(10.1)

$$\mathcal{H} = \frac{\pi^2}{2} + \frac{m^2\phi^2}{2} + \frac{(\vec{\nabla}\phi)^2}{2} - g\frac{\phi^3}{3!}$$

$$\mathcal{H}_1 = -g\frac{\phi^3}{3!} \quad ; \quad \mathcal{H}_I = -\frac{g}{3!} e^{iH_0 t} \phi^3 e^{-iH_0 t}$$

$$\langle 0|T\{\phi(x_4)\phi(x_3)\phi(x_2)\phi(x_1)\}|0\rangle$$

$$= \frac{\langle \emptyset|T\{\phi_I(x_4)\phi_I(x_3)\phi_I(x_2)\phi_I(x_1)\}\left[1 + \frac{ig}{3!}\int d^4x\, \phi_I^3(x) + \mathcal{O}(g^2)\right]|\emptyset\rangle}{\langle \emptyset|T\{1 + \frac{ig}{3!}\int d^4x\, \phi_I^3(x) + \mathcal{O}(g^2)\}|\emptyset\rangle}$$

$$= \left[\langle \emptyset|T\{\phi_I(x_4)\phi_I(x_3)\phi_I(x_2)\phi_I(x_1)\}|\emptyset\rangle + \frac{ig}{3!}\langle \emptyset|T\{\phi_I(x_4)\phi_I(x_3)\phi_I(x_2)\phi_I(x_1)\int d^4x\,\phi_I^3(x)\}|\emptyset\rangle\right.$$

$$\times\left[1 - \frac{ig}{3!}\int d^4x\, \cancelto{0}{\langle \emptyset|\phi_I^3(x)|\emptyset\rangle}\right]$$

parte desconexa

$$= \langle \emptyset|T\{\cdots\}|\emptyset\rangle + \frac{ig}{3!}\int d^4x \cancelto{0}{\langle \emptyset|T\{\phi_{4321}\,\phi_I^3(x)\}|\emptyset\rangle} + \left(\frac{ig}{3!}\right)^2 \int d^4x\, d^4y \langle \emptyset|T\{\phi_{4321}\,\phi_I^3(x)\phi_I^3(y)\}|\emptyset\rangle$$

$$= [\cdots] + \left(\frac{ig}{3!}\right)^2 \int d^4x\, d^4y\, \frac{3!^2}{i^5}\Delta(x-y)\Big[\Delta(x_1-y)\Delta(x_2-y)\Delta(x_3-x)\Delta(x_4-x)$$
$$+ \Delta(x_1-y)\Delta(x_3-y)\Delta(x_2-x)\Delta(x_4-x)$$
$$+ \Delta(x_1-y)\Delta(x_4-y)\Delta(x_3-x)\Delta(x_2-x)\Big]$$

$$= [\cdots] + \frac{(ig)^2}{i^5}\int d^4x\, d^4y\, \Delta(x-y)\Big[\Delta(x_1-y)\Delta(x_2-y)\Delta(x_3-x)\Delta(x_4-x)$$
$$+ \Delta(x_1-y)\Delta(x_3-y)\Delta(x_2-x)\Delta(x_4-x)$$
$$+ \Delta(x_1-y)\Delta(x_4-y)\Delta(x_3-x)\Delta(x_2-x)\Big]$$

$$+ \mathcal{O}(g^4).$$

(10.2) A lagrangiana:

$$\mathcal{L} = -Z_\phi \partial^\mu \phi^\dagger \partial_\mu \phi - Z_m m^2 \phi^\dagger \phi - \frac{Z_\lambda}{4} \lambda (\phi^\dagger \phi)^2$$

as regras são:

(1) Desenhe linhas para cada partícula que entra e sai; se a fonte é J a ~~linha~~ possui uma seta ▶ apontando para fora da fonte e para a fonte se $J^\dagger$.

