First-Order Coalition Logic

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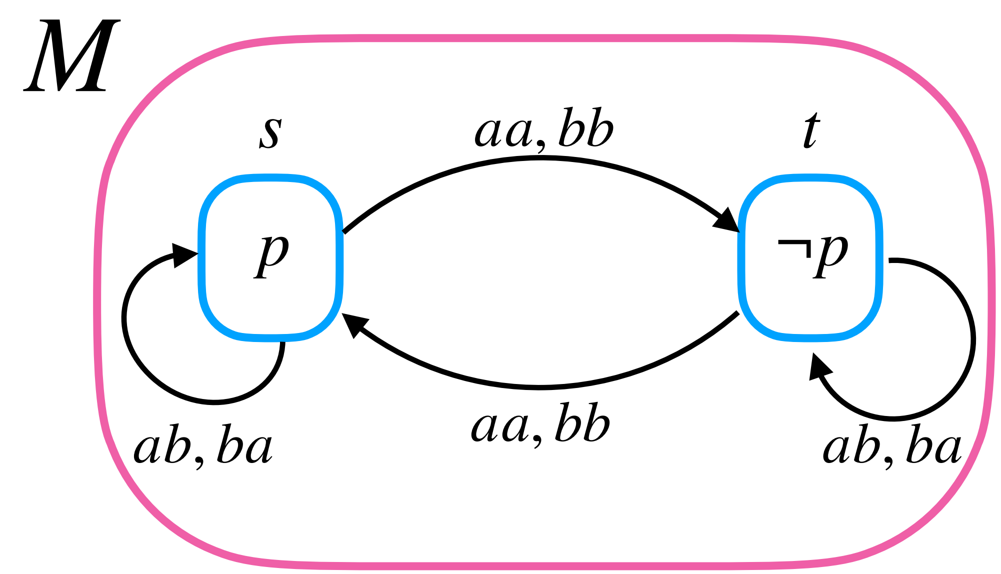
Concurrent Game Models

Alur, Henzinger, Kupferman *Alternating-time temporal logic*, 2002

Concurrent Game Models

A CGM M is $\langle n, \mathtt{Ac}, \mathscr{D}, S, R, \mathscr{V} \rangle$, where $n \geqslant 1$ is the number of agents, $\mathtt{Ac} \neq \varnothing$ is a set of action, $\mathscr{D} = \mathtt{Act}^n$ is a set of decision, $S \neq \varnothing$ is a set of states, $R : S \times \mathscr{D} \to S$ is a transition function, $\mathscr{V} : \mathtt{Ap} \to 2^S$ is a valuation function



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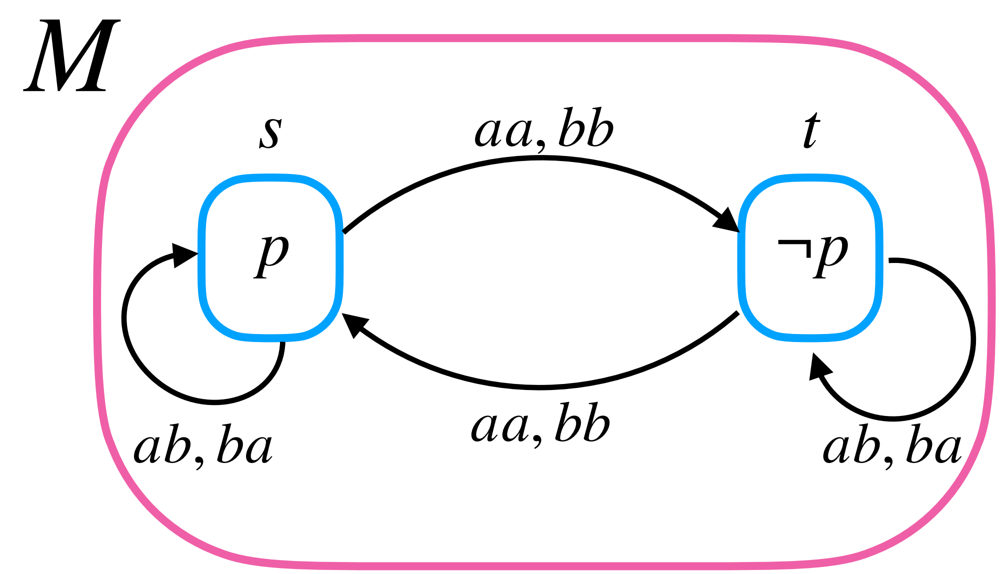

Logics interpreted on CGMs are used for specification and verification of such MAS as voting protocols, autonomous submarines, manufacturing robots, etc.

Alur, Henzinger, Kupferman *Alternating-time temporal logic*, 2002

Logics for Reasoning About Strategic Abilities

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Logics for Reasoning About Strategic Abilities

$$\mathsf{ATL} \ni \varphi := p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle\langle C \rangle\rangle \mathsf{X}\varphi \mid \langle\langle C \rangle\rangle \varphi \mathsf{U} \psi \mid \langle\langle C \rangle\rangle \varphi \mathsf{R} \psi$$

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$\langle\!\langle C \rangle\!\rangle \varphi$: coalition C has a strategy to ensure φ no matter what agents outside of the coalition do

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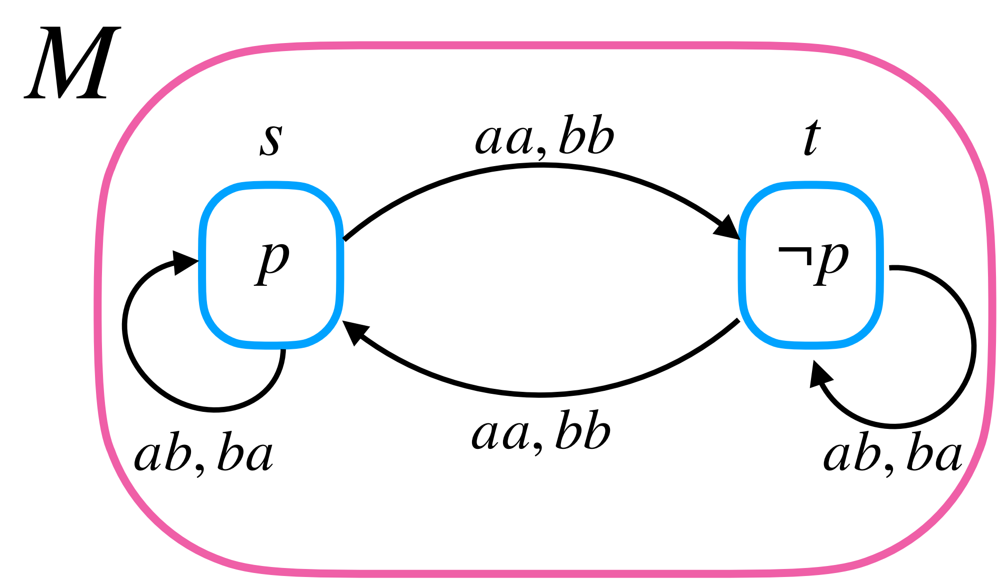

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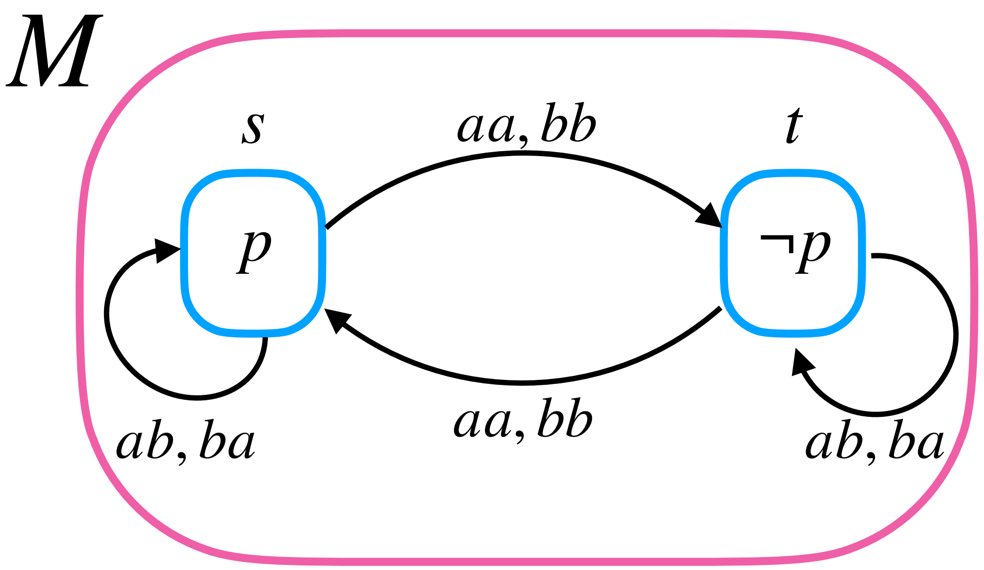

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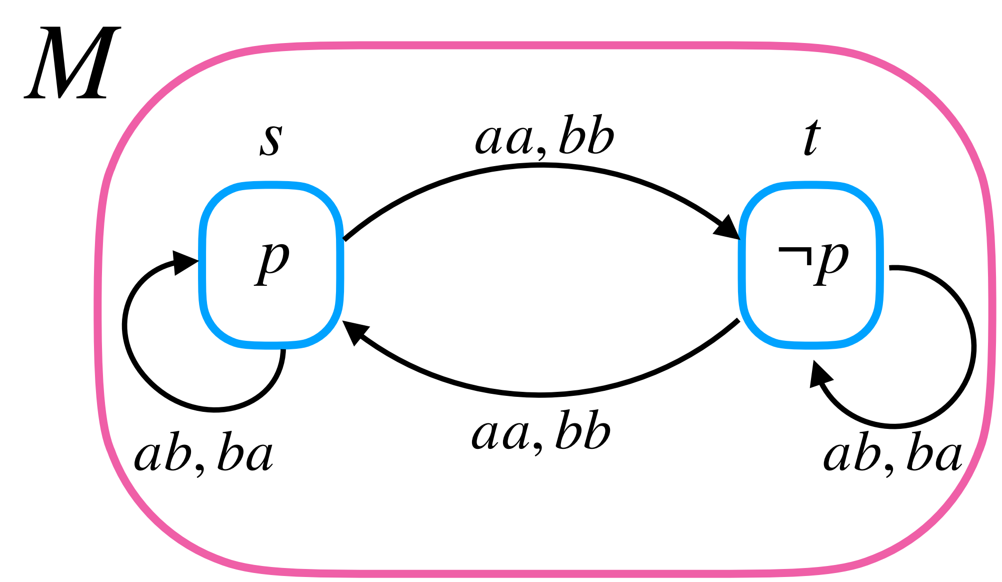

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Fixed quantification and no way to reference strategies (and hence no NE)

Strategy Logic

Mogavero, Murano, Vardi *Reasoning About Strategies*, 2010

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$$\mathsf{SL} \ni \varphi := p \,|\, \neg\varphi \,|\, (\varphi \wedge \varphi) \,|\, \mathsf{X}\varphi \,|\, \varphi \mathsf{U} \varphi \,|\, \varphi \mathsf{R} \varphi \mid \forall x \varphi \mid \exists x \varphi \mid (i, x)\varphi$$

