Numerical method of dipole domains

Governing equations:

Dipole Density Equation:

$$\frac{\partial \mathbf{D}}{\partial t} = \mathbf{V} \times \mathbf{\Omega} - \nabla\left(\mathbf{V}\mathbf{D} - \nu\nabla\mathbf{D}\right)$$

$$\frac{\partial}{\partial t}\left(\nabla \times \mathbf{D}\right) = \nabla \times \left(\mathbf{V} \times \mathbf{\Omega}\right)$$

The Euler equation:

$$\frac{\partial \mathbf{V}}{\partial t} = \mathbf{V} \times \mathbf{\Omega} - \nabla\left(\frac{p}{\rho} + V^2\right), \quad \nabla \cdot \mathbf{V} = 0$$

$$\frac{\partial \mathbf{\Omega}}{\partial t} = \nabla \times \left(\mathbf{V} \times \mathbf{\Omega}\right)$$

$$\nabla \times \mathbf{D} = \mathbf{\Omega}$$

Biot-Savarat formula $\mathbf{V}\left(\mathbf{R}\right) = \frac{1}{4\pi}\int \frac{\mathbf{r} - \mathbf{R}}{\left|\mathbf{r} - \mathbf{R}\right|^3} \times \left(\nabla \times \mathbf{D}\right) \mathrm{d}\tau + \mathbf{V}_\infty, \quad \mathbf{r} \in \tau.$

$$\mathbf{V}\left(\mathbf{R}\right) = \frac{2}{3}\mathbf{D}\left(\mathbf{R}\right) + \frac{1}{4\pi}\int\left(-\frac{\mathbf{D}}{\left|\mathbf{r} - \mathbf{R}\right|^3} + \frac{3\left(\mathbf{r} - \mathbf{R}\right)\left(\left(\mathbf{r} - \mathbf{R}\right) \cdot \mathbf{D}\right)}{\left|\mathbf{r} - \mathbf{R}\right|^5}\right)\mathrm{d}\tau.$$

Numerical scheme

Parameters of *i*-th dipole particle:

strength (dipole moment) ς_i

coordinates $\mathbf{r}_i$

the particle radius ε_i

Dipole density distribution $$\mathbf{D}(\mathbf{r}) = \sum_i \mathbf{D}_i(\mathbf{r})$$

$$\mathbf{D}_i(\mathbf{r}) = \begin{cases} \dfrac{\varsigma_i}{\varepsilon_i^3} f(\xi_i), & \xi < 1, \\ 0, & \xi \geq 1, \end{cases} \qquad \xi = \frac{\left|\mathbf{r} - \mathbf{r}_i\right|}{\varepsilon_i}, \quad f(\xi_i) = \frac{105}{16\pi}(1+3\xi)(1-\xi)^3$$

$$\frac{\partial \mathbf{r}_i}{\partial t} = \mathbf{V}(\mathbf{r}_i), \qquad \frac{\partial \varsigma_i}{\partial t} = -(\varsigma_i \nabla)\mathbf{V} - \varsigma_i \times \mathbf{\Omega}$$

The dipole particles interaction

$$\frac{\partial \mathbf{r}_i}{\partial t} = \sum_j \mathbf{v}_{ij} + \mathbf{V}_\infty, \qquad \frac{\partial \boldsymbol{\varsigma}_i}{\partial t} = -\sum_j \boldsymbol{\delta}_{ij}$$

$$\mathbf{v}_{ij} = -\frac{\boldsymbol{\varsigma}_j a}{r'^3} + \frac{3b\left(\boldsymbol{\varsigma}_j \mathbf{r}_{ij}\right)\mathbf{r}_{ij}}{r'^5}, \quad \mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j,$$

$$a = \eta_j - \sigma_j, \quad b = \eta_j - \frac{\sigma_j}{3}, \quad \eta_j = \int_0^{r'/\varepsilon} \xi^2 f(\xi)\, d\xi, \quad \sigma_j = \frac{r_{ij}^3}{\varepsilon^3} f\left(\frac{r_{ij}}{\varepsilon}\right).$$

$$\boldsymbol{\delta}_{ij} = \frac{\left(\boldsymbol{\varsigma}_i \boldsymbol{\varsigma}_n\right)\left(3a - a' r_{ij}\right)}{r^5}\mathbf{r} + \frac{3b\left(\left(\boldsymbol{\varsigma}_i \mathbf{r}_{ij}\right)\boldsymbol{\varsigma}_n + \left(\boldsymbol{\varsigma}_n \mathbf{r}_{ij}\right)\boldsymbol{\varsigma}_i\right)}{r^5} - \frac{3\left(5b - b' r_{ij}\right)\left(\boldsymbol{\varsigma}_n \mathbf{r}_{ij}\right)\left(\boldsymbol{\varsigma}_i \mathbf{r}_{ij}\right)}{r^7}\mathbf{r}_{ij}$$

