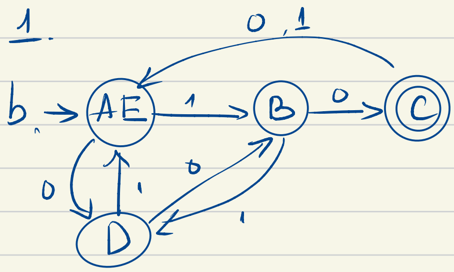
Vu Nguyen

1.

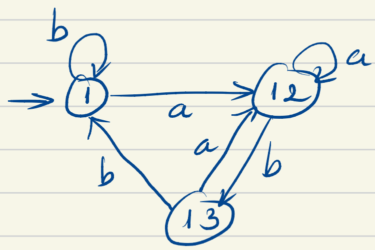


a

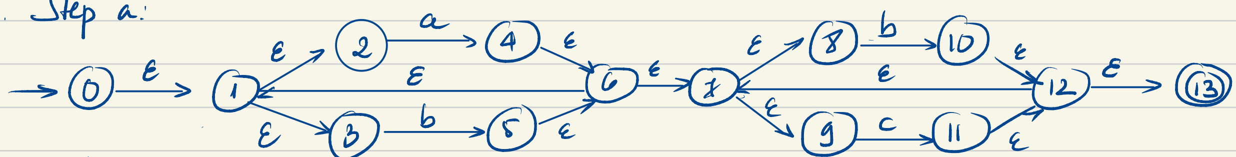
B	X			
C	X	X		
D	X	X	X	
E		X	X	X
	A	B	C	D

2.

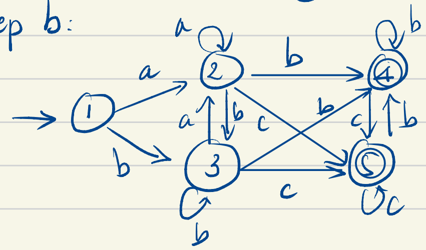
	a	b
→ 1	12	1
12	12	13
* 13	12	1



3. Step a:



Step b:



Step c:

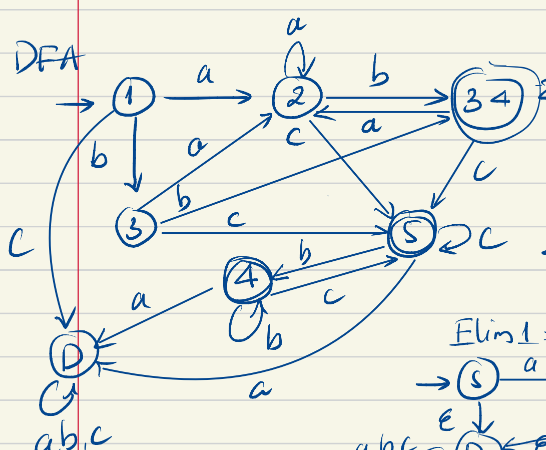
NFA

	a	b	c
→ 1	{2}	{3}	∅
2	{2}	{3,4}	{5}
3	{2}	{3,4}	{5}
* 4	∅	{4}	{5}
* 5	∅	{4}	{5}

DFA

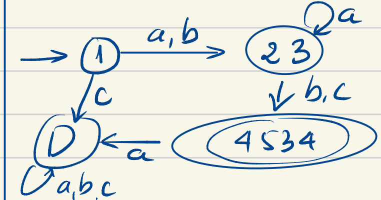
	a	b	c
→ 1	2	3	D
2	2	34	5
3	2	34	5
* 4	D	4	5
* 5	D	4	5
* 34	2	34	5
D	D	D	D

→ DFA

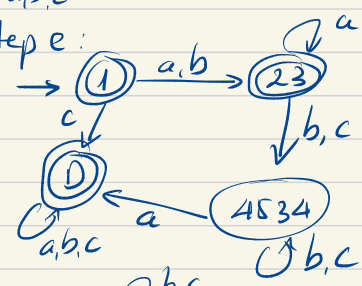


Step d:

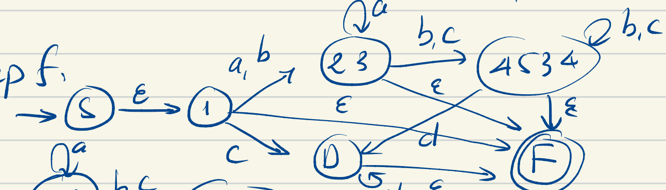
2						
3						
4	X	X	X			
5	X	X	X			
34	X	X	X			
D	X	X	X	X	X	X
	1	2	3	4	5	34