(2) Deixar sempre um ~~dos~~ extremos das linhas externas livre e ligar outro com um vértice com 4 linhas ligadas. cada vértice deve possuir 2 partículas J e 2 partículas $J^\dagger$. Desenhar todos os diagramas inequivalentes ~~[illegible]~~ respeitando os vértices.

(3) Para cada partícula que entra desenha uma seta ▷ apontando ~~[illegible]~~ para o vértice e para cada linha saindo uma seta apontando para fora do vértice ▶. Para cada linha interna a seta ▷ tem direção arbitrária.

(4) Amarrar os quadrimomentos.

(5) Os vértices devem conservar os momentos.

(6) Os valores dos diagramas são dados por:

(i) para cada linha externa; $1$

(ii) para cada linha de momento; $\frac{-i}{k^2+m^2-i\epsilon}$

(iii) para cada vértice; $-iZ_\lambda \lambda$

(7) Integrar sobre loops fechados

(8) Dividir pelos fatores de simetrias de loops internos

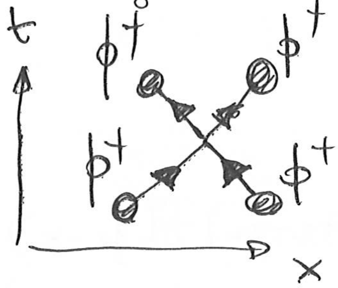
(9) Adicionar vértices de contratermos com valor:

$$-i\left[(Z_\phi-1)k^2+(Z_m-1)m^2\right]$$

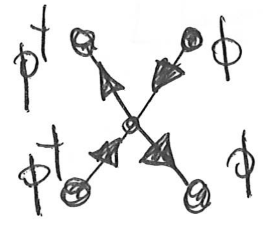
(10) Valores de diagramas somados dão $iT$.

---

outro método para utilizar apenas uma linha é introduzir o ordenamento de espaço e tempo.




se a reta de momento aponta na direção positiva de $t$ é $\phi^\dagger$ se não é $\phi$.

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(10.3) A teoria é $\mathcal{L} = -\partial_\mu\phi^\dagger\partial^\mu\phi - m_\phi\phi^\dagger\phi + g\chi\phi^\dagger\phi Z_g$

$$-\frac{\partial_\mu\chi\partial^\mu\chi}{2} - \frac{m_\chi\chi^2}{2} + \mathcal{L}_{ct}$$

$$Z_0(J^\dagger,J,K) = \int \mathcal{D}\phi^\dagger \mathcal{D}\phi\, \mathcal{D}\chi \, \exp\left[ i\int d^4x \left\{ \mathcal{L}_0 + J^\dagger\phi + \phi^\dagger J + K\chi \right\} \right]$$

$$= \exp\left[ i\int d^4x\, d^4y\, J^\dagger(x)\,\Delta_\phi(x-y)\,J(y) + \frac{i}{2}\int d^4x\, d^4y\, K(x)\,\Delta_\chi(x-y)\,K(y)\right]$$

$$Z_1 \propto \exp\left[ Z_g\, g\, i\int d^4x \left(\frac{1}{i^3}\frac{\delta}{\delta K(x)}\frac{\delta}{\delta J(x)}\frac{\delta}{\delta J^\dagger(x)}\right)\right] Z_0(J^\dagger,J,K)$$

$$Z_1 \propto \sum_{V=0}^{\infty} \frac{1}{V!}\sum_{P=0}^{\infty} \frac{1}{P!}\frac{(igZ_g)^V}{i^{3V}}\left[\int d^4x\, \frac{\delta}{\delta K(x)}\frac{\delta}{\delta J(x)}\frac{\delta}{\delta J^\dagger(x)}\right]^V i^P \left[\int d^4z\, d^4y \left\{ J^\dagger(z)\,\Delta_\phi(z-y)\,J(y) + \frac{K(z)\,\Delta_\chi(z-y)\,K(y)}{2}\right\}\right]^P$$

$$Z_1 \propto \overset{V=P=0}{1} + \overset{V=0\ \ P=1}{i\int d^4z\, d^4y \left\{ J^\dagger(z)\,\Delta_\phi(z-y)\,J(y) + \frac{K(z)\,\Delta_\chi(z-y)\,K(y)}{2}\right\}}$$

