Reference Frames and Time Derivatives of a Vector

AERSP 304 - Dynamics and Control of Aerospace Systems

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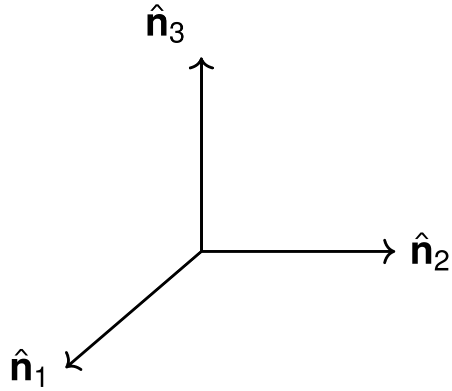
Goals for Today

- Review the definition of Frame of References
- Review the definition of DCMs
- Understand time derivatives of vectors in different frames of reference

Reference Frames

A *frame* is a language used to convey dynamical information.



Reference Frames Continued

How is *B* oriented w.r.t. *N*?

Direction Cosine Matrix (DCM)

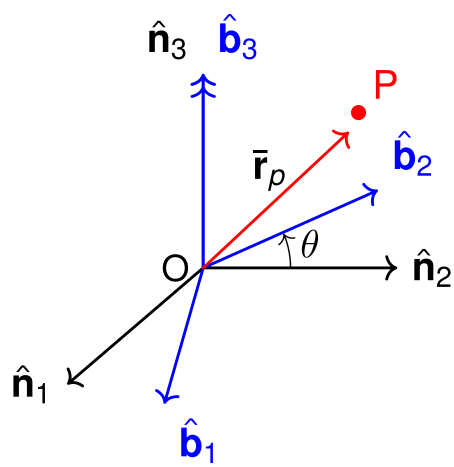
What is the orientation of N w.r.t. B?

What about another frame (A) rotating w.r.t. B?

Reference Frames and Independent Parameters

Reference Frames and Independent Parameters

Time Derivative of a Vector



Transport Theorem

Summary

Basis vectors: orthonormal, dextral (right-handed)

$$\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2 = \hat{\mathbf{n}}_3$$

DCM: maps coordinates *to B from N*

$$\begin{bmatrix} \mathbf{x}_B \\ \mathbf{y}_B \\ \mathbf{z}_B \end{bmatrix} = \underbrace{\begin{bmatrix} \hat{\mathbf{b}}_1 \cdot \hat{\mathbf{n}}_1 & \hat{\mathbf{b}}_1 \cdot \hat{\mathbf{n}}_2 & \hat{\mathbf{b}}_1 \cdot \hat{\mathbf{n}}_3 \\ \hat{\mathbf{b}}_2 \cdot \hat{\mathbf{n}}_1 & \hat{\mathbf{b}}_2 \cdot \hat{\mathbf{n}}_2 & \hat{\mathbf{b}}_2 \cdot \hat{\mathbf{n}}_3 \\ \hat{\mathbf{b}}_3 \cdot \hat{\mathbf{n}}_1 & \hat{\mathbf{b}}_3 \cdot \hat{\mathbf{n}}_2 & \hat{\mathbf{b}}_3 \cdot \hat{\mathbf{n}}_3 \end{bmatrix}}_{C_{BN}} \begin{bmatrix} \mathbf{x}_N \\ \mathbf{y}_N \\ \mathbf{z}_N \end{bmatrix}$$

$$\hat{\mathbf{b}} = C_{BN}\,\hat{\mathbf{n}}, \quad \hat{\mathbf{n}} = C_{NB}\,\hat{\mathbf{b}}$$

$$C_{NB} = (C_{BN})^{-1}, \qquad C_{NB} = (C_{BN})^T.$$

$$\hat{\mathbf{a}} = \underbrace{C_{AB}\,C_{BN}}_{C_{AN}}\,\hat{\mathbf{n}}.$$

Summary Continued

Let

$$\mathbf{r}_p = x\,\hat{\mathbf{b}}_1 + y\,\hat{\mathbf{b}}_2 + z\,\hat{\mathbf{b}}_3.$$

Differentiate:

$$\frac{d\mathbf{r}_p}{dt} = \dot{x}\,\hat{\mathbf{b}}_1 + x\,\dot{\hat{\mathbf{b}}}_1 + \dot{y}\,\hat{\mathbf{b}}_2 + y\,\dot{\hat{\mathbf{b}}}_2 + \dot{z}\,\hat{\mathbf{b}}_3 + z\,\dot{\hat{\mathbf{b}}}_3.$$

Transport theorem (applied to $\hat{\mathbf{b}}_i$):

$$^{N}\frac{d\hat{\mathbf{b}}}{dt} = {}^{B}\frac{d\hat{\mathbf{b}}}{dt} + \boldsymbol{\omega}_{B/N} \times \hat{\mathbf{b}}.$$

Hence,

$$\boxed{{}^{N}\dot{\mathbf{r}}_p = {}^{B}\dot{\mathbf{r}}_p + \boldsymbol{\omega}_{B/N} \times \mathbf{r}_p}$$