Homework II

Vicente V. Figueira — NUSP 11809301

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Contents

Problem 1

1.A)

Let M be our $D > 2$ dimensional C^∞ manifold and $\phi : \mathbb{R} \times M \to M$ be a one parameter family of diffeomorphisms, that is, $\forall t \in \mathbb{R}|\ \phi_t : M \to M$ is a diffeomorphism such that,

- $\forall p \in M|\ \phi_0(p) = p$
- $\forall p \in M; \forall t, s \in \mathbb{R}|\ \phi_{t+s}(p) = (\phi_t \circ \phi_s)(p)$
- $\forall p \in M|\ \phi(p) : \mathbb{R} \to M$ is at least C^1

Then this family of diffeomorphisms define in a natural manner a vector which generate these transformations, we define at each point $p \in M$ this vector by it's action in a function $f : M \to \mathbb{R}$,

$$\xi_p(f) = \frac{\mathrm{d}}{\mathrm{d}t}((f \circ \phi_t)(p))\bigg|_{t=0}$$

And from this we define ξ as a vector field in M, this vector field has as integral curves exactly ϕ. Let's open a little bit more in some chart $x : M \to \mathbb{R}^D$,

$$\begin{aligned}
\xi_p(f) &= \frac{\mathrm{d}}{\mathrm{d}t}\big((f \circ x^{-1} \circ x \circ \phi_t)(p)\big)\bigg|_{t=0} \\
\xi_p(f) &= \frac{\mathrm{d}}{\mathrm{d}t}\big((f \circ x^{-1}) \circ (x \circ \phi_t)(p)\big)\bigg|_{t=0} \\
\xi_p(f) &= \partial_\mu\big(f \circ x^{-1}\big)\bigg|_{x\circ\phi_0(p)} \frac{\mathrm{d}}{\mathrm{d}t}((x \circ \phi_t)^\mu(p))\bigg|_{t=0} \\
\xi_p^\mu \partial_\mu\big(f \circ x^{-1}\big)\bigg|_{x(p)} &= \partial_\mu\big(f \circ x^{-1}\big)\bigg|_{x(p)} \frac{\mathrm{d}}{\mathrm{d}t}((x \circ \phi_t)^\mu(p))\bigg|_{t=0}
\end{aligned}$$

Where of curse ∂_μ is to be interpreted as the derivative of the μ-th component in the chart x. Here we have a clear definition of the values of the ξ vector field in a chart x,

$$\xi_p^\mu = \frac{\mathrm{d}}{\mathrm{d}t}((x \circ \phi_t)^\mu(p))\bigg|_{t=0}$$

The term inside the derivative is just the pullback of the chart x — the chart can be seen as a $\mathbb{R}^D$-valued function —, which in it's own can be seen as a new chart x'_t defined by the transformations of the diffeomorphism family ϕ, that is,

$$x'_t = \phi_t^* x = x \circ \phi_t : M \to \mathbb{R}^D$$

All of this is consistent with our interpretation of the diffeomorphisms being a '*coordinate change*', in principle, with enough derivability of ϕ we can actually write,

$$\begin{aligned}
x'_t &= x'_0 + t\frac{\mathrm{d}}{\mathrm{d}t}(x'_t)\bigg|_{t=0} + \mathcal{O}(t^2) \\
{x'_1}^\mu =: x'^\mu &= x^\mu + \xi^\mu + \cdots
\end{aligned}$$

We just restored the index to not confuse the components of the vector field ξ in the basis x with the vector field itself. That is, we showed that the transformation done by ϕ_1 is equivalent to a '*infinitesimal coordinate change*' by ξ^μ. Actually, all this we did is the special case of a more general type of derivative, the Lie Derivative, given a vector field ξ and it's family of integral curves ϕ, it's defined in terms of the pushforward of the object under analysis,

$$\pounds_\xi T = \frac{\mathrm{d}}{\mathrm{d}t}(\phi_{-t*}T)\bigg|_{t=0}$$

For a $(0,2)$ tensor, that is, for the metric,

$$\pounds_\xi g_{\mu\nu} = \nabla_\xi g_{\mu\nu} + g_{\mu\alpha}\nabla_\nu \xi^\alpha + g_{\alpha\nu}\nabla_\mu \xi^\alpha$$

And as the connection is metric compatible,

$$\pounds_\xi g_{\mu\nu} = \nabla_\nu \xi_\mu + \nabla_\mu \xi_\nu = 2\nabla_{(\mu}\xi_{\nu)}$$