$$- \int \frac{d^4z_1\, d^4y_1\, d^4z_2\, d^4y_2}{2} \Big\{ J^\dagger(z_1)\,\Delta_\phi(z_1-y_1)\,J(y_1)\,J^\dagger(z_2)\,\Delta_\phi(z_2-y_2)\,J(y_2) \qquad V=0\ \ P=2$$

$$+ \frac{1}{4} K(z_1)\,\Delta_\chi(z_1-y_1)\,K(y_1)\,K(z_2)\,\Delta_\chi(z_2-y_2)\,K(y_2)$$

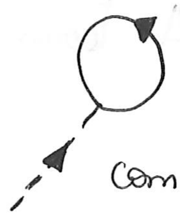
$$+ J^\dagger(z_1)\,\Delta_\phi(z_1-y_1)\,J(y_1)\,K(z_2)\,\Delta_\chi(z_2-y_2)\,K(y_2) \Big\}$$

$$+ \frac{gZ_g}{2} \int^{d^4z_1\, d^4z_2\, d^4y_1\, d^4y_2} d^4x\, \delta(z_1-x)\,\Delta_\phi(z_1-y_1)\,\delta(y_1-x)\Big[K(z_2)\,\Delta_\chi(z_2-y_2)\,\delta(y_2-x) + \delta(z_2-x)\,\Delta_\chi(z_2-y_2)\,K(y_2)\Big]$$

$\cancel{+gZ_g}$

contribuição de ordem $g$ é:

$$gZ_g \int d^4x\, d^4y\, \Delta_\phi(x-x)\,\Delta_\chi(x-y)\,K(y)$$

que corresponde ao diagrama:  com fator $igZ_g$

a próximo contribuição em $g$ é com $P=3$

$V=1$ e $P=3$:

$$\frac{1}{3!}\frac{ig Z_g}{\cancel{\hbar^3}}\cdot \cancel{\hbar^3}\int d^4x\left(\frac{\delta}{\delta K(x)}\frac{\delta}{\delta J(x)}\frac{\delta}{\delta J^\dagger(x)}\right)\left[\int d^4z_1 d^4z_2 d^4z_3\, d^4y_1 d^4y_2 d^4y_3\right]\Bigg\{$$

$$3J^\dagger(z_1)\Delta_\phi(z_1-y_1)J(y_1)J^\dagger(z_2)\Delta_\phi(z_2-y_2)J(y_2)\frac{K(z_3)}{2}\Delta_\chi(z_3-y_3)K(y_3)$$

$$+3J^\dagger(z_1)\Delta_\phi(z_1-y_1)J(y_1)K(z_2)\Delta_\chi(z_2-y_2)\frac{K(y_2)}{4}K(z_3)\Delta_\chi(z_3-y_3)K(y_3)\Bigg\}$$

$$=\frac{igZ_g}{2}\left[\int d^4x\, d^4z_1 d^4z_2 d^4z_3\, d^4y_1 d^4y_2 d^4y_3\right]\Big\{2\,\delta(z_1-x)\Delta_\phi(z_1-y_1)\delta(y_1-x)J^\dagger(z_2)\Delta_\phi(z_2-y_2)J(y_2)$$

$$\frac{K(z_3)}{2}\Delta_\chi(z_3-y_3)\delta(y_3-x)$$

$$+2\,\delta(z_1-x)\Delta_\phi(z_1-y_1)J(y_1)J^\dagger(z_2)\Delta_\phi(z_2-y_2)\delta(y_2-x)$$

$$\frac{K(z_3)\Delta_\chi(z_3-y_3)\delta(y_3-x)}{2}$$

$$+\frac{2}{2}J^\dagger(z_1)\Delta_\phi(z_1-y_1)J(y_1)\delta(z_2-x)\Delta_\phi(z_2-y_2)\delta(y_2-x)$$

