Non Linear Programming: Exam 1

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1.1 Problem setup

$Pr(x = a_i) = p_i$; $a_1 < a_2 .. < a_n$. P is the probability simplex, formed by all p : $1^T p = 1, p \geq 0$.

1.2 a

$\alpha \leq E[f(x)] = \sum_i p_i f(a_i) \leq \beta$. This is convex: intersection of halfspaces.

1.3 b

$Pr(x > \alpha) = \sum_{a_i > \alpha} p_i \leq \beta$. Convex.

1.4 c

$var(X) = E[X^2] - (E[X]^2) \leq \alpha$. Not convex.

1.5 d

$var(X) = E[X^2] - (E[X]^2) \geq \alpha$. Not convex.

1.6 e

$quartile(x) = \inf \{b : Pr(x \leq b) \geq 0.25\} \geq \alpha \equiv Pr(x < \alpha) \leq 0.25$: convex.

1.7 f

$quartile(x) \leq \alpha$. Convex.

1.8 g

$E[f(x)] = \sum p_i f(a_i)$: convex in p.

1.9 h

$Pr(\alpha \leq x \leq \beta)$: convex in p.

1.10 i

var(x) : doesn't look as if it fits any category. [**Check**]

1.11 j

quartile(x) : quasilinear.

2

$f^*(y) = \sup_{x \in dom(f)} y^T x - f(x)$.

2.1 a

$$\begin{aligned}
h_1(x) &= f(x) + c^T x + d \\
h_1^*(y) &= \sup_{x \in dom(f)} y^T x - f(x) - c^T x - d \\
&= \sup_{x \in dom(f)} (y - c)^T x - f(x) - d \\
&= f^*(y - c) - d \ \texttt{if} \ y \neq c \\
&= \sup_{x \in dom(f)} -f(x) - d \ \texttt{if} \ y = c
\end{aligned}$$

2.2 b

$$\begin{aligned}
h_2(x,t) &= tf(x/t), dom(h_2) = \{(x,t) | t > 0, x/t \in dom(f)\} \\
h_2^*(y) &= \sup_{(x,t) \in dom(h_2)} y^T x - tf(x/t) \\
&= \sup_{(x,t) \in dom(h_2)} y^T x - tf(x/t) \\
&= \sup_{(x,t) \in dom(h_2)} t(y^T (x/t) - f(x/t)) \\
&= \sup_{t>0} t \sup_{x/t \in dom(f)} (y^T (x/t) - f(x/t)) \\
&= \sup_{t>0} tf^*(y)
\end{aligned}$$

$dom(h_2^*) = \{y | \exists k : \forall t : tf^*(y) \leq k\}$.

2.3 c

$$
\begin{aligned}
h_3(x) &= \inf_z \{f(z) | Az + b = x\} \\
h_2^*(y) &= \sup_{x \in dom(h_3)} y^T x - \inf_{z : Az+b=x} f(z) \\
&= \sup_{x=Az+b, z \in dom(f)} y^T (Az + b) - \inf_z f(z) \\
&= y^T b + \sup_{z \in dom(f)} y^T Az - f(z) \\
&= y^T b + f^*(A^T y)
\end{aligned}
$$

2.4 d

$$
\begin{aligned}
h_4(x) &= f(Ax + b) \\
h_4^*(y) &= \sup_{x \in dom(h_4)} y^T x - f(Ax + b)
\end{aligned}
$$

Consider the optimization problem: $\min_{w,x} f_0(w, x) = f(w) - y^T x : Ax + b = w$. The optimal value attained for this problem is $-h_4^*(y)$.

Consdier the dual of this problem: $\sup_m -f_0^*(A^T m - m) - bm$. As strong duality holds, we can say that $-h_4^*(y) = \sup_m -f_0^*(A^T m - m) - bm$.

Now, consider the relationship betweeen $f(y)^*$ and $f_0^*(y)$.

$$
\begin{aligned}
f^*(t) &= \sup_x t^T x - f_0(x) - y^T x \\
&= f_0^*(t - y) \\
\therefore -h_4^*(y) &= \sup_m -f^*(A^T m - m + y) - bm \\
\therefore h_4^*(y) &= \sup_m f^*(A^T m - m + y) + bm
\end{aligned}
$$

2.5 e

Assumption 2.5.1. *$dom(f_1) = dom(f_2)$.*

$$
\begin{aligned}
h_5(x) &= f_1(x) + f_2(x) \\
h_5^*(y) &= \sup_{x \in dom(f_1) \cap dom(f_2)} y^T x - f_1(x) - f_2(x) \\
\texttt{Let } y &= y_1 + y_2 \\
h_5^*(y) &= \sup_x y_1^T x - f_1(x) + y_2^T x - f_2(x)
\end{aligned}
$$

[**Incomplete**]

2.6 f

$$\begin{aligned} h_6(x) &= \max\{f_1(x), f_2(x)\} \\ h_6^*(y) &= \sup_x y^T x - \max\{f_1(x), f_2(x)\} \end{aligned}$$

[**Incomplete**]

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Submitted handwritten.

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Submitted handwritten.

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