# SREDNICKI CAPÍTULO 1

VICENTE V. FIGUEIRA

## Sumário



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## 1. Exercício 1

$$
\begin{aligned}
H_{ab}H_{bc} &= \Big(\mathrm{c}P_j\alpha^j_{ab} + m\mathrm{c}^2\beta_{ab}\Big)\big(\mathrm{c}P_k\alpha^k_{bc} + m\mathrm{c}^2\beta_{bc}\big) \\
&= \mathrm{c}^2P_j\alpha^j_{ab}P_k\alpha^k_{bc} + m\mathrm{c}^3P_j\alpha^j_{ab}\beta_{bc} + m\mathrm{c}^3P_k\beta_{ab}\alpha^k_{bc} + m^2\mathrm{c}^4\beta_{ab}\beta_{bc} \\
&= \frac{\mathrm{c}^2}{2}P_jP_k\big(\alpha^j\alpha^k + \alpha^k\alpha^j\big)_{ac} + m\mathrm{c}^3P_j\big(\alpha^j\beta + \beta\alpha^j\big)_{ac} + m^2\mathrm{c}^4\beta^2_{ac} \\
\Rightarrow &\big(\alpha^j\alpha^k + \alpha^k + \alpha^j\big)_{ac} = \big\{\alpha^j, \alpha^k\big\}_{ac} = 2\delta^{jk}\delta_{ac} \\
&\big(\alpha^j\beta + \beta\alpha^j\big)_{ac} = \big\{\alpha^j, \beta\big\}_{ac} = 0 \\
&\beta^2_{ac} = \frac{1}{2}\{\beta, \beta\}_{ac} = \delta_{ac}
\end{aligned}
$$

Os auto-valores de $\beta$ são,

$$
\begin{aligned}
&\beta v = \lambda v \Rightarrow \beta^2 v = \lambda v = \lambda^2 v \\
&\Rightarrow v = \lambda^2 v \Rightarrow \lambda = \pm 1
\end{aligned}
$$

Como $\alpha^j\beta = -\beta\alpha^j \Rightarrow \beta = -\alpha^j\beta\alpha^j$, temos que,

$$
\begin{aligned}
\mathrm{Tr}\,[\beta] &= -\,\mathrm{Tr}\,\big[\alpha^j\beta\alpha^j\big] = -\,\mathrm{Tr}\,\big[\alpha^j\alpha^j\beta\big] \\
&= -\,\mathrm{Tr}\,[\beta] \Rightarrow \mathrm{Tr}\,[\beta] = 0
\end{aligned}
$$

Logo $\beta$ possui número igual de auto-valores $+1$ e $-1$, logo, tem dimensão par.
Para $\alpha^j$,

$$
\begin{aligned}
&\alpha^j v = \lambda v \Rightarrow {\alpha^j}^2 v = \lambda v = \lambda^2 v \\
&\Rightarrow v = \lambda^2 v \Rightarrow \lambda = \pm 1
\end{aligned}
$$

Como $\alpha^j\beta = -\beta\alpha^j \Rightarrow \alpha^j = -\beta\alpha^j\beta$, temos que,

$$
\begin{aligned}
\mathrm{Tr}\,\big[\alpha^j\big] &= -\,\mathrm{Tr}\,\big[\beta\alpha^j\beta\big] = -\,\mathrm{Tr}\,\big[\alpha^j\beta^2\big] \\
&= -\,\mathrm{Tr}\,\big[\alpha^j\big] \Rightarrow \mathrm{Tr}\,\big[\alpha^j\big] = 0
\end{aligned}
$$

Logo pelo mesmo argumento $\alpha^j$ tem dimensão Par! ∎

## 2. Exercício 2

Para $n = 1$,

$$
\begin{aligned}
H\left|\phi;t\right\rangle &= \int \mathrm{d}^3\mathbf{x}\, a^\dagger(\mathbf{x})\left[-\frac{\hbar^2}{2m}\nabla_x^2 + U(\mathbf{x})\right]a(\mathbf{x})\int \mathrm{d}^3\mathbf{x}_1\,\phi(\mathbf{x}_1;t)a^\dagger(\mathbf{x}_1)\left|0\right\rangle \\
&+ \frac{1}{2}\int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y}\,V(\mathbf{x}-\mathbf{y})a^\dagger(\mathbf{x})a^\dagger(\mathbf{y})a(y)a(\mathbf{x})\int \mathrm{d}^3\mathbf{x}_1\,\phi(\mathbf{x}_1;t)a^\dagger(\mathbf{x}_1)\left|0\right\rangle \\
&= \int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{x}_1\, a^\dagger(\mathbf{x})\left[-\frac{\hbar^2}{2m}\nabla_x^2 + U(\mathbf{x})\right]\phi(\mathbf{x};t)\Big(a^\dagger(\mathbf{x}_1)a(\mathbf{x}) + \delta^{(3)}(\mathbf{x}-\mathbf{x}_1)\Big)\left|0\right\rangle \\
&+ \frac{1}{2}\int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y}\,V(\mathbf{x}-\mathbf{y})\phi(\mathbf{x}_1;t)a^\dagger(\mathbf{x})a^\dagger(\mathbf{y})a(\mathbf{y})\Big(a^\dagger(\mathbf{x}_1)a(\mathbf{x}) + \delta^{(3)}(\mathbf{x}-\mathbf{x}_1)\Big)\left|0\right\rangle \\
&= \int \mathrm{d}^3\mathbf{x}\, a^\dagger(\mathbf{x})\left[-\frac{\hbar}{2m}\nabla_x^2 + U(\mathbf{x})\right]\phi(\mathbf{x};t)\left|0\right\rangle \\
&= \int \mathrm{d}^3\mathbf{x}\, a^\dagger(\mathbf{x})\mathrm{i}\hbar\frac{\partial}{\partial t}\phi(\mathbf{x};t)\left|0\right\rangle = \mathrm{i}\hbar\frac{\partial}{\partial t}\left|\phi;t\right\rangle
\end{aligned}
$$