$$\times K(z_3)\Delta_\chi(z_3-y_3)\delta(y_3-x)$$

$$+\frac{2}{2}J^\dagger(z_1)\Delta_\phi(z_1-y_1)\delta(y_1-x)\delta(z_2-x)\Delta_\phi(z_2-y_2)J(y_2)$$

$$K(z_3)\Delta_\chi(z_3-y_3)\delta(y_3-x)$$

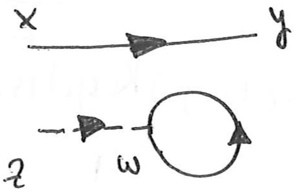
$$+\delta(z_1-x)\Delta_\phi(z_1-y_1)\delta(y_1-x)K(z_2)\Delta_\chi(z_2-y_2)K(y_2)K(z_3)\Delta_\chi(z_3-y_3)\delta(y_3-x)\Big\}$$

$$= i g Z_g \Big[ \int d^4x\, d^4y\, d^4z\, d^4w\; \Delta_\phi(0)\, J^\dagger(x)\, \Delta_\phi(x-y)\, J(y)\, K(z)\, \Delta_\chi(z-w)$$

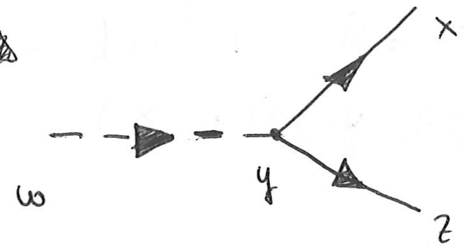
$$+ \int d^4x\, d^4y\, d^4z\, d^4w\; J^\dagger(x)\, \Delta_\phi(x-y)\, \Delta_\phi(y-z)\, J(z)\, K(w)\, \Delta_\chi(w-y)$$

$$+ \frac{\Delta_\phi(0)}{2} \int d^4x\, d^4y\, d^4z\, d^4w\; K(x)\, \Delta_\chi(x-y)\, K(y)\, K(z)\, \Delta_\chi(z-w) \Big]$$

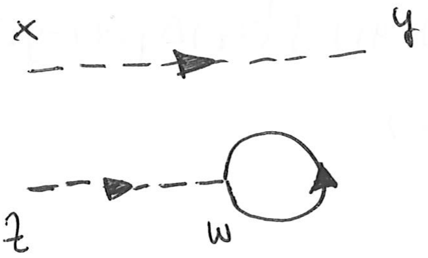
diagrama:




fator $igZ_g$




fator $igZ_g$




fator $igZ_g$ com fator de simetria 2.

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(10.4) $$\mathcal{L} = -\frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2} + \frac{gZ_g}{2}\phi\,\partial_\mu\phi\,\partial^\mu\phi + \mathcal{L}_{ct}$$

$$Z_0(J) = \exp\left[\frac{i}{2}\int d^4x\, d^4y\; J(x)\,\Delta(x-y)\,J(y)\right]$$

$$Z_1 \propto \exp\left[\frac{Z_g g i}{2}\int d^4x \frac{1}{i^3}\frac{\delta}{\delta J(x)}\,\partial_\mu\left[\frac{\delta}{\delta J(x)}\right]\partial^\mu\left[\frac{\delta}{\delta J(x)}\right]\right] Z_0(J)$$

$$Z_1 \propto \sum_{V=0}^{\infty}\sum_{P=0}^{\infty} \frac{(Z_g g)^V i^V}{i^{3V}\, 2^V} \frac{i^P}{2^P V!\, P!} \left\{ \int d^4x \left( \frac{\delta}{\delta J(x)} \partial_\mu \left[\frac{\delta}{\delta J(x)}\right] \partial^\mu \frac{\delta}{\delta J(x)} \right) \right\}^V \left[ \int d^4z_1 d^4y_1\, J(z_1)\Delta(z_1-y_1)J(y_1)\right]^P$$

