

# Reference Frames and Time Derivatives of a Vector

AERSP 304 - Dynamics and Control of  
Aerospace Systems

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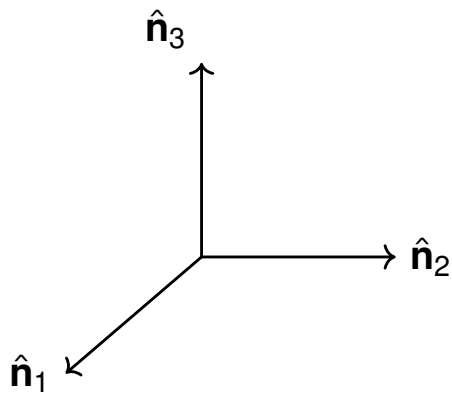
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## Goals for Today

- ▶ Review the definition of Frame of References
- ▶ Review the definition of DCMs
- ▶ Understand time derivatives of vectors in different frames of reference

## Reference Frames

A *frame* is a language used to convey dynamical information.



## Reference Frames Continued

How is  $B$  oriented w.r.t.  $N$ ?

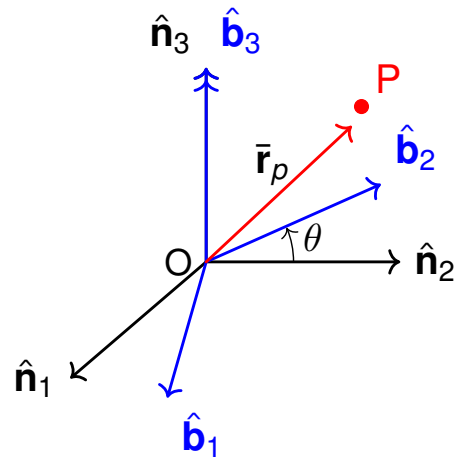
## Direction Cosine Matrix (DCM)

What is the orientation of  $N$  w.r.t.  $B$ ?

What about another frame ( $A$ ) rotating w.r.t.  $B$ ?

Reference Frames and Independent Parameters

## Time Derivative of a Vector



## Transport Theorem

## Summary

**Basis vectors:** orthonormal, dextral (right-handed)

$$\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2 = \hat{\mathbf{n}}_3$$

**DCM:** maps coordinates *to B from N*

$$\begin{bmatrix} \mathbf{x}_B \\ \mathbf{y}_B \\ \mathbf{z}_B \end{bmatrix} = \underbrace{\begin{bmatrix} \hat{\mathbf{b}}_1 \cdot \hat{\mathbf{n}}_1 & \hat{\mathbf{b}}_1 \cdot \hat{\mathbf{n}}_2 & \hat{\mathbf{b}}_1 \cdot \hat{\mathbf{n}}_3 \\ \hat{\mathbf{b}}_2 \cdot \hat{\mathbf{n}}_1 & \hat{\mathbf{b}}_2 \cdot \hat{\mathbf{n}}_2 & \hat{\mathbf{b}}_2 \cdot \hat{\mathbf{n}}_3 \\ \hat{\mathbf{b}}_3 \cdot \hat{\mathbf{n}}_1 & \hat{\mathbf{b}}_3 \cdot \hat{\mathbf{n}}_2 & \hat{\mathbf{b}}_3 \cdot \hat{\mathbf{n}}_3 \end{bmatrix}}_{C_{BN}} \begin{bmatrix} \mathbf{x}_N \\ \mathbf{y}_N \\ \mathbf{z}_N \end{bmatrix}$$

$$\hat{\mathbf{b}} = C_{BN} \hat{\mathbf{n}}, \quad \hat{\mathbf{n}} = C_{NB} \hat{\mathbf{b}}$$

$$C_{NB} = (C_{BN})^{-1}, \quad C_{NB} = (C_{BN})^T.$$

$$\hat{\mathbf{a}} = \underbrace{C_{AB} C_{BN}}_{C_{AN}} \hat{\mathbf{n}}.$$

## Summary Continued

Let

$$\mathbf{r}_p = x \hat{\mathbf{b}}_1 + y \hat{\mathbf{b}}_2 + z \hat{\mathbf{b}}_3.$$

Differentiate:

$$\frac{d\mathbf{r}_p}{dt} = \dot{x} \hat{\mathbf{b}}_1 + x \dot{\hat{\mathbf{b}}}_1 + \dot{y} \hat{\mathbf{b}}_2 + y \dot{\hat{\mathbf{b}}}_2 + \dot{z} \hat{\mathbf{b}}_3 + z \dot{\hat{\mathbf{b}}}_3.$$

Transport theorem (applied to  $\hat{\mathbf{b}}_i$ ):

$${}^N \frac{d\hat{\mathbf{b}}}{dt} = {}^B \frac{d\hat{\mathbf{b}}}{dt} + \boldsymbol{\omega}_{B/N} \times \hat{\mathbf{b}}.$$

Hence,

$$\boxed{{}^N \dot{\mathbf{r}}_p = {}^B \dot{\mathbf{r}}_p + \boldsymbol{\omega}_{B/N} \times \mathbf{r}_p}$$