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Tut 5: Evaluation and Measurement- Hypothesis Testing

Make Assumptions about values when it is necessary in a consistent manner. Refer to the necessary table from the following link when necessary.

https://www.sheffield.ac.uk/polopoly_fs/1.43999!/file/tutorial-10-reading-tables.pdf

Testing a Proportion of small samples

1. H_0: $p = p_0$

2. One of the alternatives H\: $p < p_{n,}$ $p > p_{o,}$ or $p \neq p_o$

3. Choose a level of significance equal to a.

4. Test statistic: Binomial variable *X* with $p = p_0$.

5. Computations: Find x, the number of successes, and compute the appropriate P-value.

6. Decision: Draw appropriate conclusions based on the P-value

Ex. 1

A builder claims that air-conditions are installed in 70% of all homes being constructed today in the city of Mumbai. Would you agree with this claim if a random survey of new homes in this city shows that 8 out of 15 had air-conditions installed?

1) $H_0 = p = 0.7$ $\quad H_1: p \neq 0.7$

Significance level $\alpha = 0.1$

Test:

Binomial variable x with $p = 0.7$ & $n = 15$

$x = 8$ & $np_0 = 15 \times 0.7 = 10.5$

$$\therefore \quad P = 2P(X \leq 8 \text{ when } p = 0.7)$$

$$= 2 \sum_{x=0}^{8} b(x; 15, 0.7)$$

$$= 2 \times 0.1311$$

$$= 0.2622$$

$\therefore \; P > 0.1$ i.e. $P > \alpha$

Therefore, can't reject H_0. (unsufficient reason)

Ex.2

A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use a 0.05 level of significance.

2) $H_0 : p = 0.6$ $\quad H_1 : p > 0.6$

Significance level $= \alpha = 0.05$

$x = 70$, $n = 100$, $P_0 = 0.6$

$$\therefore \quad z = \frac{x - np_0}{\sqrt{np_0 q_0}}$$

$$= \frac{70 - 100 \times 0.6}{\sqrt{100 \times 0.6 \times 0.4}} = \frac{10}{\sqrt{24}} = 2.04$$

$p = P(z > 2.04)$ $\quad p = 0.0207$

$\therefore \quad p < \alpha$

$\therefore$ Reject H_0 & Hence new drug is superior

Ex.3

A vote is to be taken among the residents of Mumbai and the surrounding area to determine whether a proposed Nuclear plant should be constructed. The construction site is within the Mumbai limits, and for this reason many voters in the surrounding area feel that the proposal will pass because of the large proportion of Mumbai voters who favor the construction. To determine if there is a significant difference in the proportion of Mumbai voters and surrounding area voters favoring the proposal, a poll is taken. If 120 of 200 Mumbai voters favor the proposal and 240 of 500 surrounding area residents favor it, would you agree that the proportion of Mumbai voters favoring the proposal is higher than the proportion of surrounding area voters? Use an a = 0.05 level of significance.

3) p_1 - proportion Mumbai voters
p_2 - surrounding area residential proportion
$\alpha = 5\% = 0.05$
$\hat{p}_1 = 120/200 = 0.6$ $\quad \hat{p}_2 = 240/300 = 0.8$
$\hat{p}_p = 120+240/500 = 0.514$

$H_0: p_1 \leq p_0$ $\quad H_1: p_1 > p_2$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_p - (1-\hat{p}_0)} / \left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \frac{0.6-0.48}{\sqrt{0.514 \times 0.486 \times \left(\frac{1}{200}+\frac{1}{500}\right)}}$$

$= 2.869$

$p = P(Z > 2.869) = 0.0044$ $\therefore p < \alpha$
Rejecting H_0 $\therefore$ proportion is higher

Ex.4

State the null and alternative hypotheses to be used in testing the following claims, and determine generally where the critical region is located:
(a) At most, 20% of next year's wheat crop will be exported to Russia..
(b) On the average, Indian homemakers drink 3 cups of tea per day.
(c) The proportion of graduates in engineering this year majoring in the computer sciences is at least. 0.15.
(d) The average donation to the Indian Autism Association is no more than 500 INR.
(e) Residents in suburban Mumbai commute, on the average, 15 kilometers to their place of employment.

4) a) Null hypo $H_0: p = 0.20$
Alternate $H_1: p > 0.20$
Critical region in right-tail

b) $H_0: \mu = 3$ $H_1: \mu \neq 3$
Both tails

c) $H_0: p = 0.15$ $H_1: p < 0.15$
Left tail

d) $H_0: \mu = 500$ $H_1: \mu > 500$
Right tail

e) $H_0: \mu = 15$ $H_1: \mu \neq 15$
Both tails

<u>Ex.5</u>

In a study conducted by the Department of computer Engineering and analyzed by the Statistics Consulting Center at SPIT the laptops supplied by two different companies were compared. Ten sample laptops were made out of the Intel chips supplied by each company and the "robustness" was studied. The data are as follows:

Company A: 9.3 8.8 6.8, 8.7 8.5 6.7 8.0 6.5 9.2 7.0

Company B: 11.0 9.8 9.9 10.2,10.1 9.7 11.0 11.1 10.2 9.6

Can you conclude that there is virtually no difference in means between the laptops supplied by the two companies? Use a P-value to reach your conclusion. Should variances be pooled here?

5) μ_1 & μ_2 are population mean robustness of laptops supplied by A & B resp

$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$

$\alpha = 0.05$

$$\overline{x_1} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_{1i} = \frac{9.3 + 8.8 + 6.8 + 8.7 + 8.5 + 6.7 + 8 + 6.5 + 9.2 + 7}{10}$$

$= 7.95$

$\bar{x}_2 = \frac{1}{n_2} = 11 + 9.8 + 9.9 + 10.2 + 10.1 + 9.7 + 11 + 11.1 + 10.2 + 9.6 \,/\, 10$

$= 10.26$

$s_1^2 = \frac{10.65}{9} = 1.207$

$s_2^2 = 0.325$

Use unpooled t-test because sample variances are different

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$

test statistic used to test hypothesis:

$$T = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

follows approximate t-destruction with $\nu = 10$ degrees of freedom

$\mu_1 - \mu_2 = 0$

$$\therefore T = \frac{7.95 - 10.26}{\sqrt{\frac{12.07}{10} + \frac{0.325}{10}}} = -5.9$$

$|t| = 5.9$

$p = 2P(T \geq |t|) = 2P(T \geq 5.9)$

$t_{0.005}(10) = 4.582$

& $|t| = 5.9 > P(T \geq 5.9) < 0.0005$

$p < 0.001$

$p < \alpha$

Null hypothesis rejected

Mean robustness is not the same.