$$\boldsymbol{\delta}_{ij} = -\boldsymbol{\delta}_{ji}$$

Computing cycle

1. Calculation of the dipole moments of the attached particles satisfying the boundary conditions $\mathbf{n} \cdot \mathbf{V} = \mathbf{n} \cdot \mathbf{V}_s$
2. The calculation of the particles velocity
3. Calculation of changes in the dipole moments as a result of particles interactions
4. Displacement of the particles, including particles at the trailing edge

The force calculation

$$\mathbf{F} = -\frac{d\mathbf{I}}{dt}$$

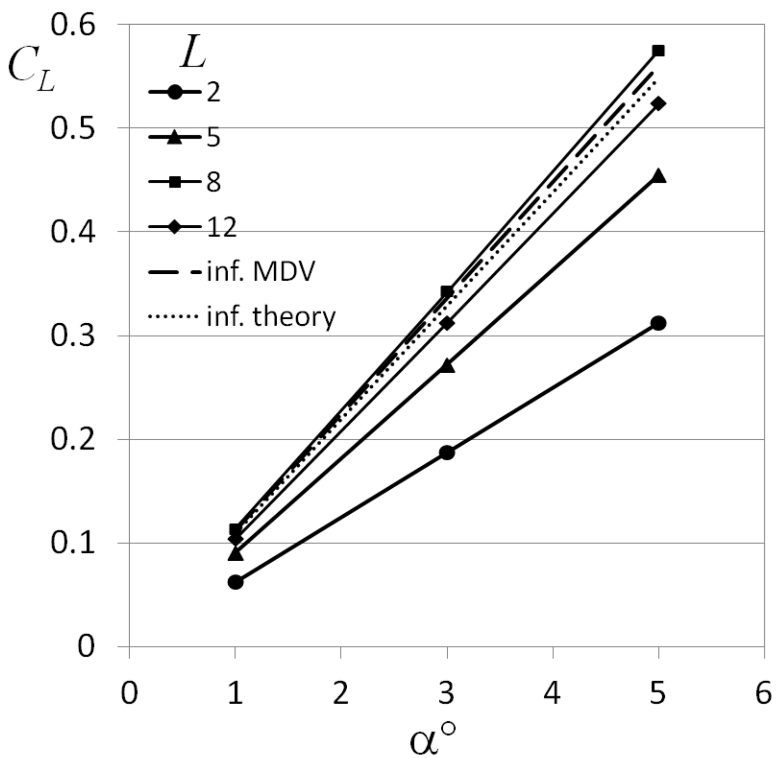
$\mathbf{I}$ is the hydrodynamic impulse of the fluid

$$\mathbf{I} = \frac{1}{2}\int_{\tau} \mathbf{r} \times \boldsymbol{\Omega}\, d\tau = \frac{1}{2}\int_{\tau} \mathbf{r} \times \left(\nabla \times \mathbf{D}\right) d\tau =$$

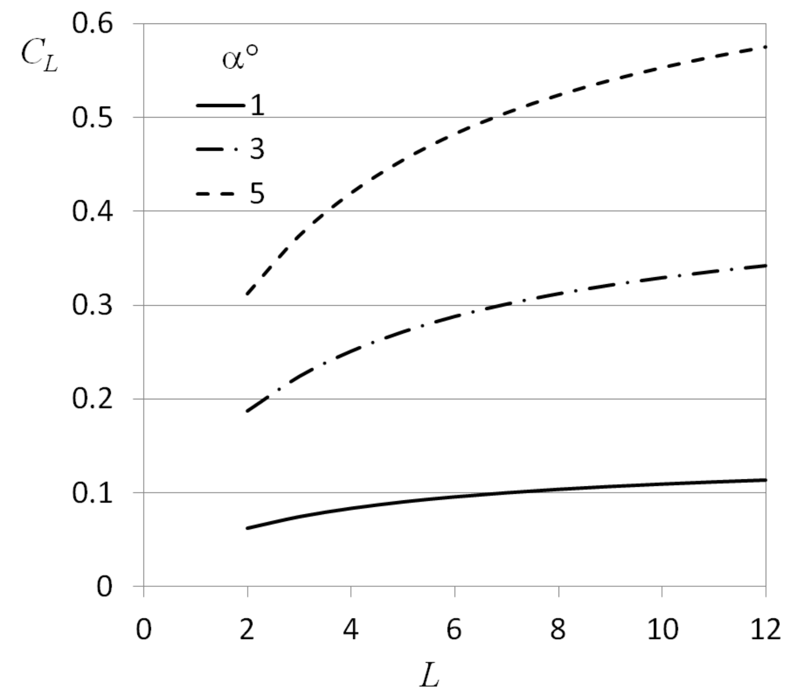
$$= \frac{1}{2}\sum_{i}\int_{\tau} \mathbf{r} \times \left(\nabla \times \mathbf{D}_i\right) d\tau = \frac{1}{2}\sum_{i}\int_{\tau} \mathbf{r} \times \left(\nabla f(\xi_i) \times \frac{\boldsymbol{\varsigma}_i}{\varepsilon_i^3}\right) d\tau = \sum_{i} \boldsymbol{\varsigma}_i$$

