(B.1) $$\Delta(x-x') = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-x')}}{k^2+m^2-i\epsilon}$$

$$\partial_{\mu_x} \Delta(x-x') = \int \frac{d^4k}{(2\pi)^4} \frac{ik_\mu e^{ik(x-x')}}{k^2+m^2-i\epsilon}$$

$$\partial_x^2 \Delta(x-x') = -\int \frac{d^4k}{(2\pi)^4} \frac{k^2 e^{ik(x-x')}}{m^2+k^2-i\epsilon}$$

$$(-\partial_x^2+m^2)\,\Delta(x-x') = \int \frac{d^4k}{(2\pi)^4} \frac{(m^2+k^2)\, e^{ik(x-x')}}{m^2+k^2}$$

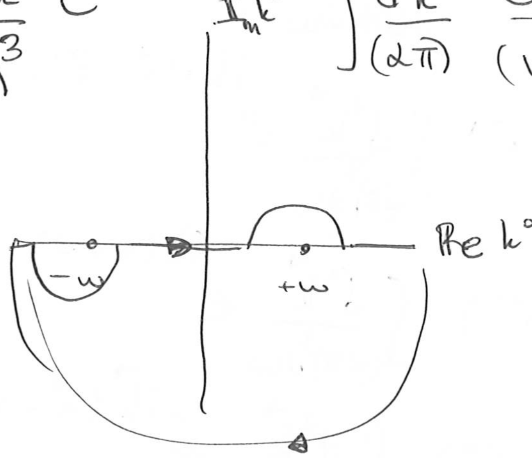
$$= \delta^4(x-x')$$

(B.2) $$\Delta(x-x') = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik^0(x^0-x'^0)}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}}{m^2+\vec{k}^2-i\epsilon-k^{0\,2}}$$

$$\omega^2 = \vec{k}^2+m^2 \ ;$$

$$\Delta(x-x') = \int \frac{d^3\vec{k}}{(2\pi)^3}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \int \frac{dk^0}{(2\pi)} \frac{e^{-ik^0(x^0-x'^0)}}{(\omega-i\epsilon)^2-k^{0\,2}}$$

se $x^0-x'^0>0$.



$$-\oint \frac{dk^0}{2\pi}\frac{e^{-ik^0(x^0-x^{0\prime})}}{(\omega-i\epsilon)^2-k^{0\,2}} = \operatorname{Res}\left[\frac{1}{2\pi}\frac{e^{-ik^0(x^0-x^{0\prime})}}{(\omega-i\epsilon)^2-k^{0\,2}}\right]_{k^0=\omega-i\epsilon}$$

$$= -\frac{i}{2\omega}e^{-i\omega(x^0-x^{0\prime})}$$

$$= -\int_{-\infty}^{+\infty}\frac{dk^0}{2\pi}\frac{e^{-ik^0(x^0-x^{0\prime})}}{(\omega-i\epsilon)^2-k^{0\,2}} \underbrace{-i\int_0^{-\pi}\frac{d\theta}{2\pi}Re^{i\theta}\frac{e^{-ie^{i\theta}R(x^0-x^{0\prime})}}{(\omega-i\epsilon)^2-e^{2i\theta}R^2}}_{R\to\infty \;\Rightarrow\; 0}$$

$$= -\int_{-\infty}^{+\infty}\frac{dk^0}{2\pi}\frac{e^{-ik^0(x^0-x^{0\prime})}}{(\omega-i\epsilon)^2-k^{0\,2}}$$

mesmo resultado para $x^0-x^{0\prime}<0$.

logo: $$\int\frac{dk^0}{2\pi}\frac{e^{-ik^0(x^0-x^{0\prime})}}{(\omega-i\epsilon)^2-k^{0\,2}} = \frac{i}{2\omega}e^{-i\omega|(x^0-x^{0\prime})|}$$

logo $$\Delta(x-x') = i\int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}e^{i\vec{k}\cdot(\vec{x}-\vec{x}')-i\omega|x^0-x^{0\prime}|}$$

$$= i\,\theta(x^0-x^{0\prime})\int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}e^{i\vec{k}\cdot(\vec{x}-\vec{x}')-i\omega(x^0-x^{0\prime})}$$

