

SREDNICKI CAPÍTULO 2

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1. EXERCÍCIO 1

$$\begin{aligned}
 g_{\mu\nu}(\delta^\mu_\rho + \delta\omega^\mu_\rho)(\delta^\nu_\sigma + \delta\omega^\nu_\sigma) &= g_{\rho\sigma} \\
 g_{\mu\nu}(\delta^\mu_\rho\delta^\nu_\sigma + \delta^\mu_\rho\delta\omega^\nu_\sigma + \delta^\nu_\sigma\delta\omega^\mu_\rho) &= g_{\rho\sigma} \\
 g_{\rho\sigma} + g_{\rho\nu}\delta\omega^\nu_\sigma + g_{\mu\sigma}\delta\omega^\mu_\rho &= g_{\rho\sigma} \\
 \delta\omega_{\rho\sigma} &= -\delta\omega_{\sigma\rho}
 \end{aligned}$$

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2. EXERCÍCIO 2

$$\begin{aligned}
 U^{-1}(\Lambda)U(\Lambda')U(\Lambda) &= U(\Lambda^{-1}\Lambda'\Lambda) \\
 U^{-1}\left(\mathbb{1} + \frac{i}{2\hbar}\delta\omega'_{\mu\nu}M^{\mu\nu}\right)U(\Lambda) &= U(\Lambda^{-1}(\mathbb{1} + \delta\omega')\Lambda) \\
 \mathbb{1} + \frac{i}{2\hbar}\delta\omega'_{\mu\nu}U^{-1}(\Lambda)M^{\mu\nu}U(\Lambda) &= \mathbb{1} + \frac{i}{2\hbar}(\Lambda^{-1})_{\alpha}^{\mu}\delta\omega'_{\mu\nu}\Lambda^{\nu}_{\beta}M^{\alpha\beta} \\
 \delta\omega'_{\mu\nu}U^{-1}(\Lambda)M^{\mu\nu}U(\Lambda) &= \delta\omega'_{\mu\nu}\Lambda^{\nu}_{\beta}\Lambda^{\mu}_{\alpha}M^{\alpha\beta} \\
 U^{-1}(\Lambda)M^{\mu\nu}U(\Lambda) &= \Lambda^{\nu}_{\beta}\Lambda^{\mu}_{\alpha}M^{\alpha\beta}
 \end{aligned}$$

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3. EXERCÍCIO 3

$$\begin{aligned}
 \left(\mathbb{1} - \frac{i}{2\hbar} \delta\omega_{\rho\sigma} M^{\rho\sigma} \right) M^{\mu\nu} \left(\mathbb{1} - \frac{i}{\hbar} \delta\omega_{\rho\sigma} M^{\rho\sigma} \right) &= (\delta^\nu_\beta + \delta\omega^\nu_\beta) (\delta^\mu_\alpha + \delta\omega^\mu_\alpha) M^{\alpha\beta} \\
 M^{\mu\nu} + \frac{i}{2\hbar} \delta\omega_{\rho\sigma} [M^{\mu\nu}, M^{\rho\sigma}] &= M^{\mu\nu} + \delta\omega^\nu_\beta M^{\mu\beta} + \delta\omega^\mu_\alpha M^{\alpha\nu} \\
 \frac{i}{2\hbar} \delta\omega_{\rho\sigma} [M^{\mu\nu}, M^{\rho\sigma}] &= \frac{\delta\omega_{\rho\sigma}}{2} (g^{\nu\rho} M^{\mu\sigma} + g^{\nu\sigma} M^{\mu\rho} + g^{\nu\rho} M^{\mu\sigma} - g^{\nu\sigma} M^{\mu\rho}) \\
 &\quad + \frac{\delta\omega_{\rho\sigma}}{2} (g^{\mu\rho} M^{\sigma\nu} + g^{\mu\sigma} M^{\rho\nu} + g^{\mu\rho} M^{\sigma\nu} - g^{\mu\sigma} M^{\rho\nu}) \\
 [M^{\mu\nu}, M^{\rho\sigma}] &= i\hbar (g^{\mu\rho} M^{\nu\sigma} - g^{\nu\rho} M^{\mu\sigma} + g^{\nu\sigma} M^{\mu\rho} - g^{\mu\sigma} M^{\nu\rho})
 \end{aligned}$$