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Temporal goal Nash Equilibrium

$$\exists x_1 \ldots \exists x_n (1, x_1) \ldots (n, x_n) \left(\bigwedge_{i=1}^{n} \exists y (i, y) \psi_i \rightarrow \psi_i \right)$$

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$$\exists x(1,x)(2,x)\mathsf{X}\neg p$$

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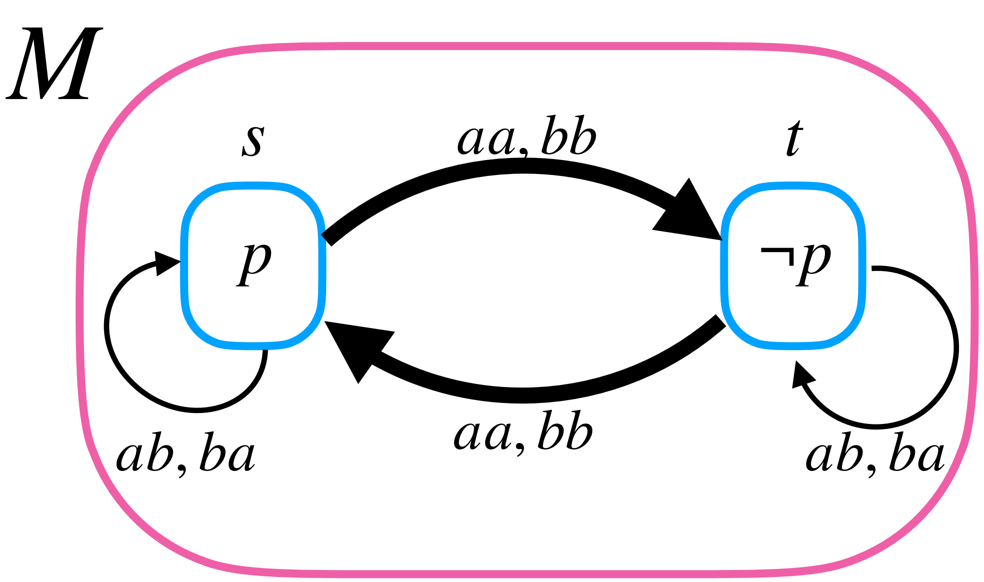

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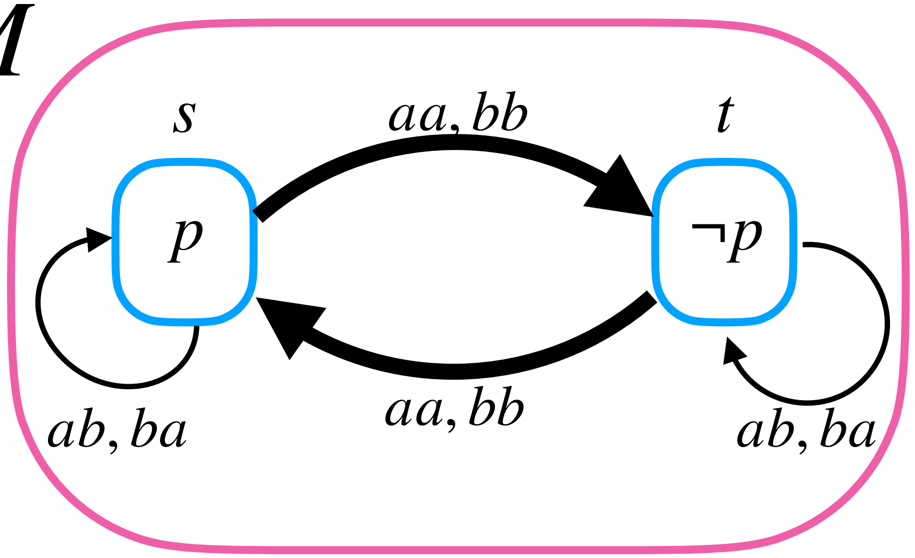

