# RDBMS AND SQL
# RELATIONAL ALGEBRA


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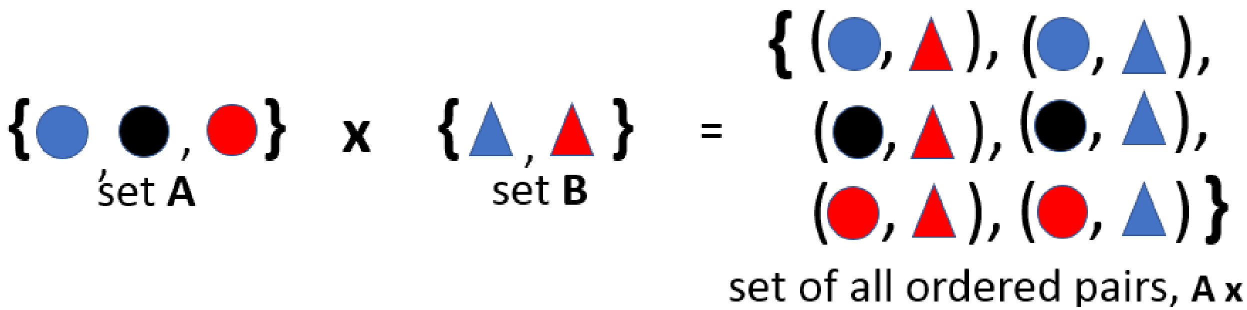
The primary goal of a DBMS is to provide a way to store and retrieve database information that is both *convenient* and *efficient*. - Silberschatz, Korth and Sudarshan.

# Preliminaries

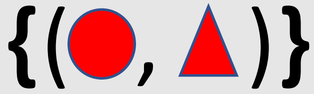
# Quiz

- A relation R from a set A to set B is a subset of the cartesian product A x B. True/False?

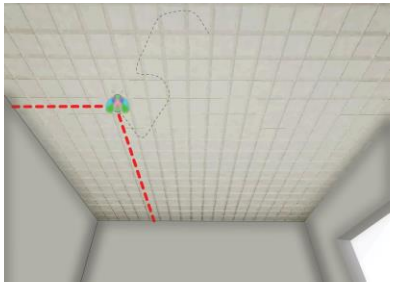



$$A \times B = \{\, (a, b) \mid a \in A \text{ and } b \in B \,\}$$

Let's say a relation exists between the reds:

**Relation R =** 

# The Fly on the Ceiling - Descartes



French mathematician René Descartes (1596-1650)

Image Source:
http://sites.psu.edu/solvingproblems
https://wild.maths.org/ren%C3%A9-descartes-and-fly-ceiling
vectorstock.com

# A Relation

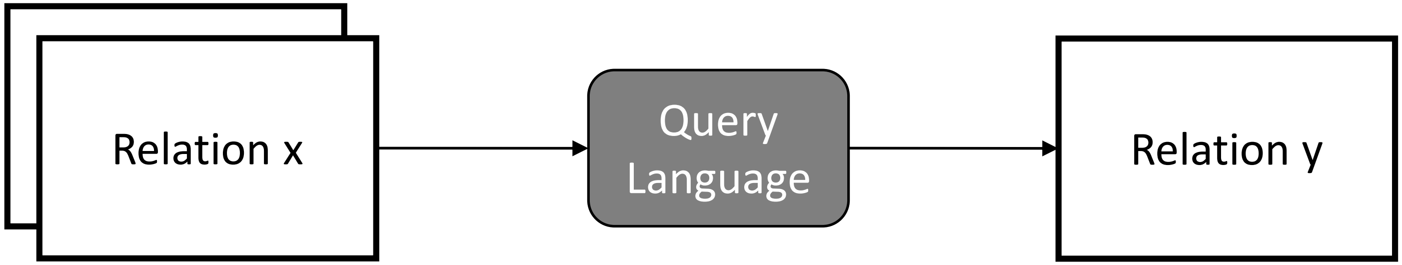
- Let the set, id = {1,2,3}
- Let the set, names = {vv, sd}
- What is id x names?
- We have a relation if we assign a sequential id to each name.