$$+ i\,\theta(x^{0\prime}-x^0)\int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}e^{i\vec{k}\cdot(\vec{x}-\vec{x}')+i\omega(x^0-x^{0\prime})}$$

$$= i\,\theta(x^0-x^{0\prime})\int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}e^{ik(x-x')}$$

$$+ i\,\theta(x^{0\prime}-x^0)\int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}e^{-i\vec{k}(\vec{x}-\vec{x}')+i\omega(x^0-x^{0\prime})}$$

$$\Delta(x-x') = i\theta(x^0-x^{0\prime})\int \frac{d^3\vec{k}}{(2\pi)^3 2\omega} e^{ik(x-x')}$$

$$+ i\theta(x^{0\prime}-x^0)\int \frac{d^3\vec{k}}{(2\pi)^3 2\omega} e^{-ik(x-x')}$$

(6.3) Note que: $\partial_x^2 e^{ik(x-x')} = \partial_x^2 e^{-ik(x-x')} = \omega^2 - \vec{k}^2 = m^2$

logo $(-\partial_x^2+m^2)e^{ik(x-x')} = (-\partial_x^2+m^2)e^{-ik(x-x')} = 0$

basta avaliar: $-i\int \frac{d^3\vec{k}}{(2\pi)^3 2\omega} e^{ik(x-x')} \partial_x^2\theta(x^0-x^{0\prime})$

$$-i\int \frac{d^3\vec{k}\, e^{-ik(x-x')}}{(2\pi)^3 2\omega} \partial_x^2\theta(x^{0\prime}-x^0)$$

$$= i\int \frac{d^3\vec{k}}{(2\pi)^3 2\omega} e^{ik(x-x')} \partial_{x^0}^2\theta(x^0-x^{0\prime}) + i\int \frac{d^3\vec{k}}{(2\pi)^3 2\omega} e^{-ik(x-x')} \partial_{x^0}^2\theta(x^{0\prime}-x^0)$$

$$= i\int \frac{d^3\vec{k}\, e^{ik(x-x')}}{(2\pi)^3 2\omega} \delta'(x^0-x^{0\prime}) + i\int \frac{d^3\vec{k}}{(2\pi)^3 2\omega} e^{-ik(x-x')} \delta'(x^{0\prime}-x^0)$$

$$= i\,\partial_{x^0}\left[\int \frac{d^3\vec{k}\, e^{ik(x-x')}}{(2\pi)^3 2\omega} \delta(x^0-x^{0\prime})\right] + i\delta(x^0-x^{0\prime})\int \frac{d^3\vec{k}(-i\omega)e^{ik(x-x')}}{(2\pi)^3 2\omega}$$

$$+ i\,\partial_{x^{0\prime}}\left[\int \frac{d^3\vec{k}\, e^{ik(x'-x)}}{(2\pi)^3 2\omega} \delta(x^{0\prime}-x^0)\right] + i\delta(x^{0\prime}-x^0)\int \frac{d^3\vec{k}(-i\omega)e^{ik(x'-x)}}{(2\pi)^3 2\omega}$$

$$= i\partial_{x^0}\left[\int \frac{d^3\vec{k}\, e^{i\vec{k}(\vec{x}-\vec{x}')}}{(2\pi)^3 2\omega} \delta(x^0-x^{0\prime})\right] + \frac{\delta(x^0-x^{0\prime})}{2}\int \frac{d^3\vec{k}\, e^{i\vec{k}(\vec{x}-\vec{x}')}}{(2\pi)^3}$$

$$+ i\partial_{x^{0\prime}}\left[\int \frac{d^3\vec{k}}{(2\pi)^3 2\omega} e^{-i\vec{k}(\vec{x}-\vec{x}')}\delta(x^{0\prime}-x^0)\right] + \frac{\delta(x^{0\prime}-x^0)}{2}\int\frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}(\vec{x}'-\vec{x})}$$

$$= i\int\frac{d^3\vec{k}}{(2\pi)^3 2\omega} e^{i\vec{k}(\vec{x}-\vec{x}')}\delta'_{x^0}(x^0-x^{0\prime}) + i\int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}e^{i\vec{k}(\vec{x}-\vec{x}')}\delta'_{x^{0\prime}}(x^{0\prime}-x^0)$$

$$+ \frac{\delta(x^{0\prime}-x^0)}{2}\int\frac{d^3\vec{k}}{(2\pi)^3}e^{-i\vec{k}(\vec{x}-\vec{x}')} + \frac{\delta(x^0-x^{0\prime})}{2}\int\frac{d^3\vec{k}}{(2\pi)^3}e^{i\vec{k}(\vec{x}-\vec{x}')}$$

$$= \delta(x^0-x^{0\prime})\,\delta^3(\vec{x}-\vec{x}') = \delta^4(x-x') \quad \blacksquare$$

