Locomotion of the fish-like foil under own effort

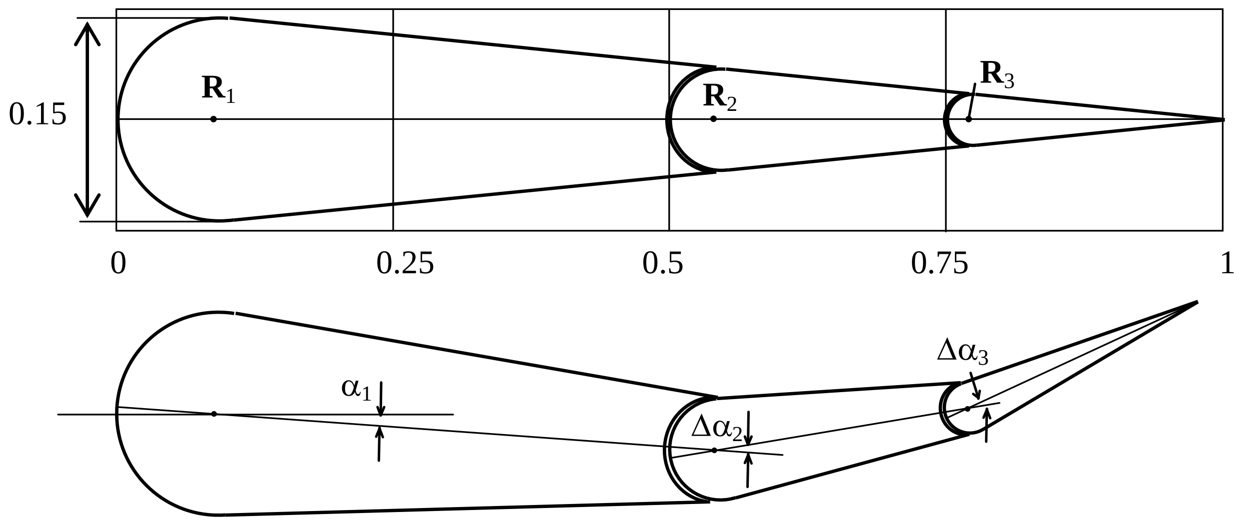
Yaroslav A. Dynnikov, Galina Ya. Dynnikova, Tatyana V. Malakhova

Lomonosov Moscow State University

Barselona, Spain, 28-30 October 2019

The fish-like model

The moment of force $\mathbf{M}_f$ is applied between the first and second sections by harmonic law, resulting in the bending of the fish body. This simulates the muscular effort of a fish. The second hinge is elastic and passive. The flow-structure task is solved.



$$\frac{d\mathbf{R}_i}{dt} = \mathbf{U}_i$$

$$\frac{d\alpha_i}{dt} = \Omega_i$$

$$\mathbf{R}_i = \mathbf{R}_{i-1} + a_{i-1}\begin{pmatrix}\cos\alpha_{i-1} \\ \sin\alpha_{i-1}\end{pmatrix}, \quad a_{i-1} = \left|\mathbf{R}_i - \mathbf{R}_{i-1}\right| = const$$

Numerical method of Viscous Vortex Domains

Governing equations:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} - \nu \nabla^2 \mathbf{u} = -\frac{1}{\rho}\nabla p, \quad \nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \omega}{\partial t} = \nabla(\mathbf{u}\omega), \quad \mathbf{u} = \mathbf{V} + \mathbf{V}_d, \quad \mathbf{V}_d = -\nu \frac{\nabla \omega}{\omega},$$

$$\boldsymbol{\omega} = \omega \mathbf{e}_z = \nabla \times \mathbf{V},$$

Dynnikova G. Ya. (2004). Doklady Physics. Vol. 49, no. 11. p. 648–652.

Ogami, Y. and Akamatsu, T. (1991). Computers and Fluids, Vol. 19, ¾, p. 433-441.