This amounts for the first term in an expansion in t of the diffeomorphism transformed metric $\phi_{-t*}g = g'_{t\mu\nu}$, that is,

$$\begin{aligned} \phi_{-t*}g_{\mu\nu} =: g'_{t\mu\nu} &= g_{\mu\nu} + t\pounds_\xi g_{\mu\nu} + \mathcal{O}\big(t^2\big) \\ g'_{t\mu\nu} &= g_{\mu\nu} + 2t\nabla_{(\mu}\xi_{\nu)} + \mathcal{O}\big(t^2\big) \end{aligned}$$

Which also can be interpreted as the '*infinitesimal transformation*' of the metric. Imposing that the initial and transformed metric are conformally flat,

$$\begin{aligned} \exp{(2\omega'_t)}\eta_{\mu\nu} &= \exp{(2\omega)}\eta_{\mu\nu} + 2t\nabla_{(\mu}\xi_{\nu)} + \mathcal{O}\big(t^2\big) \\ \exp\left(2\omega'_0 + 2t\frac{\mathrm{d}}{\mathrm{d}t}(\omega'_t)\bigg|_{t=0} + \mathcal{O}\big(t^2\big)\right)\eta_{\mu\nu} &= \exp{(2\omega)}\eta_{\mu\nu} + 2t\nabla_{(\mu}\xi_{\nu)} + \mathcal{O}\big(t^2\big) \\ \exp{(2\omega)}\exp\left(2t\frac{\mathrm{d}}{\mathrm{d}t}(\omega'_t)\bigg|_{t=0} + \mathcal{O}\big(t^2\big)\right)\eta_{\mu\nu} &= \exp{(2\omega)}\eta_{\mu\nu} + 2t\nabla_{(\mu}\xi_{\nu)} + \mathcal{O}\big(t^2\big) \\ \exp{(2\omega)}\left(1 + 2t\frac{\mathrm{d}}{\mathrm{d}t}(\omega'_t)\bigg|_{t=0} + \mathcal{O}\big(t^2\big)\right)\eta_{\mu\nu} &= \exp{(2\omega)}\eta_{\mu\nu} + 2t\nabla_{(\mu}\xi_{\nu)} + \mathcal{O}\big(t^2\big) \\ \exp{(2\omega)}2t\frac{\mathrm{d}}{\mathrm{d}t}(\omega'_t)\bigg|_{t=0}\eta_{\mu\nu} &= 2t\nabla_{(\mu}\xi_{\nu)} \\ \frac{\mathrm{d}}{\mathrm{d}t}(\omega'_t)\bigg|_{t=0} g_{\mu\nu} &= \nabla_{(\mu}\xi_{\nu)} \end{aligned}$$

The term $\frac{\mathrm{d}}{\mathrm{d}t}(\omega'_t)\big|_{t=0}$ is fully determined by ξ, to see this just contract both sides with the metric,

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}(\omega'_t)\bigg|_{t=0} g^{\mu\nu}g_{\mu\nu} &= g^{\mu\nu}\nabla_{(\mu}\xi_{\nu)} \\ \frac{\mathrm{d}}{\mathrm{d}t}(\omega'_t)\bigg|_{t=0} D &= \nabla_\mu \xi^\mu \end{aligned}$$

Substituting this back in the original equation,

$$\nabla_\alpha \xi^\alpha g_{\mu\nu} = D\nabla_{(\mu}\xi_{\nu)} \tag{1.1}$$

This is the condition upon ξ that ensures the diffeomorphism maintain the conformally flatness of the metric. To solve it in it's full generality is hard, so, we'll make some use of symmetry, first, let's write it again in a coordinate free form,

$$\mathrm{Div}_g(\xi)g = \frac{D}{2}\pounds_\xi g \tag{1.2}$$

Where $\mathrm{Div}_g(\xi)$ is just $\nabla_\alpha \xi^\alpha$, but with the connection ∇ defined with respect with the metric g, now, suppose we have a function $f : M \to \mathbb{R}$, define $\tilde{g} = \exp(2f)g$, thus,

$$\begin{aligned}
\pounds_\xi \tilde{g} &= \pounds_\xi(\exp(2f)g) \\
\pounds_\xi \tilde{g} &= \exp(2f)\pounds_\xi g + g\pounds_\xi \exp(2f) \\
\pounds_\xi \tilde{g} &= \exp(2f)\pounds_\xi g + g\xi(\exp(2f)) \\
\pounds_\xi \tilde{g} &= \exp(2f)\pounds_\xi g + g\xi(f)\exp(2f) \\
\pounds_\xi \tilde{g} &= \exp(2f)\pounds_\xi g + \tilde{g}\xi(2f)
\end{aligned} \tag{1.3}$$

