2+1 Dimensional Gravity as a Gauge Theory

An Exploration of Complexity

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Introduction

Gravity, at its core, is a theory of spacetime geometry — but there are strong motivations to reformulate it as a gauge theory. Gauge theories describe interactions as consequences of local symmetries, and this perspective has proven remarkably powerful in the Standard Model of particle physics. Writing gravity as a gauge theory places it on the same conceptual footing as the other fundamental forces: the metric or vielbein becomes the gauge field of local translations, and the spin connection gauges local Lorentz transformations. This approach clarifies the role of symmetries, makes the counting of degrees of freedom more transparent, and provides a natural framework for coupling gravity to matter with spin. It also opens the door to techniques from Yang–Mills theory — such as BRST quantization, loop variables, and color-kinematics duality — which can be crucial in the search for a quantum theory of gravity.

Course of Steps

1. Demand only metric compatibility from the connection.
2. Rewrite the EH action in form language.
3. Interpret vielbein and spin connection as gauge fields.
4. Pose the EH action as being a trace in the flat spacetime isometry algebra.

Spin connection and vielbein

$$\eta_{\mu\nu} = \mathbf{g}(\tilde{\mathbf{e}}_\mu, \tilde{\mathbf{e}}_\nu) \Leftrightarrow \eta_{\mu\nu}\mathbf{e}^\mu \otimes \mathbf{e}^\nu = \mathbf{g}(\tilde{\mathbf{e}}_\mu, \tilde{\mathbf{e}}_\nu)\mathbf{e}^\mu \otimes \mathbf{e}^\nu = \mathbf{g} \tag{1}$$

Form language

Einstein-Hilbert action in form language

$$S_{\text{EH}} = \frac{1}{2\kappa}\int\limits_M \star\mathbf{R}_{\beta\rho} \wedge \mathbf{e}^\beta \wedge \mathbf{e}^\rho \tag{2}$$

Redundancies of Gravity

Diffs + Local Lorentz:

$$\begin{cases} \mathbf{e}^\mu & \to \Lambda^\mu{}_\nu \mathbf{e}^\nu \\ \boldsymbol{\omega}^\alpha{}_\beta & \to \Lambda^\alpha{}_\rho \boldsymbol{\omega}^\rho{}_\sigma \Lambda^{-1\sigma}{}_\beta + \Lambda^\alpha{}_\sigma \mathbf{d}\Lambda^{-1\sigma}{}_\beta \end{cases} \text{AND} \begin{cases} \mathbf{e}^\mu & \to \phi_* \mathbf{e}^\mu \\ \boldsymbol{\omega}^\alpha{}_\beta & \to \phi_* \boldsymbol{\omega}^\alpha{}_\beta \end{cases} \tag{3}$$

2+1 Gravity as a gauge theory

$$S_{\text{EH}} = \frac{1}{2\kappa}\epsilon_{\mu\alpha\beta}\int\limits_M \mathbf{e}^\mu \wedge \mathbf{R}^{\alpha\beta} = \frac{1}{\kappa}\int\limits_M \left\langle \mathbf{e} \mathbin{\dot{\wedge}} \left(\mathbf{d}\boldsymbol{\omega} + \frac{1}{2}[\boldsymbol{\omega} \mathbin{\dot{\wedge}} \boldsymbol{\omega}]\right)\right\rangle \tag{4}$$

Comparison with Chern-Simons theory,

$$S_{\text{CS}}[\mathbf{A}] = \frac{k}{4\pi}\int\limits_M \left\langle \mathbf{A} \mathbin{\dot{\wedge}} \left(\mathbf{dA} + \frac{1}{3}[\mathbf{A} \mathbin{\dot{\wedge}} \mathbf{A}]\right)\right\rangle \tag{5}$$

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Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table 1: Table caption

Nulla ut porttitor enim. Suspendisse venenatis dui eget eros gravida tempor. Mauris feugiat elit et augue placerat ultrices. Morbi accumsan enim nec tortor consectetur non commodo. Pellentesque condimentum dui. Etiam sagittis purus non tellus tempor volutpat. Donec et dui non massa tristique adipiscing. Quisque vestibulum eros eu. Phasellus imperdiet, tortor vitae congue bibendum, felis enim sagittis lorem, et volutpat ante orci sagittis mi. Morbi rutrum laoreet semper. Morbi accumsan enim nec tortor consectetur non commodo nisi sollicitudin.

