(4.1) Suponho : $(x-x')^2 = r^2 > 0$, tomo referencial tal que $t = t' \Rightarrow (\vec{x}-\vec{x}')^2 = r^2 > 0$.

$$C(r) = \int \frac{d^3\vec{k}}{(2\pi)^3 2\omega_{\vec{k}}} e^{ik(x-x')}$$

$$= \int \frac{d^3\vec{k}}{(2\pi)^3 2\omega_{\vec{k}}} e^{i\vec{k}(\vec{x}-\vec{x}')}$$

$$= \int_0^\infty dk \int_0^\pi d\theta \int_0^{2\pi} d\phi \, \frac{k^2 \sin\theta}{(2\pi)^3 2\sqrt{k^2+m^2}} e^{ikr\cos\theta}$$

$$= \int_0^\infty dk \int_0^\pi d\theta \, \frac{k\, i}{r} \cdot \frac{1}{(2\pi)^2 2\sqrt{k^2+m^2}} \frac{d}{d\theta} e^{ikr\cos\theta}$$

$$= \int_0^\infty dk \, \frac{ik}{(2\pi)^2 2r\sqrt{k^2+m^2}} \left( e^{-ikr} - e^{ikr} \right)$$

$$= \int_0^\infty dk \, \frac{k}{r(2\pi)^2} \cdot \frac{1}{\sqrt{k^2+m^2}} \cdot \sin(kr)$$

$$= \frac{1}{4\pi^2 r^2} \int_0^\infty \frac{(-k)\, d(\cos kr)}{\sqrt{k^2+m^2}} = \frac{1}{4\pi^2 r^2} \int_0^\infty \cos kr \, d\left(\frac{k}{\sqrt{k^2+m^2}}\right)$$

$$= \frac{m^2}{4\pi^2 r^2} \int_0^\infty \frac{\cos(kr)}{\left(\sqrt{k^2+m^2}\right)^3} dk = \frac{m^2}{4\pi^2} \int_0^\infty dt \, \frac{\cos t}{\left(\sqrt{t^2+m^2r^2}\right)^3}$$

como : $K_1(mr) = mr \int_0^\infty \frac{dt \cos t}{(t^2+m^2r^2)^{3/2}}$

$$C(r) = \frac{m}{4\pi^2 r} K_1(mr) .$$

como $K_1(mr) \sim \dfrac{1}{mr}$ , $mr \to 0$

$$C(r) \xrightarrow{m \to 0} \frac{1}{4\pi^2 r^2}$$

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