Diffusion velosity

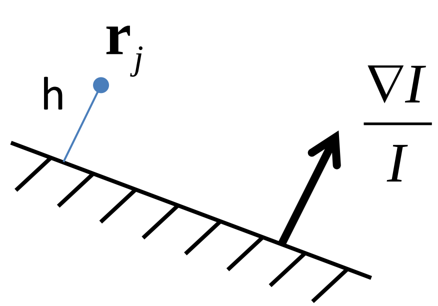
$$\boldsymbol{\omega}\left(\mathbf{r}_j\right)=\frac{1}{2\pi\varepsilon I\left(\mathbf{r}_j\right)}\lim_{\varepsilon\to 0}\int_s \boldsymbol{\omega}(\mathbf{r})\exp\left(-\frac{\left|\mathbf{r}_j-\mathbf{r}\right|}{\varepsilon}\right)\mathrm{d}s\approx\frac{1}{2\pi\varepsilon_j I\left(\mathbf{r}_j\right)}\sum_i \gamma_i \exp\left(-\frac{\left|\mathbf{r}_{ji}\right|}{\varepsilon_j}\right)$$

$$I\left(\mathbf{r}_j\right)=\frac{1}{2\pi\varepsilon_j}\int_s \exp\left(-\frac{\left|\mathbf{r}_j-\mathbf{r}\right|}{\varepsilon_j}\right)\mathrm{d}s,\quad \mathbf{r}\in s,\quad \mathbf{r}_{ji}=\mathbf{r}_j-\mathbf{r}_i,$$

$$h\gg\varepsilon_j \quad\Rightarrow\quad I\left(\mathbf{r}_j\right)=1$$

$$\frac{h}{\varepsilon_j}\to 0 \quad\Rightarrow\quad I\left(\mathbf{r}_j\right)\to 0.5$$

$$\mathbf{V}_d\left(\mathbf{r}_j\right)=-\frac{\nu}{\varepsilon_j}\left(\frac{-\sum_i \frac{\gamma_i \mathbf{r}_{ji}}{\left|\mathbf{r}_{ji}\right|}\exp\left(-\frac{\left|\mathbf{r}_{ji}\right|}{\varepsilon_j}\right)}{\sum_i \gamma_i \exp\left(-\frac{\left|\mathbf{r}_{ji}\right|}{\varepsilon_j}\right)}-\frac{\nabla I}{I}\right)$$



Boundary conditions

Biot-Savart Law

$$\mathbf{V}(\mathbf{R}) = \int_{S_f} \omega(\mathbf{r})\mathbf{e}_z \times \mathbf{K}(\mathbf{R},\mathbf{r})\,ds + \sum_j \int_{C_j} \left(\mathbf{V}_c^{(j)} \times \mathbf{n}\right) \times \mathbf{K}\,dl - \sum_j \int_{C_j} \mathbf{K}\left(\mathbf{n}\cdot\mathbf{V}_c^{(j)}\right) dl$$

$$\mathbf{K}(\mathbf{R},\mathbf{r}) = \frac{1}{2\pi}\frac{\mathbf{R}-\mathbf{r}}{(\mathbf{R}-\mathbf{r})^2}, \quad \mathbf{V}_c^{(j)} = \mathbf{U}_j + \mathbf{\Omega}_j \times \left(\mathbf{r}-\mathbf{R}_j\right),$$ j is the section number

$$\mathbf{n}\cdot\mathbf{V}(\mathbf{r}_c) = \mathbf{n}\cdot\mathbf{V}_c \quad \Rightarrow \quad \mathbf{V}(\mathbf{r}_c) = \mathbf{V}_c$$ no-sleep condition

$$\sum_j \sum_k a_{lk}^{(j)} \gamma_{kj}^{\text{new}} + \mathbf{b}_l^{(j)} \cdot \mathbf{U}_j + c_l^{(j)} \mathbf{\Omega}_j = d_l^{(j)}; \quad l = 1, N_j; \quad k = 1, N_j$$

