Examples and Intuitions II

The $\Theta^{(1)}$ matrices for AND, NOR, and OR are:

$$egin{aligned} AND: \ \Theta^{(1)} &= [-30 \quad 20 \quad 20] \ NOR: \ \Theta^{(1)} &= [10 \quad -20 \quad -20] \ OR: \ \Theta^{(1)} &= [-10 \quad 20 \quad 20] \end{aligned}$$

We can combine these to get the XNOR logical operator (which gives 1 if x_1 and x_2 are both 0 or both 1).

$$egin{bmatrix} x_0 \ x_1 \ x_2 \end{bmatrix}
ightarrow egin{bmatrix} a_1^{(2)} \ a_2^{(2)} \end{bmatrix}
ightarrow egin{bmatrix} a^{(3)} \ a^{(3)} \end{bmatrix}
ightarrow h_\Theta(x)$$

For the transition between the first and second layer, we'll use a $\Theta^{(1)}$ matrix that combines the values for AND and NOR:

$$\Theta^{(1)} = egin{bmatrix} -30 & 20 & 20 \ 10 & -20 & -20 \end{bmatrix}$$

For the transition between the second and third layer, we'll use a $\Theta^{(2)}$ matrix that uses the value for OR:

$$\Theta^{(2)} = egin{bmatrix} -10 & 20 & 20 \end{bmatrix}$$

Let's write out the values for all our nodes:

$$egin{aligned} a^{(2)} &= g(\Theta^{(1)} \cdot x) \ a^{(3)} &= g(\Theta^{(2)} \cdot a^{(2)}) \ h_{\Theta}(x) &= a^{(3)} \end{aligned}$$

And there we have the XNOR operator using a hidden layer with two nodes! The following summarizes the above algorithm:

