NYU, Tandon School of Engineering

Extended Bridge to CS

Homework #1

Submission instructions:

1. For this assignment you should turn in a '.pdf' file with your answers.

   Name your file 'YourNetID_hw1.pdf'

2. **Each question should start on a new page.**

3. **Typing your solutions would grant you 5 extra points.**

4. **You should submit your homework in the Gradescope system.**

   Note that when submitting the pdf file, you would be asked to assign the pages from your file to their corresponding questions.

5. **You can work and submit in groups of up to 4 people. If submitting as a group, make sure to associate all group members to the submission on gradescope.**

6. **You are expected to justify all your answers (not just to give the final answer).** As a rule of thumb, for questions taken from zyBooks, the format of your

   answers, should be like the format demonstrated in the sample solutions we exposed.

Question 1:

A. Convert the following numbers to their decimal representation. Show your work.

1. 10011011_2 = 155_{10}

$$1*2^0+1*2^1+0*2^2+1*2^3+1*2^4+0*2^5+0*2^6+1*2^7$$
$$1*1+1*2+0*4+1*8+1*16+0*32+0*64+1*128$$
$$1+2+0+8+16+0+0+128=155_{10}$$

2. 456_7 = 237_{10}

$$6*7^0+5*7^1+4*7^2$$
$$6*1+5*7+4*49$$
$$6+35+196=237_{10}$$

3. $38A_{16}$ = 906_{10}

$$A*16^0+8*16^1+3*16*2$$
$$10*16^0+8*16^1+3*16^2$$
$$10*1+8*16+3*256$$
$$10+128+768=906_{10}$$

4. 2214_5 = 309_{10}

$$4*5^0+1*5^1+2*5^2+2*5^3$$
$$4*1+1*5+2*25+2*125$$
$$4+5+50+250=309_{10}$$

B. Convert the following numbers to their binary representation:

1. $69_{10} = 1000101_2$

$$69 \div 2 = 34 \text{ R1}$$
$$34 \div 2 = 17 \text{ R0}$$
$$17 \div 2 = 8 \text{ R1}$$
$$8 \div 2 = 4 \text{ R0}$$
$$4 \div 2 = 2 \text{ R0}$$
$$2 \div 2 = 1 \text{ R0}$$
$$1 \div 2 = 0 \text{ R 1}$$

2. $485_{10} = 111100101_2$

$$485 \div 2 = 242 \text{ R 1}$$
$$242 \div 2 = 121 \text{ R 0}$$
$$121 \div 2 = 60 \text{ R1}$$
$$60 \div 2 = 30 \text{ R 0}$$
$$30 \div 2 = 15 \text{ R 0}$$
$$15 \div 2 = 7 \text{ R1}$$
$$7 \div 2 = 3 \text{ R1}$$
$$3 \div 2 = 1 \text{ R1}$$
$$1 \div 2 = 0 \text{ R1}$$

3. $6D1A_{16}$ = 0110 1101 0001 1010

Reviewed chart from video

6 = 0110
D =1101
1 = 0001
A = 1010

C. Convert the following numbers to their hexadecimal representation:

4. $1101011_2 = 6B_{16}$

$$0110 = 6_{16}$$
$$1011 = b_{16}$$

5. $895_{10} = 37F_{16}$

$$895 \div 16 = 55 \text{ R F}$$
$$55 \div 16 = 3 \text{ R 7}$$
$$3 \div 16 = 0 \text{ R 3}$$

Question 2:

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 = 14403_8$

$$\begin{array}{r} 7566_8 \\ + \quad 4515_8 \\ \hline 14403_8 \end{array}$$

2. $10110011_2 + 1101_2 = 11000000_2$

$$\begin{array}{r} 10110011_2 \\ + \qquad 1101_2 \\ \hline 11000000_2 \end{array}$$

3. $7A66_{16} + 45C5_{16} = C03B_{16}$

$$\begin{array}{r} 7A66_{16} \\ + \, 45C5_{16} \\ \hline C03B_{16} \end{array}$$

4. $3022_5 - 2433_5 = 34c_5$

$$\begin{array}{r} 3022_5 \\ - \, 2433_5 \\ \hline 34c_5 \end{array}$$

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. 124_{10} = 01111100_2

$$124 \div 2 = 62 \text{ R } 0$$
$$62 \div 2 = 31 \text{ R } 0$$
$$31 \div 2 = 15 \text{ R } 1$$
$$15 \div 2 = 7 \text{ R } 1$$
$$7 \div 2 = 3 \text{ R } 1$$
$$3 \div 2 = 1 \text{ R } 1$$
$$1 \div 2 = 0 \text{ R } 1$$