Unknown values: $\gamma_{kj}^{\text{new}}, \mathbf{U}_j, \mathbf{\Omega}_j$. N_j is the amount of points on the j-th contour

Amount of the equations is equal to $N_\Sigma = \sum_j N_j$

Amount of the unknown values is equal to $N_\Sigma + 9$

S. N. Kempka, M. W. Glass, J. S. Peery et al. 1996. 52 pp. SANDIA report. SAND–96-0583.

The equations of motion of the sections

$$m_1\dot{\mathbf{U}}_{m,1} = \mathbf{F}_{H,1} - \mathbf{F}_{h,2},;$$

$$m_2\dot{\mathbf{U}}_{m,2} = \mathbf{F}_{H,2} + \mathbf{F}_{h,2} - \mathbf{F}_{h,3};$$

$$m_3\dot{\mathbf{U}}_{m,3} = \mathbf{F}_{H,3} + \mathbf{F}_{h,3};$$

$$I_1\dot{\mathbf{\Omega}}_1 = \mathbf{M}_{H,1} - \mathbf{M}_f - \mathbf{M}_{h,2} - (\mathbf{R}_2 - \mathbf{R}_1)\times\mathbf{F}_{h,2} - m_1\dot{\mathrm{U}}_1 \times \left(\mathbf{r}_{m,1} - \mathbf{R}_1\right);$$

$$I_2\dot{\mathbf{\Omega}}_2 = \mathbf{M}_{H,2} + \mathbf{M}_f + \mathbf{M}_{h,2} - \mathbf{M}_{h,3} - (\mathbf{R}_3 - \mathbf{R}_2)\times\mathbf{F}_{h,3} - m_2\dot{\mathrm{U}}_2 \times \left(\mathbf{r}_{m,2} - \mathbf{R}_2\right);$$

$$I_3\dot{\mathbf{\Omega}}_3 = \mathbf{M}_{H,3} + \mathbf{M}_{h,3} - m_3\dot{\mathrm{U}}_3 \times \left(\mathbf{r}_{m,3} - \mathbf{R}_3\right);$$

$$\mathbf{U}_1 + \mathbf{\Omega}_1 \times \left(\mathbf{R}_2 - \mathbf{R}_1\right) = \mathbf{U}_2$$

$$\mathbf{U}_2 + \mathbf{\Omega}_2 \times \left(\mathbf{R}_3 - \mathbf{R}_2\right) = \mathbf{U}_3$$

$\mathbf{F}_{H,j}, \mathbf{M}_{H,j}$ Hydrodynamic forces and moments

$\mathbf{F}_{h,j}, \mathbf{M}_{h,j}$ Forces and moments in the hinges

$$\mathbf{M}_{h,j} = -k_j\left(\alpha_j - \alpha_{j-1}\right)$$

k_j is spring constant

The expression of force and moment via the vortex flux from the surfaces

$$\mathbf{F}_H = \mathbf{F}_p + \mathbf{F}_w, \quad \mathbf{M}_H = \mathbf{M}_p + \mathbf{M}_w$$

$$\frac{\mathbf{F}_p}{\rho} = \mathbf{e}_z \times \oint \mathbf{r}\left(\mathbf{J}_d \mathbf{n}\right) \mathrm{d}l + 2\dot{\mathbf{\Omega}} \times \mathbf{r}_m S + \ddot{\mathbf{r}}_m S \approx$$

$$\approx -\frac{1}{\Delta t} \sum_k \mathbf{r}_k \times \mathbf{e}_z \gamma_k^{new} + \frac{2\left(\mathbf{\Omega} - \mathbf{\Omega}^{old}\right) \times \mathbf{r}_m + \left(\mathbf{U} - \mathbf{U}^{old}\right)}{\Delta t} S$$

$$\frac{\mathbf{M}_p}{\rho} = \frac{\mathbf{e}_z}{2} \oint \mathbf{r}^2 \left(\mathbf{J}_d \mathbf{n}\right) \mathrm{d}l + 2\dot{\mathbf{\Omega}} I_m - \ddot{\mathbf{r}}_m \times \mathbf{r}_m S \approx$$

$$\approx \frac{\mathbf{e}_z}{\Delta t} \sum_k r_k^2 \gamma_k^{new} + \frac{2\left(\mathbf{\Omega} - \mathbf{\Omega}^{old}\right) I_m - \left(\mathbf{U} - \mathbf{U}^{old}\right) \times \mathbf{r}_m S}{\Delta t}$$

