Non Linear Programming: Homework 1

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1 1: 2.1: Convex combinations of points in a convex set

1.1 To prove

Let C be a convex set. Show that convex combinations of any number of points is also in C.

1.2 Proof by induction

1.2.1 Notation

Below, we assume that $x_i \in C, t_i \geq 0$.

1.2.2 Base case

By definition, if C is a convex set, for $t_1 + t_2 = 1$, $t_1x_1 + t_2x_2 \in C$.

1.2.3 Induction

Assume that, for some $k \geq 2, \sum_{i=1}^{k} t_i = 1, \sum_{i=1}^{k} t_i x_i \in C$ for any set of k points in C. Then, we will show that, if $\sum_{i=1}^{k+1} t_i = 1$, $\sum_{i=1}^{k+1} t_i x_i \in C$ for any set of k+1 points in C.

$\sum_{i=1}^{k+1} t_i x_i = \frac{\sum_{i=1}^{k} t_i}{\sum_{i=1}^{k} t_i} \sum_{i=1}^{k} t_i x_i + t_{k+1}x_{k+1}$. But, by our inductive assumption, as $(\sum_{i=1}^{k} t_i)^{-1} \sum_{i=1}^{k} t_i x_i$ is some $x' \in C$. So, $\sum_{i=1}^{k+1} t_i x_i = (\sum_{i=1}^{k} t_i)x' + t_{k+1}x_{k+1} \in C$, again by our inductive assumption.

2 2: 2.5: Distance between hyperplanes

Take 2 parallel hyperplanes: $\left\{x \in R^n : a^T x = b_1\right\}, \left\{x \in R^n : a^T x = b_2\right\}$. The vector a is perpendicular to both planes, so the distance should be measured along this vector. Considering the point along a which lies on these planes, we see that the distance from the origin of these planes is $\frac{b_i}{\|a\|}$. So, the distance between these planes is $(\|a\|^{-1} |b_1 - b_2|)$.

3 3: 2.6: containment of halfspaces

Consider halfspaces $H_1 = \left\{x \in R^n : a_1^T x \leq b_1\right\}, H_2 = \left\{x \in R^n : a_2^T x \leq b_2\right\}$, with $a_i \neq 0$.

3.1 Condition for containment

By geometric intuition, we see that one halfspace contains the other only if the hyperplanes defining the halfspaces are parallel. a_i is the vector which is perpendicular to the hyperplane defining halfspace H_i. So, for hyperplanes to be parallel, a_1, a_2 should be collinear; ie: $a_1 = ka_2$ for some scalar k. Furthermore, both half spaces should lie on the same side of the defining hyperplanes. To ensure this, we need the additional condition that $sign(b_1 b_2) = sign(k)$.

3.2 Condition for equality

For two halfspaces to be equal, they should contain each other. So, the **conditions mentioned earlier should hold**. As the hyperplane defining both halfspaces is exactly the same, we need the following condition to ensure that these hyperplanes are of equal distance from the origin: $\frac{b_1}{\|a_1\|} = \frac{b_2}{\|a_2\|}$. Using the condition that $a_1 = ka_2$, we can state this as: $\frac{b_1}{b_2} = k$.

4 4: CVX installation proof

```
CVX␣version␣1.2␣(build␣711)
MATLAB␣version␣7.5␣(R2007b)␣on␣GLNXA64
Executed␣on␣2010/1/13,␣19:24:54
Solving␣a␣randomly-generated␣CVX␣problem:
norm(A*x-b,1):
␣␣␣ans␣␣␣=␣␣␣21.5652
Optimal␣vector:
␣␣␣x␣␣␣␣␣=␣[␣␣0.0854␣␣0.1064␣␣0.6850␣-0.0390
␣␣␣0.5163␣␣0.0453␣-0.2020␣␣0.0034␣␣0.0739␣␣0.0875␣]
Residual␣vector:
␣␣␣A*x-b␣=␣[␣␣1.7350␣␣2.8750␣␣0.4650␣␣1.4400
␣␣␣-0.4363␣-0.0000␣␣1.9215␣-2.0069␣-0.6220␣-0.0000␣-1.3788
␣␣␣-1.9207␣-0.0000␣-0.0000␣-0.1832␣␣0.8469␣-3.3315␣-2.2507
␣␣␣0.0000␣␣0.1517␣]
Equality␣constraints:
␣␣␣C*x␣␣␣=␣[␣␣0.3668␣-0.1412␣-0.6425␣-1.2374␣-0.1964␣]
␣␣␣d␣␣␣␣␣=␣[␣␣0.3668␣-0.1412␣-0.6425␣-1.2374␣-0.1964␣]
Lagrange␣multiplier␣for␣C*x==d:
␣␣␣y␣␣␣␣␣=␣[␣␣6.8734␣␣3.6399␣-9.3291␣-1.2074␣-4.8483␣]
cvxtest.m␣finished␣successfully.
```