Add a zero because it's positive

2. -124_{10} =10000100

01111100 turn the 0's into 1's and 1's into 0's then add 1.

$$\begin{array}{r} 10000011 \\ + \qquad\quad 1 \\ 10000100 \end{array}$$

3. 109_{10} = 01111100

$$109 \div 2 = 62 \text{ R } 0$$
$$62 \div 2 = 31 \text{ R } 0$$
$$31 \div 2 = 15 \text{ R } 1$$
$$15 \div 2 = 7 \text{ R } 1$$
$$7 \div 2 = 3 \text{ R } 1$$
$$3 \div 2 = 1 \text{ R } 1$$
$$1 \div 2 = 0 \text{ R } 1$$

Add a zero because it's positive

4. -79_{10} = 10110001

$$79 \div 2 = 39 \text{ R } 1$$
$$39 \div 2 = 19 \text{ R } 1$$
$$19 \div 2 = 9 \text{ R } 1$$
$$9 \div 2 = 4 \text{ R } 1$$
$$4 \div 2 = 2 \text{ R } 0$$
$$2 \div 2 = 1 \text{ R } 0$$
$$1 \div 2 = 0 \text{ R } 1$$

79 is 01001111 then turn the 0s into 1's and 1's into 0s and add 1.

$$\begin{array}{r} 10110000 \\ \underline{+ \qquad\quad 1} \\ 10110001 \end{array}$$

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8\text{ bit 2's comp}}$ = 30

$$\frac{0}{128}\ \frac{0}{64}\ \frac{0}{32}\ \frac{1}{16}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{2}\ \frac{0}{1}$$

$$16 + 8 + 4 + 2 = 30$$

2. $11100110_{8\text{ bit 2's comp}}$ = -26

$$\frac{1}{128}\ \frac{1}{64}\ \frac{1}{32}\ \frac{0}{16}\ \frac{0}{8}\ \frac{1}{4}\ \frac{1}{2}\ \frac{0}{1}$$

$$-128 + 64 + 32 + 4 + 2 = -26$$

3. $00101101_{8\text{ bit 2's comp}}$ = 45

$$\frac{0}{128}\ \frac{0}{64}\ \frac{1}{32}\ \frac{0}{16}\ \frac{1}{8}\ \frac{1}{4}\ \frac{0}{2}\ \frac{1}{1}$$

$$32 + 8 + 4 + 1 = 45$$

4. $10011110_{8\text{ bit 2's comp}}$ = -98

$$\frac{1}{128}\ \frac{0}{64}\ \frac{0}{32}\ \frac{1}{16}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{2}\ \frac{0}{1}$$

$$-128 + 16 + 8 + 4 + 2 = -98$$

Question 4:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4, sections b, c

B.

p	q	$\neg(p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

C.

p	q	r	$r \vee (p \wedge \neg q)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

2. Exercise 1.3.4, sections b, d

B.

p	q	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T
T	F	T
F	T	F
F	F	T

D.

p	q	$(p \iff q) \oplus (p \iff \neg q)$
T	T	T
T	F	T
F	T	T
F	F	T

Question 5:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.7, sections b, c

B. $(B \land D) \lor (B \land M) \lor (D \land M)$

C. $B \lor (D \land M)$

2. Exercise 1.3.7, sections b – e

B. $(s \lor v) \to p$

C. $p \to y$

D. $p \iff (s \land y)$

E. $p \to (s \lor y)$

3. Exercise 1.3.9, sections c, d

C. $c \to p$

D. $c \to p$

Question 6:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.3.6, sections b - d

B. If Joe maintains a B average, then he is eligible for the honors program.
C. If Rajiv can go on the roller coaster then he must be at least four feet tall.
D. If Rajiv is at least four feet tall then he can go on the roller coaster.

2. Exercise 1.3.10, sections c – f

p is true, *q* is false, *r* is unknown

C.

$$(p \lor r) \iff (q \land r)$$ Answer: False

$(q \land r)$ *q AND r* is unknown. It becomes F either way because *q* is F and all combinations are false. The expression True if and only if False is False.

D.

$$(p \land r) \iff (q \land r)$$ Answer: Unknown

$(p \land r)$ *p AND r* can be false if *r* is False and that would make this expression True because False if only if False is True.