| id | name |
|---|---|
| 1 | sd |
| 1 | vv |
| 2 | sd |
| 2 | vv |
| 3 | sd |
| 3 | vv |

| id | name |
|---|---|
| 1 | sd |
| 2 | vv |

# Relational Algebra and Query Languages

# Query Languages

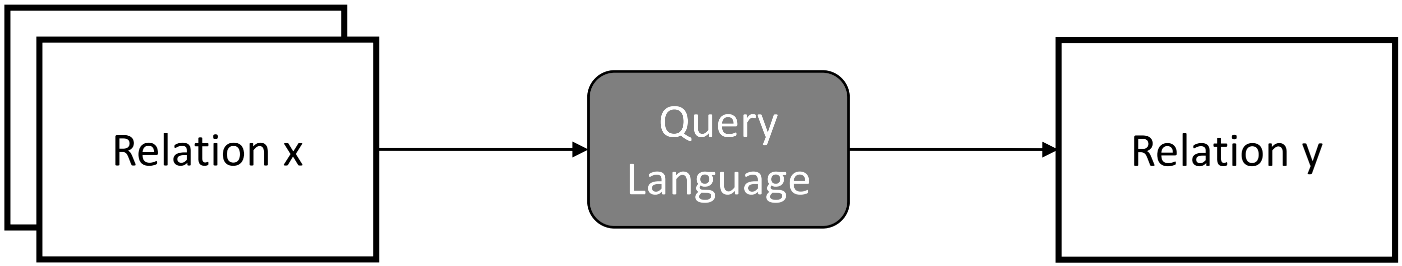



**Procedural Language**
Relational Algebra

**Declarative Languages**
Tuple Relational Calculus
Domain Relational Calculus

**Popular Language**
SQL

# Relational Algebra




**Fundamental Operations**
select, project, rename
set difference, cartesian product, union

**More Operations**
set intersection, natural join, assignment

# Select Operation

- Notation: $\sigma_p(r)$ where p is called the selection predicate

| *ID* | *name* | *dept_name* | *salary* |
|---|---|---|---|
| 22222 | Einstein | Physics | 95000 |
| 12121 | Wu | Finance | 90000 |
| 32343 | El Said | History | 60000 |
| 45565 | Katz | Comp. Sci. | 75000 |
| 98345 | Kim | Elec. Eng. | 80000 |
| 76766 | Crick | Biology | 72000 |
| 10101 | Srinivasan | Comp. Sci. | 65000 |
| 58583 | Califieri | History | 62000 |
| 83821 | Brandt | Comp. Sci. | 92000 |
| 15151 | Mozart | Music | 40000 |
| 33456 | Gold | Physics | 87000 |
| 76543 | Singh | Finance | 80000 |

(a) The *instructor* table

- Example of selection:

$$\sigma_{\text{dept\_name=“Physics”}}(\text{instructor})$$

# Select Operation

| *ID* | *name* | *dept_name* | *salary* |
|---|---|---|---|
| 22222 | Einstein | Physics | 95000 |
| 12121 | Wu | Finance | 90000 |
| 32343 | El Said | History | 60000 |
| 45565 | Katz | Comp. Sci. | 75000 |
| 98345 | Kim | Elec. Eng. | 80000 |
| 76766 | Crick | Biology | 72000 |
| 10101 | Srinivasan | Comp. Sci. | 65000 |
| 58583 | Califieri | History | 62000 |
| 83821 | Brandt | Comp. Sci. | 92000 |
| 15151 | Mozart | Music | 40000 |
| 33456 | Gold | Physics | 87000 |
| 76543 | Singh | Finance | 80000 |