(B.4) $$\phi(x) = \int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}\left(a(\vec{k})e^{ikx} + a^\dagger(\vec{k})e^{-ikx}\right)$$

$$\langle 0|T\{\phi(x_1)\phi(x_2)\}|0\rangle \quad ; \quad x_1^0 > x_2^0$$

$$= \langle 0|\int\frac{d^3\vec{k}\,d^3\vec{k}'}{(2\pi)^6 4\omega\omega'}\left[a(\vec{k})e^{ikx_1} + a^\dagger(\vec{k})e^{-ikx_1}\right]\left(a(\vec{k}')e^{ik'x_2} + a^\dagger(\vec{k}')e^{-ik'x_2}\right)|0\rangle$$

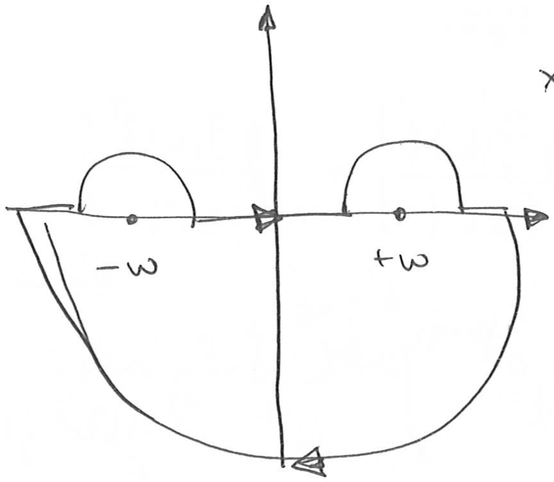
$$= \int\frac{d^3\vec{k}\,d^3\vec{k}'}{(2\pi)^6 4\omega\omega'}\langle 0|\Big(\cancelto{0}{a(\vec{k})a(\vec{k}')}e^{ikx_1+ik'x_2} + a(\vec{k})a^\dagger(\vec{k}')e^{ikx_1-ik'x_2}$$
$$+ \cancelto{0}{a^\dagger(\vec{k})a(\vec{k}')}e^{-ikx_1+ik'x_2} + \cancelto{0}{a^\dagger(\vec{k})a^\dagger(\vec{k}')}e^{-ikx_1-ikx_2}\Big)|0\rangle$$

$$= \int\frac{d^3\vec{k}\,d^3\vec{k}'}{(2\pi)^6 4\omega\omega'}e^{ikx_1-ik'x_2}\delta^3(\vec{k}-\vec{k}')(2\pi)^3 2\omega'$$

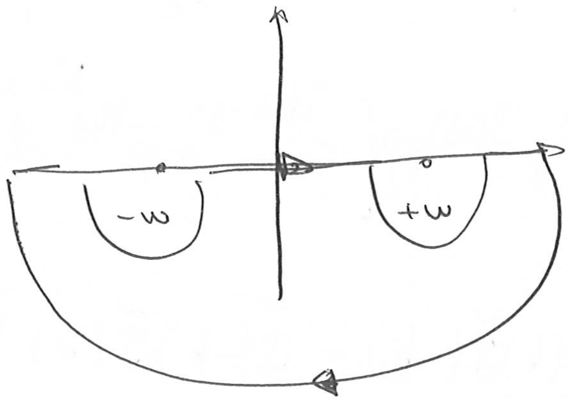
$$= \int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}e^{ik(x_1-x_2)} \equiv \int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}e^{i\vec{k}(\vec{x}_1-\vec{x}_2)}e^{-i\omega|x_1^0-x_2^0|}$$
$$= \frac{1}{i}\Delta(x_1-x_2) \quad \blacksquare$$

(6.5) $$\Delta(x-x') = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-x')}}{k^2+m^2}$$

escolhas possíveis são:



$x^0 - x^{0\prime} > 0$ neste caso $\Delta_{avan}(x^0 - x^{0\prime}) = 0$ para $x^0 - x^{0\prime} > 0$



$x^0 - x^{0\prime} < 0$ neste caso $\Delta_{ret}(x^0 - x^{0\prime}) = 0$ para $x^0 - x^{0\prime} > 0$

motivo é conter ou não polos ▨

(6.6) $$Z_0(J) = \exp[\,i W_0(J)\,]$$

$$= \exp\left[\frac{i}{2}\int \frac{d^4k}{(2\pi)^4} \frac{\tilde{J}(k)\,\tilde{J}(-k)}{k^2+m^2-i\epsilon}\right]$$

$$\frac{1}{z-i\epsilon} = P\frac{1}{z} + i\pi\,\delta(z)$$

$$W_0(J) = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} \frac{\tilde{J}(k)\,\tilde{J}(-k)}{k^2+m^2-i\epsilon} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} \frac{\|\tilde{J}(k)\|^2}{k^2+m^2-i\epsilon}$$

$$W_0(J) = \frac{P1}{k^2+m^2}\left(\frac{1}{2}\frac{1}{(2\pi)^4}\|\tilde{J}(k)\|^2\right) + \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\|\tilde{J}(k)\|^2 i\pi\,\delta(k^2+m^2)$$

$$= \frac{P1}{k^2+m^2}\left(\frac{1}{2}\frac{\|\tilde{J}(k)\|^2}{(2\pi)^4}\right) + \frac{i\pi}{2}\int\frac{d^3\vec{k}}{(2\pi)^3}\int\frac{dk^0}{(2\pi)}\|\tilde{J}(k)\|^2\delta(\omega^2-k^{0\,2})$$

$$= \frac{P1}{k^2+m^2}\left(\frac{1}{2}\frac{\|\tilde{J}(k)\|^2}{(2\pi)^4}\right) + \frac{i\pi}{2}\int\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2\pi}\left[\left.\frac{\|\tilde{J}(k)\|^2}{|2k^0|}\right|_{k^0=\omega} + \left.\frac{\|\tilde{J}(k)\|^2}{|2k^0|}\right|_{k^0=-\omega}\right]$$

$$= \frac{P1}{k^2+m^2}\left(\frac{1}{2}\frac{\|\tilde{J}(k)\|^2}{(2\pi)^4}\right) + \frac{i}{4}\int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}\left[\int d^4x\,d^4y\,e^{ik(x-y)}J(x)J(y)\Big|_{k^0=\omega} + \int d^4x\,d^4y\,J(x)J(y)\,e^{-ik^0(x^0-y^0)}\times e^{i\vec{k}(\vec{x}-\vec{y})}\right]$$

$$= \frac{P1}{k^2+m^2}\left(\frac{1}{2}\frac{\|\tilde{J}(k)\|^2}{(2\pi)^4}\right) + \frac{i}{4}\int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}\int d^4x\,d^4y\,J(x)J(y)\left[e^{ik(x-y)} + e^{-ik(x-y)}\right]_{k^0=-\omega}$$

$$= \frac{P1}{k^2+m^2}\left(\frac{1}{2}\frac{\|\tilde{J}(k)\|^2}{(2\pi)^4}\right) + \frac{i}{4}\int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}\left[\tilde{J}(k)\tilde{J}(-k) + \tilde{J}(-k)\tilde{J}(k)\right]$$

$$= \frac{P1}{k^2+m^2}\left(\frac{1}{2}\frac{\|\tilde{J}(k)\|^2}{(2\pi)^4}\right) + \frac{i}{2}\int\frac{d^3k}{(2\pi)^3 2\omega}\|\tilde{J}(k)\|^2$$

$$\mathrm{Re}\,W_0(J) = \frac{P1}{k^2+m^2}\left(\frac{1}{2}\frac{\|\tilde{J}(k)\|^2}{(2\pi)^4}\right)$$

$$\mathrm{Im}\,W_0(J) = \frac{1}{2}\int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}\|\tilde{J}(k)\|^2 \qquad \blacksquare$$

(B.7) A lagrangiana é:

$$\mathcal{L}_0 = -\partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi$$; que implica na Hamiltoniana:

$$\mathcal{H}_0 = \pi^\dagger \pi + (\vec{\nabla}\phi^\dagger)\cdot(\vec{\nabla}\phi) + m^2 \phi^\dagger \phi .$$

