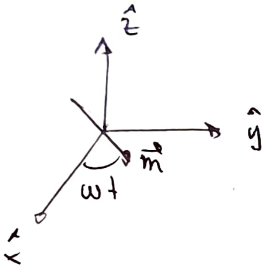

$\vec m = m(\cos\omega t\,\hat x + \operatorname{sen}\omega t\,\hat y)$, $\vec B = \dfrac{\mu_0}{4\pi r^3}\left[3(\vec m\cdot\hat r)\hat r - \vec m\right]$

$$\vec B = \frac{\mu_0 m}{4\pi r^3}\left[\begin{array}{l} 3(\cos\omega t\,\hat x + \operatorname{sen}\omega t\,\hat y)\cdot\begin{pmatrix}\operatorname{sen}\theta\cos\phi\,\hat x \\ +\operatorname{sen}\theta\operatorname{sen}\phi\,\hat y \\ +\cos\theta\,\hat z\end{pmatrix}\begin{pmatrix}\operatorname{sen}\theta\cos\phi\,\hat x \\ +\operatorname{sen}\theta\operatorname{sen}\phi\,\hat y \\ +\cos\theta\,\hat z\end{pmatrix} \\ -\cos\omega t\,\hat x - \operatorname{sen}\omega t\,\hat y \end{array}\right]$$

$U = -\vec m'\cdot\vec B - z Mg$, $\vec m' = m'(\operatorname{sen}\theta\cos\phi\,\hat x + \operatorname{sen}\theta\operatorname{sen}\phi\,\hat y + \cos\theta\,\hat z)$

$$= -\frac{\mu_0 m m'}{4\pi r^3}\left[\begin{array}{l} 3\operatorname{sen}\theta\cos\phi\operatorname{sen}\theta'\cos\phi'(\cos\omega t\cos\phi\operatorname{sen}\theta + \operatorname{sen}\omega t\operatorname{sen}\theta\operatorname{sen}\phi) \\ 3\operatorname{sen}\theta\operatorname{sen}\phi\operatorname{sen}\theta'\operatorname{sen}\phi'(\cos\omega t\cos\phi\operatorname{sen}\theta + \operatorname{sen}\omega t\operatorname{sen}\theta\operatorname{sen}\phi) \\ 3\cos\theta\cos\theta'(\cos\omega t\cos\phi\operatorname{sen}\theta + \operatorname{sen}\omega t\operatorname{sen}\theta\operatorname{sen}\phi) \\ -\operatorname{sen}\theta'\cos\phi'\cos\omega t - \operatorname{sen}\theta'\operatorname{sen}\phi'\operatorname{sen}\omega t \end{array}\right]$$

$$-r\cos\theta\, Mg.$$

$$U = -\frac{\mu_0 m m'}{4\pi r^3}\left[\begin{array}{l} 3\operatorname{sen}\theta'\operatorname{sen}^2\theta\cos(\phi'-\phi)\cos(\phi-\omega t) \\ +3\cos\theta'\cos\theta\operatorname{sen}\theta\cos(\phi-\omega t) - \operatorname{sen}\theta'\cos(\phi'-\omega t)\end{array}\right]$$

$$-r\cos\theta\, Mg.$$

$$T = \frac{M\dot r^2}{2} + \frac{M}{2}r^2\operatorname{sen}^2\theta'\dot\phi'^2 + \frac{M}{2}r^2\dot\theta'^2 ,$$

pequenas oscilações : $\operatorname{sen}\theta' \simeq \theta'$, $\operatorname{sen}\theta \simeq \theta$, $\dot\theta'^2 \simeq 0$.

