The famous Hungarian mathematician Paul Erd˝os was particularly renowned for his social practice of mathematics, in which he engaged with more than 500 different collaborators on research papers. His collaboration was so prolific in the mathematics community that it prompted the creation of the “Erd˝os number”, which describes the “collaborative distance” between Paul Erd˝os and another person.

The calculation of Erd˝os numbers can be aided through the construction of a graph, where each node or vertex represents a distinct person, and edges between persons indicate a paper co-authorship. The length of the shortest path between a person and Paul Erd˝os is that person’s Erd˝os number.

For this question, you will implement the following parts in the class ErdosNumbers.java.

(a) Implement the constructor for ErdosNumbers, which takes as input a list of papers for use in the subsequent parts. In addition, implement
- The method getPapers, which returns the set of papers an author has written.
- The method getCollaborators, which returns the set of unique co-authors an author has written a paper with before.

(b) In isErdosConnectedToAll, implement an efficient algorithm to determine if every author has an Erd˝os number. Consult the Javadoc for more specific details.

(c) In calculateErdosNumber, implement an efficient algorithm to determine the Erd˝os number of a given author. Consult the Javadoc for more specific details.

(d) In averageErdosNumber, implement the code to determine the average Erd˝os number of all the co-authors on a given paper.

(e) Multiple variants of the Erd˝os number exist that consider more information about co-authorships. One variant, which we will call here the “weighted Erd˝os number”, weights each edge in the graph between two authors a and b as w(a, b) = 1 c(a,b) , where c(a, b) is the number of papers the two authors have published together.

Then, the “weighted Erd˝os number” is the shortest weighted path between a person and Erd˝os in this graph - and will hence be a real number instead of an integer.

In calculateWeightedErdosNumber, implement an efficient algorithm to calculate the weighted Erd˝os number of a given author. Consult the Javadoc for more specific details.

Notes and Hints:

• A constant has been defined in ErdosNumbers.java for you to test against for Erd˝os’s name in the datasets.

• Authors and papers have unique names (e.g. you can consider authors with the exact same name to be the exact same author).

• It may make more sense to do some pre-processing in your constructor in order to make other methods more efficient (as they may be called multiple times).