$$\mathbf{I} = \sum_{i} \boldsymbol{\varsigma}_i$$

Testing of the method.
The flow around the thin unmoving plate.

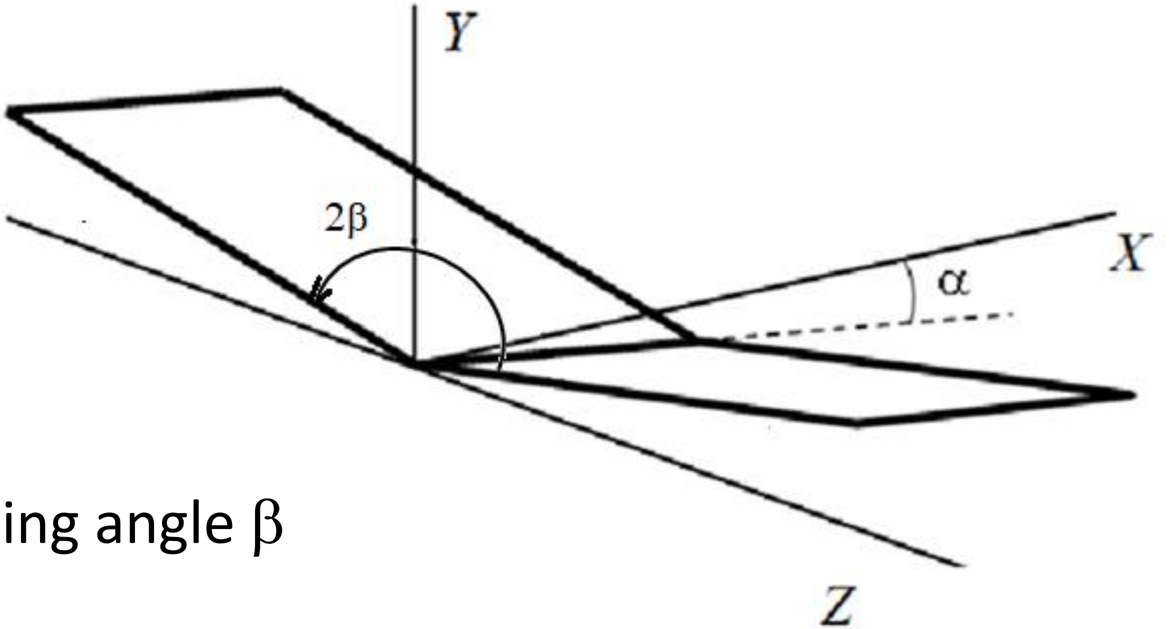


The lift coefficient dependence on the attack angle at different span *L*