Substituting these expressions into the equations of motion of bodies, we obtain the equations, which, together with the equations of the boundary conditions, form a closed linear system of equations. The solution of this system gives us the values of the velocities of all sections and circulations of new particles simultaneously.

$$\mathbf{F}_w = -\nu\rho \int_C \mathbf{n} \times \boldsymbol{\omega}\, \mathrm{d}l, \quad \mathbf{M}_w = -\nu\rho \left(4\mathbf{\Omega} S + \int_C \left(\mathbf{r} - \mathbf{R}\right) \times \left(\mathbf{n} \times \boldsymbol{\omega}\right) \mathrm{d}l \right)$$

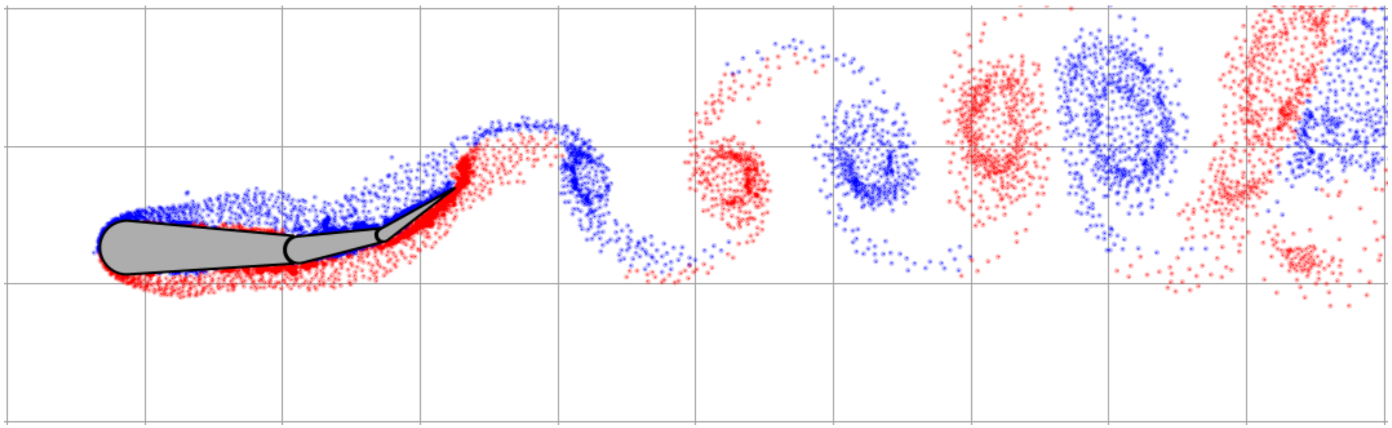
Dynnikova G. Y., Andronov P. R. Expressions of force and moment exerted on a body in a viscous flow via the flux of vorticity generated on its surface // *European Journal of Mechanics, B/Fluids*. — 2018. — Vol. 72, no. Nov-Dec. — P. 293–300.