Also,

$$\begin{aligned}
\mathrm{Div}_g(\xi) &= \partial_\alpha \xi^\alpha + \Gamma^\alpha_{\alpha\lambda}\xi^\lambda \\
\mathrm{Div}_g(\xi) &= \partial_\alpha \xi^\alpha + \frac{1}{2}g^{\alpha\beta}(\partial_\alpha g_{\beta\lambda} + \partial_\lambda g_{\beta\alpha} - \partial_\beta g_{\alpha\lambda})\xi^\lambda \\
\mathrm{Div}_g(\xi) &= \partial_\alpha \xi^\alpha + g^{\alpha\beta}(\partial_\alpha \omega g_{\beta\lambda} + \partial_\lambda \omega g_{\beta\alpha} - \partial_\beta \omega g_{\alpha\lambda})\xi^\lambda \\
\mathrm{Div}_g(\xi) &= \partial_\alpha \xi^\alpha + (\partial_\lambda \omega + D\partial_\lambda \omega - \partial_\lambda \omega)\xi^\lambda \\
\mathrm{Div}_g(\xi) &= \partial_\alpha \xi^\alpha + D\partial_\lambda \omega \xi^\lambda
\end{aligned}$$

And as $\tilde{g} = \exp(2f)g = \exp(2(f+w))\eta$,

$$\begin{aligned}
\mathrm{Div}_{\tilde{g}}(\xi) &= \partial_\alpha \xi^\alpha + D\partial_\lambda(f+\omega)\xi^\lambda \\
\mathrm{Div}_{\tilde{g}}(\xi) &= \partial_\alpha \xi^\alpha + D\partial_\lambda \omega \xi^\lambda + D\xi^\lambda \partial_\lambda f \\
\mathrm{Div}_{\tilde{g}}(\xi) &= \mathrm{Div}_g(\xi) + D\xi(f)
\end{aligned} \tag{1.4}$$

Multiplying 1.4 by g and subtracting 1.3,

$$\begin{aligned}
\mathrm{Div}_{\tilde{g}}(\xi)\tilde{g} - \frac{D}{2}\pounds_\xi \tilde{g} &= \mathrm{Div}_g(\xi)\tilde{g} + D\tilde{g}\xi(f) - \frac{D}{2}\exp(2f)\pounds_\xi g - \frac{D}{2}\tilde{g}\xi(2f) \\
\mathrm{Div}_{\tilde{g}}(\xi)\tilde{g} - \frac{D}{2}\pounds_\xi \tilde{g} &= \exp(2f)\mathrm{Div}_g(\xi)g - \exp(2f)\frac{D}{2}\pounds_\xi g + D\tilde{g}\xi(f) - D\tilde{g}\xi(f) \\
\mathrm{Div}_{\tilde{g}}(\xi)\tilde{g} - \frac{D}{2}\pounds_\xi \tilde{g} &= \exp(2f)\left[\mathrm{Div}_g(\xi)g - \frac{D}{2}\pounds_\xi g\right]
\end{aligned}$$

That is, as long as f is sufficiently well behaved, we have,

$$\mathrm{Div}_g(\xi)g = \frac{D}{2}\pounds_\xi g \Leftrightarrow \mathrm{Div}_{\tilde{g}}(\xi)\tilde{g} = \frac{D}{2}\pounds_\xi \tilde{g} \tag{1.5}$$

In other words, the vector field ξ which generates diffeomorphisms that preserves the conformally flat condition, does not depend on ω of our conformally flat metric $g = \exp(2\omega)\eta$, so, we can choose $f = -\omega$, such that $\tilde{g} = \eta$, and by 1.5, we just have to solve for,

$$\partial_\alpha \xi^\alpha \eta_{\mu\nu} = D\partial_{(\mu}\xi_{\nu)} \tag{1.6}$$

