1.1 $H_{ab}H_{bc} = (cP_j \alpha^j_{ab} + mc^2 \beta_{ab})(cP_k \alpha^k_{bc} + mc^2 \beta_{bc})$

$$= c^2 P_j \alpha^j_{ab} P_k \alpha^k_{bc} + mc^3 P_j \alpha^k_{ab} \beta_{bc} + mc^3 P_k \beta_{ab} \alpha^k_{bc}$$

$$+ m^2 c^4 \beta_{ab} \beta_{bc}$$

$$= \frac{c^2}{2} P_j P_k (\alpha^j \alpha^k + \alpha^k \alpha^j)_{ac} + m c^3 P_j (\alpha^j \beta + \beta \alpha^j)_{ac}$$

$$+ m^2 c^4 (\beta^2)_{ac}$$

$$\Rightarrow (\alpha^j \alpha^k + \alpha^k \alpha^j)_{ac} = (\{\alpha^j, \alpha^k\})_{ac} = 2\delta^{jk} \delta_{ac}$$

$$(\alpha^j \beta + \beta \alpha^j)_{ac} = (\{\alpha^j, \beta\})_{ac} = 0$$

$$(\beta^2)_{ac} = \frac{1}{2}(\{\beta, \beta\})_{ac} = \delta_{ac}$$

Auto valores de β:

$$\beta v = \lambda v \Rightarrow \beta^2 v = \lambda \beta v = \lambda^2 v$$

$$\Rightarrow v = \lambda^2 v \Rightarrow \lambda = \pm 1$$

como: $\alpha^j \beta = -\beta \alpha^j \Rightarrow \beta = -\alpha^j \beta \alpha^j$

Temos: $T_r \beta = -T_r(\alpha^j \beta \alpha^j) = -T_r(\alpha^j \alpha^j \beta)$

$$= -T_r(\beta) \Rightarrow T_r \beta = 0.$$

logo β possui # igual de auto valores +1 e −1, logo tem dimensão par.

Para α^j : $\quad \alpha^j v = \lambda v \Rightarrow \alpha^{j\,2} v = \lambda \alpha^j v$

$$v = \lambda^2 v \Rightarrow \lambda = \pm 1$$

$$\alpha^j \beta = -\beta \alpha^j \Rightarrow \alpha^j = -\beta \alpha^j \beta$$

$$Tr(\alpha^j) = -Tr(\beta\alpha^j\beta) = -Tr(\beta^2 \alpha^j)$$

$$= -Tr(\alpha^j) \Rightarrow Tr(\alpha^j) = 0$$

logo, pelo mesmo argumento α^j tem dimensão par!

(1.2) Para $n=1$:

$$H|\phi,t\rangle = \int d^3\vec{x}\, a^\dagger(\vec{x}) \left[-\frac{\hbar^2 \nabla_x^2}{2m} + U(\vec{x}) \right] a(\vec{x}) \int d^3\vec{x}_1 \phi(\vec{x}_1;t)\, a^\dagger(\vec{x}_1)|0\rangle$$

$$+ \frac{1}{2}\int d^3\vec{x}\, d^3\vec{y}\, V(\vec{x}-\vec{y})\, a^\dagger(\vec{x}) a^\dagger(\vec{y}) a(\vec{y}) a(\vec{x}) \int d^3\vec{x}_1 \phi(\vec{x}_1;t)\, a^\dagger(\vec{x}_1)|0\rangle$$

$$= \int d^3\vec{x}\, d^3\vec{x}_1\, a^\dagger(\vec{x}) \left(-\frac{\hbar^2 \vec{\nabla}_x^2}{2m} + U(\vec{x}) \right) \phi(\vec{x}_1;t) \left(a^\dagger(\vec{x}_1) a(\vec{x}) + \delta^3(\vec{x}-\vec{x}_1) \right)|0\rangle$$

$$+ \frac{1}{2}\int d^3\vec{x}\, d^3\vec{y}\, V(\vec{x}-\vec{y})\, \phi(\vec{x}_1;t)\, a^\dagger(\vec{x}) a^\dagger(\vec{y}) a(y) \left(a^\dagger(\vec{x}_1) a(\vec{x}) + \delta^3(\vec{x}-\vec{x}_1) \right)|0\rangle$$

$$= \int d^3\vec{x}\, a^\dagger(\vec{x}) \left(-\frac{\hbar^2 \nabla_x^2}{2m} + U(\vec{x}) \right) \phi(\vec{x};t)|0\rangle$$

$$= \int d^3\vec{x}\, a^\dagger(\vec{x})\, i\hbar \frac{\partial}{\partial t} \phi(\vec{x};t)|0\rangle = i\hbar \frac{\partial}{\partial t}|\phi,t\rangle$$