E.

$$p \rightarrow (r \lor q)$$ Answer: Unknown

If r is True then the expression is if it's false then it's False. Expression is unknown.

F.

$$p \rightarrow (r \lor q)$$ Answer: True

p AND *q* is False so no matter what *r* is the expression will always be True because *r* is the conclusion.

Question 7:

Solve Exercise 1.4.5, sections b – d, from the Discrete Math zyBook:

B.

- If Sally did not get the job, then she was late for interview or did not update her resume.
- If Sally updated her resume and was not late for her interview, then she got the job.

Answer. Logically equivalent.

- $\neg j \to (l \vee \neg r)$
- $(r \wedge \neg l) \to j$

j	l	r	$\neg j \to (l \vee \neg r)$	$(r \wedge \neg l) \to j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

C.

- If Sally got the job then she was not late for her interview.
- If Sally did not get the job, then she was late for her interview.

Answer. Not logically equivalent.

- $j \to \neg l$
- $\neg j \to l$

j	l	$j \to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

D.

- If Sally updated her resume or she was not late for her interview, then she got the job.
- If Sally got the job, then she updated her resume and was not late for her interview.

Answer. Not logically equivalent

- $(r \vee \neg l) \rightarrow j$
- $j \rightarrow (r \wedge \neg l)$

j	l	r	$(r \vee \neg l) \rightarrow j$	$j \rightarrow (r \wedge \neg l)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

Question 8:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2, sections c, f, i

C.
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
Answer.

$(\neg p \vee q) \wedge (\neg p \vee r)$	Conditional Identity
$\neg p \vee (q \wedge r)$	Distributive Law
$p \to (q \wedge r)$	Conditional Identity

F.
$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$
Answer.

$\neg((p \vee \neg p) \wedge (p \vee q))$	Distributive Law
$\neg(T \wedge (p \vee q))$	Complement Law
$\neg((p \vee q) \wedge T)$	Commutative Law
$\neg(p \vee q)$	Identity Law
$\neg p \wedge \neg q$	De Morgan's Law

I.
$(p \wedge q) \rightarrow r \equiv (p \wedge \neg r) \rightarrow \neg q$
Answer:

$(p \wedge q) \rightarrow r$	
$\neg(p \wedge q) \vee r$	Conditional identity
$(\neg p \wedge \neg q) \vee r$	De Morgan's Law
$r \vee (\neg p \vee \neg q)$	Commutative Law
$(r \vee \neg p) \vee \neg q$	Associative Law
$(\neg\neg r \wedge \neg p) \vee \neg q$	Double Negation Law
$\neg(\neg r \wedge p) \vee \neg q$	De Morgan's Law
$(\neg r \wedge p) \rightarrow \neg q$	Conditional Identity
$(p \wedge \neg r) \rightarrow \neg q$	Commutative Law

2. Exercise 1.5.3, sections c, d

C.
$\neg r \vee (\neg r \rightarrow p)$

$\neg r \vee (r \vee p)$	Conditional identity
$(\neg r \vee r) \vee p$	Associative Law
$(r \vee \neg r) \vee p$	Commutative Law
$T \vee p$	Complement Law
$p \vee T$	Commutative Law
T	Domination Law

D.
$\neg(p \vee q) \rightarrow \neg q$

$\neg(\neg p \vee q) \rightarrow \neg q$	Conditional identity
$(\neg\neg p \wedge \neg q) \rightarrow \neg q$	De Morgan's Law
$(p \wedge \neg q) \rightarrow \neg q$	Double Negation Law
$\neg(p \wedge \neg q) \vee \neg q$	Conditional identity
$\neg p \vee q \vee \neg q$	De Morgan's Law
$\neg p \vee T$	Complement Law
T	Domination Law

Question 9:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.6.3, sections c, d

C. $\exists x \left(x = x^2 \right)$
D. $\forall x \left(x \leq x^2 + 1 \right)$

2. Exercise 1.7.4, sections b - d

B. $\forall x(\neg S(x) \land W(x))$
C. $\forall x(S(x) \rightarrow \neg W(x))$
D. $\exists x(S(x) \land W(x))$

Question 10:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9, sections c - i