The numerical results

The moment of force imitating the muscular efforts of the fish $\mathbf{M}_f = \mathbf{M}_{f0} \sin(2\pi f t)$

Dimensionless variables: $\bar{t} = tf, \quad \bar{V} = V/(L f), \quad \bar{M} = M/(\rho_f L^4 f^2),$

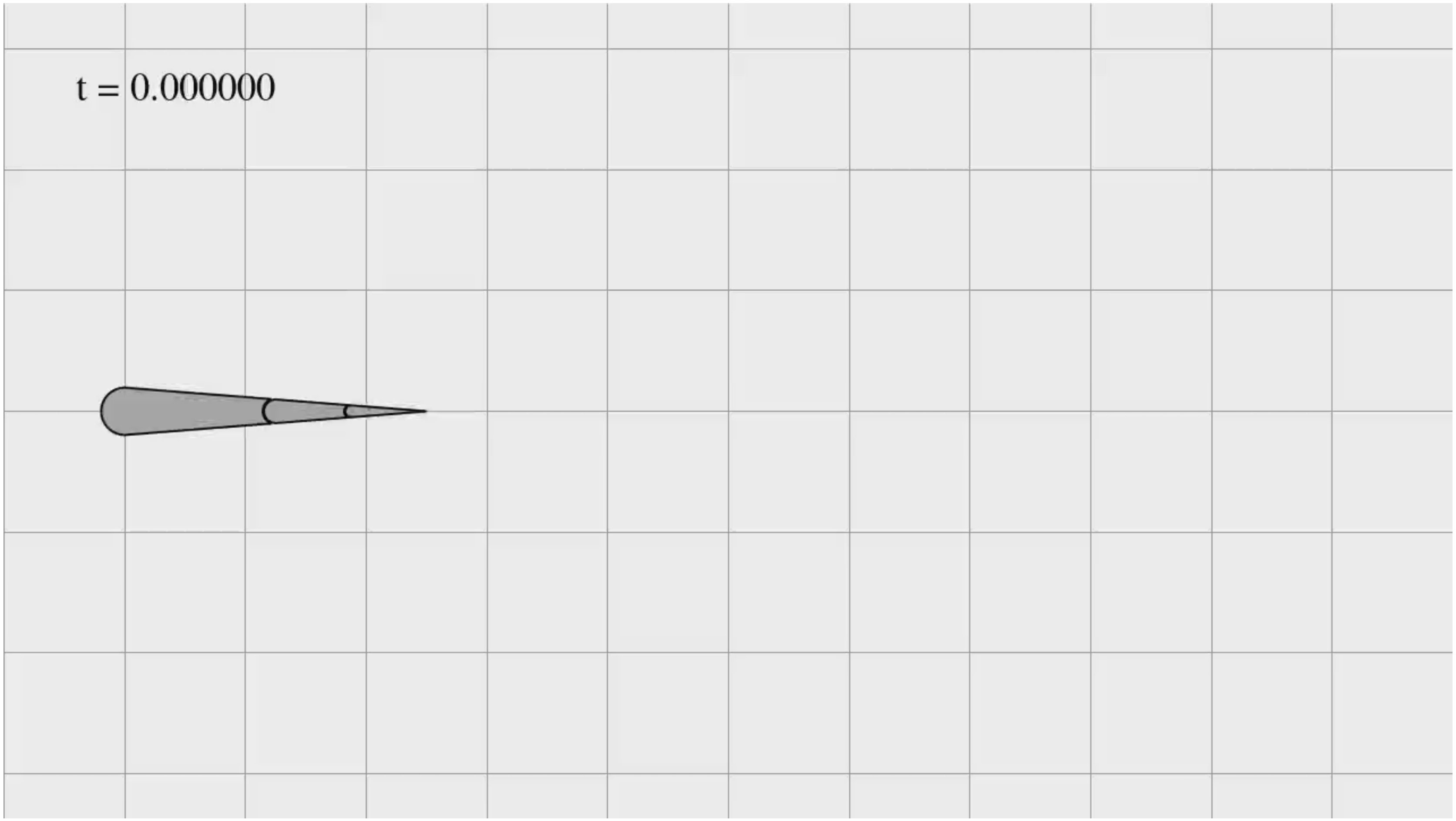
$$\mathrm{Re} = L^2 f / \nu, \quad \bar{k} = k/(\rho_f L^4 f^2)$$



