

SREDNICKI CAPÍTULO 1

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1. EXERCÍCIO 1

$$\begin{aligned}
H_{ab}H_{bc} &= (cP_j\alpha_{ab}^j + mc^2\beta_{ab})(cP_k\alpha_{bc}^k + mc^2\beta_{bc}) \\
&= c^2P_j\alpha_{ab}^jP_k\alpha_{bc}^k + mc^3P_j\alpha_{ab}^j\beta_{bc} + mc^3P_k\beta_{ab}\alpha_{bc}^k + m^2c^4\beta_{ab}\beta_{bc} \\
&= \frac{c^2}{2}P_jP_k(\alpha^j\alpha^k + \alpha^k\alpha^j)_{ac} + mc^3P_j(\alpha^j\beta + \beta\alpha^j)_{ac} + m^2c^4\beta_{ac}^2 \\
&\Rightarrow (\alpha^j\alpha^k + \alpha^k\alpha^j)_{ac} = \{\alpha^j, \alpha^k\}_{ac} = 2\delta^{jk}\delta_{ac} \\
&(\alpha^j\beta + \beta\alpha^j)_{ac} = \{\alpha^j, \beta\}_{ac} = 0 \\
\beta_{ac}^2 &= \frac{1}{2}\{\beta, \beta\}_{ac} = \delta_{ac}
\end{aligned}$$

Os auto-valores de β são,

$$\begin{aligned}
\beta v &= \lambda v \Rightarrow \beta^2 v = \lambda v = \lambda^2 v \\
&\Rightarrow v = \lambda^2 v \Rightarrow \lambda = \pm 1
\end{aligned}$$

Como $\alpha^j\beta = -\beta\alpha^j \Rightarrow \beta = -\alpha^j\beta\alpha^j$, temos que,

$$\begin{aligned}
\text{Tr}[\beta] &= -\text{Tr}[\alpha^j\beta\alpha^j] = -\text{Tr}[\alpha^j\alpha^j\beta] \\
&= -\text{Tr}[\beta] \Rightarrow \text{Tr}[\beta] = 0
\end{aligned}$$

Logo β possui número igual de auto-valores $+1$ e -1 , logo, tem dimensão par.

Para α^j ,

$$\begin{aligned}
\alpha^j v &= \lambda v \Rightarrow \alpha^{j^2} v = \lambda v = \lambda^2 v \\
&\Rightarrow v = \lambda^2 v \Rightarrow \lambda = \pm 1
\end{aligned}$$

Como $\alpha^j\beta = -\beta\alpha^j \Rightarrow \alpha^j = -\beta\alpha^j\beta$, temos que,

$$\begin{aligned}
\text{Tr}[\alpha^j] &= -\text{Tr}[\beta\alpha^j\beta] = -\text{Tr}[\alpha^j\beta^2] \\
&= -\text{Tr}[\alpha^j] \Rightarrow \text{Tr}[\alpha^j] = 0
\end{aligned}$$

Logo pelo mesmo argumento α^j tem dimensão Par! ■

2. EXERCÍCIO 2

Para $n = 1$,

$$\begin{aligned}
 H |\phi; t\rangle &= \int d^3\mathbf{x} a^\dagger(\mathbf{x}) \left[-\frac{\hbar^2}{2m} \nabla_x^2 + U(\mathbf{x}) \right] a(\mathbf{x}) \int d^3\mathbf{x}_1 \phi(\mathbf{x}_1; t) a^\dagger(\mathbf{x}_1) |0\rangle \\
 &+ \frac{1}{2} \int d^3\mathbf{x} d^3\mathbf{y} V(\mathbf{x} - \mathbf{y}) a^\dagger(\mathbf{x}) a^\dagger(\mathbf{y}) a(\mathbf{y}) a(\mathbf{x}) \int d^3\mathbf{x}_1 \phi(\mathbf{x}_1; t) a^\dagger(\mathbf{x}_1) |0\rangle \\
 &= \int d^3\mathbf{x} d^3\mathbf{x}_1 a^\dagger(\mathbf{x}) \left[-\frac{\hbar^2}{2m} \nabla_x^2 + U(\mathbf{x}) \right] \phi(\mathbf{x}; t) \left(a^\dagger(\mathbf{x}_1) a(\mathbf{x}) + \delta^{(3)}(\mathbf{x} - \mathbf{x}_1) \right) |0\rangle \\
 &+ \frac{1}{2} \int d^3\mathbf{x} d^3\mathbf{y} V(\mathbf{x} - \mathbf{y}) \phi(\mathbf{x}_1; t) a^\dagger(\mathbf{x}) a^\dagger(\mathbf{y}) a(\mathbf{y}) \left(a^\dagger(\mathbf{x}_1) a(\mathbf{x}) + \delta^{(3)}(\mathbf{x} - \mathbf{x}_1) \right) |0\rangle \\
 &= \int d^3\mathbf{x} a^\dagger(\mathbf{x}) \left[-\frac{\hbar^2}{2m} \nabla_x^2 + U(\mathbf{x}) \right] \phi(\mathbf{x}; t) |0\rangle \\
 &= \int d^3\mathbf{x} a^\dagger(\mathbf{x}) i\hbar \frac{\partial}{\partial t} \phi(\mathbf{x}; t) |0\rangle = i\hbar \frac{\partial}{\partial t} |\phi; t\rangle
 \end{aligned}$$

