SCALAR PROXY

VICENTE V. FIGUEIRA

1. Introduction

We will work most with the scalar proxy given by the lagrangian,

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2M^2}\Box\phi\Box\phi - \frac{\kappa}{2}\Box\phi\phi^2 \tag{1.1}$$

The idea here is reintegrate the higher derivative term, in order to obtain a lower derivative term, but in terms of additional fields. This is easily done by,

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \Box\phi\eta + \frac{M^2}{2}\eta^2 - \frac{\kappa}{2}\Box\phi\phi^2 \tag{1.2}$$

The new lagrangian has mixed propagator terms, to diagonalize it is also easy, we just open in terms of $\phi = h - \eta$,

$$\mathcal{L} = -\frac{1}{2}\partial_\mu h\partial^\mu h + \partial_\mu h\partial^\mu\eta - \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{\kappa}{2}\Box(h-\eta)(h-\eta)^2 + \eta\Box(h-\eta) + \frac{M^2}{2}\eta^2 \tag{1.3}$$

$$\mathcal{L} = -\frac{1}{2}\partial_\mu h\partial^\mu h + \partial_\mu h\partial^\mu\eta - \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{\kappa}{2}\Box(h-\eta)(h-\eta)^2 - \partial_\mu\eta\partial^\mu(h-\eta) + \frac{M^2}{2}\eta^2 \tag{1.4}$$

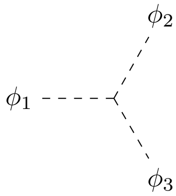
$$\mathcal{L} = -\frac{1}{2}\partial_\mu h\partial^\mu h + \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{M^2}{2}\eta^2 - \frac{\kappa}{2}\Box(h-\eta)(h-\eta)^2 \tag{1.5}$$

The Feynman rules are easily red as,

- h - - - - - - - - - - h $= \frac{1}{\mathrm{i}}\frac{1}{p^2}$
- η - - - - - - - - - - η $= -\frac{1}{\mathrm{i}}\frac{1}{p^2+M^2}$
- h_1, h_2, h_3 $= \mathrm{i}\kappa\big(p_1^2+p_2^2+p_3^2\big)$
- h_1, h_2, η_3 $= -\mathrm{i}\kappa\big(p_1^2+p_2^2+p_3^2\big)$
- h_1, η_2, η_3 $= \mathrm{i}\kappa\big(p_1^2+p_2^2+p_3^2\big)$
- η_1, η_2, η_3 $= -\mathrm{i}\kappa\big(p_1^2+p_2^2+p_3^2\big)$

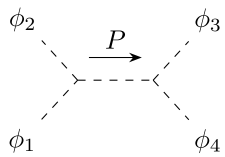
Which can also be seen directly from the Feynman rules of the ϕ field,

- ϕ - - - - - - - - - - ϕ $= \frac{1}{\mathrm{i}}\frac{1}{p^2+\frac{p^4}{M^2}}$

- 

$$= \mathrm{i}\kappa\left(p_1^2+p_2^2+p_3^2\right) = \mathrm{i}\kappa(p_1+p_2+p_3)^2 - 2\mathrm{i}\kappa(p_1\cdot p_2 + p_2\cdot p_3 + p_3\cdot p_1) = -\mathrm{i}\kappa(\langle 12\rangle[12]+\langle 23\rangle[23]+\langle 31\rangle[31])$$

So that the four point amplitude can be computed by,



$$= \frac{1}{\mathrm{i}}(-\mathrm{i}\kappa)^2\frac{1}{P^2+\frac{P^4}{M^2}}(\langle 12\rangle[12]+\langle 2P\rangle[2P]+\langle P1\rangle[P1])(\langle 34\rangle[34]-\langle 4P\rangle[4P]-\langle P3\rangle[P3]) \tag{1.6}$$

$$= \frac{1}{\mathrm{i}}(-\mathrm{i}\kappa)^2\frac{1}{P^2+\frac{P^4}{M^2}}(\langle 12\rangle[12]-\langle P2\rangle[2P]-\langle P1\rangle[1P])(\langle 34\rangle[34]+\langle P4\rangle[4P]+\langle P3\rangle[3P]) \tag{1.7}$$

