Reference Frames Continued and Examples

AERSP 304 - Dynamics and Control of Aerospace Systems

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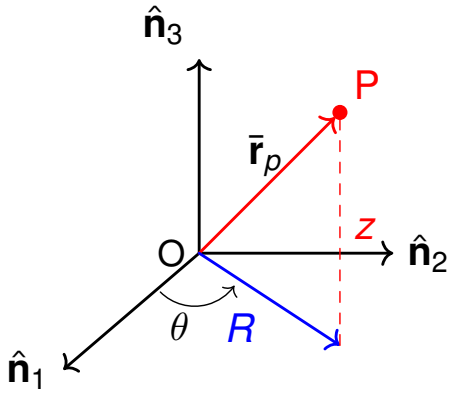
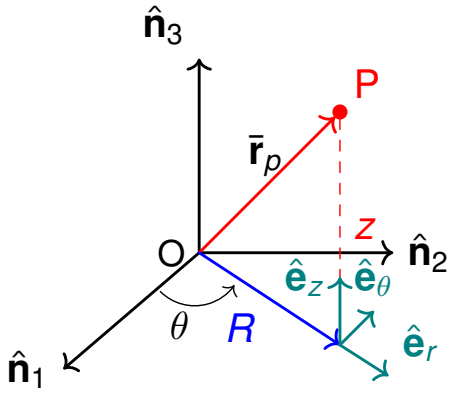
Goals for Today

- Continue review of Frame of References
- Analyze velocity and acceleration in different frames
- Examples

Reference Frames and Velocity

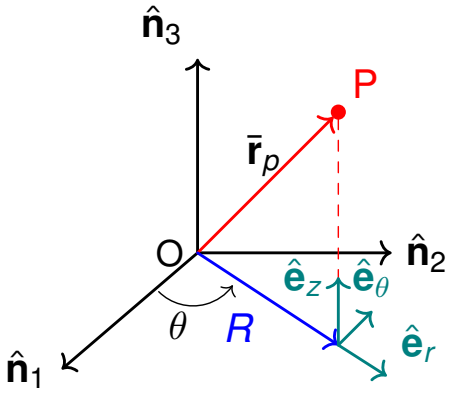




Writing a DCM for a Simple Rotation

Steps for a simple rotation DCM:

1. Identify the axis of rotation (here: $\hat{\mathbf{n}}_3 \equiv \hat{\mathbf{e}}_z$).
2. Visualize in 2D by looking through the axis of rotation.



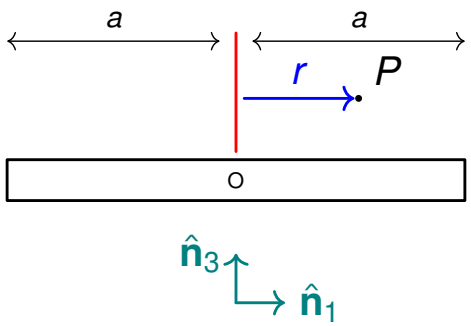
What about Acceleration?

Example: Tug-of-War on a Rotating Platform

In a game of Tug-of-War, participants are constrained to move only along a narrow platform. Moreover, the platform is rotating at angular velocity $\dot{\theta}\,\hat{\mathbf{n}}_3$. They move back and forth so that the distance from the center of the platform to a point P (point mass) is given by

$$r(t) = \frac{a}{2}\Big(1 + \sin(\omega t)\Big)$$

Find the *inertial* acceleration of P.



Step 1: Define Frames

Step 2: Write the DCM

1. Identify the axis of rotation (here: $\hat{\mathbf{n}}_3$).
2. Visualize in 2D by looking through the axis of rotation.

Step 3: Position, Velocity, and Acceleration of P

Step 3: ...Continued

Summary

- Reference frames are essential to describe motion
- Direction Cosine Matrices (DCMs) allow us to convert vectors between frames
    1. Identify the axis of rotation
    2. Visualize in 2D by looking through the axis of rotation
- Velocity and acceleration in rotating frames have additional terms