(a) The *instructor* table

- Example of selection:

$$\sigma_{\text{dept\_name="Physics"}}(\text{instructor})$$

| *ID* | *name* | *dept_name* | *salary* |
|---|---|---|---|
| 22222 | Einstein | Physics | 95000 |
| 33456 | Gold | Physics | 87000 |

# Project Operation

- Notation: $\prod_{A1, A2,..,Ak}$ (r) where $A_i$ are attribute names

| *ID* | *name* | *dept_name* | *salary* |
|---|---|---|---|
| 22222 | Einstein | Physics | 95000 |
| 12121 | Wu | Finance | 90000 |
| 32343 | El Said | History | 60000 |
| 45565 | Katz | Comp. Sci. | 75000 |
| 98345 | Kim | Elec. Eng. | 80000 |
| 76766 | Crick | Biology | 72000 |
| 10101 | Srinivasan | Comp. Sci. | 65000 |
| 58583 | Califieri | History | 62000 |
| 83821 | Brandt | Comp. Sci. | 92000 |
| 15151 | Mozart | Music | 40000 |
| 33456 | Gold | Physics | 87000 |
| 76543 | Singh | Finance | 80000 |

(a) The *instructor* table

- Example of projection:

$$\prod_{ID,\ name,\ salary} (instructor)$$

- Duplicate rows removed from result, since relations are sets

# Projection

| ID | name | dept_name | salary |
|---|---|---|---|
| 22222 | Einstein | Physics | 95000 |
| 12121 | Wu | Finance | 90000 |
| 32343 | El Said | History | 60000 |
| 45565 | Katz | Comp. Sci. | 75000 |
| 98345 | Kim | Elec. Eng. | 80000 |
| 76766 | Crick | Biology | 72000 |
| 10101 | Srinivasan | Comp. Sci. | 65000 |
| 58583 | Califieri | History | 62000 |
| 83821 | Brandt | Comp. Sci. | 92000 |
| 15151 | Mozart | Music | 40000 |
| 33456 | Gold | Physics | 87000 |
| 76543 | Singh | Finance | 80000 |

(a) The *instructor* table

$\Pi_{ID,\ name,\ salary}$ (*instructor*)

| ID | name | salary |
|---|---|---|
| 10101 | Srinivasan | 65000 |
| 12121 | Wu | 90000 |
| 15151 | Mozart | 40000 |
| 22222 | Einstein | 95000 |
| 32343 | El Said | 60000 |
| 33456 | Gold | 87000 |
| 45565 | Katz | 75000 |
| 58583 | Califieri | 62000 |
| 76543 | Singh | 80000 |
| 76766 | Crick | 72000 |
| 83821 | Brandt | 92000 |
| 98345 | Kim | 80000 |

# Union Operation

- Notation: $r \cup s$. Defined as:

$$r \cup s = \{t \mid t \in r \text{ or } t \in s\}$$

- For $r \cup s$ to be valid:
  - $r$, $s$ must have the *same* **arity** (same number of attributes)
  - The attribute domains must be **compatible** (example: 2nd column of $r$ deals with the same type of values as does the 2nd column of $s$)

- Example: to find all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or in both

$$\Pi_{course\_id}\,(\sigma_{semester=\text{“Fall”}\ \wedge\ year=2009}\,(section))\ \cup$$

$$\Pi_{course\_id}\,(\sigma_{semester=\text{“Spring”}\ \wedge\ year=2010}\,(section))$$