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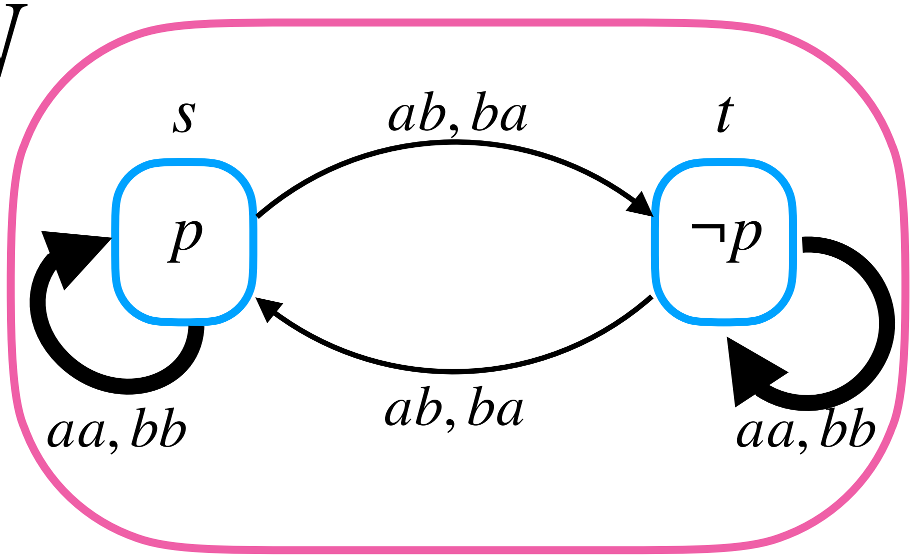

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Why axiomatising (fragments of) SL is hard

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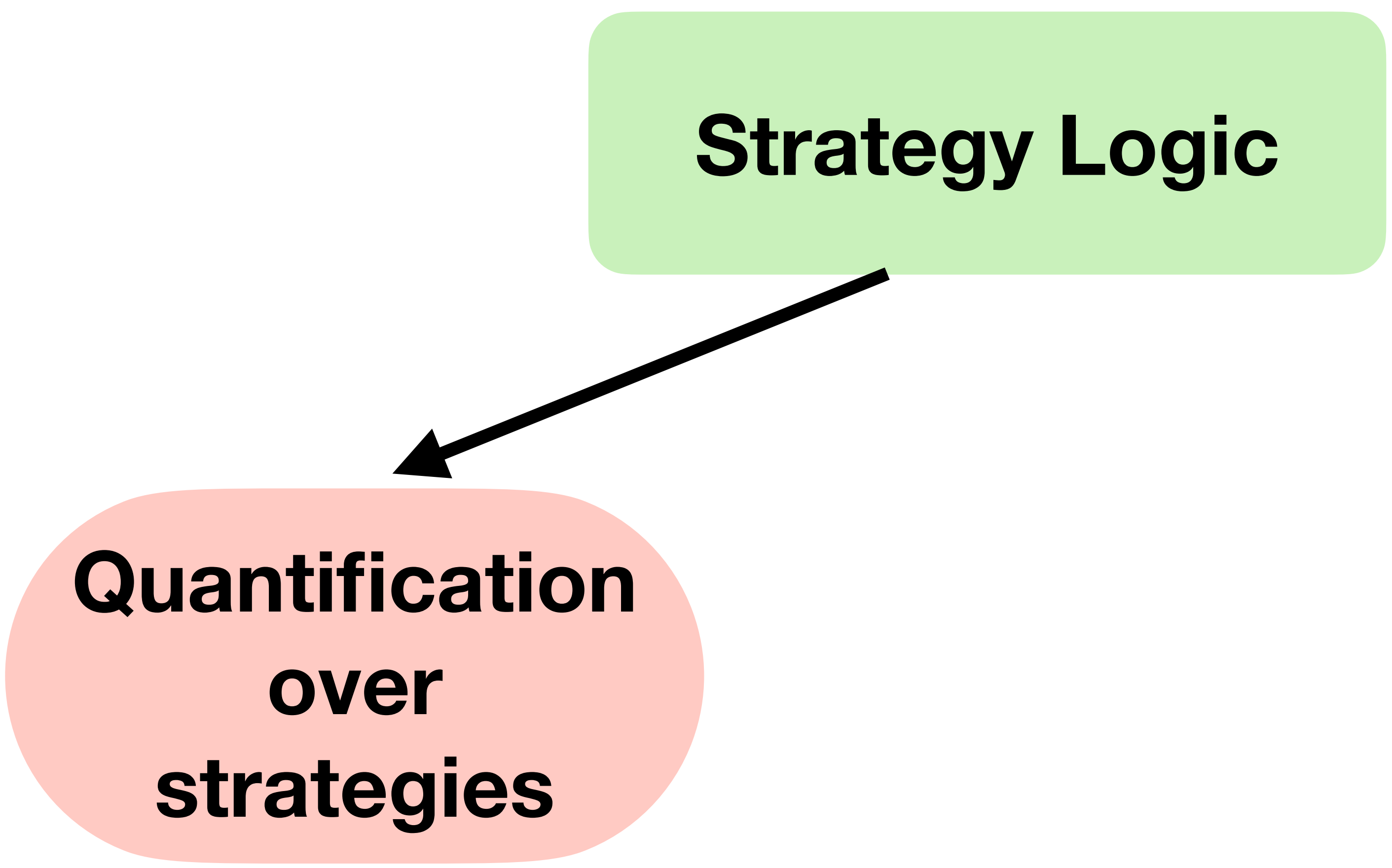