Which is a lot easier then 1.1, first, we apply ∂^ν to the both sides, relabel the index, apply ∂_ν, symmetrize $\mu \leftrightarrow \nu$ and use 1.6,

$$\begin{aligned}
\partial^\nu\partial_\alpha\xi^\alpha\eta_{\mu\nu} &= D\partial^\nu\partial_{(\mu}\xi_{\nu)} \\
\frac{2}{D}\partial_\mu\partial_\alpha\xi^\alpha &= \partial^\nu\partial_\mu\xi_\nu + \partial^\nu\partial_\nu\xi_\mu \\
\frac{2}{D}\partial_\mu\partial_\alpha\xi^\alpha &= \partial^\alpha\partial_\mu\xi_\alpha + \partial^\alpha\partial_\alpha\xi_\mu \\
\frac{2}{D}\partial_\nu\partial_\mu\partial_\alpha\xi^\alpha &= \partial_\nu\partial^\alpha\partial_\mu\xi_\alpha + \partial_\nu\partial^\alpha\partial_\alpha\xi_\mu \\
\frac{2}{D}\partial_{(\nu}\partial_{\mu)}\partial_\alpha\xi^\alpha &= \partial_{(\nu}\partial_{\mu)}\partial^\alpha\xi_\alpha + \partial^\alpha\partial_\alpha\partial_{(\nu}\xi_{\mu)} \\
\frac{2}{D}\partial_\nu\partial_\mu\partial_\alpha\xi^\alpha &= \partial_\nu\partial_\mu\partial_\alpha\xi^\alpha + \frac{\eta_{\mu\nu}}{D}\partial^\alpha\partial_\alpha\partial_\beta\xi^\beta \\
(2-D)\partial_\nu\partial_\mu\partial_\alpha\xi^\alpha &= \eta_{\mu\nu}\partial^\alpha\partial_\alpha\partial_\beta\xi^\beta
\end{aligned} \tag{1.7}$$

Contracting with $\eta^{\mu\nu}$ gives,

$$\begin{aligned}
(2-D)\partial^\mu\partial_\mu\partial_\alpha\xi^\alpha &= D\partial^\alpha\partial_\alpha\partial_\beta\xi^\beta \\
2(1-D)\partial^\mu\partial_\mu\partial_\alpha\xi^\alpha &= 0 \\
\partial^\mu\partial_\mu\partial_\alpha\xi^\alpha &= 0
\end{aligned} \tag{1.8}$$

The last step is justified by merely $D \geq 2$, so that up to now, we haven't fully used the hypothesis of being in $D > 2$. So, we'll invoke it now, by using 1.8 in equation 1.7, this is only justified if $D \neq 2$, because, for $D = 2$ the left-hand side of 1.7 is identically zero, but, for $D > 2$ the use of 1.8 in 1.7 results in another constraint,

$$\begin{aligned}
(2-D)\partial_\mu\partial_\nu\partial_\alpha\xi^\alpha &= 0 \\
\partial_\mu\partial_\nu\partial_\alpha\xi^\alpha &= 0
\end{aligned} \tag{1.9}$$

It's clear that equation 1.7 allows for much more solutions than 1.9, for example, $\partial \cdot \xi = \cos(k_\mu x^\mu)$ with $k_\mu k^\mu = 0$ is a solution for 1.8, but, isn't for 1.9. To integrate 1.9 is also a lot easier than 1.8,

$$\begin{aligned}
\partial_\mu\partial_\nu\partial_\alpha\xi^\alpha &= 0 \\
\partial_\nu\partial_\alpha\xi^\alpha &= a_\nu
\end{aligned}$$

$$\partial_\alpha \xi^\alpha = a_\nu x^\nu + b \tag{1.10}$$

With a_ν, b arbitrary constants, this is also automatically a solution of 1.8. To integrate 1.10 is not trivial, as there might be some divergenceless current contributing to ξ^α, that is, the integration gives,

$$\begin{aligned}\partial_\alpha \xi^\alpha &= a_\nu x^\nu + b \\ \xi^\alpha &= \frac{1}{2} a^\alpha{}_{\mu\nu} x^\mu x^\nu + b^\alpha{}_\mu x^\mu + c^\alpha + f^\alpha \end{aligned} \tag{1.11}$$