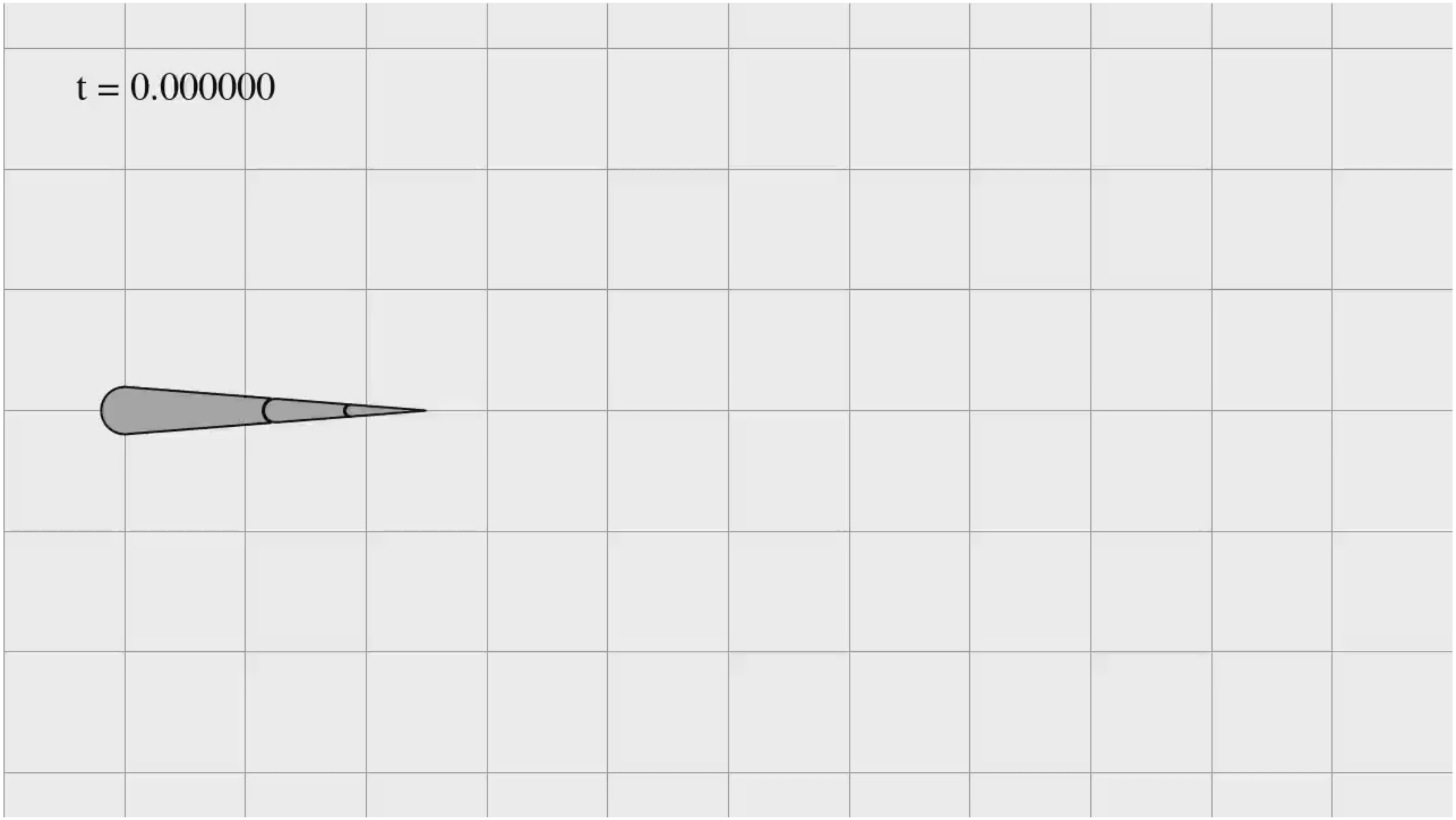
$$\bar{\mathbf{M}}_{f0} = 5.5, \ \mathrm{Re} = 1000, \ \bar{k}_2 = 13.4, \ \bar{k}_3 = 3.33$$

$$\bar{U} = 0.96$$

$$\bar{\mathbf{M}}_{f0} = 5.5, \ \mathrm{Re} = 1000, \ \bar{k}_2 = 13.4, \ \bar{k}_3 = 3.3$$

