%comment not required. Just for understanding

1 Draw the contours of (i) $f(x,y) = (x+y)^2$ (ii) $f(x,y) = x^2 + y^2$

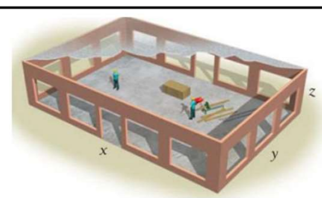
```
[x,y]=meshgrid(-100:2:100,-100:2:100);

z=(x+y).^2;

surf(x,y,z) %graph of function
surfc(x,y,z) %graph with contour map
contour(x,y,z) %only contour map

z=x.^2 + y.^2; %paraboloid
% geometry of function
surf(x,y,z) %graph of function
surfc(x,y,z) %graph with contour map
contour(x,y,z) %only contour map
```

2 The Beverage-Can Problem. The standard beverage can holds 12 fl. oz, or has a volume of 21.66 what dimensions yield the minimum surface area? Find the minimum surface area. (Assume that the shape of the can is a right circular cylinder


```
syms r h
eq=pi*r*r*h-21.66==0

h=solve(eq,h)

A=2*pi*r*(r+h)

Ar=diff(A,r)
r=solve(Ar,r,'Real',true)

h=subs(h)
h=double(h)

A=subs(A)
A=double(A)

fprintf("Min surface area of can is %f",A)
```

3 <u>Minimizing construction costs:</u> A company is planning to construct a warehouse whose interior volume is to be 252,000 Construction costs per square foot are estimated to be as follows:

Walls: \$3.00 Floor: \$4.00 Ceiling: 3.00




```
syms x y z
V=x*y*z-252000==0
c=4*x*y + 3*x*y + 6*z*(x+y)
z=solve(V,z)
c=4*x*y + 3*x*y + 6*z*(x+y)
cx=diff(c,x);
cy=diff(c,y);
[x,y]=solve(cx,cy,x,y,'real',true)
z=subs(z)
c=subs(c)
fprintf("minimum cost of construction of warehouse is %d",c)
```

5 Use double integral to find area bounded by the parabola $\mathbf{y} = \mathbf{x}^2$ and the line y=2x+3.

```
syms x y
f=1
eq=x^2-2*x-3==0
x1=solve(eq,x)
A=int(int(f,y,x^2,2*x+3),x,-1,3)
```

6 Use double integral to find area of the region inside the circle $r = 4\sin\theta$ and outside the circle r=2

```
syms r t
f=1
eq=4*sin(t)-2==0
tt=solve(eq,t)
A=int(int(f,r,2,4*sin(t)),t,pi/6,5*pi/6)
Area=double(A)
```

7 An agricultural sprinkler distributes water in a circular pattern of radius 100ft. It supplies water to a depth of e^{-r} feet per hour at a distance of r feet from the sprinkler. If $0 < R < 100$, what is the total amount of water supplied per hour to the region between the circle of radius R=5 and R=100, centered at the sprinkler?

```
syms t r
f=exp(-r)
fprintf("amount of water supplied to region of 5ft radius\n");
a1=double(int(int(f*r,r,0,5),t,0,2*pi))
fprintf("amount of water supplied to region of 5ft radius\n");
a2=double(int(int(f*r,r,0,100),t,0,2*pi))
fprintf("total amt of h20 at region b/w 5 and 100ft radius circles is\n");
A=a2-a1
```

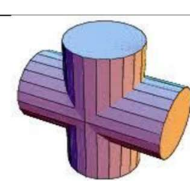
8 The population density of fire flies in a field is given by $f = \frac{1}{100}x^2y$ where $0 \le x \le 30$ and $0 \le y \le 20$, x and y are in feet, and f is the number of fireflies per square foot. Determine the total population of fireflies in the field.

```
syms x y
f=(1/100)*x^2*y
fprintf("total population of fireflies in field is")
A=int(int(f,x,0,30),y,0,20)
```

9 The density of students living near a university is modeled by $p(x, y) = 9 - x^2 - y^2$ where x and y are in miles and p is the number of students per square mile, in hundreds. Assume the university is located at (0, 0). Find the number of students who live in the shaded region shown below.

