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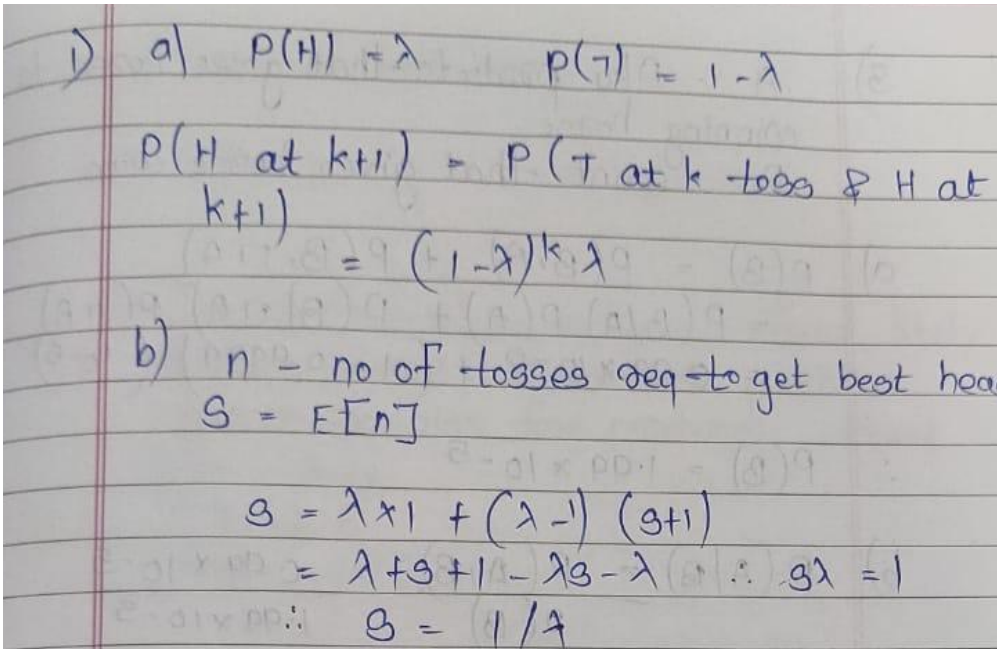
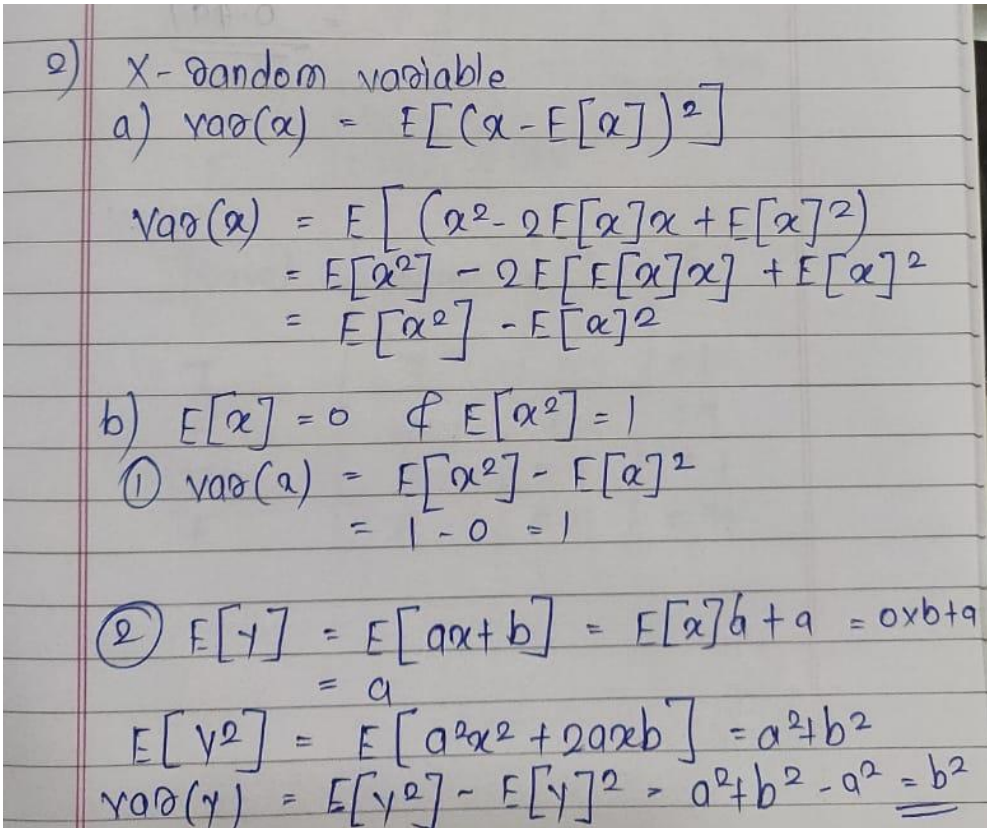
Tut 6: Machine Learning 1

1. [Probability] Assume that the probability of obtaining heads when tossing a coin is λ.
a. What is the probability of obtaining the first head at the (k + 1)-th toss?

b. What is the expected number of tosses needed to get the first head?



2. [Probability] Assume X is a random variable.
a. We define the variance of X as: $Var(X) = E[(X - E[X])^2]$. Prove that $Var(X) = E[X^2] - E[X]^2$.
b. If $E[X] = 0$ and $E[X^2] = 1$, what is the variance of X? If $Y = a + bX$, what is the variance of Y ?



3. [Probability] Your friend Aku is a great predictor about winning a horse race. Assume that we know three facts: 1) If Aku tells you that a horse name black beauty will win, it will win with probability 0.99. 2) If Aku tells you that a black beauty will not win, it will not win with probability 0.99999. 3) With probability 10^{-5}, Aku predicts that a black beauty is a winning horse. This also means with probability $1 - 10^{-5}$, Aku predicts that a black beauty will not win.

a. Given a horse, what is the probability that it wins?

b. What is the probability that Aku correctly predicts a black beauty is winning ?

3) A : Aku predicts that given horse is winning horse
B : event that given horse wins

a) $P(B) = P(B, A) + P(B, \neg A)$
$= P(B/A)\, P(A) + P(B/\neg A)\, P(\neg A)$
$= 0.99 \times 10^{-5} + (1 - 0.9999)(10^{-5})$

$\therefore P(B) = 1.99 \times 10^{-5}$

b) $P(A/B) = \frac{P(A, B)}{P(B)} = \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$
$= 0.497$