$V=P=0$ $\quad$ $V=0\ \ P=1$

$$Z_1 \propto 1 + \frac{i}{2}\int d^4z_1 d^4y_1\, J(z_1)\Delta(z_1-y_1)J(y_1)$$

$V=0\ \ P=2$

$$-\frac{1}{2\cdot 4}\int d^4z_1 d^4z_2 d^4y_1 d^4y_2\, J(z_1)\Delta(z_1-y_1)J(y_1)J(z_2)\Delta(z_2-y_2)J(y_2)$$

$$+\frac{g Z_g}{2!\,6}\int d^4x\, d^4z_1 d^4z_2 d^4y_1 d^4y_2 \left\{ 4\frac{\delta}{\delta J(x)}\partial_\mu\frac{\delta}{\delta J(x)}\partial^\mu\left[\delta(x-z_1)\Delta(z_1-y_1)J(y_1)\, J(z_2)\Delta(z_2-y_2)J(y_2)\right]\right\}$$

$$\frac{gZ_g}{2!\,2}\int d^4x\, d^4z_2 d^4y_1 d^4y_2 \left\{\frac{\delta}{\delta J(x)}\partial_\mu\frac{\delta}{\delta J(x)}\left[\partial^\mu\Delta(x-y_1)J(y_1)J(z_2)\Delta(z_2-y_2)J(y_2)\right]\right\}$$

$$\frac{gZ_g}{2!\,2}\int d^4x\, d^4z_2 d^4y_1 d^4y_2 \left\{\frac{\delta}{\delta J(x)}\partial_\mu\left[\partial^\mu\Delta(x-y_1)\delta(y_1-x)J(z_2)\Delta(z_2-y_2)J(y_2) + 2\,\partial^\mu\Delta(x-y_1)J(y_1)\delta(z_2-x)\Delta(z_2-y_2)J(y_2)\right]\right\}$$

$$\frac{gZ_g}{2!\,2}\int d^4x\, d^4y_1 d^4y_2 \left\{\frac{\delta}{\delta J(x)}\left[+2\,\partial_\mu\partial^\mu\Delta(x-y_1)J(y_1)\Delta(x-y_2)J(y_2) + 2\,\partial^\mu\Delta(x-y_1)J(y_1)\partial_\mu\Delta(x-y_2)J(y_2)\right]\right\}$$

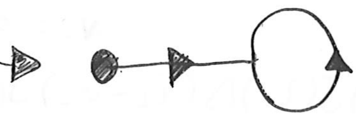
$$\frac{gZ_g}{2!}\int d^4x\, d^4y_1 d^4y_2 \left\{ +\partial_\mu\partial^\mu\Delta(x-y_1)J(y_1)\Delta(x-y_2)\delta(y_2-x) \right\}$$

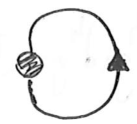
$$\frac{gZ_g}{2!}\int d^4x\, d^4y\, \underbrace{\partial_\mu\partial^\mu\Delta(x-y)}_{=\, m^2\Delta(x-y)-\delta(x-y)} J(y)\Delta(0)$$

$$\frac{gZ_g}{2!}\int d^4x\, d^4y\, m^2\Delta(x-y)J(y)\Delta(0) - \frac{gZ_g}{3!}\int d^4x\, d^4y\, \delta(x-y)J(y)\Delta(0)$$

contribuição em $g$ com 2 propagadores:

$$= \frac{g Z_g m^2}{2!} \Delta(0) \int d^4x\, d^4y\, \Delta(x-y)\, J(y) - \frac{g Z_g}{2!} \int d^4x\, J(x)\, \Delta(0)$$





com fator $\frac{i g Z_g m^2}{2}$ com fator $-\frac{i g Z_g}{2}$

para 3 propagadores: $V = 1$ $P = 3$

$$-\frac{g Z_g}{2}\, \frac{i^3}{8} \cdot \frac{1}{3!} \int d^4x\, d^4z_1\, d^4z_2\, d^4z_3\, d^4y_1\, d^4y_2\, d^4y_3 \left[\cancel{J(z_1)\Delta(z_1-y_1)J(y_1)\, J(z_2)\Delta(z_2-y_2)J(y_2)} \right.$$