Construindo o gerador funcional:

$$Z_0(J,J^\dagger) = \int \mathcal{D}\phi \mathcal{D}\phi^\dagger \exp\left[i\int d^4x \left\{ \mathcal{L}_0 + J^\dagger \phi + J\phi^\dagger \right\} \right]$$

$$S_0 = \int d^4x \left\{ -\partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi + J^\dagger \phi + J\phi^\dagger \right\}$$

$$\phi(x) = \int \frac{d^4k}{(2\pi)^4} e^{ikx} \tilde{\phi}(k)$$

$$S_0 = \int \frac{d^4x\, d^4k\, d^4p}{(2\pi)^6} \Big\{ -\partial_\mu \left(e^{-ipx} \tilde{\phi}^\dagger(p)\right) \partial^\mu \left(e^{ikx} \tilde{\phi}(k)\right)$$
$$- m^2 e^{-ipx+ikx} \tilde{\phi}^\dagger(p) \phi(k)$$
$$+ \tilde{J}^\dagger(p) \phi(k) e^{-ipx+ikx}$$
$$+ \tilde{J}(k) \phi^\dagger(p) e^{-ipx+ikx} \Big\}$$

$$S_0 = \int \frac{d^4k\, d^4p}{(2\pi)^4} \Big\{ -p_\mu k^\mu \tilde{\phi}^\dagger(p) \phi(k) \delta^4(p-k)$$
$$- m^2 \phi^\dagger(p) \phi(k) \delta^4(p-k)$$

$$+ \tilde{J}^\dagger(p)\,\tilde{\phi}(k)\,\delta^4(p-k) + \tilde{J}(k)\,\tilde{\phi}^\dagger(k)\,\delta^4(p-k) \Big\}$$

$$S_0 = \int \frac{d^4k}{(2\pi)^4}\Big[-\tilde{\phi}^\dagger(k)(k^2+m^2)\tilde{\phi}(k) + \tilde{J}^\dagger(k)\tilde{\phi}(k) + \tilde{\phi}^\dagger(k)\tilde{J}(k)\Big]$$

$$= \int \frac{d^4k}{(2\pi)^4}\Big[-\Big(\tilde{\phi}^\dagger(k) - \frac{\tilde{J}^\dagger(k)}{(k^2+m^2)}\Big)(k^2+m^2)\Big(\tilde{\phi}(k) - \frac{\tilde{J}(k)}{k^2+m^2}\Big) + \frac{\tilde{J}^\dagger(k)\tilde{J}(k)}{k^2+m^2}\Big]$$

$$= \int \frac{d^4k}{(2\pi)^4}\Big[\frac{\tilde{J}^\dagger(k)\tilde{J}(k)}{k^2+m^2} - \tilde{\chi}^\dagger(k)(k^2+m^2)\tilde{\chi}(k)\Big]$$

$$\tilde{\chi}(k) = \tilde{\phi}(k) - \frac{\tilde{J}(k)}{k^2+m^2}$$

$$Z_0(J,J^\dagger) =$$

$$= \int \mathcal{D}\tilde{\chi}\,\mathcal{D}\tilde{\chi}^\dagger \exp\Big[-i\int \frac{d^4k}{(2\pi)^4}\tilde{\chi}^\dagger(k)(k^2+m^2)\tilde{\chi}(k)\Big] \times$$

$$\times \exp\Big[i\int \frac{d^4k}{(2\pi)^4}\Big[\frac{\tilde{J}^\dagger(k)\tilde{J}(k)}{k^2+m^2}\Big]\Big]$$

$$Z_0(J,J^\dagger) = \exp\Big[i\int \frac{d^4k}{(2\pi)^4}\frac{\tilde{J}^\dagger(k)\tilde{J}(k)}{k^2+m^2}\Big]$$

$$= \exp\Big[i\int d^4x\,d^4y\, J^\dagger(x)\,\Delta(x-y)\,J(y)\Big]$$

Agora podemos calcular:

$$\langle 0|T\{\phi(x_1)\phi(x_2)\}|0\rangle = \frac{1}{i}\frac{\delta}{\delta J^\dagger(x_1)}\frac{1}{i}\frac{\delta}{\delta J^\dagger(x_2)} Z_0(J,J^\dagger)\Big|_{J=J^\dagger=0}.$$