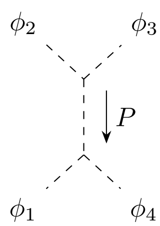
$$= \frac{1}{\mathrm{i}}(-\mathrm{i}\kappa)^2\frac{1}{P^2+\frac{P^4}{M^2}}(\langle 12\rangle[12]+\langle P|\,1+2|P])(\langle 34\rangle[34]-\langle P|\,3+4|P]) \tag{1.8}$$

$$= \frac{1}{\mathrm{i}}(-\mathrm{i}\kappa)^2\frac{1}{P^2+\frac{P^4}{M^2}}(\langle 12\rangle[12]-\langle P|\,P|P])(\langle 34\rangle[34]-\langle P|\,P|P]) \tag{1.9}$$

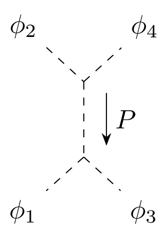
$$= \frac{1}{\mathrm{i}}(-\mathrm{i}\kappa)^2\frac{1}{P^2+\frac{P^4}{M^2}}\left(\langle 12\rangle[12]-2P^2\right)\left(\langle 34\rangle[34]-2P^2\right) \tag{1.10}$$

$$= -\mathrm{i}\frac{(\kappa M)^2}{s(M^2-s)}(\langle 12\rangle[12]+2s)(\langle 34\rangle[34]+2s) \tag{1.11}$$

It's trivial to read the t and u channels from this expression,



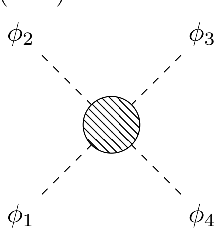
$$= -\mathrm{i}\frac{(\kappa M)^2}{t(M^2-t)}(\langle 23\rangle[23]+2t)(\langle 41\rangle[41]+2t) \tag{1.12}$$



$$= -\mathrm{i}\frac{(\kappa M)^2}{u(M^2-u)}(\langle 24\rangle[24]+2u)(\langle 31\rangle[31]+2u) \tag{1.13}$$

So that the full 4–point amplitude is,

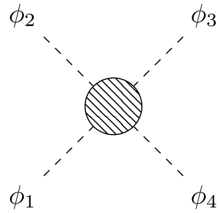
(1.14)



$$= -\mathrm{i}\frac{(\kappa M)^2}{stu(M^2-s)(M^2-t)(M^2-u)}\Big[(\langle 12\rangle[12]+2s)(\langle 34\rangle[34]+2s)tu\big(M^2-t\big)\big(M^2-u\big)+(\langle 23\rangle[23]+2t)(\langle 41\rangle[41]+2t$$

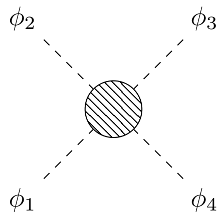
Let us specialize when $1, 2$ are massless and $3, 4$ are massive, then,

(1.15)



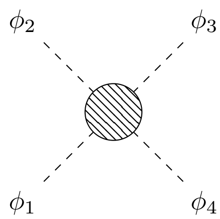
$$= -\mathrm{i}\frac{(\kappa M)^2}{stu(M^2-s)(M^2-t)(M^2-u)}\Big[s\big(-s+2M^2+2s\big)tu\big(M^2-t\big)\big(M^2-u\big)+\big(-t+M^2+2t\big)\big(-t+M^2+2t\big)su\big(M^2$$

(1.16)



$$= -\mathrm{i}\frac{(\kappa M)^2}{stu(M^2-s)(M^2-t)(M^2-u)}\Big[stu\big(2M^2+s\big)\big(M^2-t\big)\big(M^2-u\big)+su\big(M^2+t\big)\big(M^2+t\big)\big(M^2-s\big)\big(M^2-u\big)+st$$

(1.17)



$$= -\mathrm{i}\frac{(\kappa M)^2}{tu(M^2-s)(M^2-t)(M^2-u)}\Big[tu\big(2M^2+s\big)\big(M^2-t\big)\big(M^2-u\big)+u\big(M^2+t\big)^2\big(M^2-s\big)\big(M^2-u\big)+t\big(M^2+u\big)^2\big($$

2. Conformal Toy Model

Consider the following Lagrangian,

$$\mathcal{L} = -\frac{1}{2}\Box\phi\Box\phi - \frac{g}{2}\phi^2\Box\phi - \frac{g^2}{8}\phi^4 + m^2\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{g}{3!}\phi^3\right)$$