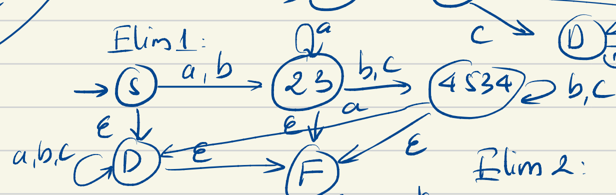
Step e:



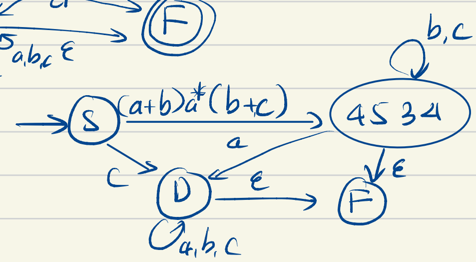
Step f:



Elim 1:



Elim 2:



Elim 4534 & D

$$\rightarrow (S) \xrightarrow[(a+b+c)^* + ((a+b)a^*(b+c)(b+c^*)a)]{(a+b)a^*(b+c)(b+c)^* +} (F)$$

4. Let Σ be the alphabet. Let D be a DFA recognizing L with start state q_0 and transition function δ. We modify D to an ϵ-NFA N recognizing $ps(L)$ as follows:

a. Add a new state s to D & make it the start state of N

b. For every $a \in \Sigma$, add an ϵ-transition from s to $\delta(q_0, a)$.

Alternatively, for any regex r over Σ, we define a regex r' over Σ such that $L(r') = ps(L(r))$ via the following recursive rules

r	r'
$\emptyset$	$\emptyset$
a	ε
$s+t$	$s'+t'$
st	if $\varepsilon \in L(s)$ then $s't+t'$ else $s't$
s^*	$s's^*$

8. a. Given any $p>0$. Let $s := (^p)^p$ for any x, y, z with $|xy| \leq p$ & $|y|>0$

Let $i := 2$

We have $xy^2z = (^p)^{p+|y|} \notin L$. Because there are more opening parentheses than the closing ones $\rightarrow$ L is unbalanced $\rightarrow$ not pumpable

b. Given any $p>0$. Let $s := 0^p 1^p$ for any x, y, z with $xyz = s$, $|xy| \leq p$ & $|y|>0$

Let $i := 2$

We have $xy^2z = 0^{p+|y|} 1^p \notin L$. Because there are more 0's than 1's $\rightarrow$ L is not regular $\rightarrow$ not pumpable

c. Given any $p>0$. Let $s := 0^p 1^p 2^p$ for any x, y, z with $xyz = s$, $|xy| \leq p$ & $|y|>0$

Let $i := 0$

We have $xy^0z = xz = 0^{p-|y|} 1^p 2^p \notin L$. Because the # of 0's & 2's are not the same $\rightarrow$ L is not regular $\rightarrow$ not pumpable

d. Given any $p>0$, Let $s := 0^p 1^{2p}$. for any x, y, z with $xyz = s$, $|xy| \leq p$ & $|y|>0$

Let $i := 2$

We have $xy^2z = xyyz = 0^{p+|y|} 1^{2p} \notin L$. Because the # 0's > # of 1's $\rightarrow$ L is not regular $\rightarrow$ not pumpable

9. $S \rightarrow \underset{1}{(S)S} \mid \underset{2}{\varepsilon}$

a. Left most derivation of $(())$:

$S \xrightarrow{1} (S)S \xrightarrow{1} ((S)S)S \xrightarrow{2} (()S)S \xrightarrow{2} (())S \xrightarrow{2} (())$

b. Rightmost derivation of $()(()())$:

$S \xrightarrow{1} (S)S \xrightarrow{1} (S)(S)S \xrightarrow{2} ()(S)S \xrightarrow{1} ()((S)(S))S \xrightarrow{2} ()(()(S))S$

$\xrightarrow{2} ()(()())S \xrightarrow{2} ()(()())$

10. $G = (\{A, B\}, \{a, b, c\}, A, P)$ CFG

$A \to aAc \mid B$
$B \to \varepsilon \mid Bc$
$L(G) = \{a^m b^n c^m \mid m, n \geq 0\}$
So $L(G)$ encompass all strings starting with some # of a's, followed by some # of b's & ending with some # of c's as many as a's

11. $L_1 := \{a^l b^m c^n \mid l \leq m\}$ $L_2 := \{a^l b^m c^n \mid m \leq n\}$

$S \to T \mid U$
$T \to Tc \mid A$
$A \to Ab \mid aAb \mid \varepsilon$
$U \to aU \mid B$
$B \to Bc \mid bBc$

T generates L_1
U generates L_2

12. abba
$S \to A$
$A \to ab$
$A \to bba$
$S \to A \to ab \to bba$

while

aba
$S \to A \to aba$

cannot generate the production rules for aba