$$\left. \cancel{\times J(z_3)\Delta(z_3-y_3)J(y_3)} \right]$$

$$\left[\frac{\delta}{\delta J(x)}\, \partial_\mu \frac{\delta}{\delta J(x)}\, \partial^\mu \frac{\delta}{\delta J(x)}\right] \left[J(z_1)\Delta(z_1-y_1)J(y_1)\, J(z_2)\Delta(z_2-y_2)J(y_2)\, J(z_3)\Delta(z_3-y_3)J(y_3)\right]$$

$$= \frac{i g Z_g}{16 \cdot \cancel{3!}} \int d^4x\, d^4z_2\, d^4z_3\, d^4y_1\, d^4y_2\, d^4y_3 \left[\frac{\delta}{\delta J(x)}\, \partial_\mu \frac{\delta}{\delta J(x)}\right] \cancel{3!} \left[\partial^\mu \Delta(x-y_1)\, J(y_1)\, J(z_2)\, \Delta(z_2-y_2)\, J(y_2)\, J(z_3)\, \Delta(z_3-y_3)\, J(y_3)\right]$$

$$= \frac{i g Z_g}{4} \int d^4x\, d^4z_3\, d^4y_1\, d^4y_2\, d^4y_3 \left[\frac{\delta}{\delta J(x)}\right] \left[\partial^\mu \Delta(x-y_1)\, J(y_1)\, \partial^\mu \Delta(x-y_2)\, J(y_2)\, J(z_3)\, \Delta(z_3-y_3)\, J(y_3) + \partial_\mu \partial^\mu \Delta(x-y_1)\, J(y_1)\, \Delta(x-y_2)\, J(y_2)\, J(z_3)\, \Delta(z_3-y_3)\, J(y_3)\right]$$

$$= \frac{i g Z_g}{2} \int d^4x\, d^4y\, d^4z\, d^4w$$

$$\left[\partial_\mu \partial^\mu \Delta(x-y)\, J(y)\, \Delta(0)\, J(z)\, \Delta(z-w)\, J(w)\right.$$

3)

$$+ 2\, \partial_\mu \partial^\mu \Delta(x-y)\, J(y)\, \Delta(x-z)\, J(z)\, \Delta(x-w)\, J(w)$$

$$+ 2\, \partial_\mu \Delta(x-y)\, J(y)\, \partial^\mu \Delta(x-z)\, J(z)\, \Delta(x-w)\, J(w) \Big]$$

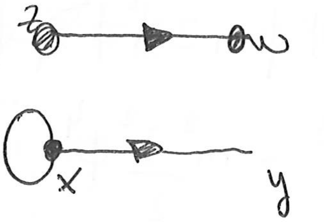
$$i g Z_g \int d^4x\, d^4y\, d^4z\, d^4w \Big[ \frac{m^2}{2} \Delta(x-y)\, J(y)\, \Delta(0)\, J(z)\, \Delta(z-w)\, J(w)$$

$$+ m^2\, \Delta(x-y)\, J(y)\, \Delta(x-z)\, J(z)\, \Delta(x-w)\, J(w)$$

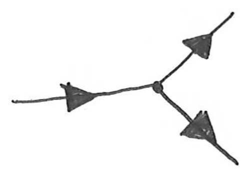
$$+ \partial_\mu \Delta(x-y)\, J(y)\, \partial^\mu \Delta(x-z)\, J(z)\, \Delta(x-w)\, J(w) \Big]$$

$$- i g Z_g \int d^4x\, d^4z\, d^4w \Big[ \frac{J(x)}{2} \Delta(0)\, J(z)\, \Delta(z-w)\, J(w)$$

$$+ J(x)\, \Delta(x-z)\, J(z)\, \Delta(x-w)\, J(w) \Big]$$

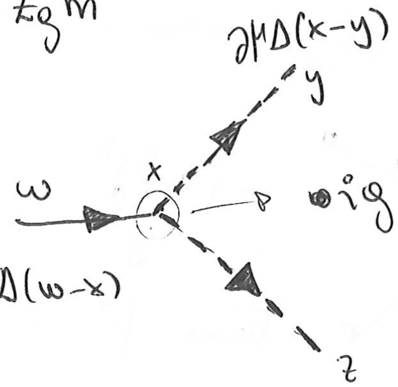



$i g Z_g m^2$ fator de simetria 2.