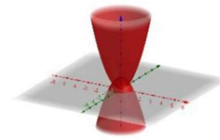
```
syms x y
p=9-x^2-y^2
A=int(int(p,y,0,2-x),x,0,2)
A=double(A)
```

11 Find the volume of the solid common to the two cylinder $x^2 + y^2 = a^2,\ x^2 + z^2 = a^2$



```
syms x y z a
f=1
eq=int(int(int(f,z,0,sqrt(a^2-x^2)),y,0,sqrt(a^2-x^2)),x,0,a)
ans=8*eq
```

12	Find the volume of the solid formed by two paraboloids $z_1 = x^2 + y^2$ and $z_2 = 1 - x^2 - y^2$



```
syms r t z
f=1
eq=int(int(int(f,z,r^2,1-r^2),r,0,1/(sqrt(2))),t,0,2*pi)
ans=double(eq)
```

15	Plot gradient field of (i) $f = \frac{1}{2}(x^2 + y^2)$ (ii) $f(x, y) = x^2 + y^2$

```
%i)
syms x y real
f1=0.5*(x^2 + y^2)
F1=gradient(f1,[x,y])'
[x,y]=meshgrid([-3:0.4:3,-3:0.4:3]);
u=x;v=y;
figure;
grid on
hold on
quiver(x,y,u,v,'r')
plot([0,0],[-3,3],'k','linewidth',0.5)
plot([-3,3],[0,0],'k','linewidth',0.5)
axis equal
axis([-3,3,-3,3])
xlabel('x')
ylabel('y')

%ii)

syms x y real
f2=x^2 + y^2
F2=gradient(f2,[x,y])'
[x,y]=meshgrid([-3:0.4:3,-3:0.4:3]);
u=2*x;
v=2*y;
figure;
grid on
hold on
quiver(x,y,u,v,'b')
plot([0,0],[-3,3],'k','linewidth',0.5)
plot([-3,3],[0,0],'k','linewidth',0.5)
axis equal
axis([-3,3,-3,3])
xlabel('x')
ylabel('y')
```

16	Suppose that over a certain region of space the electrical potential V is given by $V(x, y, z) = 5x^2 - 3xy + xyz$ .Find the rate of change of the potential at P(3,4,5) in the direction of the vector $a = i + j - k$

```
syms x y z real
v=5*x^2-3*x*y+x*y*z
a=[1,1,-1]
gradient_v=gradient(v,[x,y,z])'
P=sym([3,4,5])
gradient_v=subs(gradient_v,[x,y,z],P)
unit_a=a/norm(a)
Rate_change_v=dot(gradient_v,unit_a)
Rate_change_v=double(dot(gradient_v,unit_a))
```

17	Evaluate line integral $\int_c y^2dx + xdy$, where $C = C_1$ is the line segment from (-5,-3) to (0,2)

```
syms x y t real
F=[y^2,x]
p=sym([-5,-3])
q=sym([0,2])
r=(1-t)*p+t*q
dr=diff(r,t)
x=5*t-5
y=5*t-3
F=subs(F)
Integrand=dot(F,dr)
line_integral=int(Integrand,t,0,1)
```

20	Using Green's theorem, find the area of the region in the first quadrant bounded by the curves $y = x, y = \frac{1}{x}, y = \frac{x}{4}$

```
syms x y t real
f=[-y/2 x/2]


r1=[t t/4]
dr1=diff(r1,t)
Int1=dot(f,dr1)
Int1=subs(Int1,[x,y],[t,t/4])
ic1=int(Int1,t,0,2)


r2=[t 1/t]
dr2=diff(r2,t)
Int2=dot(f,dr2)
Int2=subs(Int2,[x,y],[t,1/t])
ic2=int(Int2,t,2,1)


r3=[t t]
dr3=diff(r3, t)
Int3=dot(f,dr3)
Int3=subs(Int3,[x,y],[t,t])
ic3=int(Int3,t,1,0)


ic=ic1+ic2+ic3
ic=double(ic)
```

22	Use Stokes theorem to evaluate surface integral $F = yzi + xzj + xyk$; S is surface of paraboloid $z = 9 - x^2 - y^2$ that lies above the plane z=5

```
syms x y z t real
f=[y*z, x*z, x*y]
r=[2*cos(t), 2*sin(t), 5]
dr=diff(r,t)
Int=dot(f,dr)
Int=subs(Int,[x,y,z],[2*cos(t),2*sin(t),5])
ic=int(Int,t,0,2*pi)
```