Deriving Matchup Win Probability from Winning Percentage

Vasco Villas-Boas

September 5, 2022

Suppose it's the start of a tennis season. There are two tennis players (i, j) and they both talk smack that they will be the better player at the end of the season and schedule a match for that time. For sake of argument, suppose that the tennis season consists of a league of N players and they all play each other twice so that they face the same strength of schedule. The tennis smack talk becomes so contentious that at the end of the season, it's unsafe for the two to play. Nonetheless they want to know who is the better player. Player k jumps in, he was an average player with a winning percentage of 50% over the season, and proposes that both i and j play versus k with the following rules:

- If i beats k and k beats j, i is declared the better player.

- If j beats k and k beats i, j is declared the better player.

- If (i beats k and j beats k) or (k beats i and k beats j), they do the matches again.

Suppose all matches are independent and player i posed a winning percentage of WP_i and player j posed a winning percentage of WP_j over the long season. Then, since k has a winning percentage of 50%, that means:

$$\Pr[\text{i beats k}] = WP_i, \;\; \Pr[\text{j beats k}] = WP_j$$

since player i posed a winning percentage of WP_i versus a group that has an average winning percentage of 50%[1] and player k has a winning percentage of 50%, vis-a-vis for player j. In other words, while WP_i is a measure of the percentage of games A won over the season, it's also a measure of how likely i is to beat an average player in the league.

As a result, we can say

$$\Pr[\text{A declared better player}] = \frac{(WP_i)(1 - WP_j)}{(WP_i)(1 - WP_j) + (1 - WP_i)(WP_j)} \tag{1}$$

[1] Technically i probably didn't face an average winning percentage of 50% over the season. It's only true that $\frac{1}{N}\sum_{l=0}^{N} WP_l = 50\%$. In fact i faced an average winning percentage of $\frac{1}{N-1}\sum_{l=0, l\neq i}^{N} WP_l$. Thus, if i had a winning percentage above 50%, he actually faced a winning percentage below 50% over the season. However, for large enough N, 50% is a reasonable approximation of the winning percentage he faced.