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Tut 6: Machine Learning 1

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1. [Probability] Assume that the probability of obtaining heads when tossing a coin is  $\lambda$ .

a. What is the probability of obtaining the first head at the  $(k + 1)$ -th toss?

b. What is the expected number of tosses needed to get the first head?

Handwritten solution for problem 1:

a)  $P(H) = \lambda$        $P(T) = 1 - \lambda$

$P(H \text{ at } k+1) = P(T \text{ at } k \text{ tosses} \& H \text{ at } k+1)$   
 $= (1 - \lambda)^k \cdot \lambda$

b)  $n$  - no of tosses req to get best head  
 $S = E[n]$

$S = \lambda \times 1 + (1 - \lambda)(S + 1)$   
 $= \lambda + S + 1 - \lambda S - \lambda$        $\therefore S\lambda = 1$   
 $\therefore S = 1/\lambda$

2. [Probability] Assume  $X$  is a random variable.

a. We define the variance of  $X$  as:  $\text{Var}(X) = E[(X - E[X])^2]$ . Prove that  $\text{Var}(X) = E[X^2] - E[X]^2$ .

b. If  $E[X] = 0$  and  $E[X^2] = 1$ , what is the variance of  $X$ ? If  $Y = a + bX$ , what is the variance of  $Y$ ?

2)  $X$  - random variable

$$a) \text{var}(X) = E[(X - E[X])^2]$$

$$\begin{aligned} \text{var}(X) &= E[(X^2 - 2E[X]X + E[X]^2)] \\ &= E[X^2] - 2E[E[X]X] + E[X]^2 \\ &= E[X^2] - E[X]^2 \end{aligned}$$

$$b) E[X] = 0 \quad \& \quad E[X^2] = 1$$

$$\begin{aligned} \textcircled{1} \text{var}(X) &= E[X^2] - E[X]^2 \\ &= 1 - 0 = 1 \end{aligned}$$

$$\textcircled{2} E[Y] = E[aX + b] = E[X]a + a = 0 \times b + a = a$$

$$\begin{aligned} E[Y^2] &= E[a^2X^2 + 2aXb] = a^2 + b^2 \\ \text{var}(Y) &= E[Y^2] - E[Y]^2 = a^2 + b^2 - a^2 = \underline{\underline{b^2}} \end{aligned}$$

3. [Probability] Your friend Aku is a great predictor about winning a horse race. Assume that we know three facts: 1) If Aku tells you that a horse name black beauty will win, it will win with probability 0.99.

2) If Aku tells you that a black beauty will not win, it will not win with probability 0.99999. 3)

With probability  $10^{-5}$ , Aku predicts that a black beauty is a winning horse. This also means with probability  $1 - 10^{-5}$ , Aku predicts that a black beauty will not win.

a. Given a horse, what is the probability that it wins?

b. What is the probability that Aku correctly predicts a black beauty is winning ?

3) A: Aku predicts that given horse is winning horse

B: count that given horse wins

$$\begin{aligned} a) \quad P(B) &= P(B|A) + P(B|\neg A) \\ &= P(B|A) P(A) + P(B|\neg A) P(\neg A) \\ &= 0.99 \times 10^{-5} + (1 - 0.9999) (10^{-5}) \end{aligned}$$

$$\therefore P(B) = 1.99 \times 10^{-5}$$

$$\begin{aligned} b) \quad P(A|B) &= \frac{P(A, B)}{P(B)} = \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}} \\ &= \underline{\underline{0.497}} \end{aligned}$$