Exercícios:

1.1 —

$$(H)_{ab}(H)_{bc} = \left(cP_j(\alpha^j)_{ab} + mc^2(\beta)_{ab}\right)\left(cP_k(\alpha^k)_{bc} + mc^2(\beta)_{bc}\right)$$

$$= c^2 P_j(\alpha^j)_{ab} P_k(\alpha^k)_{bc} + mc^3 P_j(\alpha^k)_{ab}(\beta)_{bc}$$

$$+ mc^3 P_k(\beta)_{ab}(\alpha^k)_{bc} + m^2c^4 (\beta)_{ab}(\beta)_{bc}$$

$$= c^2 \frac{P_j P_k}{2}\left(\alpha^j\alpha^k + \alpha^k\alpha^j\right)_{ac} + mc^3 P_j\left(\alpha^j\beta + \beta\alpha^j\right)_{ac}$$

$$+ m^2c^4(\beta^2)_{ac}$$

$$\Rightarrow (\alpha^j\alpha^k + \alpha^k\alpha^j)_{ac} = (\{\alpha^j, \alpha^k\})_{ac} = 2\delta^{jk}\delta_{ac}$$

$$(\alpha^j\beta + \beta\alpha^j)_{ac} = (\{\alpha^j, \beta\})_{ac} = 0$$

$$(\beta^2)_{ac} = \delta_{ac}$$

autovalores de β:

$$\beta v = \lambda v \Rightarrow \beta^2 v = \lambda \beta v = \lambda^2 v$$

$$\Rightarrow v = \lambda^2 v \Rightarrow \lambda = \pm 1.$$

mas $\alpha^j\beta = -\beta\alpha^j \Rightarrow \beta = -\alpha^j\beta\alpha^j$

$$Tr(\beta) = -Tr(\alpha^j\beta\alpha^j) = -T(\beta\alpha^j\alpha^j) = -Tr(\beta)$$

$\Rightarrow Tr(\beta) = 0$, mas como $Tr(\beta) = \sum_\lambda \lambda$

segue que β deve possuir o mesmo número de $+1$ e -1 como autovalores. Isto é, tem dimensão par.

Para α^j:

$$\alpha^j v = \lambda v \Rightarrow \alpha^{j2} v = \lambda \alpha^j v = \lambda^2 v$$

$$\Rightarrow v = \lambda^2 v \Rightarrow \lambda = \pm 1.$$

mas $\alpha^j \beta = -\beta \alpha^j \Rightarrow \alpha^j = -\beta \alpha^j \beta$

$$Tr(\alpha^j) = -Tr(\beta \alpha^j \beta) = -Tr(\alpha^j \beta^2) = -Tr(\alpha^j)$$

$$\Rightarrow Tr(\alpha^j) = 0$$

pelo mesmo argumento segue que a dimensão de α^j e β é par.

1.2 —

Caso $n = 1$:

$$H|\phi,t\rangle = \int d^3x \, a^\dagger(x) \left(-\frac{\hbar^2}{2m} \nabla^2 + U(x) \right) a(x) \int d^3x_1 \, \phi(x_1;t) \, a^\dagger(x_1) |0\rangle$$

$$+ \frac{1}{2} \int d^3x \int d^3y \, V(x-y) \, a^\dagger(x) \, a^\dagger(y) \, a(y) \, a(x) \int d^3x_1 \phi(x_1;t) \, a^\dagger(x_1) |0\rangle$$

$$= \int d^3x \int d^3x_1 \, a^\dagger(x) \left(-\frac{\hbar^2}{2m} \nabla^2 + U(x) \right) \phi(x_1;t) \left(a^\dagger(x_1) \, a(x) + \delta^3(x_1 - x) \right) |0\rangle$$

$$+ \frac{1}{2} \int d^3x \int d^3y \, V(x-y) \, a^\dagger(x) \, a^\dagger(y) \, a(y) \left(a^\dagger(x_1) \, a(x) + \delta^3(x_1 - x) \right) \phi(x_1;t) |0\rangle$$