# Union, Selection and Projection

- $\Pi_{course\_id}\,(\sigma_{semester=\text{“Fall”}\ \land\ year=2009}\,(section))\ \cup$
  $\Pi_{course\_id}\,(\sigma_{semester=\text{“Spring”}\ \land\ year=2010}\,(section))$

| course_id | sec_id | semester | year | building | room_number | time_slot_id |
|---|---|---|---|---|---|---|
| BIO-101 | 1 | Summer | 2009 | Painter | 514 | B |
| BIO-301 | 1 | Summer | 2010 | Painter | 514 | A |
| CS-101 | 1 | Fall | 2009 | Packard | 101 | H |
| CS-101 | 1 | Spring | 2010 | Packard | 101 | F |
| CS-190 | 1 | Spring | 2009 | Taylor | 3128 | E |
| CS-190 | 2 | Spring | 2009 | Taylor | 3128 | A |
| CS-315 | 1 | Spring | 2010 | Watson | 120 | D |
| CS-319 | 1 | Spring | 2010 | Watson | 100 | B |
| CS-319 | 2 | Spring | 2010 | Taylor | 3128 | C |
| CS-347 | 1 | Fall | 2009 | Taylor | 3128 | A |
| EE-181 | 1 | Spring | 2009 | Taylor | 3128 | C |
| FIN-201 | 1 | Spring | 2010 | Packard | 101 | B |
| HIS-351 | 1 | Spring | 2010 | Painter | 514 | C |
| MU-199 | 1 | Spring | 2010 | Packard | 101 | D |
| PHY-101 | 1 | Fall | 2009 | Watson | 100 | A |

| course_id |
|---|
| CS-101 |
| CS-315 |
| CS-319 |
| CS-347 |
| FIN-201 |
| HIS-351 |
| MU-199 |
| PHY-101 |

# Set Difference Operation

- Notation: $r - s$. Defined as

$$r - s = \{t \mid t \in r \textbf{ and } t \notin s\}$$

- Example: to find all courses taught in the Fall 2009 semester, but not in the Spring 2010 semester

$$\Pi_{course\_id}\,(\sigma_{semester=\text{“Fall”}\ \land\ year=2009}\,(section))\ -$$
$$\Pi_{course\_id}\,(\sigma_{semester=\text{“Spring”}\ \land\ year=2010}\,(section))$$

# Union, Selection and Projection

- $\Pi_{course\_id}\,(\sigma_{semester=\text{“Fall”}\ \land\ year=2009}\,(section))\ -$
  $\Pi_{course\_id}\,(\sigma_{semester=\text{“Spring”}\ \land\ year=2010}\,(section))$

| course_id | sec_id | semester | year | building | room_number | time_slot_id |
|---|---|---|---|---|---|---|
| BIO-101 | 1 | Summer | 2009 | Painter | 514 | B |
| BIO-301 | 1 | Summer | 2010 | Painter | 514 | A |
| CS-101 | 1 | Fall | 2009 | Packard | 101 | H |
| CS-101 | 1 | Spring | 2010 | Packard | 101 | F |
| CS-190 | 1 | Spring | 2009 | Taylor | 3128 | E |
| CS-190 | 2 | Spring | 2009 | Taylor | 3128 | A |
| CS-315 | 1 | Spring | 2010 | Watson | 120 | D |
| CS-319 | 1 | Spring | 2010 | Watson | 100 | B |
| CS-319 | 2 | Spring | 2010 | Taylor | 3128 | C |
| CS-347 | 1 | Fall | 2009 | Taylor | 3128 | A |
| EE-181 | 1 | Spring | 2009 | Taylor | 3128 | C |
| FIN-201 | 1 | Spring | 2010 | Packard | 101 | B |
| HIS-351 | 1 | Spring | 2010 | Painter | 514 | C |
| MU-199 | 1 | Spring | 2010 | Packard | 101 | D |
| PHY-101 | 1 | Fall | 2009 | Watson | 100 | A |

| course_id |
|---|
| CS-347 |
| PHY-101 |

# Set-Intersection Operation

- Notation: $r \cap s$
- Defined as:
- $r \cap s = \{ t \mid t \in r \textbf{ and } t \in s \}$

