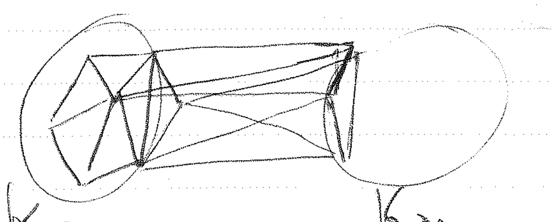
assume K_X, K_Y respective polihedra homeomorphic to X, Y are triangular;



K_X K_Y

Removing triangle from K_X and K_Y

makes:

$\chi(X\#Y) = \chi(X) + \chi(Y)$

$-2 + 3 - 3$

$$\Rightarrow \chi(X\#Y) = \chi(X) + \chi(Y) - 2 .$$

or: $\Sigma_g = T^2\#\cdots\#T^2$:

$$\chi(\Sigma_g) = -2(g-1) = 2-2g.$$

The euler characteristic is a topological invariant. But stronger than that if X is the same homotopy type as Y, $\chi(X) = \chi(Y)$.

→ Chapter 3: Homology groups is a way of characterizing Topological spaces, ~~that~~ are related to a refinement of the Euler characteristic. The guiding principle in classifying spaces with the homology groups is to find a region without boundaries, ~~that~~ is not itself a boundary of some region.

• 3.1 — Given groups G_1, G_2, a homomorphism (structure preserving map) is:

$$f: G_1 \to G_2; \quad f(x +_1 y) = f(x) +_2 f(y), \ \forall x, y \in G_1.$$

if a homomorphism is also a bijection we say that f is an isomorphism and ~~that~~ $G_1 \cong G_2$, ~~that~~ is, G_1 and G_2 are isomorphic.

• Example: $f: \mathbb{Z} \to \mathbb{Z}_2$:

$f(2n) = 0$, $f(2n+1) = 1$. Is indeed a homomorphism.

A subset $H \subseteq G$ is said a subgroup if H is a group with the group operation of G. Let $H \subseteq G$ be a subgroup of G, we say ~~that~~ $x, y \in G$ are equivalent if: $xy^{-1} \in H$. ($x \sim y$). This defines a equivalence ~~class~~ relation, and equivalence classes. Let $G/H = \{[x] \mid x \in G\}$ be the quotient space. The group operation on G can naturally extends to G/H as:

$$[x]\cdot[y] = [xy].$$

~~This~~ Care must be taken as this should be independent of representation.