \in \Sigma$ is any symbol, we can sometimes eliminate multiple arrows and labels by using $\overline{a}$ to mean **any symbol except a**. Thus, the machine shown in Figure 2(a) scans its tape to the right until it finds a blank. We shall denote this most useful machine by $R_{\sqcup}$.

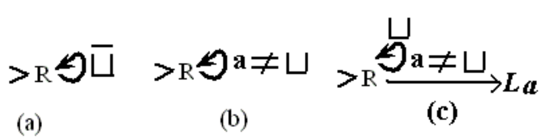

Figure 2: Demonstration of example B

Another shorthand version of the same machine as in Figure 2(a) is shown in Figure 2(b). Here, $a \neq \sqcup$ is read **any symbol a other than** $\sqcup$. The advantage of this notation is that a may then be used elsewhere in the diagram as the name of a machine. To illustrate, Figure 2(c) depicts a machine that scans to the right until it finds a nonblank square, then copies the symbol in that square onto the square immediately to the left of where it was found.

Example C: Machines to find marked or unmarked squares are illustrated in Figure 3. They are the following:

(a) $R_{\sqcup}$ - finds the first blank square to the right of the currently scanned square.

(b) $L_{\sqcup}$ - finds the first blank square to the left of the currently scanned square.

(c) $R_{\overline{\sqcup}}$ - finds the first nonblank square to the right of the currently scanned square.

(d) $L_{\overline{\sqcup}}$ - finds the first nonblank square to the left of the currently scanned square.

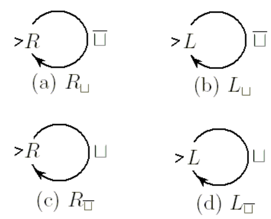

Figure 3: Demonstration of example C

Example D: The **copying machine (CM)** performs the following function: If CM starts with input w, that is, if string w, containing only nonblank symbols but possibly empty, is put on an otherwise blank tape with one blank square to its left, and the head is put on the blank square to the left of w, then the machine will eventually stop with $w \sqcup w$ on an otherwise blank tape. We say that CM **transforms** $\sqcup\underline{\alpha}w'\sqcup$ **into** $\sqcup w\underline{\sqcup}w\sqcup$, where $\alpha \in \Sigma - \{\triangleright, \sqcup\}$ and $w = \alpha w'$. A diagram for CM is given in Figure 4.

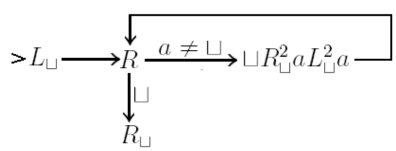

Figure 4: Demonstration of example D

Example E: The right shifting machine $S_{\rightarrow}$, transforms $\sqcup w\underline{\sqcup}$, where w contains no blanks, into $\sqcup \sqcup w\underline{\sqcup}$. It is illustrated in Figure 5.

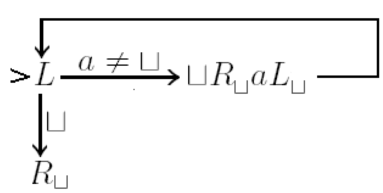

Figure 5: Demonstration of example E

Example F: Figure 6 is the machine defined in Example 1, which erases the a's in its tape.

As a matter of fact, the fully developed transition table of this machine will differ from that of the machine given in Example 1 in ways that are subtle, inconsequential,

$$\begin{tikzcd} R \arrow[r, "a"] & \sqcup \arrow[l, bend right] \end{tikzcd}$$

Figure 6: Demonstration of example F

and explored in the problem given below. The machine in Figure 6 will also contain certain extra states, which are final states of its constituents machines.

Problem: Give the full details of the Turing machines illustrated.

$$>LL \qquad \begin{tikzcd} >R \arrow[loop above] \end{tikzcd}$$

$$>L \xrightarrow{\sqcup} R$$