SCALAR PROXY

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1. DF^2 Theory

The $(DF)^2 + \text{YM}$ theory is given by the following Lagrangian,

$$\mathcal{L} = -\frac{1}{2}D_\mu F^{a\mu\nu}D_\alpha F_a{}^\alpha{}_\nu + \frac{1}{3}f_{abc}F^a{}_\mu{}^\nu F^b{}_\nu{}^\alpha F^c{}_\alpha{}^\mu - \frac{1}{2}D_\mu\phi^I D^\mu\phi_I + \frac{g}{2}C^{Iab}\phi_I F_{a\mu\nu}F_b{}^{\mu\nu} + \frac{g}{6}d^{IJK}\phi_I\phi_J\phi_K - \frac{m^2}{2}\phi_I\phi^I - \frac{m^2}{4}F_{a\mu\nu}F^{a\mu\nu}$$

Where of course,

$$\begin{aligned}
F^a{}_{\mu\nu} &= \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu + gf^a{}_{bc}A^b{}_\mu A^c{}_\nu \\
(D_\alpha F^{\mu\nu})^a &= \partial_\alpha F^a{}_{\mu\nu} + gf^a{}_{bc}A^b{}_\alpha F^c{}_{\mu\nu} \\
(D_\alpha\phi)^I &= \partial_\alpha\phi^I - \mathrm{i}g\mathrm{T}^{a}_{\mathrm{R}}{}^I{}_J A_{a\alpha}\phi^J
\end{aligned}$$

We also have to incorporate the gauge fixing part,

$$\mathcal{L}_{\text{GF}} = -\frac{1}{2}\partial_\mu A^{a\mu}(-\Box + m^2)\partial_\nu A_a{}^\nu$$

1.1. **quadratic piece.** Let us collect all the quadratic pieces,

$$\begin{aligned}
-\frac{1}{2}D_\mu F^{a\mu\nu}D_\alpha F_a{}^\alpha{}_\nu &= -\frac{1}{2}\left(\partial_\mu F^{a\mu\nu} + gf^a{}_{bc}A^b{}_\mu F^{c\mu\nu}\right)\left(\partial_\alpha F_a{}^\alpha{}_\nu + gf_{ade}A^d{}_\alpha F^{e\alpha}{}_\nu\right) \\
&= -\frac{1}{2}(\partial_\mu F^{a\mu\nu})(\partial_\alpha F_a{}^\alpha{}_\nu) \\
&= -\frac{1}{2}\left(\partial_\mu\left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + gf^a{}_{bc}A^{b\mu}A^{c\nu}\right)\right)\left(\partial_\alpha\left(\partial^\alpha A_{a\nu} - \partial_\nu A_a{}^\alpha + gf_{ade}A^d{}_\alpha A^e{}_\nu\right)\right) \\
&= -\frac{1}{2}(\partial_\mu(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}))(\partial_\alpha(\partial^\alpha A_{a\nu} - \partial_\nu A_a{}^\alpha)) \\
&= -\frac{1}{2}(\Box A^{a\nu} - \partial^\nu\partial_\mu A^{a\mu})(\Box A_{a\nu} - \partial_\nu\partial_\alpha A_a{}^\alpha) \\
&= -\frac{1}{2}\Box A^{a\nu}\Box A_{a\nu} + \frac{1}{2}\Box A^{a\nu}\partial_\nu\partial_\alpha A_a{}^\alpha + \frac{1}{2}\partial^\nu\partial_\mu A^{a\mu}\Box A_{a\nu} - \frac{1}{2}\partial^\nu\partial_\mu A^{a\mu}\partial_\nu\partial_\alpha A_a{}^\alpha \\
&= -\frac{1}{2}\Box A^{a\nu}\Box A_{a\nu} + \Box A^{a\nu}\partial_\nu\partial_\alpha A_a{}^\alpha - \frac{1}{2}\Box A^{a\mu}\partial_\mu\partial_\alpha A_a{}^\alpha \\
&= -\frac{1}{2}\Box A^{a\nu}\Box A_{a\nu} + \frac{1}{2}\Box A^{a\nu}\partial_\nu\partial_\alpha A_a{}^\alpha \\
&= \frac{1}{2}A^{a\mu}\delta_{ab}\left(-\eta_{\mu\nu}\Box^2 + \partial_\mu\partial_\nu\Box\right)A^{b\nu}
\end{aligned}$$