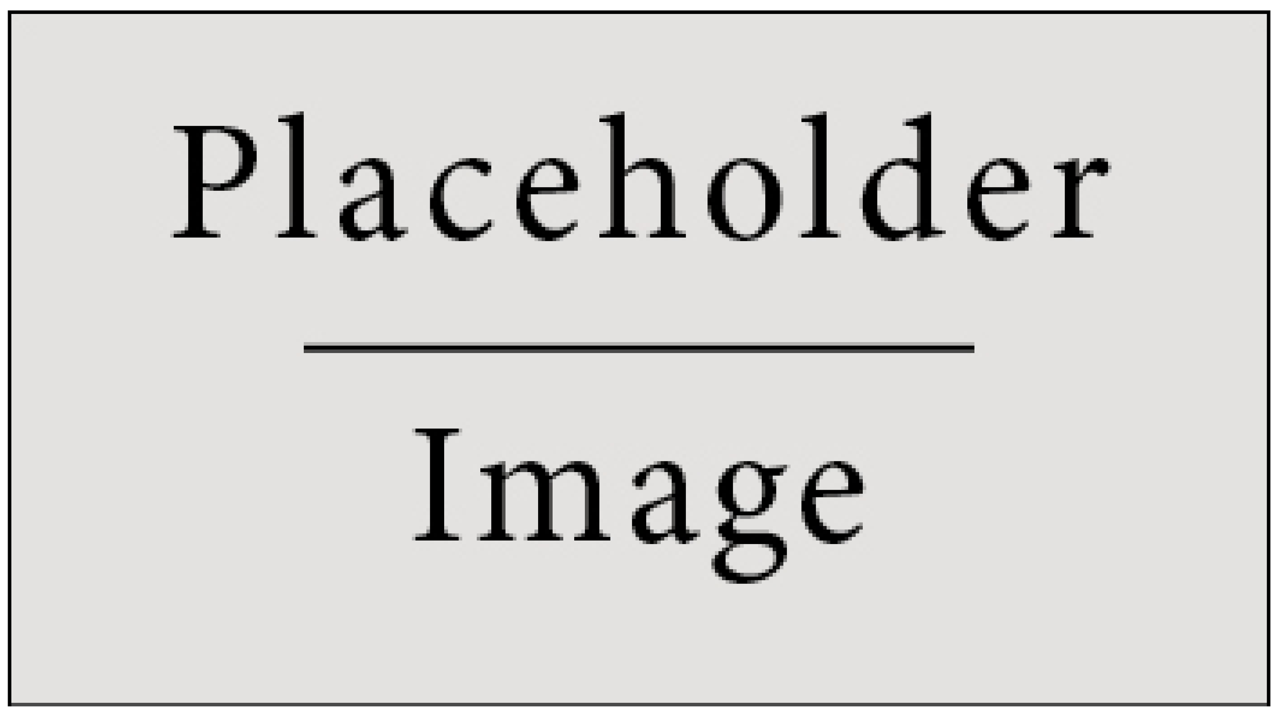

Figure 1: Figure caption

In hac habitasse platea dictumst. Etiam placerat, risus ac. Adipiscing lectus in magna blandit:

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
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Table 2: Table caption

Vivamus sed nibh ac metus tristique tristique a vitae ante. Sed lobortis mi ut arcu fringilla et adipiscing ligula rutrum. Aenean turpis velit, placerat eget tincidunt nec, ornare in nisl. In placerat.