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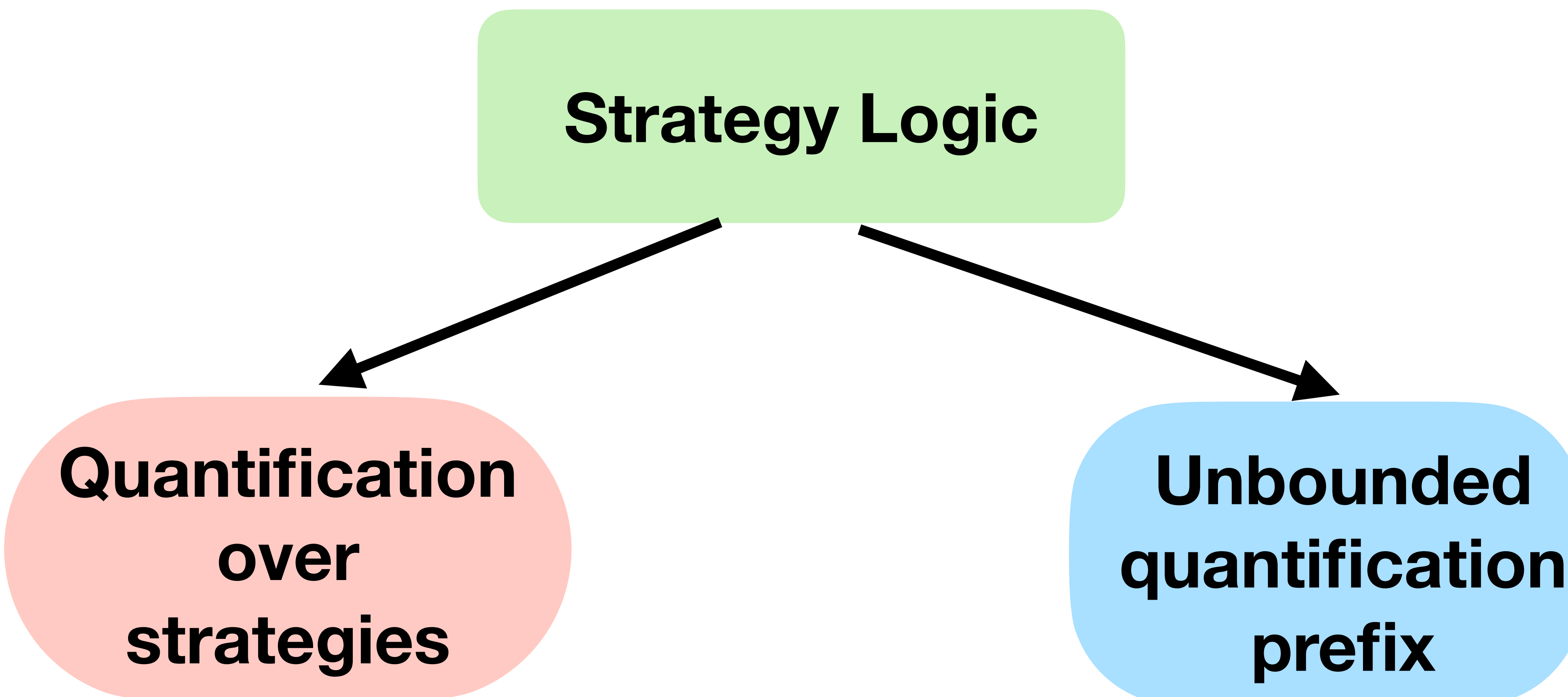