**Quiz: True/False?**
**$r \cap s = r - (r - s)$**

# Cartesian-Product Operation

- Notation *r* x *s*

| *ID* | *name* | *dept_name* | *salary* |
|---|---|---|---|
| 10101 | Srinivasan | Comp. Sci. | 65000 |
| 12121 | Wu | Finance | 90000 |
| 15151 | Mozart | Music | 40000 |
| 22222 | Einstein | Physics | 95000 |
| 32343 | El Said | History | 60000 |
| 33456 | Gold | Physics | 87000 |
| 45565 | Katz | Comp. Sci. | 75000 |
| 58583 | Califieri | History | 62000 |
| 76543 | Singh | Finance | 80000 |
| 76766 | Crick | Biology | 72000 |
| 83821 | Brandt | Comp. Sci. | 92000 |
| 98345 | Kim | Elec. Eng. | 80000 |

**instructor relation**

| *ID* | *course_id* | *sec_id* | *semester* | *year* |
|---|---|---|---|---|
| 10101 | CS-101 | 1 | Fall | 2009 |
| 10101 | CS-315 | 1 | Spring | 2010 |
| 10101 | CS-347 | 1 | Fall | 2009 |
| 12121 | FIN-201 | 1 | Spring | 2010 |
| 15151 | MU-199 | 1 | Spring | 2010 |
| 22222 | PHY-101 | 1 | Fall | 2009 |
| 32343 | HIS-351 | 1 | Spring | 2010 |
| 45565 | CS-101 | 1 | Spring | 2010 |
| 45565 | CS-319 | 1 | Spring | 2010 |
| 76766 | BIO-101 | 1 | Summer | 2009 |
| 76766 | BIO-301 | 1 | Summer | 2010 |
| 83821 | CS-190 | 1 | Spring | 2009 |
| 83821 | CS-190 | 2 | Spring | 2009 |
| 83821 | CS-319 | 2 | Spring | 2010 |
| 98345 | EE-181 | 1 | Spring | 2009 |

**teaches relation**

# The *instructor* X *teaches table*

| *instructor.ID* | *name* | *dept_name* | *salary* | *teaches.ID* | *course_id* | *sec_id* | *semester* | *year* |
|---|---|---|---|---|---|---|---|---|
| 10101 | Srinivasan | Comp. Sci. | 65000 | 10101 | CS-101 | 1 | Fall | 2017 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 10101 | CS-315 | 1 | Spring | 2018 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 10101 | CS-347 | 1 | Fall | 2017 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 12121 | FIN-201 | 1 | Spring | 2018 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 15151 | MU-199 | 1 | Spring | 2018 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 22222 | PHY-101 | 1 | Fall | 2017 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 12121 | Wu | Finance | 90000 | 10101 | CS-101 | 1 | Fall | 2017 |
| 12121 | Wu | Finance | 90000 | 10101 | CS-315 | 1 | Spring | 2018 |
| 12121 | Wu | Finance | 90000 | 10101 | CS-347 | 1 | Fall | 2017 |
| 12121 | Wu | Finance | 90000 | 12121 | FIN-201 | 1 | Spring | 2018 |
| 12121 | Wu | Finance | 90000 | 15151 | MU-199 | 1 | Spring | 2018 |
| 12121 | Wu | Finance | 90000 | 22222 | PHY-101 | 1 | Fall | 2017 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 15151 | Mozart | Music | 40000 | 10101 | CS-101 | 1 | Fall | 2017 |
| 15151 | Mozart | Music | 40000 | 10101 | CS-315 | 1 | Spring | 2018 |
| 15151 | Mozart | Music | 40000 | 10101 | CS-347 | 1 | Fall | 2017 |
| 15151 | Mozart | Music | 40000 | 12121 | FIN-201 | 1 | Spring | 2018 |
| 15151 | Mozart | Music | 40000 | 15151 | MU-199 | 1 | Spring | 2018 |
| 15151 | Mozart | Music | 40000 | 22222 | PHY-101 | 1 | Fall | 2017 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 22222 | Einstein | Physics | 95000 | 10101 | CS-101 | 1 | Fall | 2017 |
| 22222 | Einstein | Physics | 95000 | 10101 | CS-315 | 1 | Spring | 2018 |
| 22222 | Einstein | Physics | 95000 | 10101 | CS-347 | 1 | Fall | 2017 |
| 22222 | Einstein | Physics | 95000 | 12121 | FIN-201 | 1 | Spring | 2018 |
| 22222 | Einstein | Physics | 95000 | 15151 | MU-199 | 1 | Spring | 2018 |
| 22222 | Einstein | Physics | 95000 | 22222 | PHY-101 | 1 | Fall | 2017 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |

# Join Operation

- The Cartesian-Product *instructor X teaches* associates every tuple of instructor with every tuple of teaches.
- Most of the resulting rows have information about instructors who did NOT teach a particular course.
- To get only those tuples of instructor X teaches that pertain to instructors and the courses that they taught, we write:

$$\sigma_{\,instructor.id\, =\, teaches.id}\ (instructor\ \mathrm{x}\ teaches\,))$$

# Join Operation (Cont.)

- $\sigma_{instructor.id = teaches.id}(instructor \times teaches))$

| *instructor.ID* | *name* | *dept_name* | *salary* | *teaches.ID* | *course_id* | *sec_id* | *semester* | *year* |
|---|---|---|---|---|---|---|---|---|
| 10101 | Srinivasan | Comp. Sci. | 65000 | 10101 | CS-101 | 1 | Fall | 2017 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 10101 | CS-315 | 1 | Spring | 2018 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 10101 | CS-347 | 1 | Fall | 2017 |
| 12121 | Wu | Finance | 90000 | 12121 | FIN-201 | 1 | Spring | 2018 |
| 15151 | Mozart | Music | 40000 | 15151 | MU-199 | 1 | Spring | 2018 |
| 22222 | Einstein | Physics | 95000 | 22222 | PHY-101 | 1 | Fall | 2017 |
| 32343 | El Said | History | 60000 | 32343 | HIS-351 | 1 | Spring | 2018 |
| 45565 | Katz | Comp. Sci. | 75000 | 45565 | CS-101 | 1 | Spring | 2018 |
| 45565 | Katz | Comp. Sci. | 75000 | 45565 | CS-319 | 1 | Spring | 2018 |
| 76766 | Crick | Biology | 72000 | 76766 | BIO-101 | 1 | Summer | 2017 |
| 76766 | Crick | Biology | 72000 | 76766 | BIO-301 | 1 | Summer | 2018 |
| 83821 | Brandt | Comp. Sci. | 92000 | 83821 | CS-190 | 1 | Spring | 2017 |
| 83821 | Brandt | Comp. Sci. | 92000 | 83821 | CS-190 | 2 | Spring | 2017 |
| 83821 | Brandt | Comp. Sci. | 92000 | 83821 | CS-319 | 2 | Spring | 2018 |
| 98345 | Kim | Elec. Eng. | 80000 | 98345 | EE-181 | 1 | Spring | 2017 |

# Join Operation (Cont.)

- The **join** operation allows us to combine a select operation and a Cartesian-Product operation into a single operation.
- Consider relations *r* (*R*) and *s* (*S*). Let "theta" be a predicate on attributes in the schema R "union" S. The join operation r $\bowtie_\theta$ s is defined as follows:

$$r \bowtie_\theta s = \sigma_\theta \, (r \times s)$$

- Thus

$$\sigma_{instructor.id = teaches.id} \, (instructor \times teaches))$$

- Can equivalently be written as

$$instructor \bowtie_{instructor.id = teaches.id} teaches.$$

# Rename Operation

- Allows us to refer to a relation by more than one name.
- Example:

$$\rho_x (E)$$

  returns the expression *E* under the name *X*
- If a relational-algebra expression *E* has arity *n*, then

$$\rho_{x(A_1, A_2, \ldots, A_n)} (E)$$

  returns the result of expression *E* under the name *X*, and with the attributes renamed to $A_1, A_2, \ldots, A_n$.