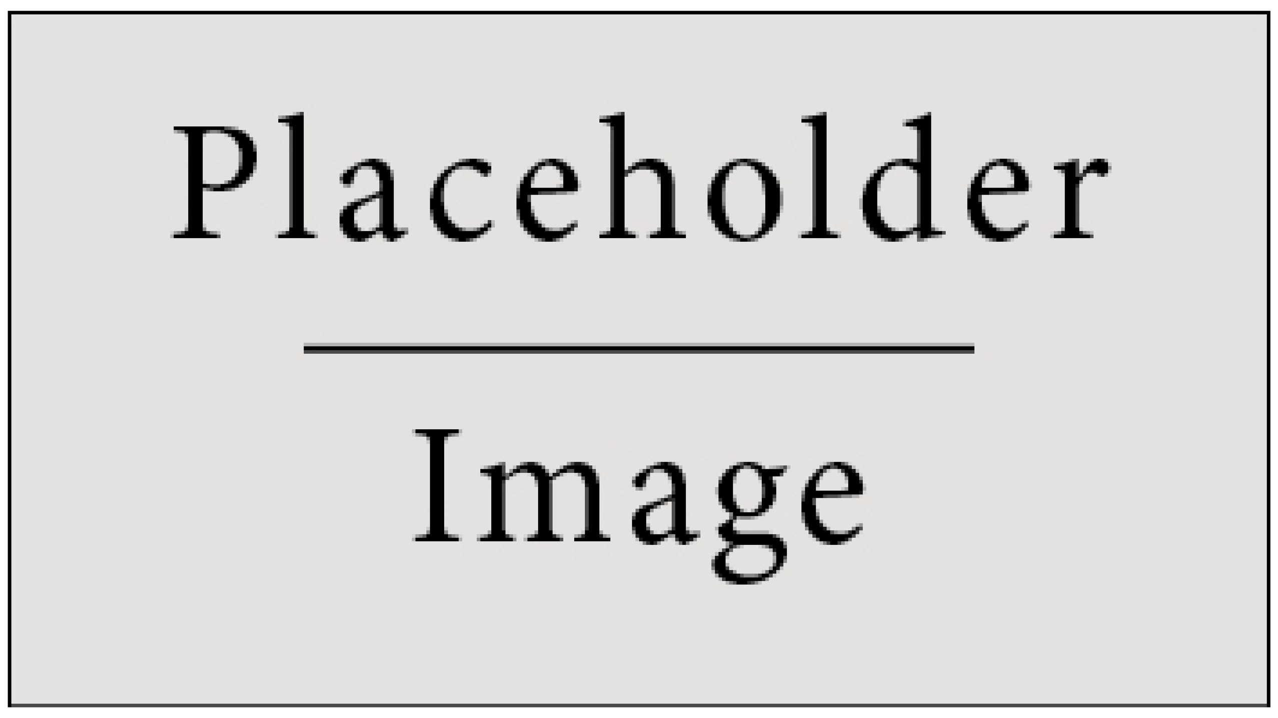

Figure 2: Figure caption

Conclusions

- Pellentesque eget orci eros. Fusce ultricies, tellus et pellentesque fringilla, ante massa luctus libero, quis tristique purus urna nec nibh. Phasellus fermentum rutrum elementum. Nam quis justo lectus.
- Vestibulum sem ante, hendrerit a gravida ac, blandit quis magna.
- Donec sem metus, facilisis at condimentum eget, vehicula ut massa. Morbi consequat, diam sed convallis tincidunt, arcu nunc.
- Nunc at convallis urna. isus ante. Pellentesque condimentum dui. Etiam sagittis purus non tellus tempor volutpat. Donec et dui non massa tristique adipiscing.

Forthcoming Research

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Acknowledgements

Etiam fermentum, arcu ut gravida fringilla, dolor arcu laoreet justo, ut imperdiet urna arcu a arcu. Donec nec ante a dui tempus consectetur. Cras nisi turpis, dapibus sit amet mattis sed, laoreet.

CAPES