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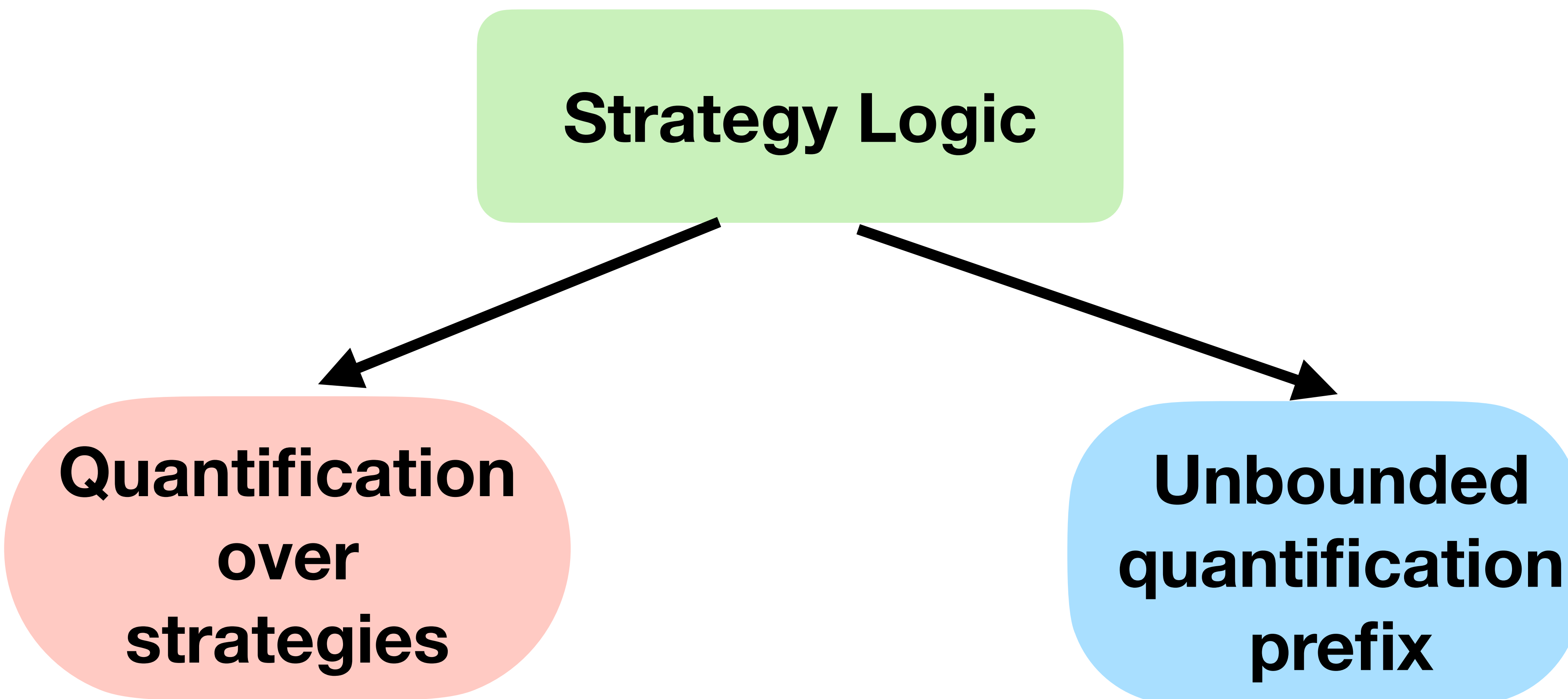

