SCALAR PROXY

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1. Cut solutions

Of course in each amplitude we have different cut solutions. Now let us solve them,

1.1. **all massless.** The cut condition is,

$$k_1^2 = k_2^2 = (3 - k_1)^2 = (3 - k_1 - k_2)^2 = (3 + 4 - k_1 - k_2)^2 = 0$$

The first and third condition enforces $k_1 = -|k_1]\langle 3|$. But the fourth and fifth conditions enforces $3 - k_1 - k_2 = n$, with $n \cdot 4 = 0$ & $n^2 = 0$. Lastly, the second condition imposes $(3 - k_1 - n)^2 = -23 \cdot n + 2k_1 \cdot n = 0$, that is,

$$[3n]\langle n3 \rangle = [k_1 n]\langle n3 \rangle$$

which has two solutions, $|n] = |k_1] - |3]$ & $|n\rangle = z|4\rangle$ or $|n\rangle = |3\rangle$ & $|n] = z|4]$. When working with scalar particles it's better to choose the first solution, as this avoids singularities in denominators such as $(k_1 \cdot k_2)^{-1}$. Hence, the solution we're going to choose is,

$$\begin{cases} k_1 &= -|k_1]\langle 3| \\ k_2 &= -|3]\langle 3| + |k_1]\langle 3| + z(|k_1] - |3])\langle 4| \end{cases}$$

1.2. **massive legs first topology.** Our approach to massive legs is to shift the solution with massless, in order to obtain a well behaved solution in the $m^2 \to 0$ limit. For this topology the cut constrains are,

$$l_1^2 = l_2^2 = (3 - l_1)^2 = -m^2 \ \& \ (3 - l_1 - l_2)^2 = (3 + 4 - l_1 - l_2)^2 = 0$$

The ideia here is to define, $l_i = k_i + \alpha_i q_i$ (no sum), with $q_i^2 = 0$ and $\alpha_i = -m^2(2k_i \cdot q_i)^{-1}$, then, q_i, k_i are not allowed to have any dependence on m^2. The first and second constrains are already satisfied. The third one gives,

$$-23 \cdot l_1 = 0 \to 3 \cdot (k_1 + \alpha_1 q_1) = 0 \to 3 \cdot q_1 = 0$$

As $|q_1\rangle = |3\rangle$ is forbidden, $|q_1] = |3]$. The fourth and fifth constrains imposes,

$$\begin{cases} -n \cdot (\alpha_1 q_1 + \alpha_2 q_2) + \alpha_1 \alpha_2 q_1 \cdot q_2 &= 0 \\ 4 \cdot (\alpha_1 q_1 + \alpha_2 q_2) &= 0 \end{cases}$$

This imposes actually $q_1 \cdot q_2 = 0$, for this to be true we have to options, either $|q_2] = |3]$, or $|q_2\rangle = |q_1\rangle$. If we choose the first, we can shift k_1 by 3 such to make $|q_1\rangle = |4\rangle$, this imposes further $|q_2\rangle = |4\rangle$. Hence, a possible solution is,

$$q_1 = q_2 = -|3]\langle 4|$$

1.3. **massive legs second topology.** The constrains now are slightly different,

$$l_1^2 = (3 - l_1)^2 = 0 \ \& \ l_2^2 = (3 - l_1 - l_2)^2 = (3 + 4 - l_1 - l_2)^2 = -m^2$$

Which has as solution $q_2 = -|4]\langle 3|$

1.4. **massive legs third topology.** Now the constrain is difficult to solve,

$$l_2^2 = 0 \ \& \ l_1^2 = (3 - l_1)^2 = (3 - l_1 - l_2)^2 = (3 + 4 - l_1 - l_2)^2 = -m^2$$

The first, second and third constrains give, $l_2 = -|l_2]\langle l_2|$, $l_1 = -|l_1]\langle 3| - \alpha|3]\langle l_1|$, which is get by shifting $|l_1]$. Now, the fourth constrain gives,

$$l_2 \cdot (3 - l_1) = 0$$

Expanding $|l_2] = x|l_1] + y|3]$,

$$x[l_1 3]\langle 3l_2 \rangle - y[3l_1]\langle 3l_2 \rangle - x\alpha[l_1 3]\langle l_1 l_2 \rangle = 0$$
$$(x + y)\langle 3l_2 \rangle = x\alpha\langle l_1 l_2 \rangle$$

The fifth constrain gives,

$$4 \cdot (3 - l_1 - l_2) = 0$$
$$[4|(-|3]\langle 3| + |l_1]\langle 3| + \alpha|3]\langle l_1| + x|l_1]\langle l_2| + y|3]\langle l_2|)|4\rangle = 0$$
$$-[43]\langle 34 \rangle + [4l_1]\langle 34 \rangle + \alpha[43]\langle l_1 4 \rangle + x[4l_1]\langle l_2 4 \rangle + y[43]\langle l_2 4 \rangle = 0$$
$$[4l_1](\langle 34 \rangle + x\langle l_2 4 \rangle) + [43](y\langle l_2 4 \rangle - \langle 34 \rangle + \alpha\langle l_1 4 \rangle) = 0$$

This fixes, $x\langle l_2 4 \rangle = -\langle 34 \rangle = -y\langle l_2 4 \rangle - \alpha\langle l_1 4 \rangle$, hence,

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