With of course $a^\alpha{}_{\mu\nu} = a^\alpha{}_{(\mu\nu)}, a^\alpha{}_{\alpha\nu} = a_\nu, b^\alpha{}_\alpha = b, c^\alpha$ being constants, with exception of f^α, which is some non constant divergenceless vector field $\partial_\alpha f^\alpha = 0$. To know what choice of f^α is the correct one we have to go back to equation 1.6, and apply to it ∂_λ,

$$\frac{2}{D} \partial_\lambda \partial_\alpha \xi^\alpha \eta_{\mu\nu} = \partial_\lambda \partial_\mu \xi_\nu + \partial_\lambda \partial_\nu \xi_\mu$$

Now we permute the index,

$$\begin{aligned}\frac{2}{D} \partial_\lambda \partial_\alpha \xi^\alpha \eta_{\mu\nu} &= \partial_\lambda \partial_\mu \xi_\nu + \partial_\nu \partial_\lambda \xi_\mu \\ \frac{2}{D} \partial_\mu \partial_\alpha \xi^\alpha \eta_{\nu\lambda} &= \partial_\mu \partial_\nu \xi_\lambda + \partial_\lambda \partial_\mu \xi_\nu \\ \frac{2}{D} \partial_\nu \partial_\alpha \xi^\alpha \eta_{\lambda\mu} &= \partial_\nu \partial_\lambda \xi_\mu + \partial_\mu \partial_\nu \xi_\lambda \end{aligned}$$

Sum the two first equations and subtract the third one,

$$\frac{2}{D}(\partial_\lambda \partial_\alpha \xi^\alpha \eta_{\mu\nu} + \partial_\mu \partial_\alpha \xi^\alpha \eta_{\nu\lambda} - \partial_\nu \partial_\alpha \xi^\alpha \eta_{\lambda\mu}) = \partial_\lambda \partial_\mu \xi_\nu + \partial_\nu \partial_\lambda \xi_\mu + \partial_\mu \partial_\nu \xi_\lambda + \partial_\lambda \partial_\mu \xi_\nu - \partial_\nu \partial_\lambda \xi_\mu - \partial_\mu \partial_\nu \xi_\lambda$$

By use of 1.10,

$$\frac{2}{D}(a_\lambda \eta_{\mu\nu} + a_\mu \eta_{\nu\lambda} - a_\nu \eta_{\lambda\mu}) = 2 \partial_\lambda \partial_\mu \xi_\nu$$

And substituting 1.11 in the right-hand side,

$$\frac{2}{D}(a_\lambda \eta_{\mu\nu} + a_\mu \eta_{\nu\lambda} - a_\nu \eta_{\lambda\mu}) = 2 a_{\nu\mu\lambda} + 2 \partial_\lambda \partial_\mu f_\nu$$

We can thus use this to fix f_λ, it's trivial to carry out the integration,

$$\begin{aligned}\partial_\lambda \partial_\mu f_\nu &= \frac{1}{D}(a_\lambda \eta_{\mu\nu} + a_\mu \eta_{\nu\lambda} - a_\nu \eta_{\lambda\mu}) - a_{\nu\mu\lambda} \\ \partial_\mu f_\nu &= \frac{1}{D}(a \cdot x \eta_{\mu\nu} + a_\mu x_\nu - a_\nu x_\mu) - a_{\nu\mu\lambda} x^\lambda + A_{\nu\mu} \\ f_\nu &= \frac{1}{D}\left(a \cdot x x_\nu - \frac{1}{2} a_\nu x \cdot x\right) - \frac{1}{2} a_{\nu\mu\lambda} x^\lambda x^\mu + A_{\nu\mu} x^\mu + B_\nu \\ \partial_\mu f^\mu &= \frac{1}{D}(a \cdot x D + a \cdot x - a \cdot x) - a \cdot x + A^\mu{}_\mu = A^\mu{}_\mu \end{aligned} \tag{1.12}$$

Where we just computed also the divergence, which implies the constraint $A^{\mu}{}_{\mu} = 0$. Substituting this back in 1.11 get us,

$$\begin{aligned}
\xi^{\alpha} &= \frac{1}{2}a^{\alpha}{}_{\mu\nu}x^{\mu}x^{\nu} + b^{\alpha}{}_{\mu}x^{\mu} + c^{\alpha} + \frac{1}{D}\left(a \cdot x x^{\alpha} - \frac{1}{2}a^{\alpha}x \cdot x\right) - \frac{1}{2}a^{\alpha}{}_{\mu\lambda}x^{\lambda}x^{\mu} + A^{\alpha}{}_{\mu}x^{\mu} + B^{\alpha} \\
\xi^{\alpha} &= \frac{1}{D}\left(a \cdot x x^{\alpha} - \frac{1}{2}a^{\alpha}x \cdot x\right) + \left(A^{\alpha}{}_{\mu} + b^{\alpha}{}_{\mu}\right)x^{\mu} + (B^{\alpha} + c^{\alpha}) \\
\xi^{\alpha} &= \frac{1}{D}\left(a \cdot x x^{\alpha} - \frac{1}{2}a^{\alpha}x \cdot x\right) + b^{\alpha}{}_{\mu}x^{\mu} + c^{\alpha}
\end{aligned}$$

