Aula V

→ Integrais de caminho para spins

· Não possui análogo clássico

· Base de σ^z ($|\uparrow\rangle, |\downarrow\rangle$) é discreta.

→ Estados coerentes de spin:

$|\vec{n}\rangle$ (não é autoestado de spin).

$$\langle \vec{n}| \vec{S} |\vec{n}\rangle = s\,\underbrace{(\sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k})}_{\hat{n}}$$

maior autovalor de S_z.

$$|\hat{n}\rangle = \begin{pmatrix} \cos\theta/2 \\ e^{i\phi}\sin\theta/2 \end{pmatrix}$$

· Base supercompleta.

$\langle \vec{n}'|\vec{n}\rangle \neq \delta_{\vec{n}'\vec{n}}$ mas: $\displaystyle\int \frac{d\Omega}{2\pi} |\vec{n}\rangle\langle\vec{n}| = 1$

$d\phi\, d\theta \sin\theta$.

ângulo sólido

Propagador no tempo real:

$$i\hbar\, G(\vec{n}_f, \vec{n}_i; t) = \langle \vec{n}_f | U(t) | \vec{n}_i\rangle$$

$$= \lim_{N\to\infty} \langle \vec{n}_f | U(t_N, t_{N-1}) | \vec{n}_{N-1}\rangle\langle \vec{n}_{N-1}| \ldots$$

$$|\vec{n}_1\rangle\langle\vec{n}_1| U(t_1, t_0) |\vec{n}_i\rangle$$

k-ésimo termo:

$$\langle \vec{n}_{k+1} | U(t_{k+1}, t_k) | \vec{n}_k \rangle$$

$$= \langle \vec{n}_{k+1} | U(t_k + \Delta t, t_k) | \vec{n}_k \rangle$$

$$= \langle \vec{n}_{k+1} | \left(\mathbb{1} - \frac{i H \Delta t}{\hbar} \right) | \vec{n}_k \rangle + 1 - \langle \vec{n}_k | \vec{n}_k \rangle$$

$$= 1 + \langle \vec{n}_{k+1} | \vec{n}_k \rangle - \langle \vec{n}_k | \vec{n}_k \rangle - \frac{i \Delta t}{\hbar} \langle \vec{n}_{k+1} | H | \vec{n}_k \rangle$$

$$= \exp\left(\left(\underbrace{\frac{\langle \vec{n}_{k+1} | \vec{n}_k \rangle - \langle \vec{n}_k | \vec{n}_k \rangle}{\Delta t}} \right) \Delta t - \frac{i \Delta t}{\hbar} \langle \vec{n}_{k+1} | H | \vec{n}_k \rangle \right)$$

$$i\hbar\, G(\vec{n}_f, \vec{n}_i; t) = \lim_{N \to \infty} \int \prod_{k=1}^{N-1} \frac{d\Omega_k}{(2\pi)} \exp\left(\Delta t \sum_{k=0}^{N-1} \cdots \right)$$

$$= \int \mathcal{D}[\Omega] \exp\left(- \int_{t_i}^{t_f} dt' \left(- \langle \frac{\partial \vec{n}}{\partial t'} | \vec{n} \rangle + \frac{i}{\hbar} \langle \vec{n} | H | \vec{n} \rangle \right) \right)$$

$$\partial_t | \vec{n} \rangle = \frac{\partial}{\partial t} \begin{pmatrix} \cos\theta/2 \\ e^{i\phi} \sin\theta/2 \end{pmatrix} = \begin{pmatrix} -\sin\theta/2 \, \dot{\theta}/2 \\ i\dot{\phi} e^{i\phi} \sin\theta/2 + e^{i\phi} \cos\theta/2 \, \dot{\theta}/2 \end{pmatrix}$$

$$= i\dot{\phi} \sin\theta/2 \, e^{i\phi} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \frac{\dot{\theta}}{2} \begin{pmatrix} -\sin\theta/2 \\ e^{i\phi} \cos\theta/2 \end{pmatrix}$$

$$\langle \partial_t \vec{n} | \vec{n} \rangle = \left(-\sin\theta/2 \, \dot{\theta}/2 \quad -i\dot{\phi} e^{-i\phi} \sin\theta/2 + e^{-i\phi} \cos\theta/2 \, \dot{\theta}/2 \right) \begin{pmatrix} \cos\theta/2 \\ e^{i\phi} \sin\theta/2 \end{pmatrix}$$