The lift coefficient dependence on the span at different attack angle

The butterfly model

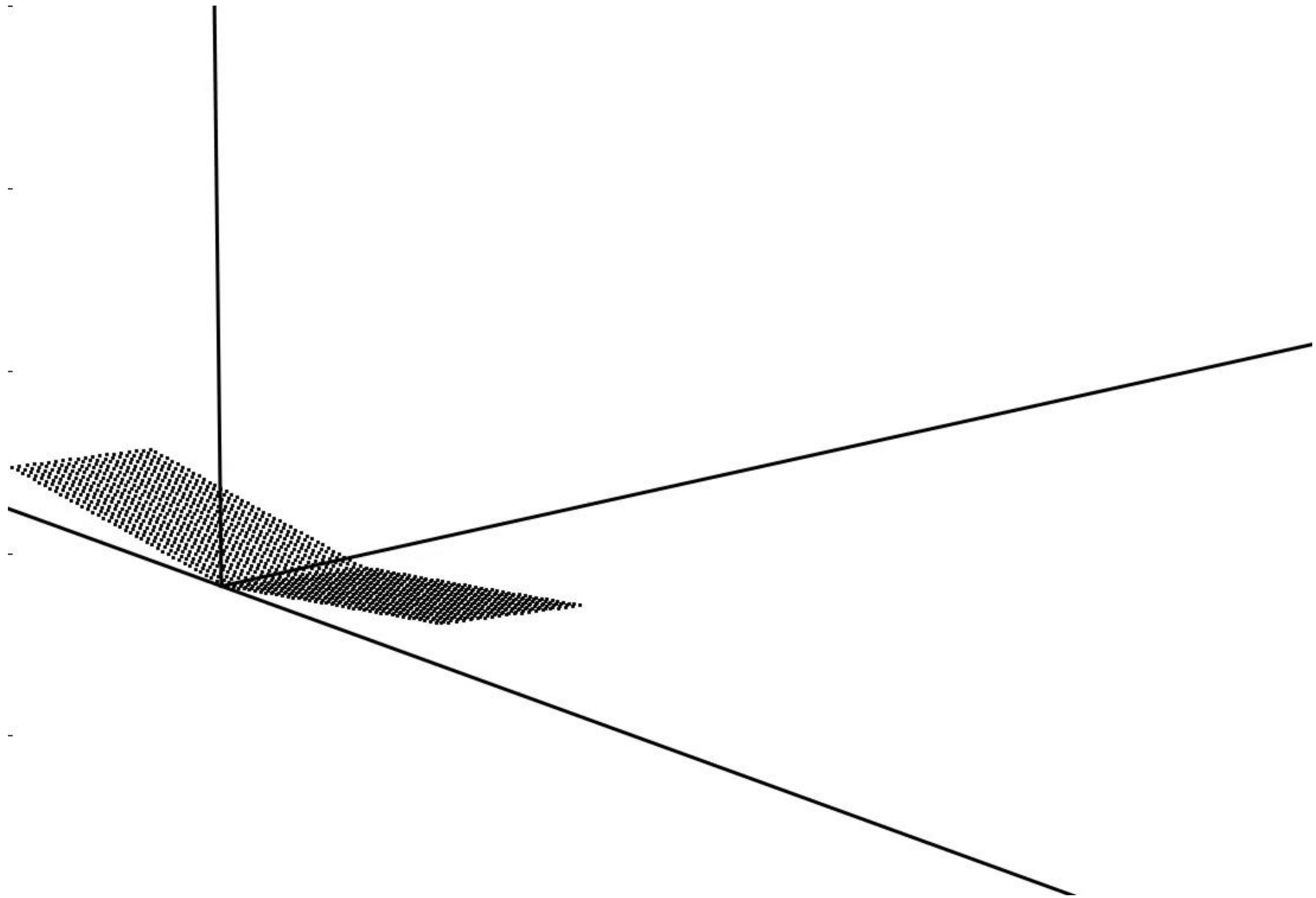


attack angle α

opening angle β