$$= \int d^3x \, a^\dagger(x) \left(-\frac{\hbar^2}{2m} \nabla^2 + U(x) \right) \phi(x;t) |0\rangle = \int d^3(x) \, a^\dagger(x) \, i\hbar \frac{\partial}{\partial t} \phi(x;t) |0\rangle$$

logo $H|\phi,t\rangle = i\hbar\frac{\partial}{\partial t}|\phi,t\rangle$,

suponho : $H|\phi,t\rangle = i\hbar\frac{\partial}{\partial t}|\phi,t\rangle$

$$\int d^3x\, a^\dagger(x)\left(-\frac{\hbar^2}{2m}\nabla^2+U(x)\right)a(x)\int d^3x_1\, \phi(x_1;t)\, a^\dagger(x_1)|0\rangle$$

$$= i\hbar\frac{\partial}{\partial t}\int d^3x\, \phi(x;t)\, a^\dagger(x)|0\rangle$$

$$\int d^3x\int d^3x_1\, a^\dagger(x)\left(-\frac{\hbar^2}{2m}\nabla^2+U(x)\right)\phi(x_1;t)\left(a^\dagger(x_1)a(x)+\delta^3(x_1-x)\right)|0\rangle$$

$$= i\hbar\frac{\partial}{\partial t}\int d^3x\, \phi(x;t)\, a^\dagger(x)|0\rangle$$

$$\int d^3x\, a^\dagger(x)\left(-\frac{\hbar^2}{2m}\nabla^2+U(x)\right)\phi(x;t)\,|0\rangle = i\hbar\frac{\partial}{\partial t}\int d^3x\, \phi(x;t)\, a^\dagger(x)|0\rangle$$

$$\Rightarrow \left(-\frac{\hbar^2}{2m}\nabla^2+U(x)\right)\phi(x;t) = i\hbar\frac{\partial}{\partial t}\phi(x;t).$$

1.3 –

$$[N,H] = \left[\int d^3x\, a^\dagger(x)a(x),\ \int d^3y\, a^\dagger(y)\left(-\frac{\hbar^2}{2m}\nabla^2+U(y)\right)a(y)\right]$$

$$= \int d^3x\int d^3y\left(a^\dagger(x)a(x)\,a^\dagger(y)\left(-\frac{\hbar^2}{2m}\nabla^2+U(y)\right)a(y)\right.$$

$$\left.-\,a^\dagger(y)\left(-\frac{\hbar^2}{2m}\nabla^2+U(y)\right)a(y)\,a^\dagger(x)a(x)\right)$$

$$= \int d^3x\int d^3y\left(a^\dagger(x)a(x)\,a^\dagger(y)\left(-\frac{\hbar^2}{2m}\nabla^2+U(y)\right)a(y)\right.$$

$$\left.-\,a^\dagger(y)\left(-\frac{\hbar^2}{2m}\nabla^2+U(y)\right)\left(a^\dagger(x)\,a(y)+\delta^3(y-x)\right)a(x)\right)$$

$$= \int d^3x \int d^3y \Bigg(a^\dagger(x) a(x) a^\dagger(y) \left(-\frac{\hbar^2}{2m} \nabla^2 + U(y) \right) a(y)$$

$$- a^\dagger(x) a^\dagger(y) a(x) \left(-\frac{\hbar^2}{2m} \nabla^2 + U(y) \right) a(y) \Bigg)$$

$$- \int d^3y \, a^\dagger(y) \left(-\frac{\hbar^2}{2m} \nabla^2 + U(y) \right) a(y) \Bigg)$$

$$= \int d^3x \int d^3y \Bigg(a^\dagger(x) a(x) a^\dagger(y) \left(-\frac{\hbar^2}{2m} \nabla^2 + U(y) \right) a(y)$$

$$- a^\dagger(x) \left(a^\dagger(y) a(x) - \delta^3(y-x) \right) \left(-\frac{\hbar^2}{2m} \nabla^2 + U(y) \right) a(y) \Bigg)$$

$$- \int d^3y \, a^\dagger(y) \left(-\frac{\hbar^2}{2m} \nabla^2 + U(y) \right) a(y) .$$

$$= 0 .$$