We focus on the unbounded quantification prefix and consider only next-time strategies

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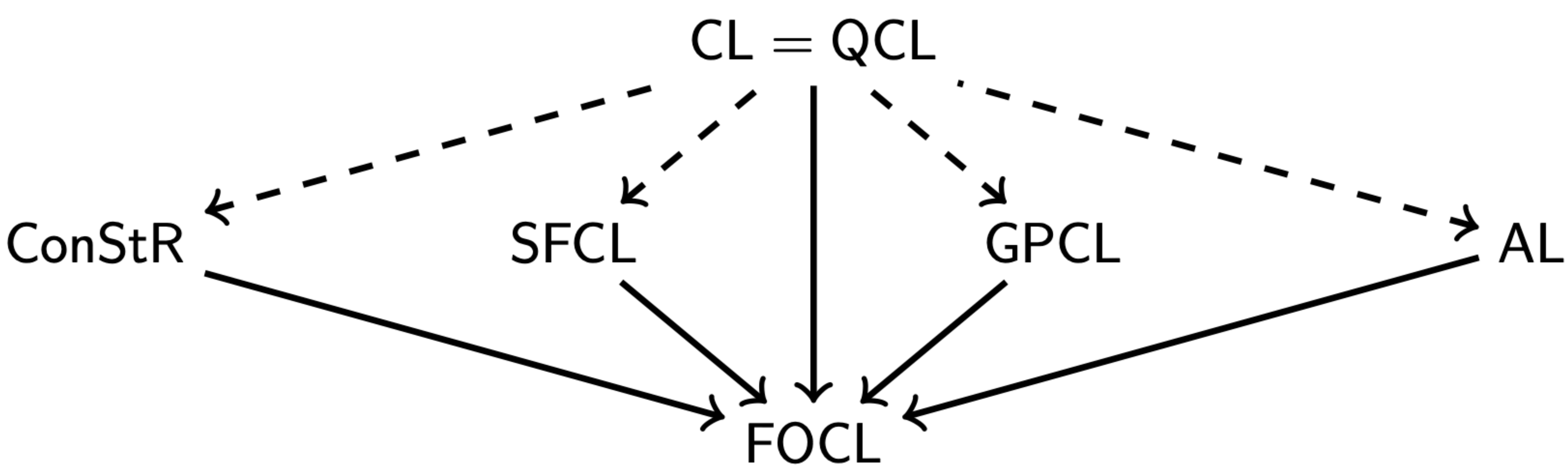

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Satisfiability: undecidable via tiling

Road Ahead

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(Re)Open(ed) question 1: is SL indeed not finitely axiomatisable?

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While proving the undecidability of SAT, we uncovered a **gap in the proof** of the high undecidability of SAT for SL

(Re)Open(ed) question 1: is SL indeed not finitely axiomatisable?

Open question 2: axiomatisations of more expressive variants of SL based on the one for FOCL