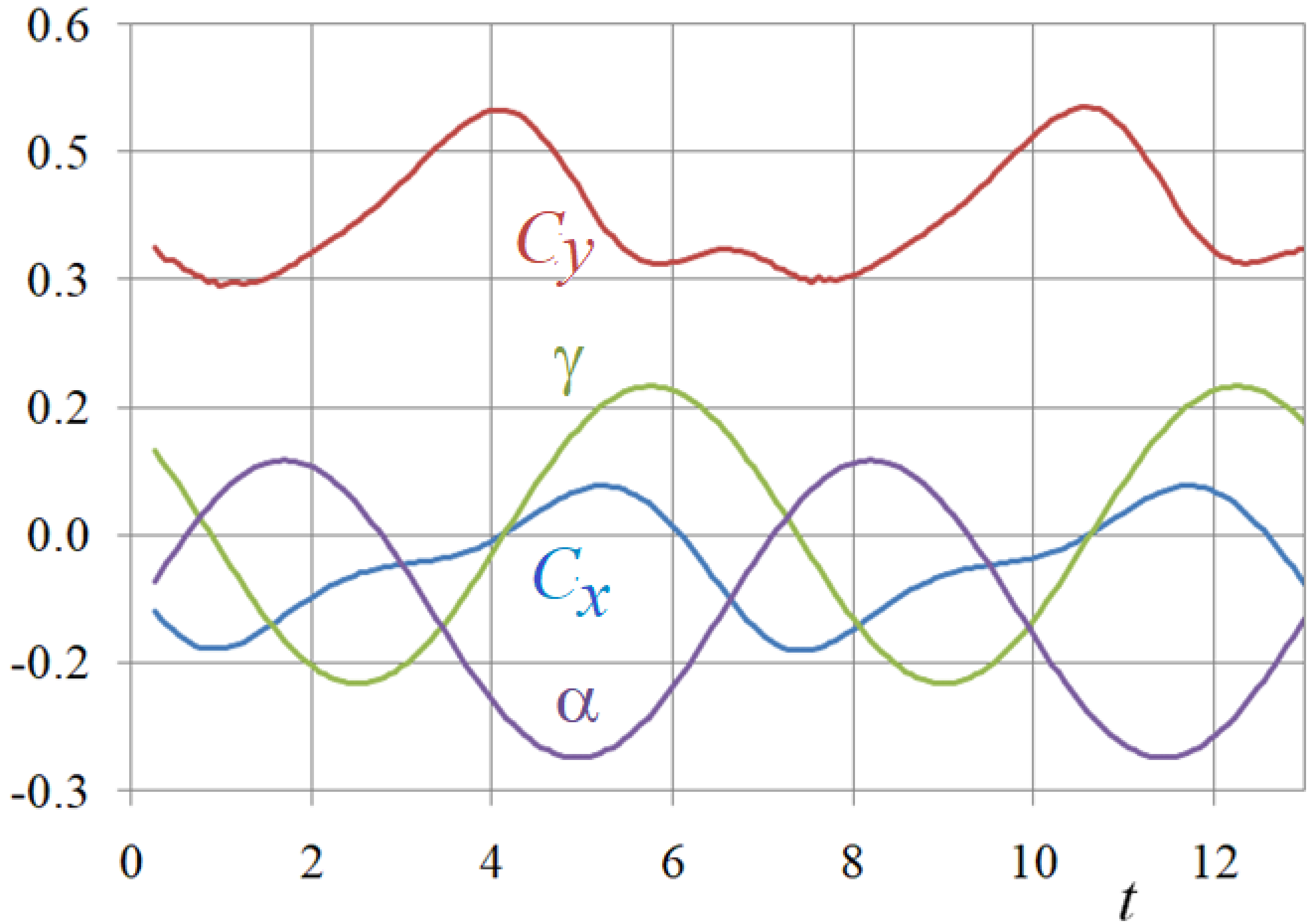
$$\beta = \pi/2 - \gamma(t), \qquad \gamma(t) = \gamma_0 \sin(2\pi f t)$$

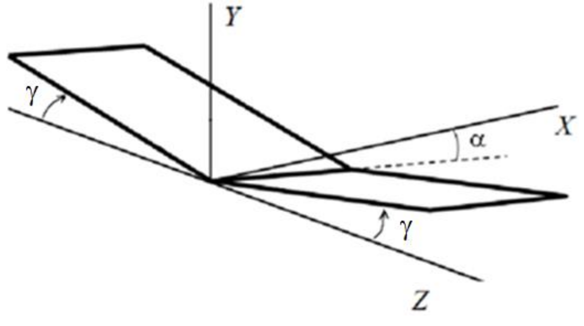
$$\alpha = \alpha_1 + \alpha_0 \sin(2\pi f t + \varphi)$$