# Find the highest salary in the university

- Steps to reach the solution:

| *ID* | *name* | *dept_name* | *salary* |
|---|---|---|---|
| 10101 | Srinivasan | Comp. Sci. | 65000 |
| 12121 | Wu | Finance | 90000 |
| 15151 | Mozart | Music | 40000 |
| 22222 | Einstein | Physics | 95000 |
| 32343 | El Said | History | 60000 |
| 33456 | Gold | Physics | 87000 |
| 45565 | Katz | Comp. Sci. | 75000 |
| 58583 | Califieri | History | 62000 |
| 76543 | Singh | Finance | 80000 |
| 76766 | Crick | Biology | 72000 |
| 83821 | Brandt | Comp. Sci. | 92000 |
| 98345 | Kim | Elec. Eng. | 80000 |

| *salary* |
|---|
| 65000 |
| 90000 |
| 40000 |
| 60000 |
| 87000 |
| 75000 |
| 62000 |
| 72000 |
| 80000 |
| 92000 |

**95000**

# Find the highest salary in the university

- Compute instructor x instructor
- Compute

$$\Pi_{instructor.salary}\,(\sigma_{instructor.salary\ <\ d.salary}\,(instructor\ \times\ \rho_d\,(instructor)))$$

- Compute

$$\Pi_{salary}\,(instructor) - \Pi_{instructor.salary}\,(\sigma_{instructor.salary\ <\ d.salary}\,(instructor\ \times\ \rho_d\,(instructor)))$$

**We use rename operation to distinguish the two salary attributes.**

# Relational Algebra

- A basic expression in the relational algebra consists of either one of the following:
  - A relation in the database
  - A constant relation
- Let $E_1$ and $E_2$ be relational-algebra expressions; the following are all relational-algebra expressions:
  - $E_1 \cup E_2$
  - $E_1 - E_2$
  - $E_1$ x $E_2$
  - $\sigma_p\,(E_1)$, $P$ is a predicate on attributes in $E_1$
  - $\prod_s(E_1)$, $S$ is a list consisting of some of the attributes in $E_1$
  - $\rho_{\,x}\,(E_1)$, x is the new name for the result of $E_1$

# Fundamental Operations

The select, project, rename, set difference, cartesian product, and union are sufficient to express queries!

# Extended Operations

- Set Intersection
  - Find the set of all courses taught in both the Fall 2017 and the Spring 2018 semesters

$$\Pi_{course\_id}\,(\sigma_{semester=\text{"Fall"}\ \land\ year=2017}\,(section))\ \cap$$
$$\Pi_{course\_id}\,(\sigma_{semester=\text{"Spring"}\ \land\ year=2018}\,(section))$$

Result

| *course_id* |
|---|
| CS-101 |

- Assignment
  - Find all instructor in the "Physics" and Music department.

$$Physics \leftarrow \sigma_{dept\_name=\text{"Physics"}}(instructor)$$
$$Music \leftarrow \sigma_{dept\_name=\text{"Music"}}(instructor)$$
$$Physics\ \cup Music$$

# Aggregate Functions

- Improves ease of use
- Most common functions:

| Functions |
| --- |
| sum |
| max |
| min |
| avg |
| count |

Calligraphic G Notation

$$\mathcal{G}_{\textbf{sum(c)}}(\text{r})$$

Compute sum of all values of attribute c on relation r.

# Summary

- Very much like normal algebra (x – y). We use relations instead of numbers as basic expressions.
- Basis for commercial query languages such as SQL.
- Three major components:
  - Fundamental Operations
  - Extended Operations
  - Aggregate Functions