1. Problem formulation

Given a network graph with a set of nodes, links between nodes, and a set of virtual network functions (VNFs), each VNFs is treated as a virtual machine. Given a set of requests named Service Function Chain (SFC), each SFC is characterized by a source, a destination, amounts of resources consumed, and a set of virtual network functions (VNFs) to be installed. The objective of the multi-objectives virtual network function placement and routing problem (VNF-PR) is to find a way to install VNFs in server nodes and the path for each request to minimize the cost of activating server nodes, the cost of installing VNFs and bandwidth utilization in links.

1.1. Input

- A network is represented as an undirected graph $G(V, E)$, where V is a set of all nodes and E is a set of all links between nodes. Nodes can be divided into server nodes V_{server} and switch nodes V_{switch}. The server nodes can run VNFs, while the switch nodes are used for forwarding the network flow. We denote m_v^{mem} and m_v^{cpu} as the memory and CPU resources of node v, respectively. Each server node has an unlimited memory resource, and each switch node has no CPU resource. For each server node v, the cost of activation is $cost_v^{server}$, the limited number of installed VNF types is m_v^{vnf}, and the delay while running VNFs is d_v. Meanwhile, each link (v, u) is associated with the bandwidth resource m_{vu}^{bw} and the delay d_{vu}.
- We denote $F = \{f_1, f_2, ..., f_{|F|}\}$ as a set of VNFs, where $|F|$ is the number of VNFs types. Each f_k has a parameter $cost_{vk}^{vnf}$, which stands for the cost install VNF f_k in server node v.
- Let $R = \{r_1, r_2, ..., r_{|R|}\}$ be the set of SFC requests, where $|R|$ is the number of SFC requests. Each SFC requests r_j starts from sn_j, traverses nodes to satisfy an ordered set of VNFs F_j, and terminates at dn_j. The memory demand, bandwidth demand, and CPU demand of the SFC are respectively $w_j^{mem}, w_j^{bw}, w_j^{cpu}$.

1.2. Output

- The set of decision variables x_v where, $x_v = 1$ if the server node v is active, and $x_v = 0$ otherwise.
- The set of decision variables x_{vk}^j where, $x_{vk}^j = 1$ if VNF f_k is installed in server v, $x_{vk}^j = 0$ otherwise.

- The set of decision variables y^j_{vu} where $y^j_{vu} = 1$ if the SFC r_j passes through the link (v, u), otherwise its value is zero.

1.3. Constraints

- The bandwidth consumption of link (v, u) must be smaller than the maximum bandwidth:

$$c^{bw}_{vu} \leq m^{bw}_{vu}, \forall (v, u) \in E. \tag{1}$$

- The memory consumption of switch node v must be smaller than the maximum memory:

$$c^{mem}_{v} \leq m^{mem}_{v}, \forall v \in \ V_{switch}. \tag{2}$$

- The CPU consumption of server node v must be smaller than the maximum CPU:

$$c^{cpu}_{v} \leq m^{cpu}_{v}, \forall v \in V_{server}. \tag{3}$$

- The number of installed VNF types in each node does not exceed the maximum number of VNF types:

$$c^{vnf}_{v} \leq m^{vnf}_{v}, \forall v \in V_{server}. \tag{4}$$

- All SFC requests must be satisfied

1.4. Objectives

- Minimize the latency of all requests.

$$DL = \frac{\sum\limits_{r_j \in R} \left(\sum\limits_{(v,u) \in E} y^j_{vu} \times d_{vu} + \sum\limits_{v \in V} \sum\limits_{f_k \in F} x^j_{vk} \times d_v \right)}{\left(\sum\limits_{(v,u) \in E} d_{vu} + \sum\limits_{v \in V_{server}} d_v \right) \times q} \rightarrow \min . \tag{5}$$

In (5), the latency of a request is calculated as the summation of the delay on links that the request passes through and the delay for running VNFs on allocated servers. Subsequently, the normalized objective is calculated as the ratio of the latency of all requests and the maximum possible latency.

- Minimize the placement cost

$$CS = \frac{\sum\limits_{v \in V_{server}} x_v \times cost_v^{server}}{\sum\limits_{v \in V_{server}} cost_v^{server}} \rightarrow \min . \tag{6}$$

  Equation 6 considers the cost of activating server nodes. This value is normalized by the server activation cost ratio and all servers' total installation costs.

- Minimize the cost of installing VNFs

$$CV = \frac{\sum\limits_{v \in V_{server}} \sum\limits_{f_k \in F} x_v \times x_{vk} \times cost_{vk}^{vnf}}{\sum\limits_{v \in V_{server}} \sum\limits_{f_k \in F} cost_{vk}^{vnf}} \rightarrow \min . \tag{7}$$

  Equation 7 are calculated similarly as Equation 6, and their value ranges are uniform between (0, 1).

Now that all objective functions DL, CS, and CV from Equation 5, Equation 6 and 7 are represented in normalized form, which is all to be minimized. The multi-objective from three objectives can be combined into a single objective function by summating these objectives.

$$DL + CS + CV \rightarrow \min . \tag{8}$$

This paper's list of important notations are provided in Table 1.

Table 1: Problem Notations

Symbol	Description
Network function virtualization	
$G = (V, E)$	G is an undirected graph with a set of nodes V and a set of links E
V_{server}	A set of server nodes
V_{switch}	A set of switch nodes
F	A set of VNF requests, $F = \{f_1, f_2, ..., f_{\|F\|}\}$
n	The number of nodes
m_{vu}^{bw}	The maximum bandwidth of link (v, u)
m_v^{mem}	The maximum memory of node v
m_v^{cpu}	The maximum CPU of node v
m_v^{vnf}	The maximum number of installed VNF types in node v
c_{vu}^{bw}	The bandwidth consumption of link (v, u)
c_v^{mem}	The memory consumption of node v
c_v^{cpu}	The CPU consumption of node v
c_v^{vnf}	The number of installed VNF types in node v
d_v	The delay of server v while running VNFs
d_{vu}	The delay of link (v, u)
$cost_v^{server}$	The cost of activating server node v
$cost_{vk}^{vnf}$	The cost of installing VNF f_k on server node v
Service Function Chain	
R	R is a set of SFC requests, $R = \{r_1, r_2, ..., r_{\|R\|}\}$
sn_j	The source node of the request r_j
dn_j	The destination node of the request r_j
w_j^{mem}	The memory demand of the request r_j
w_j^{bw}	The bandwidth demand of the request r_j
w_j^{cpu}	The CPU demand of the request r_j
F_j	The ordered set of VNFs to install
Decision Variables	
x_v	1 if server v is activated, 0 otherwise
x_{vk}^j	1 if VNF f_k of the SFC r_j is installed in server v, 0 otherwise
y_{vu}^j	1 if the SFC r_j passes through the link (v, u)