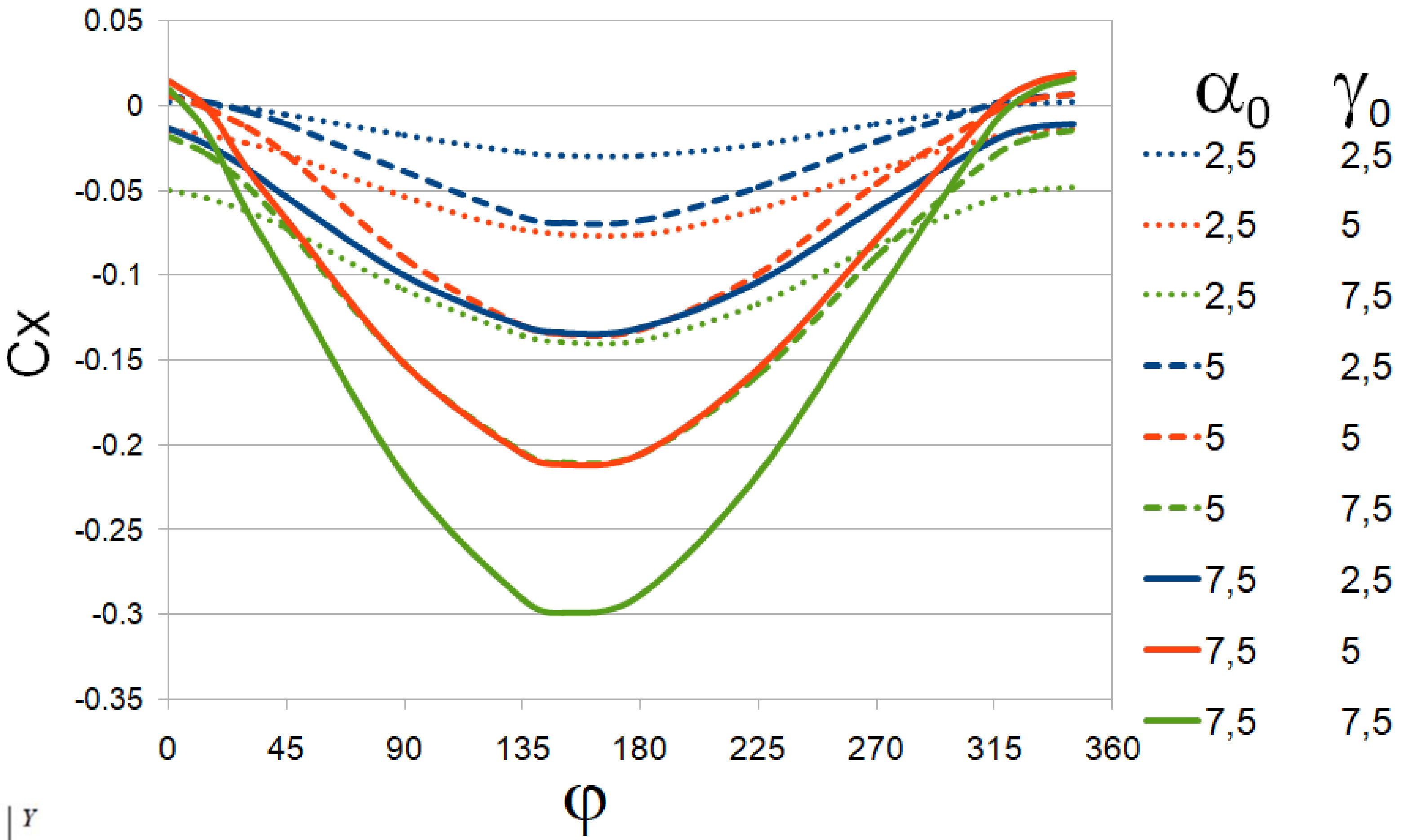
$\alpha_0 = 10°, \alpha_1 = -5°, \ \gamma_0 = 10°, \varphi = -150°$

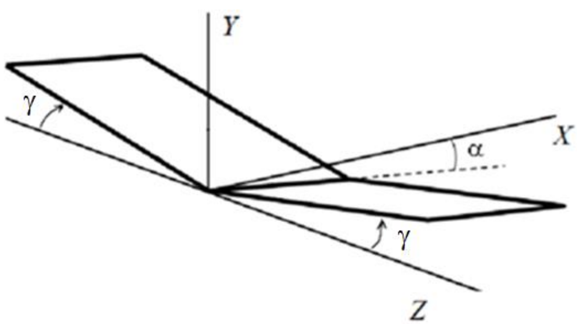
Time dependencies of the force coefficients and the angles





