Solutions to all problems in Robert Resnick's Introduction to Special Relativity

`@vvveracruz`*

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Resnick's book can be accessed for free here – you will just need to enter your email address. If that doesn't work, email me.

Problems for Chapter 1

Q1. Justify the relations $y = y'$ and $z = z'$ of Eq1-1a by symmetry arguments.

Solution. Eq1-1a describes the Galilean transformation between the two frames of reference depicted in Fig 1.

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$

The transformation between y and y' because for $y = y_0$, $y' = y_0$ (see red lines in Fig 1). Similarly for z and z'.

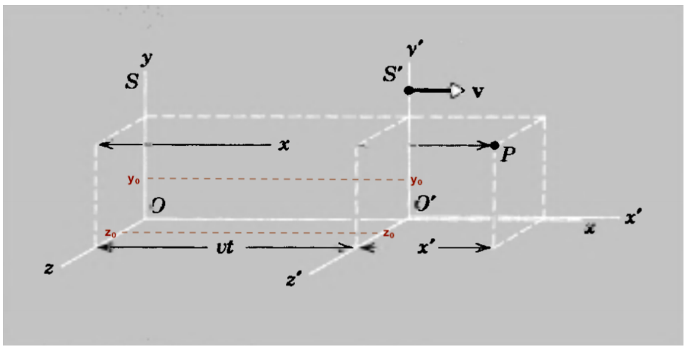

Figure 1: Resnick's diagram depicting the two inertial frames of reference S and S'. S' is moving with velocity v with respect to S. Point P is an event, whose space-time coordinates may be measured by each observer.

*Corrections to `me@vgg.cz`.

Q2. Momentum is conserved in a collision of two objects as measured by an observer on a uniformly moving train. Show that momentum is also conserved for a ground observer.

Solution. Consider two point particles of mass m_1 kg and m_2 kg travelling at speeds v_1 $\mathrm{m\,s^{-1}}$ and v_2 $\mathrm{m\,s^{-1}}$ respectively (Fig 2). Take the direction of motion of v_1 as being the positive direction. Then the total momentum for an observer inside the train before the collision is

$$p = m \cdot v,\ p_{\text{before}} = m_1 v_1 - m_2 v_2 = p_{\text{after}} \tag{1}$$

since the question tells us momentum is conserved for an observer inside the train.

An observer outside the train will see the particles moving at a different speed, namely $v_1' = v_1 + v_3$ and $-v_2' = -v_2 + v_3$. Hence,

$$p_{\text{before}} = m_1(v_1 + v_3) + m_2(-v_2 + v_3) = m_1 v_1 - m_2 v_2 + v_3(m_1 + m_2) \tag{2}$$

which is conserved since $m_1 v_1 - m_2 v_2$ is conserved by the assumption, and v_3, m_1 and m_2 are constants.

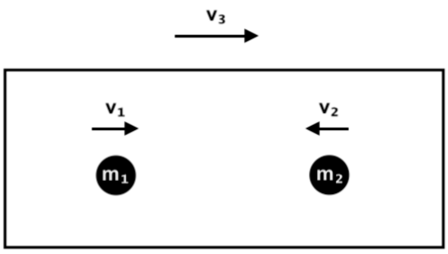

Figure 2: Two particles colliding inside a train travelling at velocity v_3 $\mathrm{m\,s^{-1}}$.

Q3. Repeat Q2 under the assumption that after the collision the masses of the two objects are different from what they were before, that is assume a transfer of mass took place in the course of the collision. Show that for momentum to be conserved for the ground observer, conservation of mass must hold true.

Solution. Consider this situation from the point of view of the observer on the train. The two objects are initially moving with velocities and masses (v_1, m_1) and $(-v_2, m_2)$ respectively. After the collision, both their velocities and masses change to (v_4, m_4) and (v_5, m_5) again respectively. Note that v_4, v_5 can be positive or negative so we don't need to worry about any negative signs. Then, since again we are told that momentum is conserved, we have:

$$p_{\text{before}} = m_1 v_1 - m_2 v_2 = m_4 v_4 + m_5 v_5 = p_{\text{after}} \tag{3}$$

We still have (2) for the total momentum before the collision from the point of view of an outside observer. After the collision, we similarly have

$$p_{\text{after}} = m_4(v_4 + v_3) + m_5(v_5 + v_3) = m_4v_4 + m_5v_5 + v_3(m_4 + m_5). \quad (4)$$

From the assumption, we know that $m_1v_1 - m_2v_2 = m_4v_4 + m_5v_5$, so for momentum to be conserved we must have $m_1 + m_2 = m_4 + m_5$, conservation of mass, as expected. s

Q4. Kinetic energy is conserved in an elastic collision by definition. Show, using the Galilean transformation equations, that if a collision is elastic in one frame of reference then it is ellastic in all intertial frames.