(5.1) Para o campo escalar complexo:

$$\phi(x) = \int \frac{d^3\vec{k}}{(2\pi)^3 2w}\left[a(\vec{k})\, e^{ikx} + b^\dagger e^{-ikx}\right]$$

$$\pi^\dagger(x) = \int \frac{d^3\vec{k}}{(2\pi)^3 2w}\left[-iw\, a(\vec{k})\, e^{ikx} + iw\, b^\dagger e^{-ikx}\right]$$

$$\int d^3\vec{x}\, e^{-ikx}\phi(x) = \int d^3\vec{x} \int \frac{d^3\vec{k}'}{(2\pi)^3 2w'}\left[a(\vec{k}')e^{ix(k'-k)} + b^\dagger(\vec{k}')e^{-ix(k'+k)}\right]$$

$$= \frac{a(\vec{k})}{2w} + \frac{b^\dagger(-\vec{k})}{2w}e^{2iwt}$$

$$\int d^3\vec{x}\, e^{-ikx}\pi^\dagger(x) = \int d^3\vec{x}\int \frac{d^3\vec{k}'}{(2\pi)^3 2w'}\left[-iw' a(\vec{k}')e^{ix(k'-k)} + iw' b^\dagger(\vec{k}')e^{-ix(k'+k)}\right]$$

$$= -\frac{i\,a(\vec{k})}{2} + \frac{i}{2}b^\dagger(-\vec{k})\,e^{i2wt}$$

$$a(\vec{k}) = \int d^3\vec{x}\, e^{-ikx}\left[w\,\phi(x) + i\,\pi^\dagger(x)\right]$$

$$\phi^\dagger(x) = \int \frac{d^3\vec{k}}{(2\pi)^3 2w}\left[b(\vec{k})e^{ikx} + a^\dagger(\vec{k})e^{-ikx}\right]$$

$$\pi(x) = \int \frac{d^3\vec{k}}{(2\pi)^3 2w}\left[-iw\, b(\vec{k})e^{ikx} + iw\, a^\dagger(\vec{k})e^{-ikx}\right]$$

$$b(\vec{k}) = \int d^3\vec{x}\, e^{-ikx}\left[w\,\phi^\dagger(x) + i\,\pi(x)\right]$$

analogamente, queremos:

$$a_1^\dagger(+\infty) - a_1^\dagger(-\infty) = \int_{-\infty}^{+\infty} dt\, \partial_0 a_1^\dagger(t)$$

$$= \int d^3\vec{k}\, f_1(\vec{k}) \int d^4x\, \partial_0 \left(e^{ikx} \left[\phi^\dagger(x) w - i\pi(x) \right] \right)$$

$$= \int d^3\vec{k}\, f_1(\vec{k}) \int d^4x \left\{ \begin{array}{l} -iw^2 e^{ikx} \phi^\dagger(x) - \cancel{w e^{ikx} \pi(x)} \\ \cancel{\dot{\phi}^\dagger w e^{ikx}} - i \ddot{\phi}^\dagger(x) e^{ikx} \end{array} \right\}$$

$$= -i \int d^3\vec{k}\, f_1(\vec{k}) \int d^4x\, e^{ikx} \left[w^2 + \partial_0^2 \right] \phi^\dagger(x)$$

$$= (-i) \int d^3\vec{k}\, f_1(\vec{k}) \int d^4x\, e^{ikx} \left[\vec{k}^2 + m^2 + \partial_0^2 \right] \phi^\dagger(x)$$

$$= (-i) \int d^3\vec{k}\, f_1(\vec{k}) \int d^4x\, e^{ikx} \left[-\partial^2 + m^2 \right] \phi^\dagger(x)$$

$$a_1^\dagger(-\infty) = a_1^\dagger(+\infty) + i \int d^3\vec{k}\, f_1(\vec{k}) \int d^4x\, e^{ikx} (-\partial^2 + m^2)\, \phi^\dagger(x)$$

$$a_1(+\infty) = a_1(-\infty) + i \int d^3\vec{k}\, f_1(\vec{k}) \int d^4x\, e^{-ikx} (-\partial^2 + m^2)\, \phi(x).$$

$$b_1^\dagger(+\infty) - b_1^\dagger(-\infty) = \int_{-\infty}^{+\infty} dt\, \partial_0 b_1^\dagger(t)$$

$$= \int d^3\vec{k}\, f_1(\vec{k}) \int d^4x\, \partial_0 \left(e^{ikx} \left[\phi(x) w - i\pi^\dagger(x) \right] \right)$$

$$= (-i) \int d^3\vec{k}\, f_1(\vec{k}) \int d^4x\, e^{ikx} (-\partial^2 + m^2)\, \phi(x)$$

$$b_1^\dagger(-\infty) = b_1^\dagger(+\infty) + i \int d^3\vec{k}\, f_1(\vec{k}) \int d^4x\, e^{ikx} (-\partial^2 + m^2)\, \phi(x)$$

$$b_1(+\infty) = b_1(-\infty) + i \int d^3\vec{k}\, f_1(\vec{k}) \int d^4x\, e^{-ikx} (-\partial^2 + m^2)\, \phi^\dagger(x)$$

portanto uma amplitude de n partículas e m anti partículas

$\to$ n' partículas e m' anti partículas :

$$\langle f|i\rangle = \langle 0|a_{1'}(+\infty)\cdots a_{n'}(+\infty)\, b_{1'}(+\infty)\cdots b_{m'}(+\infty)\, a_1^\dagger(-\infty)\cdots a_n^\dagger(+\infty)\, b_1^\dagger(-\infty)\cdots|0\rangle$$

$$= \langle 0|T\{(\cdots)\}|0\rangle$$

$$= i^{n+m+n'+m'}\int d^4x_1\, e^{ik_1x_1}(-\partial_1^2+m^2)\cdots\langle 0|T\{\phi(x_1)\cdots\phi^\dagger(x_1')\cdots\}|0\rangle$$

para partícula inicial : $i\int d^4x\, e^{ikx}\,(-\partial^2+m^2)\,\phi(x)$

(a) partícula final : $i\int d^4x\, e^{-ikx}\,(-\partial^2+m^2)\,\phi^\dagger(x)$

(b) anti partícula inicial : $i\int d^4x\, e^{ikx}\,(-\partial^2+m^2)\,\phi^\dagger(x)$

(b) anti partícula final : $i\int d^4x\, e^{-ikx}(-\partial^2+m^2)\,\phi(x)$.