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4. EXERCÍCIO 4

$$\begin{aligned}
[J_i, J_j] &= \frac{1}{4} \epsilon_{iab} \epsilon_{jcd} [M^{ab}, M^{cd}] \\
&= \frac{i\hbar}{4} \{ \delta_{ij} (\delta_{ac} \delta_{bd} - \delta_{ad} \delta_{bc}) + \delta_{ic} (\delta_{ad} \delta_{bj} - \delta_{aj} \delta_{bd}) + \delta_{id} (\delta_{aj} \delta_{bc} - \delta_{ac} \delta_{bj}) \} (g^{\mu\rho} M^{\nu\sigma} - g^{\nu\rho} M^{\mu\sigma} + g^{\nu\sigma} M^{\mu\rho} - g^{\mu\sigma} M^{\nu\rho}) \\
&= \frac{i\hbar}{2} (-g^a{}_a M_{ji} - g^b{}_b M_{ji} + g_{ja} M^a{}_i + g_{jb} M^b{}_i + g^a{}_i M_{ja} + g^b{}_i M_{jb}) \\
&= i\hbar M_{ij} = \frac{i\hbar}{2} (\delta_{ai} \delta_{bj} - \delta_{aj} \delta_{bi}) M^{ab} \\
&= i\hbar \epsilon_{ij}{}^k J_k
\end{aligned}$$

$$\begin{aligned}
[J_i, K_j] &= \frac{1}{2} \epsilon_{iab} [M^{ab}, M^{j0}] \\
&= \frac{i\hbar}{2} \epsilon_{iab} (g^{aj} M^{b0} - g^{bj} M^{a0} - g^{a0} M^{bj} + g^{b0} M^{aj}) \\
&= \frac{i\hbar}{2} \epsilon_{iab} g^{aj} M^{b0} + \frac{i\hbar}{2} \epsilon_{iab} g^{bj} M^{a0} \\
&= i\hbar \epsilon_{ijb} M^{b0} \\
&= i\hbar \epsilon_{ij}{}^k K_k
\end{aligned}$$

$$\begin{aligned}
[K_i, K_j] &= [M^{i0}, M^{j0}] \\
&= i\hbar (g^{ij} M^{00} - g^{0j} M^{i0} - g^{i0} M^{0j} + g^{00} M^{ij}) \\
&= -\frac{i\hbar}{2} (\delta_{ai} \delta_{bj} - \delta_{aj} \delta_{bi}) M^{ab} \\
&= -i\hbar \epsilon_{ij}{}^k J_k
\end{aligned}$$

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5. EXERCÍCIO 5

$$\begin{aligned}
 U^{-1}(\Lambda)P^\mu U(\Lambda) &= \Lambda^\mu{}_\nu P^\nu \\
 \left(\mathbb{1} - \frac{i}{2\hbar} \delta\omega_{\rho\sigma} M^{\rho\sigma} \right) P^\mu \left(\mathbb{1} + \frac{i}{2\hbar} \delta\omega_{\rho\sigma} M^{\rho\sigma} \right) &= (\delta^\mu{}_\nu + \delta\omega^\mu{}_\nu) P^\nu \\
 P^\mu + \frac{i}{2\hbar} \delta\omega_{\rho\sigma} [P^\mu, M^{\rho\sigma}] &= P^\mu + \delta\omega^\mu{}_\nu P^\nu \\
 \delta\omega_{\rho\sigma} [P^\mu, M^{\rho\sigma}] &= -i\hbar \delta\omega_{\rho\sigma} (g^{\mu\rho} P^\sigma - g^{\mu\sigma} P^\rho) \\
 [P^\mu, M^{\rho\sigma}] &= i\hbar (g^{\mu\sigma} P^\rho - g^{\mu\rho} P^\sigma)
 \end{aligned}$$