Analogamente,

$$i\hbar\frac{\partial}{\partial t}\int d^3\vec{x}_1\,\phi(\vec{x}_1;t)\,a^\dagger(\vec{x}_1)|0\rangle = \int d^3\vec{x}\,a^\dagger(\vec{x})\left(-\frac{\hbar^2\nabla_{\vec{x}}^2}{2m}+U(\vec{x})\right)a(\vec{x})\,\times$$

$$\times\int d^3\vec{x}_1\,\phi(\vec{x}_1;t)\,a^\dagger(\vec{x}_1)|0\rangle$$

$$=\int d^3\vec{x}\,d^3\vec{x}_1\,a^\dagger(\vec{x})\left(-\frac{\hbar^2\nabla_{\vec{x}}^2}{2m}+U(\vec{x})\right)\phi(\vec{x}_1;t)\left(a^\dagger(\vec{x}_1)a(\vec{x})+\delta^3(\vec{x}_1-\vec{x})\right)|0\rangle$$

$$\stackrel{!}{=}\int d^3\vec{x}\,a^\dagger(\vec{x})\left(-\frac{\hbar^2}{2m}\nabla_{\vec{x}}^2+U(\vec{x})\right)\phi(\vec{x};t)\,|0\rangle$$

$$\Rightarrow\quad i\hbar\frac{\partial}{\partial t}\phi(\vec{x};t)=\left(-\frac{\hbar^2}{2m}\nabla_{\vec{x}}^2+U(\vec{x})\right)\phi(\vec{x};t)$$

por indução está provado $\blacksquare$

1.3 $$[N,H]=\left[\int d^3\vec{x}\,a^\dagger(\vec{x})a(\vec{x}),\ \int d^3\vec{y}\,a^\dagger(\vec{y})\left(-\frac{\hbar^2\nabla_{\vec{y}}^2}{2m}+U(\vec{y})\right)a(\vec{y})\right]$$

$$=\int d^3\vec{x}\,d^3\vec{y}\left[a^\dagger_{\vec{x}}a_{\vec{x}}a^\dagger_{\vec{y}}\left(-\frac{\hbar^2}{2m}\nabla_y^2+U(y)\right)a(y)-a^\dagger(y)\left(-\frac{\hbar^2}{2m}\nabla_y^2+U(y)\right)a(y)\,a^\dagger(x)\,a(x)\right]$$

$$=\int d^3\vec{x}\,d^3\vec{y}\left[\ ''\ -a^\dagger(y)\left(-\frac{\hbar^2}{2m}\nabla_y^2+U(y)\right)\left(a^\dagger(x)a(x)a(\vec{y})+a^\dagger(x)a(x)\,\delta(\vec{y}-\vec{x})\right)\right]$$

$$=\int d^3\vec{x}\,d^3\vec{y}\left[\ ''\ -a^\dagger(\vec{x})\,a^\dagger(\vec{y})\,a(\vec{x})\left(-\frac{\hbar^2}{2m}\nabla_y^2+U(y)\right)a(\vec{y})\right]-\int d^3\vec{y}\,a^\dagger(\vec{y})\left(-\frac{\hbar^2\nabla_y^2}{2m}+U(y)\right)a(y)$$

$$=\int d^3\vec{x}\,d^3\vec{y}\left[\ ''\ -a^\dagger(\vec{x})\left[a(x)\,a^\dagger(y)-\delta^3(y-x)\right]\left(-\frac{\hbar^2}{2m}\nabla_y^2+U(y)\right)a(y)\right]-\quad ''$$

$$=\underline{\underline{0}}\ !$$