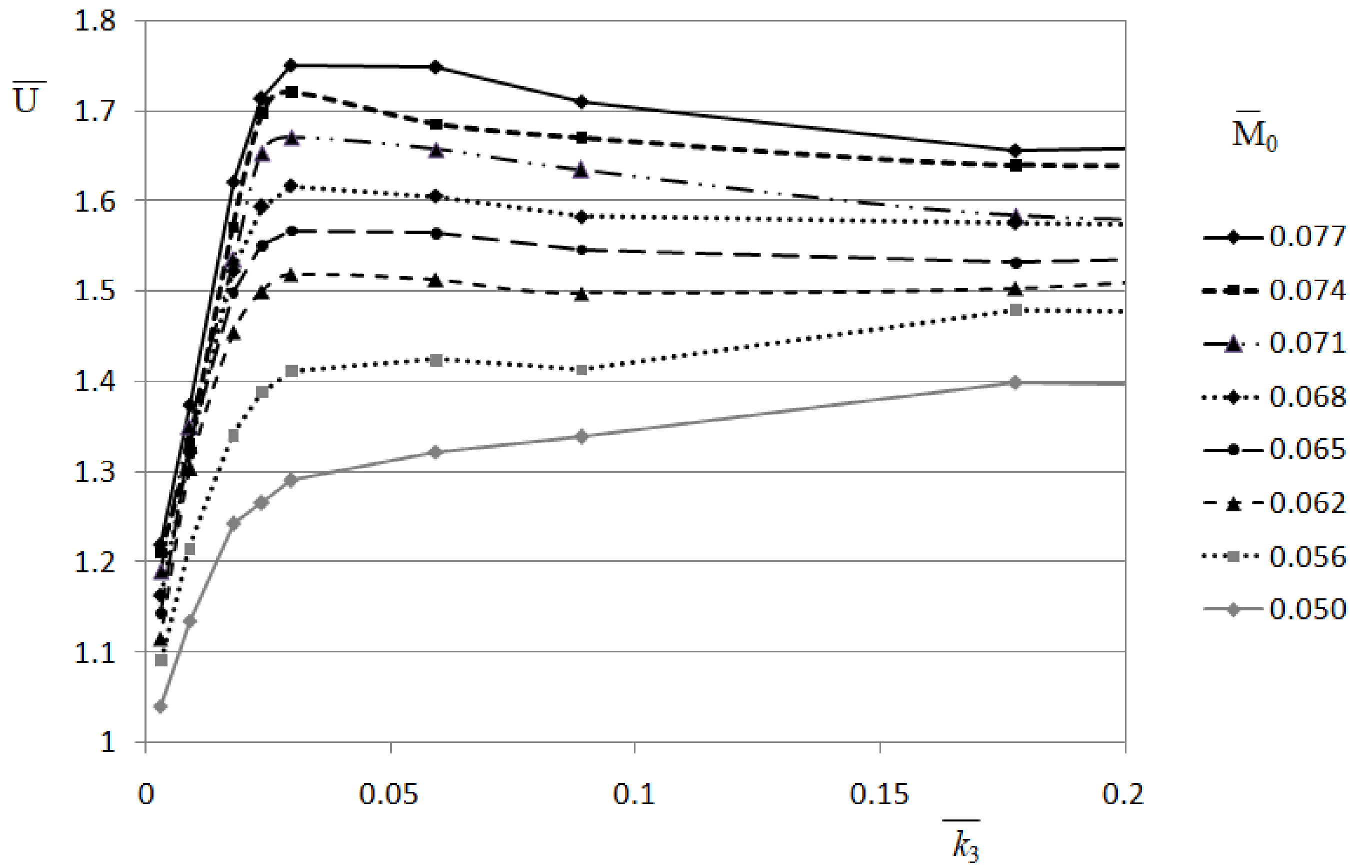


$$\bar{\mathbf{M}}_{f0} = 5.5, \ \ \mathrm{Re} = 1000, \ \overline{k_2} = 13.4, \ \overline{k_3} = 10$$