Dependence of the drag force coefficient on the phase shift

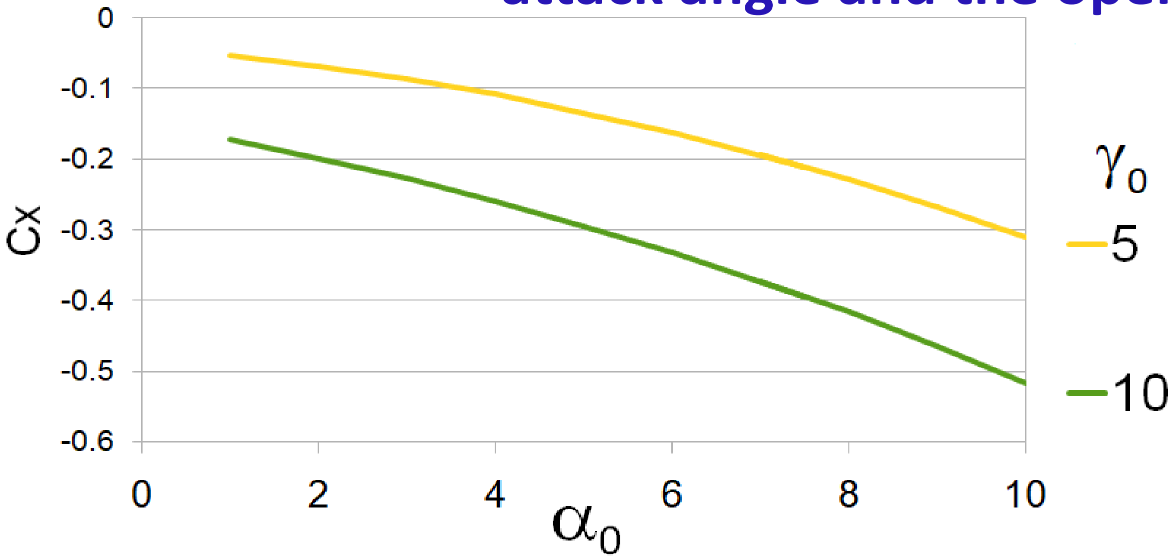


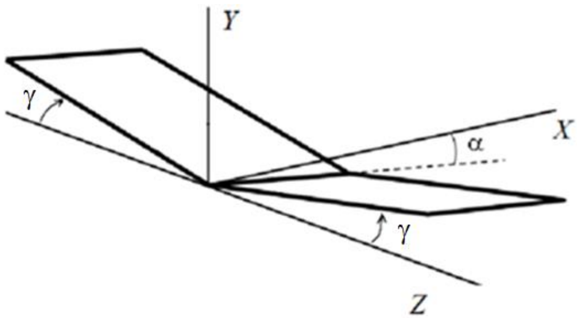


$$\gamma = \gamma_0 \sin(2\pi f t)$$

$$\alpha = \alpha_0 + \alpha_0 \sin(2\pi f t + \varphi)$$

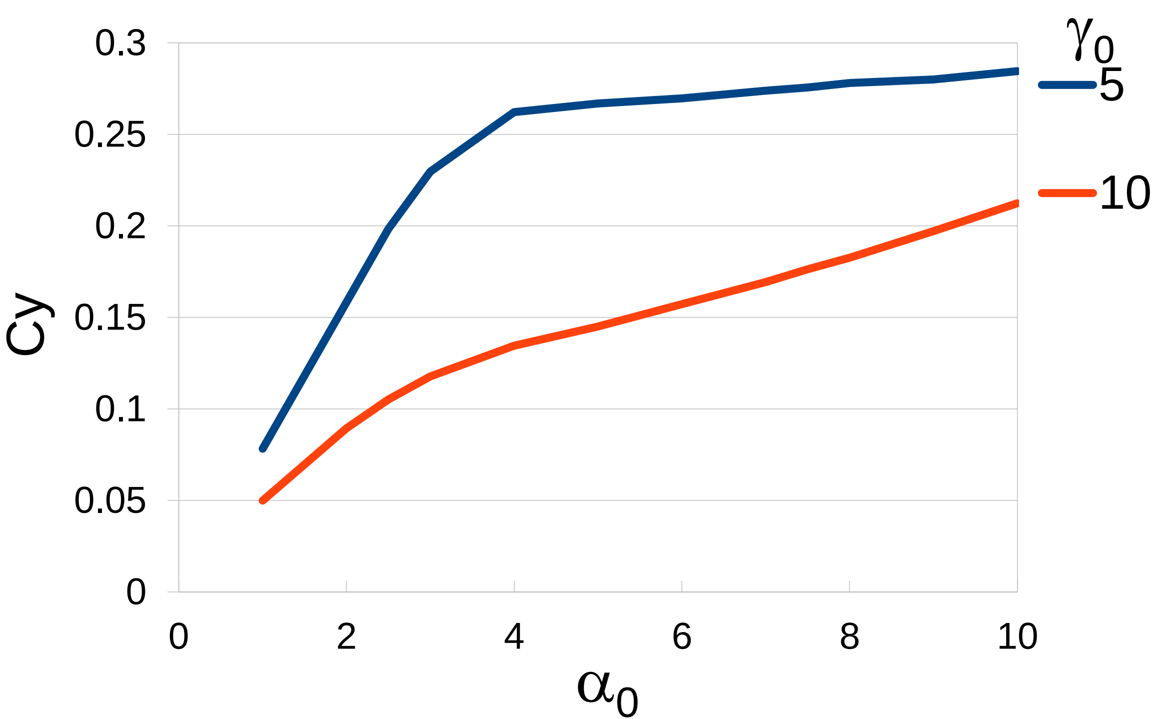
Dependencies of the average values Cx, Cy on the attack angle and the opening angle