$$-\frac{1}{2}\partial_\mu A^{a\mu}(-\Box + m^2)\partial_\nu A_a{}^\nu = \frac{1}{2}A^{a\mu}\delta_{ab}(-\Box + m^2)\partial_\mu\partial_\nu A^{b\nu}$$

$$\begin{aligned}
-\frac{m^2}{4}F_{a\mu\nu}F^{a\mu\nu} &= -\frac{m^2}{4}\left(\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} + gf_{abc}A^b{}_\mu A^c{}_\nu\right)\left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + gf^a{}_{de}A^{d\mu}A^{e\nu}\right) \\
&= -\frac{m^2}{4}(\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu})(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}) \\
&= \frac{1}{2}A^{a\mu}\delta_{ab}\left(\eta_{\mu\nu}m^2\Box - m^2\partial_\nu\partial_\mu\right)A^{b\nu}
\end{aligned}$$

Summing all the contributions, the quadratic piece of the Lagrangian is,

$$\begin{aligned}
\mathcal{L} &\supset \frac{1}{2}A^{a\mu}\delta_{ab}\left(-\eta_{\mu\nu}\Box^2 + \partial_\mu\partial_\nu\Box\right)A^{b\nu} + \frac{1}{2}A^{a\mu}\delta_{ab}(-\Box + m^2)\partial_\mu\partial_\nu A^{b\nu} + \frac{1}{2}A^{a\mu}\delta_{ab}\left(\eta_{\mu\nu}m^2\Box - m^2\partial_\nu\partial_\mu\right)A^{b\nu} \\
\mathcal{L} &\supset \frac{1}{2}A^{a\mu}\delta_{ab}\eta_{\mu\nu}\left(-\Box^2 + m^2\Box\right)A^{b\nu}
\end{aligned}$$

1.2. **cubic piece.** Now, the cubic piece,

$$
\begin{aligned}
-\frac{1}{2}D_\mu F^{a\mu\nu} D_\alpha F_a{}^\alpha{}_\nu &= -\frac{1}{2}(\Box A^{a\nu} - \partial^\nu \partial \cdot A^a) g f_{ade} \partial^\alpha \big(A^d{}_\alpha A^e{}_\nu \big) \\
&\quad - \frac{1}{2} g f^a{}_{bc} \partial_\mu \big(A^{b\mu} A^{c\nu} \big) (\Box A_{a\nu} - \partial_\nu \partial \cdot A_a) \\
&\quad - \frac{1}{2}(\Box A^{a\nu} - \partial^\nu \partial \cdot A^a) g f_{ade} A^d{}_\alpha (\partial^\alpha A^e{}_\nu - \partial_\nu A^{e\alpha}) \\
&\quad - \frac{1}{2} g f^a{}_{bc} A^b{}_\mu (\partial^\mu A^{c\nu} - \partial^\nu A^{c\mu}) (\Box A_{a\nu} - \partial_\nu \partial \cdot A_a) \\
&= -g f^a{}_{bc} \partial_\mu \big(A^{b\mu} A^{c\nu} \big) (\Box A_{a\nu} - \partial_\nu \partial \cdot A_a) \\
&\quad - g f^a{}_{bc} A^b{}_\mu (\partial^\mu A^{c\nu} - \partial^\nu A^{c\mu}) (\Box A_{a\nu} - \partial_\nu \partial \cdot A_a)
\end{aligned}
$$

$$
\frac{1}{3} f_{abc} F^a{}_\mu{}^\nu F^b{}_\nu{}^\alpha F_\alpha{}^\mu = \frac{1}{3} f_{abc} \big(\partial_\mu A^{a\nu} - \partial^\nu A^a{}_\mu \big) \big(\partial_\nu A^{b\alpha} - \partial^\alpha A^a{}_\nu \big) (\partial_\alpha A^{c\mu} - \partial^\mu A^c{}_\alpha)
$$