In the last line we just redefined the tensors. All of this with $a^{\alpha}, b^{\alpha}{}_{\mu}, c^{\alpha}$ arbitrary constants, but, we didn't really confirmed this is solution of 1.9 is a fully compatible with 1.6, we just checked it satisfy some derived equations from 1.6, now, let's put it to the real test,

$$\begin{aligned}
\partial_{\alpha}\xi^{\alpha}\eta_{\mu\nu} &= D\partial_{(\mu}\xi_{\nu)} \\
\left[\frac{1}{D}\left(a \cdot x D + a \cdot x - 2\frac{1}{2}a \cdot x\right) + b^{\alpha}{}_{\alpha}\right]\eta_{\mu\nu} &= D\left[\frac{1}{D}\big(a \cdot x\eta_{\mu\nu} + a_{(\mu}x_{\nu)} - a_{(\nu}x_{\mu)}\big) + b_{(\nu\mu)}\right] \\
a \cdot x\eta_{\mu\nu} + b^{\alpha}{}_{\alpha}\eta_{\mu\nu} &= a \cdot x\eta_{\mu\nu} + Db_{(\nu\mu)} \\
b^{\alpha}{}_{\alpha}\eta_{\mu\nu} &= Db_{(\nu\mu)} \\
b_{(\nu\mu)} &= \frac{1}{D}b^{\alpha}{}_{\alpha}\eta_{\mu\nu}
\end{aligned}$$

This constrains the symmetric part of $b_{\mu\nu}$ being a pure trace, so that the degrees of freedom reduces from $b_{\mu\nu}$ to $b_{[\mu\nu]}, b^{\alpha}{}_{\alpha}$. This is not the end! We haven't show all the solutions we found are in fact all the solutions from 1.6, as we obtained them from a different approach than direct integration of the equation 1.6, so let's show this, first, 1.6 is a set of first order partial differential equations, there are D^2 such equation, but, they're symmetric in exchange $\mu \leftrightarrow \nu$, so we have to account only for $\frac{D^2+D}{2}$ of them, still, we have to account for the possible constants which can be added to ξ without changing the equation, D of them, and also for another boundary condition on the divergence, 1, so that the full number of constants is,

$$\frac{D^2 + D}{2} + D + 1 = \frac{1}{2}(D + 1)(D + 2)$$

Now let's count the number of constants in the solution we found, namely,

$$\xi_{\alpha} = \frac{1}{D}\left(a \cdot x x_{\alpha} - \frac{1}{2}a_{\alpha}x \cdot x\right) + b_{[\alpha\mu]}x^{\mu} + \frac{1}{D}b^{\mu}{}_{\mu}x_{\alpha} + c^{\alpha}$$

Both c^{α}, a^{α} contributes with D, $b^{\mu}{}_{\mu}$ contributes with 1, and lastly $b_{[\alpha\mu]}$ contributes with $\frac{D^2-D}{2}$, hence, the total number of constants is,

$$\frac{D^2 - D}{2} + 2D + 1 = \frac{1}{2}(D + 1)(D + 2)$$

Matching exactly the number from the original equation, that is, we already got all the solutions, hence, the most general solution of 1.6, for $D > 2$, is given by,

$$\xi_\alpha = \frac{1}{D}\left(a \cdot x x_\alpha - \frac{1}{2} a_\alpha x \cdot x\right) + b_{[\alpha\mu]} x^\mu + \frac{1}{D} b^\mu{}_\mu x_\alpha + c^\alpha$$

1.B)

Problem 2

2.A)

2.B)

Problem 3

3.A)

3.B)

3.C)

3.D)

3.E)

3.F)

Problem 4

4.A)

4.B)

4.C)

4.D)

4.E)

Problem 5

5.A)

5.B)

Problem 6

6.A)

6.B)

6.C)

6.D)

A Appendix