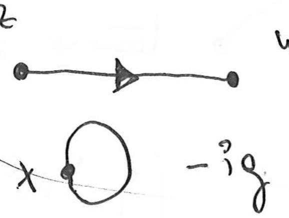


$i g Z_g m^2$

outro propagador:

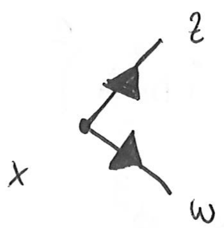



$\bullet\, i g Z_g$




$-ig$

simetria 2 $\quad -igZ_g$




$-igZ_g$

10.5

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{m^2\phi^2}{2}$$

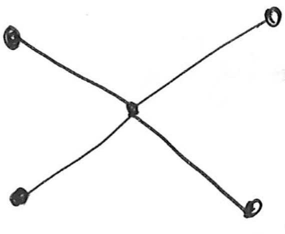
$$\mathcal{L}' = -\frac{1}{2}\partial_\mu(\phi+\lambda\phi^2)\,\partial^\mu(\phi+\phi^2\lambda) - \frac{m^2}{2}(\phi+\lambda\phi^2)^2$$

$$= -\frac{1}{2}\left[\partial_\mu\phi + \lambda 2\phi\partial_\mu\phi\right]\left[\partial^\mu\phi + \lambda 2\phi\,\partial^\mu\phi\right] - \frac{m^2}{2}(\phi^2+2\lambda\phi^3+\lambda^2\phi^4)$$

$$= -\frac{1}{2}\Big[\partial_\mu\phi\,\partial^\mu\phi + 2\lambda\phi\,\partial_\mu\phi\,\partial^\mu\phi + 2\lambda\phi\,\partial_\mu\phi\,\partial^\mu\phi + 4\lambda^2\phi^2\partial_\mu\phi\,\partial^\mu\phi\Big] - \frac{m^2\phi^2}{2} - m^2\lambda\phi^3 - \frac{m^2\lambda^2}{2}\phi^4$$

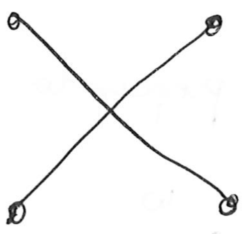
$$= -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{m^2\phi^2}{2} - \Big[2\lambda\phi\,\partial_\mu\phi\partial^\mu\phi + 2\lambda^2\phi^2\partial_\mu\phi\,\partial^\mu\phi + m^2\lambda\phi^3 + \frac{m^2\lambda^2}{2}\phi^4\Big]$$

a nível de árvore:



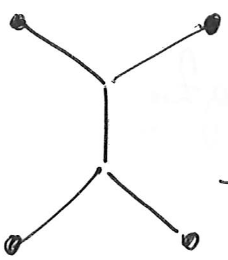
$-i\,\dfrac{m^2\lambda^2}{2}$

termo $\phi^4$



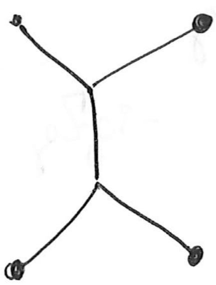
$i\,\dfrac{\lambda^2 m^2 2}{4}$

termo $\phi^2\partial_\mu\phi\,\partial^\mu\phi$



$(-i m^2\lambda)^2$

termo $\phi^3$



$\left(i\,\dfrac{\lambda m^2 2}{2}\right)^2$

termo $\phi\,\partial_\mu\phi\,\partial^\mu\phi$

logo é nulo