	P(x)	Q(x)	R(x)
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

C. True.: R(x)
D. True, example: e
E. True
F. True
G.False, counter example: c
H. True
I. True, example: a

2. Exercise 1.9.2, sections b - i

P	1	2	3
1	T	F	T
2	T	F	T
3	T	T	F

Q	1	2	3
1	F	F	F
2	T	T	T
3	T	F	F

S	1	2	3
1	F	F	F
2	F	F	F
3	F	F	F

B. True. example: x=1
C. True, example: x =1
D. False
E. False
F. True. example: y=1
G. False. Counter example: P(2,2)
H. True. example: Q(2,2)
I. True

Question 11:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.10.4, sections c - g

C. There are two numbers whose sum is equal to their product.

$$\exists x \exists y((x+y) = (x \cdot y))$$

D. The ratio of every two positive numbers is also positive.

$$\forall x \forall y\left(((x > 0) \land (y > 0)) \rightarrow \left(\frac{x}{y} > 0\right)\right)$$

E.The reciprocal of every positive number less than one is greater than one.

$$\forall x\left((1 > x > 0) \rightarrow \left(\frac{1}{x} > 1\right)\right)$$

F. There is no smallest number.

$$\neg \exists x \forall y(x \leq y)$$

G. Every number other than 0 has a multiplicative inverse.

$$\forall x \exists y((x \neq 0) \rightarrow (xy = 1))$$

2. Exercise 1.10.7, sections c - f

C. There is at least one new employee who missed the deadline.

$$\exists x(N(x) \land D(x))$$

D. Sam knows the phone number of everyone who missed the deadline.

$$\forall y(D(y) \rightarrow P(Sam, y))$$

E. There is a new employee who knows everyone's phone number.

$$\exists x \forall y(N(x) \land P(x,y))$$

F. Exactly one new employee missed the deadline.

$$\exists x((N(x) \land D(x)) \land \forall y((x \neq y)) \rightarrow (\neg N(x) \land \neg D(y)))$$

3. Exercise 1.10.10, sections c – f

C.Every student has taken at least one class other than Math 101

$$\forall x \exists y((y \neq Math\,101) \land T(x,y))$$

D. There is a student who has taken every math class other than Math 101

$$\exists x \forall y((y \neq Math\,101) \rightarrow T(x,y))$$

E. Everyone other than Sam has taken at least two different math classes.

$$\forall x \exists y \exists z(x \neq Sam \land T(x,y) \land T(x,z) \land y \neq z)$$

F. Sam has taken exactly two math classes.

$$\exists x \exists y(T(Sam, x) \land T(Sam, y) \land x \neq y \land \forall z(T(Sam, z) \rightarrow (z = x \lor z = y)))$$

Question 12:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.8.2, sections b – e

P(x): x was given the placebo
D(x): x was given the medication
M(x): x had migraines

B. Every patient was given the medication or the placebo or both.

- $\forall x(D(x) \lor P(x))$
- Negation: $\neg\forall x((D(x) \lor P(x)) \lor (D(x) \land P(x)))$
- Applying De Morgan's Law: $\exists x(\neg(D(x) \lor P(x)) \lor \neg(D(x) \land P(x)))$
- English: There exists a patient who was either not given the medication or the placebo or both.

C. There is a patient who took the medication and had migraines.

- $\exists x(D(x) \land M(x))$
- Negation: $\neg\exists x(D(x) \land M(x))$
- Applying De Morgan's Law: $\forall x(\neg D(x) \lor \neg M(x))$
- English: Every patient either didn't take the medication or didn't have migraines.

D. Every patient who took the placebo had migraines.

- $\forall x(P(x) \rightarrow M(x))$
- Negation: $\neg\forall x(P(x) \rightarrow M(x))$
- Applying Conditional Identity: $\neg\forall x(\neg P(x) \lor M(x))$
- Applying De Morgan's Law: $\exists x(P(x) \land \neg M(x))$
- English: There is a patient who took the placebo and did not have migraines.

E. There is a patient who had migraines and was given the placebo.

- $\exists x(M(x) \land P(x))$
- Negation: $\neg\exists x(M(x) \land P(x))$
- Applying De Morgan's Law: $\forall x(\neg M(x) \lor \neg P(x))$
- English: Every patient either did not have migraines or was not given the placebo.

2. Exercise 1.9.4, sections c - e

C. $\forall x\exists y(P(x,y) \land \neg Q(x,y))$
D. $\forall x\exists y((P(x,y) \land \neg P(y,x)) \lor (p(y,x) \land \neg P(x,y)))$
E. $\forall x\forall y\neg P(x,y) \lor \exists x\exists y\neg Q(x,y)$