Analogamente,

$$\begin{aligned}
 i\hbar \frac{\partial}{\partial t} \int d^3\mathbf{x}_1 \phi(\mathbf{x}_1; t) a^\dagger(\mathbf{x}_1) |0\rangle &= \int d^3\mathbf{x} a^\dagger(\mathbf{x}) \left[-\frac{\hbar^2}{2m} \nabla_x^2 + U(\mathbf{x}) \right] a(\mathbf{x}) \int d^3\mathbf{x}_1 \phi(\mathbf{x}_1; t) a^\dagger(\mathbf{x}_1) |0\rangle \\
 &= \int d^3\mathbf{x} d^3\mathbf{x}_1 a^\dagger(\mathbf{x}) \left[-\frac{\hbar^2}{2m} \nabla_x^2 + U(\mathbf{x}) \right] \phi(\mathbf{x}_1; t) \left(a^\dagger(\mathbf{x}_1) a(\mathbf{x}) + \delta^{(3)}(\mathbf{x}_1 - \mathbf{x}) \right) |0\rangle \\
 &= \int d^3\mathbf{x} a^\dagger(\mathbf{x}) \left[-\frac{\hbar^2}{2m} \nabla_x^2 + U(\mathbf{x}) \right] \phi(\mathbf{x}; t) |0\rangle \\
 \Rightarrow i\hbar \frac{\partial}{\partial t} \phi(\mathbf{x}; t) &= \left(-\frac{\hbar^2}{2m} + U(\mathbf{x}) \right) \phi(\mathbf{x}; t)
 \end{aligned}$$

Por indução está provado. ■

3. EXERCÍCIO 3

$$\begin{aligned}
[N, H] &= \left[\int d^3\mathbf{x} a^\dagger(\mathbf{x})a(\mathbf{x}), \int d^3\mathbf{y} a^\dagger(\mathbf{y}) \left[-\frac{\hbar^2}{2m} \nabla_y^2 + U(y) \right] a(\mathbf{y}) \right] \\
&= \int d^3\mathbf{x} d^3\mathbf{y} \left\{ a^\dagger(\mathbf{x})a(\mathbf{x})a^\dagger(\mathbf{y}) \left[-\frac{\hbar^2}{2m} \nabla_y^2 + U(y) \right] a(\mathbf{y}) - a^\dagger(\mathbf{y}) \left[-\frac{\hbar^2}{2m} \nabla_y^2 + U(y) \right] a(\mathbf{y})a^\dagger(\mathbf{x})a(\mathbf{x}) \right\} \\
&= \int d^3\mathbf{x} d^3\mathbf{y} \left\{ a^\dagger(\mathbf{x})a(\mathbf{x})a^\dagger(\mathbf{y}) \left[-\frac{\hbar^2}{2m} \nabla_y^2 + U(y) \right] a(\mathbf{y}) - a^\dagger(\mathbf{y}) \left[-\frac{\hbar^2}{2m} \nabla_y^2 + U(y) \right] \left(a^\dagger(\mathbf{x})a(\mathbf{y}) + \delta^{(3)}(\mathbf{y} - \mathbf{x}) \right) a(\mathbf{x}) \right\} \\
&= \int d^3\mathbf{x} d^3\mathbf{y} a^\dagger(\mathbf{x}) (a(\mathbf{x})a^\dagger(\mathbf{y}) - a^\dagger(\mathbf{y})a(\mathbf{x})) \left[-\frac{\hbar^2}{2m} \nabla_y^2 + U(y) \right] a(\mathbf{y}) - \int d^3\mathbf{y} a^\dagger(\mathbf{y}) \left[-\frac{\hbar^2}{2m} \nabla_y^2 + U(\mathbf{y}) \right] a(\mathbf{y}) \\
&= \int d^3\mathbf{x} d^3\mathbf{y} a^\dagger(\mathbf{x}) \delta^{(3)}(\mathbf{x} - \mathbf{y}) \left[-\frac{\hbar^2}{2m} \nabla_y^2 + U(y) \right] a(\mathbf{y}) - \int d^3\mathbf{y} a^\dagger(\mathbf{y}) \left[-\frac{\hbar^2}{2m} \nabla_y^2 + U(\mathbf{y}) \right] a(\mathbf{y}) \\
&= 0
\end{aligned}$$

■