$$L = T - V = \frac{M\dot r^2}{2} + \frac{\mu_0 m m'}{4\pi r^3}\left[3\theta\cos(\phi-\omega t) - \theta'\cos(\phi'-\omega t)\right] - rMg$$

$$\frac{\partial L}{\partial\theta} = \frac{3\mu_0 m m'}{4\pi r^3}\cos(\phi-\omega t) = 0 \Rightarrow \boxed{\phi = \omega t + \pi/2}$$

$$\frac{\partial L}{\partial \phi} = -\frac{3\mu_0 m m' \theta}{4\pi r^3} \operatorname{sen}(\phi - \omega t) = 0 \quad \Rightarrow \quad \boxed{\theta = 0}$$

$$\frac{\partial L}{\partial \phi'} = -\frac{\mu_0 m m' \theta'}{4\pi r^3} \operatorname{sen}(\phi' - \omega t) = 0 \quad \Rightarrow \quad \boxed{\phi' = \omega t}$$

$$\frac{\partial L}{\partial \dot{r}} = M\dot{r}, \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) = M\ddot{r} = \frac{\partial L}{\partial r} = -\frac{3\mu_0 m m'}{4\pi r^4}\left[\overset{0}{\cancel{3\theta \cos(\phi - \omega t)}} - \theta' \cos(\phi' - \omega t)\right] - Mg$$

$$\boxed{M\ddot{r} = -Mg + \frac{3\mu_0 m m'}{4\pi} \frac{\theta'}{r^4}} \quad (i)$$

$$\frac{\partial L}{\partial \dot{\theta}'} = M r^2 \dot{\theta}', \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}'}\right) = M(2r\dot{r}\dot{\theta}' + r^2 \ddot{\theta}') = \frac{\partial L}{\partial \theta'}$$

$$\frac{\partial L}{\partial \theta'} = \boxed{-\frac{\mu_0 m m'}{4\pi r^3} \cos(\phi' - \omega t) = M(2r\dot{r}\dot{\theta}' + r^2 \ddot{\theta}')} \quad (ii)$$

Supondo $r = a(1+\epsilon)$; (i) $\Rightarrow$

$$Ma\ddot{\epsilon} = -Mg + \frac{3\mu_0 m m'}{4\pi} \frac{\theta'}{a^4(1+\epsilon)^4} = -Mg + \frac{3\mu_0 m m'}{4\pi} \frac{\theta'}{a^4}(1 - 4\epsilon)$$

$$\boxed{\ddot{\epsilon} = -\frac{g}{a} + \frac{3\mu_0 m m' \theta'}{4\pi M a^5}} \quad (iii)$$

$$\frac{d}{dt}(r^2\dot{\theta}') \simeq a^2 \ddot{\theta}' = -\frac{\mu_0 m m'}{4\pi M a^3}(1 - \epsilon 3)$$

$$\boxed{\ddot{\theta}' = \frac{\mu_0 m m'}{4\pi M a^5}(3\epsilon - 1)} \quad (iv)$$

$$\Omega^2 = \frac{3\mu_0 m m'}{4\pi M a^5} , \quad \Rightarrow \quad \boxed{\begin{aligned} \ddot{\theta}' &= \Omega^2 \epsilon - \frac{\Omega^2}{3} \\ \ddot{\epsilon} &= -\frac{g}{a} + \Omega^2 \theta' \end{aligned}}$$

$$\Rightarrow \frac{d^4}{dt^4}\left(\theta' - \frac{g}{a\Omega^2}\right) = \Omega^4\left(\theta' - \frac{g}{a\Omega^2}\right).$$

$$\theta' = \frac{g}{a\Omega^2} + C_1 e^{-\Omega t} + C_2 e^{\Omega t} + C_3 \sin(\Omega t) + C_4 \cos(\Omega t)$$

$$\epsilon = \frac{1}{3} + C_1 e^{-\Omega t} + C_2 e^{\Omega t} - C_3 \sin(\Omega t) - C_4 \cos(\Omega t)$$

$$\begin{matrix} \epsilon(0) = 0 \\ \theta'(0) = 0 \end{matrix} \quad e \quad \begin{matrix} \dot{\epsilon}(0) = 0 \\ \dot{\theta}'(0) = 0 \end{matrix} \quad \rightarrow \quad C_1 = C_2 = C_3 = 0.$$

$$\boxed{\begin{aligned} \theta' &= \frac{g}{a\Omega^2} + \frac{1}{2}\left(\frac{1}{3} - \frac{g}{a\Omega^2}\right)\cos(\Omega t) \\ \epsilon &= \frac{1}{3} - \frac{1}{2}\left(\frac{1}{3} - \frac{g}{a\Omega^2}\right)\cos(\Omega t) \end{aligned}}$$