$$= \frac{1}{i^2}\frac{\delta}{\delta J^\dagger(x_1)}\left[i\int d^4x\, d^4y\, \delta(x-x_2)\Delta(x-y)J(y)Z_0(J,J^\dagger)\right]\Big|_{J=J^\dagger=0}$$

$$= \frac{1}{i}\int d^4y\, \Delta(x_2-y)J(y)\, i\int d^4x' d^4y'\, \delta(x'-x_1)\Delta(x'-y')J(y')Z_0\Big|_{J=J^\dagger=0}$$

$$= 0$$

Analogamente:

$$\langle 0|T\{\phi^\dagger(x_1)\phi^\dagger(x_2)\}|0\rangle =$$

$$= \frac{1}{i}\int d^4x\, J^\dagger(x)\Delta(x-x_2)\, i\int d^4x'\, J^\dagger(x')\Delta(x'-x_1)\, Z_0\Big|_{J=J^\dagger=0}$$

$$= 0$$

$$\langle 0|T\{\phi^\dagger(x_1)\phi(x_2)\}|0\rangle = \frac{1}{i}\frac{\delta}{\delta J(x_1)}\frac{1}{i}\frac{\delta}{\delta J^\dagger(x_2)} Z_0(J,J^\dagger)\Big|_{J=J^\dagger=0}$$

$$= \frac{1}{i^2}\frac{\delta}{\delta J(x_1)}\left[i\int d^4x\, d^4y\, \delta(x-x_2)\Delta(x-y)J(y)Z_0(J,J^\dagger)\right]\Big|_{J=J^\dagger=0}$$

$$= \frac{1}{i}\left[\Delta(x_2-x_1)Z_0(J,J^\dagger) + i^2\int d^4y\, d^4y'\, \Delta(x_2-y)J(y)J^\dagger(y')\Delta(y'-x_1)Z_0\right]\Big|_{J=J^\dagger=0}$$

$$= \frac{1}{i}\Delta(x_2-x_1)$$

verificando por: $a(\vec{k}) = i\int d^3\vec{x}\, e^{-ikx}\left(\pi^\dagger(\vec{x}) - i\omega\,\phi(x)\right)$

$$b(\vec{k}) = i\int d^3\vec{x}\, e^{-ikx}\left(\pi(x) - i\omega\,\phi^\dagger(x)\right)$$

ou: $\phi(x) = \int \frac{d^3\vec{k}}{(2\pi)^3 2\omega}\left(a(\vec{k})e^{ikx} + b^\dagger(\vec{k})e^{-ikx}\right)$

$\langle 0|T\{\phi(x_1)\phi(x_2)\}|0\rangle \quad ; \quad x_1^0 - x_2^0 > 0$

$$\langle 0|\phi(x_1)\phi(x_2)|0\rangle =$$

$$= \int \frac{d^3\vec{k}\, d^3\vec{k}'}{(2\pi)^6 4\omega\omega'} \langle 0|\left(a(\vec{k})e^{ikx_1} + b^\dagger(\vec{k})e^{-ikx_1}\right)\left(a(\vec{k}')e^{ik'x_1} + b^\dagger(\vec{k}')e^{-ik'x_2}\right)|0\rangle$$

$$= \int \frac{d^3\vec{k}\, d^3\vec{k}'}{(2\pi)^6 4\omega\omega'} \langle 0|\underbrace{a(\vec{k})\, b^\dagger(\vec{k}')}_{\text{comutam}} e^{ikx_1 - ik'x_2}|0\rangle = 0.$$

do mesmo modo: $\langle 0|T\{\phi^\dagger(x_1)\phi^\dagger(x_2)\}|0\rangle = 0.$

Por último:

$$\langle 0|T\{\phi^\dagger(x_1)\phi(x_2)\}|0\rangle =$$

$$= \int \frac{d^3\vec{k}\, d^3\vec{k}'}{(2\pi)^6 4\omega\omega'} \langle 0|\left(b(\vec{k})e^{ikx_1} + a^\dagger(\vec{k})e^{-ikx_1}\right)\left(a(\vec{k}')e^{ik'x_2} + b^\dagger(\vec{k}')e^{-ik'x_2}\right)|0\rangle$$

