

Notes on Fluid Mechanics

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1 Shear

Consider a shear testing device consisting of two plates with a small piece of material between them. We apply a force F continuously on the upper plate. The shear stress τ is defined as the amount of force per unit area,

$$\tau = \frac{F}{A} \quad (1)$$

where A is the area of the plate in contact with the fluid.

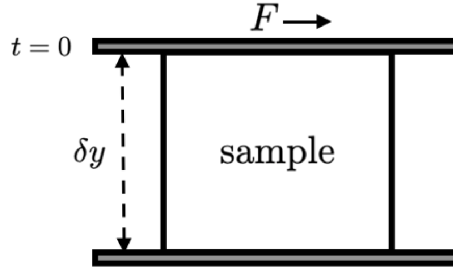


Figure 1: A shear testing device (depicted in 2D).

1.1 Shear in solids

Consider the sample in our shear testing device is a solid, Fig 1.1 shows the deformation after time t' . For any given shear τ , the shear deformation is finite in solids. We can define a new quantity, shear strain λ , to measure this deformation:

$$\lambda = \frac{\Delta x}{\delta y} \quad (2)$$

We can derive the relationship between shear stress and strain experimentally. In fact, it turns out they are related linearly. We call the constant of proportionality the *shear modulus*, G , and it is dependent on the material.

$$\tau = G\lambda \quad (3)$$

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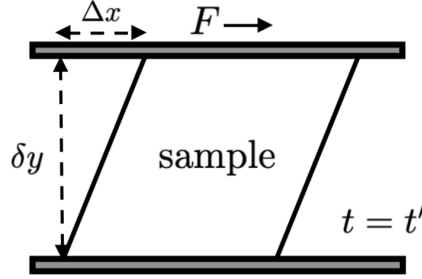


Figure 2: Shear deformation at time $t > 0$ for a solid.

1.2 Shear in fluids

In comparison to the behaviour in solids, when a continuous shear stress is applied to a fluid it continues to deform indefinitely, assuming there are no counteracting forces. We can define what a fluid is according to this behaviour.

Definition 1.1. A *fluid* is any material that is unable to prevent the deformation caused by shear stress.

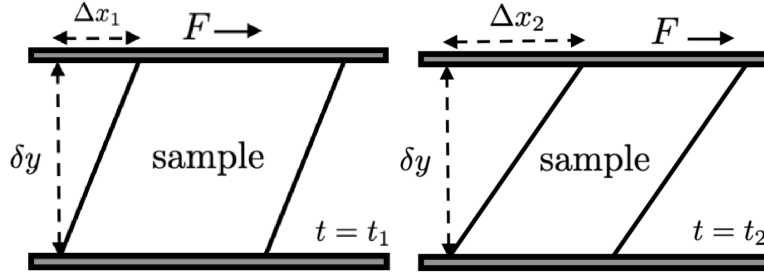


Figure 3: Shear deformation for $t > 0$ in a fluid.

Experimentally, we find that

$$\tau \propto \frac{d\lambda}{dt}. \quad (4)$$

In particular, for Newtonian fluids we find that this relationship is linear. We call the constant of proportionality the *absolute* or *dynamic viscosity*, μ .

$$\tau = \mu \frac{d\lambda}{dt} \quad (5)$$

Proposition 1.2. For a fluid with velocity $\mathbf{u} = (u, v, w)$, where u is the velocity component in the direction of the shear stress,

$$\tau = \mu \frac{du}{dy}.$$

Proof. Consider our shear testing device as $\delta y \rightarrow 0$. Then,

$$\tau = \lim_{\delta y \rightarrow 0} \frac{d\lambda}{dt} = \lim_{\delta y \rightarrow 0} \frac{d}{dt} \frac{\Delta x}{\delta y}$$

since we know that $\lambda = \Delta x / \delta y$

□