ADTGR SEMINAR

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1. Introduction

Einstein-Hilbert action:

$$S_{\mathrm{EH}} = \frac{1}{\kappa^2}\int\limits_M \mathrm{d}^4x\,\sqrt{|g|}g^{ab}R^c{}_{acb}$$

The Vierbein/Tetrad formalism:

$$\begin{aligned}
\eta_{\mu\nu} &= \mathbf{g}(\mathbf{e}_\mu, \mathbf{e}_\nu)\\
g_{ab} &= e^\mu{}_a e^\nu{}_b \eta_{\mu\nu}\\
\eta_{\mu\nu} &= e_\mu{}^a e_\nu{}^b g_{ab}
\end{aligned}$$

A connection is defined with respect to a vector basis:

$$\begin{aligned}
\nabla_{\mathbf{X}}(\mathbf{e}_\nu) &= \boldsymbol{\omega}(\mathbf{X})^\mu{}_\nu \mathbf{e}_\mu\\
\nabla_a e_\nu{}^b &= \omega_a{}^\mu{}_\nu e_\mu{}^b\\
e_\alpha{}^c g_{cb}\nabla_a e_\nu{}^b &= e_\alpha{}^c g_{cb}\omega_a{}^\mu{}_\nu e_\mu{}^b\\
e_\alpha{}^c g_{cb}\nabla_a e_\nu{}^b + e_\alpha{}^c e_\nu{}^b \nabla_a g_{cb} &= \omega_{a\alpha\nu}\\
e_\alpha{}^c \nabla_a\big(g_{cb}e_\nu{}^b\big) &= \omega_{a\alpha\nu}\\
\nabla_a\big(e_\alpha{}^c g_{cb} e_\nu{}^b\big) - g_{cb}e_\nu{}^b\nabla_a e_\alpha{}^c &= \omega_{a\alpha\nu}\\
\nabla_a(\eta_{\alpha\nu}) - g_{cb}e_\nu{}^b\nabla_a e_\alpha{}^c &= \omega_{a\alpha\nu}\\
-g_{cb}e_\nu{}^b\nabla_a e_\alpha{}^c &= \omega_{a\alpha\nu}\\
-\omega_{a\nu\alpha} &= \omega_{a\alpha\nu}, \quad \text{Metric compatibility} \qquad (1.1)
\end{aligned}$$

Riemann curvature tensor, $\mathbf{Riem}(\mathbf{X},\mathbf{Y}) : \mathfrak{X} \to \mathfrak{X}$:

$$\begin{aligned}
\mathbf{Riem}(\mathbf{X},\mathbf{Y})\mathbf{e}_\mu &= \big(\nabla_{\mathbf{X}}\nabla_{\mathbf{Y}} - \nabla_{\mathbf{Y}}\nabla_{\mathbf{X}} - \nabla_{[\mathbf{X},\mathbf{Y}]}\big)\mathbf{e}_\mu\\
\mathbf{Riem}(\mathbf{X},\mathbf{Y})\mathbf{e}_\mu &= \Big(\nabla_{\mathbf{X}}\big(Y^b\omega_b{}^\nu{}_\mu\mathbf{e}_\nu\big) - \nabla_{\mathbf{Y}}\big(X^a\omega_a{}^\nu{}_\mu\mathbf{e}_\nu\big) - [\mathbf{X},\mathbf{Y}]^b\omega_b{}^\nu{}_\mu\mathbf{e}_\nu\Big)\\
\mathbf{Riem}(\mathbf{X},\mathbf{Y})\mathbf{e}_\mu &= \Big(\nabla_{\mathbf{X}}\big(Y^b\omega_b{}^\nu{}_\mu\mathbf{e}_\nu\big) - \nabla_{\mathbf{Y}}\big(X^a\omega_a{}^\nu{}_\mu\mathbf{e}_\nu\big) - [\mathbf{X},\mathbf{Y}]^b\omega_b{}^\nu{}_\mu\mathbf{e}_\nu\Big)\\
\mathbf{Riem}(\mathbf{X},\mathbf{Y})\mathbf{e}_\mu &= \big(Y^b\nabla_{\mathbf{X}}\big(\omega_b{}^\nu{}_\mu\mathbf{e}_\nu\big) - X^a\nabla_{\mathbf{Y}}\big(\omega_a{}^\nu{}_\mu\mathbf{e}_\nu\big)\\
&\qquad + \nabla_{\mathbf{X}}\big(Y^b\big)\omega_b{}^\nu{}_\mu\mathbf{e}_\nu - \nabla_{\mathbf{Y}}\big(X^b\big)\omega_b{}^\nu{}_\mu\mathbf{e}_\nu - [\mathbf{X},\mathbf{Y}]^b\omega_b{}^\nu{}_\mu\mathbf{e}_\nu\Big)\\
\mathbf{Riem}(\mathbf{X},\mathbf{Y})\mathbf{e}_\mu &= \big(X^aY^b\nabla_a\big(\omega_b{}^\nu{}_\mu\mathbf{e}_\nu\big) - X^aY^b\nabla_b\big(\omega_a{}^\nu{}_\mu\mathbf{e}_\nu\big)\\
&\qquad + \big(\nabla_{\mathbf{X}}\big(Y^b\big) - \nabla_{\mathbf{Y}}\big(X^b\big)\big)\omega_b{}^\nu{}_\mu\mathbf{e}_\nu - [\mathbf{X},\mathbf{Y}]^b\omega_b{}^\nu{}_\mu\mathbf{e}_\nu\Big)\\
\mathbf{Riem}(\mathbf{X},\mathbf{Y})\mathbf{e}_\mu &= \big(X^aY^b\nabla_a\big(\omega_b{}^\nu{}_\mu\big)\mathbf{e}_\nu + X^aY^b\omega_b{}^\nu{}_\mu\omega_a{}^\alpha{}_\nu\mathbf{e}_\alpha - X^aY^b\nabla_b\big(\omega_a{}^\nu{}_\mu\big)\mathbf{e}_\nu - X^aY^b\omega_a{}^\nu{}_\mu\omega_b{}^\alpha{}_\nu\mathbf{e}_\alpha)\\
&\qquad + [\mathbf{X},\mathbf{Y}]^b\omega_b{}^\nu{}_\mu\mathbf{e}_\nu - [\mathbf{X},\mathbf{Y}]^b\omega_b{}^\nu{}_\mu\mathbf{e}_\nu\Big)\\
\mathbf{Riem}(\mathbf{X},\mathbf{Y})\mathbf{e}_\mu &= X^aY^b\left(\nabla_a\big(\omega_b{}^\nu{}_\mu\big) + \omega_b{}^\alpha{}_\mu\omega_a{}^\nu{}_\alpha - \nabla_b\big(\omega_a{}^\nu{}_\mu\big) - \omega_a{}^\alpha{}_\mu\omega_b{}^\nu{}_\alpha\right)\mathbf{e}_\nu\\
\mathbf{Riem}(\mathbf{X},\mathbf{Y})\mathbf{e}_\mu &= X^aY^bR_{ab}{}^\nu{}_\mu\mathbf{e}_\nu
\end{aligned}$$

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