$$= \int \frac{d^3\vec{k}\, d^3\vec{k}'}{(2\pi)^6 4\omega\omega'} \langle 0|b(\vec{k})\, b^\dagger(\vec{k}')|0\rangle e^{ik(x_1) - ik'x_2}$$

$$= \int \frac{d^3\vec{k}}{(2\pi)^3 2\omega} e^{-ik(x_2 - x_1)} \qquad x_1^0 - x_2^0 > 0. \quad \text{logo:}$$

$$\langle 0|T\{\phi^\dagger(x_1)\phi(x_2)\}|0\rangle = \frac{1}{i}\Delta(x_2 - x_1)$$

a generalização é dada por:

$$\langle 0|T\{\phi^\dagger(x_1)\phi^\dagger(x_2)\cdots\phi^\dagger(x_n)\phi(y_1)\cdots\phi(y_n)\}|0\rangle$$

$$= \frac{1}{i^n}\sum_{\substack{\text{pares}\\(x,y)}}\Delta(x_{j_1}-y_{j_2})\cdots\Delta(x_{j_{2n-1}}-y_{j_{2n}}) \qquad \blacksquare$$

(8.8) $\phi(\vec{x},0)|A\rangle = A(\vec{x})|A\rangle$, então, para ser válido:

$$[\phi(\vec{x},0),\pi(\vec{y},0)] = i\delta^3(\vec{x}-\vec{y})$$

$$\langle A|\,\phi(\vec{x})\pi(\vec{y})-\pi(\vec{y})\phi(\vec{x})\,|\Psi\rangle = i\delta^3(\vec{x}-\vec{y})\langle A|\Psi\rangle$$

$$A(\vec{x})\langle A|\pi(\vec{y})|\Psi\rangle - F[A\vec{y}]\langle A|\phi(\vec{x})|\Psi\rangle = i\,\delta^3(\vec{x}-\vec{y})\langle A|\Psi\rangle$$

$$F[A(\vec{y})] = -i\frac{\delta}{\delta A(\vec{y})}$$

logo, para o estado fundamental:

$$\langle A|a(\vec{k})|0\rangle = 0 = \langle A|\,i\int d^3\vec{x}\,e^{-i\vec{k}\cdot\vec{x}}(\pi(\vec{x})-i\omega\phi(\vec{x}))|0\rangle = 0$$

$$-i\int d^3\vec{x}\,e^{-i\vec{k}\cdot\vec{x}}\frac{\delta}{\delta A(\vec{x})}\langle A|0\rangle = i\int d^3\vec{x}\,e^{-i\vec{k}\cdot\vec{x}}\omega(\vec{k})A(\vec{x})\langle A|0\rangle$$

$$\int d^3\vec{k}\int d^3\vec{x}\,e^{i\vec{k}(\vec{y}-\vec{x})}\frac{\delta}{\delta A(\vec{x})}\langle A|0\rangle = -\int d^3\vec{k}\,e^{i\vec{k}\cdot\vec{y}}\omega(\vec{k})\tilde{A}(\vec{k})\langle A|0\rangle$$

$$\frac{\delta}{\delta A(\vec{y})}\langle A|0\rangle = -\int \frac{d^3\vec{k}}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{y}}\, \omega(\vec{k})\, \tilde{A}(\vec{k})\, \langle A|0\rangle$$

$$\langle A|0\rangle \propto \exp\left[\frac{-1}{2}\int \frac{d^3\vec{k}}{(2\pi)^3}\int d^3\vec{y}\, e^{i\vec{k}\cdot\vec{y}}\, A(\vec{y}) \int d^3\vec{x}\, e^{-i\vec{k}\cdot\vec{x}}\, A(\vec{x}) \cdot \omega(\vec{k})\right]$$

$$\langle A|0\rangle \propto \exp\left[\frac{-1}{2}\int \frac{d^3\vec{k}}{(2\pi)^3}\, \omega(\vec{k})\, \tilde{A}(\vec{k})\, \tilde{A}(-\vec{k})\right]$$

$$\propto \exp\left[\frac{-1}{2}\int \frac{d^3\vec{k}}{(2\pi)^3}\, \omega(\vec{k})\, \|\tilde{A}(\vec{k})\|^2\right] \quad \blacksquare$$