$$\langle \partial_t \vec{n} | \vec{n} \rangle = -\,\text{sen}\,\theta/2 \cos\theta/2 \;\dot\theta/2$$

$$-i\dot\phi\, \cancel{e^{-i\phi}}\, \text{sen}\,\theta/2\, \cancel{e^{i\phi}}\, \text{sen}\,\theta/2 + \cancel{e^{-i\phi}} \cos\theta/2\, \dot\theta/2\, \cancel{e^{i\phi}}\, \text{sen}\,\theta/2$$

$$= -\,\cancel{\text{sen}\,\theta/2 \cos\theta/2\, \dot\theta/2} \;-i\dot\phi\, \text{sen}^2\theta/2$$
$$+\, \cancel{\dot\theta/2 \cos\theta/2\, \text{sen}\,\theta/2}$$

$$= -i\dot\phi\,(1-\cos\theta)\cdot\frac{1}{2}$$

$$= \frac{i\dot\phi}{2}(\cos\theta - 1) \longrightarrow \text{fase de } \underline{\text{berry}}!$$

$$S_{top} = \int \langle \partial_t \vec{n} | \vec{n} \rangle$$

$$= s\int_{t_i}^{t_f} dt'\, \dot\phi\,(\cos\theta - 1) =$$

$$= s\int_{\phi_i}^{\phi_f} d\phi\,(\cos\theta - 1) \qquad \text{fase geométrica}$$

$$S_{top} = s\int_0^t dt'\; \dot{\hat{n}}\cdot\vec{A}_m\,, \qquad \dot{\hat{n}} = \dot\theta\,\hat\theta + \dot\phi\, \text{sen}\,\theta\, \hat\phi$$

$$\vec{A}_m = \frac{(\cos\theta - 1)}{\text{sen}\,\theta}\,\hat\phi$$

- Ação de uma partícula de massa m vinculada ~~a~~ em uma esfera, sob a ação de um campo magnético.

$\vec{B} = \vec\nabla \times \vec{A}$.

$$S_p = \int dt \left(A_\theta \dot{\theta} + A_\phi \dot{\phi} + \frac{1}{2} m \dot{r}'^2 \right)$$

$$S_{top} = \lim_{m \to 0} S_p$$

campo magnético fictício:

$$B_m = \vec{\nabla} \times \vec{A}_m = \hat{r}$$ (Monopolo magnético efetivo !!)

$$S_{top} = s \oint d\hat{n} \cdot \vec{A}_m$$

$$= s \iint d\vec{a} \; \underbrace{\vec{\nabla} \times \vec{A}_m}_{\hat{r}}$$

$$= s \iint da$$

$$= \boxed{s \, 4\pi} \; !!!$$

Regra de quantização de Bohr-Sommerfeld:

$$S = nh$$

$$S_{top} = s 4\pi = nh \Rightarrow s 2 = n\hbar$$

$$\boxed{s = \frac{n\hbar}{2}}$$

Spin Localizado em campo Longitudinal:

$$H = -\vec{B}\cdot\vec{S}$$

$$\langle \hat{n}|H|\hat{n}\rangle = -\vec{B}\langle \hat{n}|\vec{S}|\hat{n}\rangle$$

$$= -Bs\cos\theta$$

$$S[\theta,\phi,t] = \int_0^t dt'\left[s\dot{\phi}(\cos\theta-1) + Bs\cos\theta\right]$$

qual a trajetória clássica? $\delta S = 0$:

$\theta = \theta_{cl} + \delta\theta$, $\phi = \phi_{cl} + \delta\phi$.

variação da ação:

$$S[\theta_{cl}+\delta\theta, \phi, t] = S_{cl} + \int_0^t dt'\left[\underbrace{s\dot{\phi}(-\operatorname{sen}\theta)\delta\theta - Bs\operatorname{sen}\theta\,\delta\theta}\right]$$

$$\delta S_\theta = -\int_0^t dt'\,\delta\theta\,(s\dot{\phi}\operatorname{sen}\theta + Bs\operatorname{sen}\theta)$$

$$s\operatorname{sen}\theta(\dot{\phi}+B) = 0 \Rightarrow \overline{\left|\dot{\phi} = -B\right|}$$

$$\Rightarrow \dot{\vec{S}} = \vec{S}\times\vec{B} \quad , \quad \vec{S} = s\hat{n},$$

$\vec{B} = B\hat{z} \Rightarrow S_z =$ constante;

$S_x = s\operatorname{sen}\theta\cos(\phi_0 - Bt)$.