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6. EXERCÍCIO 6

$$\begin{aligned}
 [J_i, H] &= [J_i, P^0] \\
 &= \frac{1}{2} \epsilon_{ijk} [M^{jk}, P^0] \\
 &= -\frac{1}{2} \epsilon_{ijk} (g^{0k} P^j - g^{0j} P^k) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 [J_i, P_j] &= \frac{1}{2} \epsilon_{iab} [M^{ab}, P_j] \\
 &= -\frac{i\hbar}{2} \epsilon_{iab} (g^{ib} P^a - g^{ja} P^b) \\
 &= i\hbar \epsilon_{ijk} P^k
 \end{aligned}$$

$$\begin{aligned}
 [K_i, H] &= -[P^0, M^{i0}] \\
 &= -i\hbar (g^{00} P^i - g^{0i} P^0) \\
 &= i\hbar P^i
 \end{aligned}$$

$$\begin{aligned}
 [K_i, P_j] &= [M^{i0}, P_j] \\
 &= -i\hbar (g^{j0} P^i - g^{ji} P^0) \\
 &= i\hbar \delta_{ij} H
 \end{aligned}$$

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7. EXERCÍCIO 7

A propriedade é $T(a)T(b) = T(a + b)$,

$$\begin{aligned}
 T^{-1}(\delta a)T(b)T(\delta a) &= T(b) \\
 \left(\mathbb{1} + \frac{i\hbar}{\delta a_\mu} P^\mu \right) T(b) \left(\mathbb{1} - \frac{i}{\hbar} \delta a_\mu P^\mu \right) &= T(b) \\
 T(b) + \frac{i}{\hbar} [P^\mu, T(b)] \delta a_\mu &= T(b) \\
 [P^\mu, T(b)] &= 0 \\
 \left[P^\mu, \mathbb{1} - \frac{i}{\hbar} \delta b_\nu P^\nu \right] &= 0 \\
 [P^\mu, P^\nu] &= 0
 \end{aligned}$$

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8. EXERCÍCIO 8

a).

$$\begin{aligned}
 U^{-1}(\Lambda)\phi(x)U(\Lambda) &= \phi(\Lambda^{-1}x) \\
 \left(\mathbb{1} - \frac{i}{2\hbar}\delta\omega_{\mu\nu}M^{\mu\nu}\right)\phi(x)\left(\mathbb{1} + \frac{i}{2\hbar}\delta\omega_{\mu\nu}M^{\mu\nu}\right) &= \phi(x - \delta\omega x) \\
 \phi(x) + \frac{i}{2\hbar}[\phi(x), M^{\mu\nu}]\delta\omega_{\mu\nu} &= \phi(x) - (\delta\omega x)^\rho \partial_\rho \phi(x) \\
 \delta\omega_{\mu\nu}[\phi(x), M^{\mu\nu}] &= 2i\hbar\delta\omega_{\mu\nu}g^{\rho\mu}x^\nu \partial_\rho \phi(x) \\
 &= i\hbar\delta\omega_{\mu\nu}(g^{\rho\mu}x^\nu - g^{\rho\nu}x^\mu)\partial_\rho \phi(x) \\
 &= \frac{\hbar}{i}\delta\omega_{\mu\nu}(x^\mu \partial^\nu - x^\nu \partial^\mu)\phi(x) \\
 [\phi(x), M^{\mu\nu}] &= \mathcal{L}^{\mu\nu}\phi(x)
 \end{aligned}$$

b).

$$\begin{aligned}
 [[\phi(x), M^{\mu\nu}], M^{\rho\sigma}] &= [\mathcal{L}^{\mu\nu}\phi(x), M^{\rho\sigma}] \\
 &= \mathcal{L}^{\mu\nu}[\phi(x), M^{\rho\sigma}] \\
 &= \mathcal{L}^{\mu\nu}\mathcal{L}^{\rho\sigma}\phi(x)
 \end{aligned}$$

c).

$$\begin{aligned}
 &[[A, B], C] + [[B, C], A] + [[C, A], B] \\
 &= ABC - BAC - CAB + CBA + BCA - CBA - ABC + ACB + CAB - ACB - BCA + BAC \\
 &= 0
 \end{aligned}$$

d).

$$[\phi(x), [M^{\mu\nu}, M^{\rho\sigma}]] =$$