Analogamente,

$$
\begin{aligned}
\mathrm{i}\hbar\frac{\partial}{\partial t}\int \mathrm{d}^3\mathbf{x}_1\,\phi(\mathbf{x}_1;t)a^\dagger(\mathbf{x}_1)\left|0\right\rangle &= \int \mathrm{d}^3\mathbf{x}\, a^\dagger(\mathbf{x})\left[-\frac{\hbar^2}{2m}\nabla_x^2 + U(\mathbf{x})\right]a(\mathbf{x})\int \mathrm{d}^3\mathbf{x}_1\,\phi(\mathbf{x}_1;t)a^\dagger(\mathbf{x}_1)\left|0\right\rangle \\
&= \int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{x}_1\, a^\dagger(\mathbf{x})\left[-\frac{\hbar}{2m}\nabla_x^2 + U(\mathbf{x})\right]\phi(\mathbf{x}_1;t)\Big(a^\dagger(\mathbf{x}_1)a(\mathbf{x}) + \delta^{(3)}(\mathbf{x}_1-\mathbf{x})\Big)\left|0\right\rangle \\
&= \int \mathrm{d}^3\mathbf{x}\, a^\dagger(\mathbf{x})\left[-\frac{\hbar^2}{2m}\nabla_x^2 + U(\mathbf{x})\right]\phi(\mathbf{x};t)\left|0\right\rangle \\
\Rightarrow \mathrm{i}\hbar\frac{\partial}{\partial t}\phi(\mathbf{x};t) &= \left(-\frac{\hbar^2}{2m} + U(\mathbf{x})\right)\phi(\mathbf{x};t)
\end{aligned}
$$

Por indução está provado. ■

# 3. Exercício 3

$$\begin{aligned}
[N,H] &= \left[\int \mathrm{d}^3\mathbf{x}\, a^\dagger(\mathbf{x})a(\mathbf{x}), \int \mathrm{d}^3\mathbf{y}\, a^\dagger(\mathbf{y})\left[-\frac{\hbar^2}{2m}\nabla_y^2 + U(y)\right]a(\mathbf{y})\right] \\
&= \int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y} \left\{a^\dagger(\mathbf{x})a(\mathbf{x})a^\dagger(\mathbf{y})\left[-\frac{\hbar^2}{2m}\nabla_y^2 + U(y)\right]a(\mathbf{y}) - a^\dagger(\mathbf{y})\left[-\frac{\hbar^2}{2m}\nabla_y^2 + U(y)\right]a(\mathbf{y})a^\dagger(\mathbf{x})a(\mathbf{x})\right\} \\
&= \int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y} \left\{a^\dagger(\mathbf{x})a(\mathbf{x})a^\dagger(\mathbf{y})\left[-\frac{\hbar^2}{2m}\nabla_y^2 + U(y)\right]a(\mathbf{y}) - a^\dagger(\mathbf{y})\left[-\frac{\hbar^2}{2m}\nabla_y^2 + U(y)\right]\Big(a^\dagger(\mathbf{x})a(\mathbf{y}) + \delta^{(3)}(\mathbf{y}-\mathbf{x})\Big)a(\mathbf{x})\right\} \\
&= \int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y}\, a^\dagger(\mathbf{x})\big(a(\mathbf{x})a^\dagger(\mathbf{y}) - a^\dagger(\mathbf{y})a(\mathbf{x})\big)\left[-\frac{\hbar^2}{2m}\nabla_y^2 + U(y)\right]a(\mathbf{y}) - \int \mathrm{d}^3\mathbf{y}\, a^\dagger(\mathbf{y})\left[-\frac{\hbar^2}{2m}\nabla_y^2 + U(\mathbf{y})\right]a(\mathbf{y}) \\
&= \int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y}\, a^\dagger(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{y})\left[-\frac{\hbar^2}{2m}\nabla_y^2 + U(y)\right]a(\mathbf{y}) - \int \mathrm{d}^3\mathbf{y}\, a^\dagger(\mathbf{y})\left[-\frac{\hbar^2}{2m}\nabla_y^2 + U(\mathbf{y})\right]a(\mathbf{y}) \\
&= 0
\end{aligned}$$