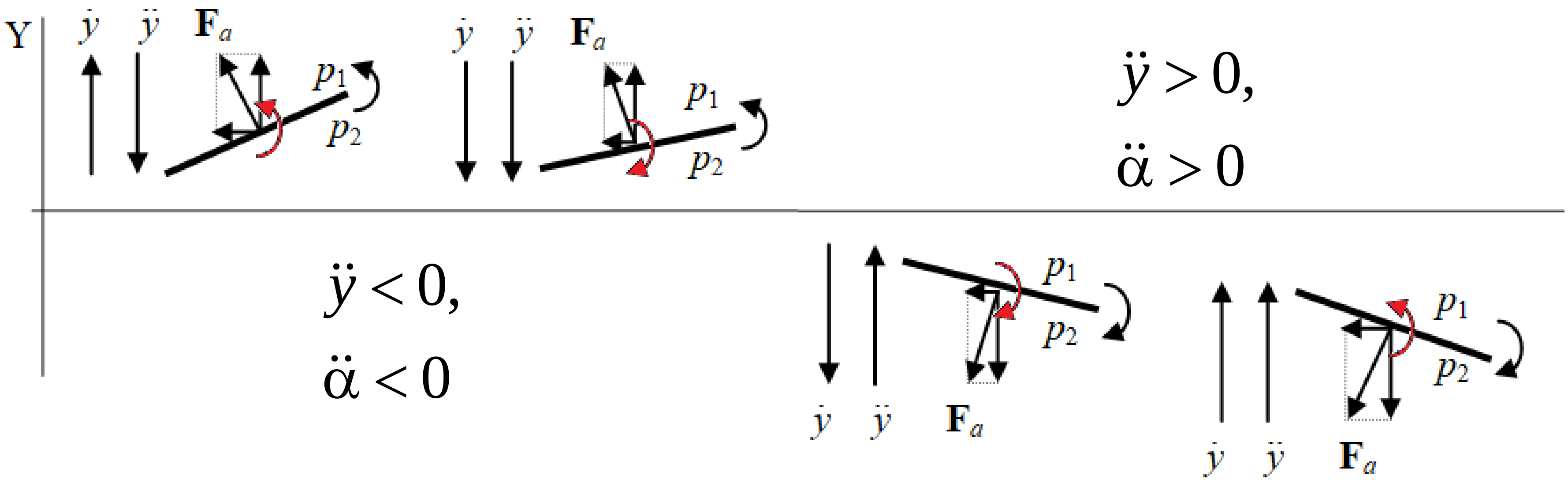
$$\alpha = \alpha_0 + \alpha_0 \sin(2\pi f t + \varphi)$$

$$\gamma = \gamma_0 \sin(2\pi f t)$$



Scheme of the forces and accelerations directions in different oscillation phase

$$y = y_0 \sin\left(\frac{2\pi}{T}t\right), \quad \ddot{y} = -y_0\left(\frac{2\pi}{T}\right)^2 \sin\left(\frac{2\pi}{T}t\right), \qquad \alpha = \alpha_0 \sin\left(\frac{2\pi}{T}t\right), \quad \ddot{\alpha} = -\alpha_0\left(\frac{2\pi}{T}\right)^2 \sin\left(\frac{2\pi}{T}t\right)$$



a)	b)	c)	d)
$0 \le t \le T/4$	$T/2 \le t \le T/4$	$T/2 \le t \le 3T/4$	$3T/4 < t < T$
$\dot{y} > 0, \ddot{y} < 0,$	$\dot{y} < 0, \ddot{y} < 0,$	$\dot{y} < 0, \ddot{y} > 0,$	$\dot{y} > 0, \ddot{y} > 0,$
$F_{a,y} > 0,\ p_2 > p_1$	$F_{a,y} > 0,\ p_2 > p_1$	$F_{a,y} < 0,\ p_2 < p_1$	$F_{a,y} < 0,\ p_2 < p_1$

CONCLUSIONS

- The mesh-free dipole particles-based methods is developed and applied for simulation of the insect flapping wings.
- It is shown that the main cause of the thrust performance is interaction of wings with an added mass of fluid at the wings acceleration.

The wake behind the plate performing angular oscillations