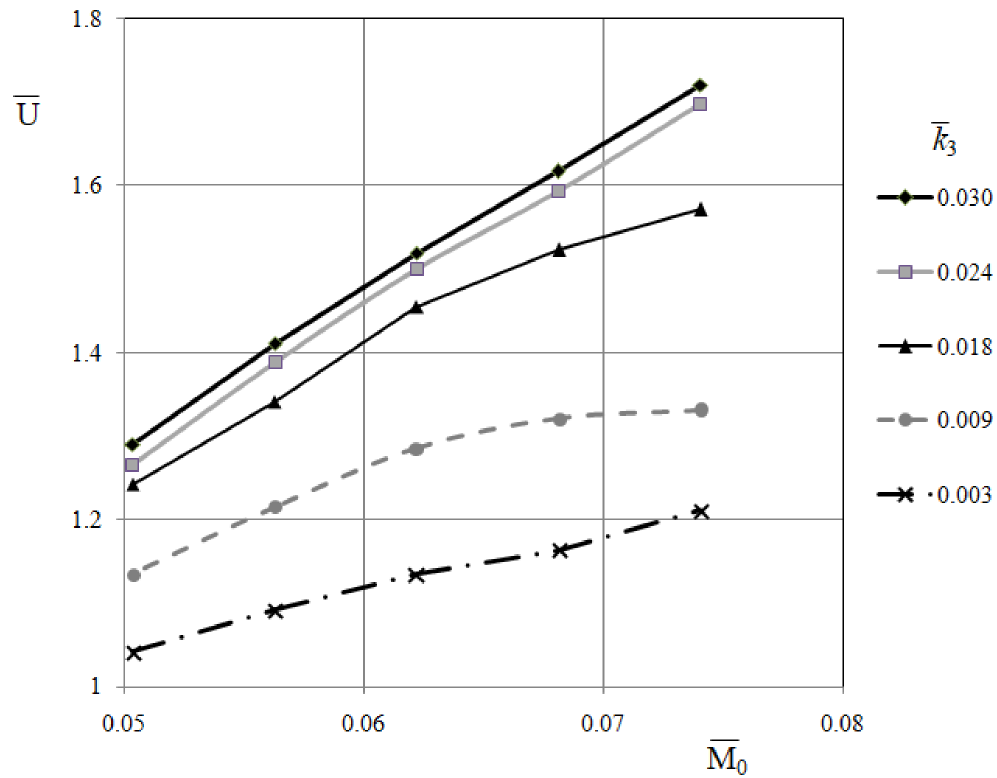


$$\bar{\mathbf{M}}_{f0} = 5.5,\ \ \mathrm{Re} = 1000,\ \bar{k}_2 = 13.4,\ \bar{k}_3 = 1.0$$

Dependency of the quasi-stationary velocity on the spring constant between the second and third sections at different amplitude of the forcing moment. Re = 1078, $\overline{k_2} = 0.06$



Dependency of the quasi-stationary velocity on the amplitude of the forcing moment at different spring constant between the second and third sections. Re = 1078, $\overline{k}_2 = 0.06$



Conclusion

The methodology of modeling the body self-locomotion under own effort is developed.

A method is applied for the fish-like model with the elastic hinges.

The calculations performed by the developed method have shown its effectiveness.

The dependency of the obtained quasi-stationary velocity on the elasticity is investigated.

It is shown that very low spring constant of the hinge is not optimal as well as very high one.