VICTORIA UNIVERSITY OF
WELLINGTON
TE HERENGA WAKA

EXAMINATIONS – 2023
TRIMESTER 2
FRONT PAGE

MURPH101

SCIENTIFIC BASIS OF MURPHY'S LAWS

OCT 1, 2023

Time allowed: **THREE HOURS**

Permitted materials: **OPEN BOOK**
Any materials except communication via electronic devices.

Instructions: Attempt ALL **5** questions
The exam will be marked out of a total of **20** marks.
You can use the formulas listed at the end without rederiving them, unless explicitly requested.

FIRST SECTION

1. Multiple choice questions. Briefly justify your answer if unsure.

(a) The reflectance of a planar multilayer structure increases with the addition of a metal layer:
- **i.** only if placed in front
- **ii.** only if placed behind
- **iii.** it depends on the structure

(b) At the Brewster angle,
- **i.** TE-polarised light is fully reflected
- **ii.** TM-polarised is fully transmitted
- **iii.** the transmitted wave is evanescent

(c) A light ray propagating between points A and B in a medium with inhomogeneous but isotropic refractive index
- **i.** finds the shortest distance between A and B
- **ii.** finds the extremum travel time between A and B
- **iii.** has a different trajectory depending on the light polarisation

[8 marks]

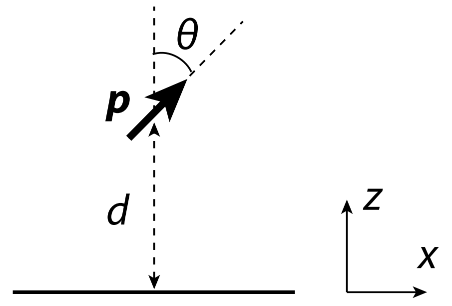
2. Solving the Laplace equation.

(a) Describe the key steps involved in the method of separation of variables for solving the Laplace equation in 3 dimensions.

(b) How does the choice of a coordinate system affect the derivation?

[5 marks]

3. Question text.
Maybe some extra lines, etc.



Maybe some more lines, etc.

[2 marks]

4. Derive the following expressions in spherical coordinates.

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$$

$$\begin{aligned}\nabla\times\mathbf{A} &= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\varphi\sin\theta) - \frac{\partial A_\theta}{\partial\varphi}\right)\hat{\mathbf{r}} \\ &+ \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right)\hat{\boldsymbol{\theta}} \\ &+ \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{\boldsymbol{\varphi}}\end{aligned}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$

[1 mark]

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APPENDIX: Misc. formulas from the lecture notes

Vector calculus identities

$$\nabla(fg) = f\nabla g + g\nabla f$$
$$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$$
$$\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$$
$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$
$$\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$$
$$\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + (\nabla\cdot\mathbf{B})\mathbf{A} - (\nabla\cdot\mathbf{A})\mathbf{B}$$

Cylindrical coordinates:

$$\begin{cases} x = s\cos\varphi \\ y = s\sin\varphi \\ z = z \end{cases}$$

$$\nabla f = \frac{\partial f}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial f}{\partial\varphi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{s}\frac{\partial}{\partial s}(sA_s) + \frac{1}{s}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{s}\left(\frac{\partial}{\partial s}(sA_\varphi) - \frac{\partial A_s}{\partial\varphi}\right)\hat{\mathbf{z}}$$

$$\nabla^2 f = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial\varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical coordinates:

$$\begin{cases} x = r\,\sin\theta\,\cos\varphi \\ y = r\,\sin\theta\,\sin\varphi \\ z = r\,\cos\theta \end{cases}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$$

$$\begin{aligned}\nabla\times\mathbf{A} &= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\varphi\sin\theta) - \frac{\partial A_\theta}{\partial\varphi}\right)\hat{\mathbf{r}} \\ &+ \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right)\hat{\boldsymbol{\theta}} \\ &+ \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{\boldsymbol{\varphi}}\end{aligned}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$