Notes on Fluid Mechanics

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[1] Corrections to me@vgg.cz.

Chapter 1

Shear

Consider a shear testing device consisting of two plates with a small piece of material between them. We apply a force F continuously on the upper plate. The shear stress τ is defined as the amount of force per unit area,

$$\tau = \frac{F}{A} \tag{1.1}$$

where A is the area of the plate in contact with the fluid.

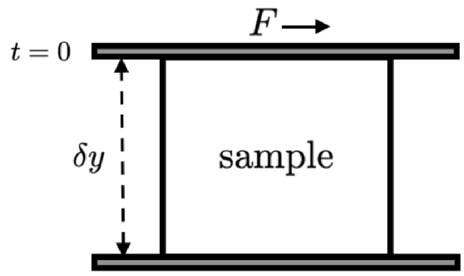

Figure 1.1: A shear testing device (depicted in 2D).

1.1 Shear in solids

Consider the sample in our shear testing device is a solid, Fig 1.1 shows the deformation after time t'. For any given shear τ, the shear deformation is finite in solids. We can define a new quantity, shear strain λ, to measure this deformation:

$$\lambda = \frac{\Delta x}{\delta y} \tag{1.2}$$

We can derive the relationship between shear stress and strain experimentally. In fact, it turns out they are related linearly. We call the constant of proportionality the *shear modulus*, G, and it is dependent on the material.

$$\tau = G\lambda \tag{1.3}$$

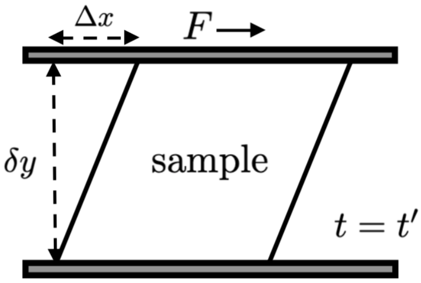

Figure 1.2: Shear deformation at time $t > 0$ for a solid.

1.2 Shear in fluids

In comparison to the behaviour in solids, when a continuous shear stress is applied to a fluid it continues to deform indefinitely, assuming there are no counteracting forces. We can define what a fluid is according to this behaviour.

Definition 1.2.1. *A* ***fluid*** *is any material that is unable to prevent the deformation caused by shear stress.*

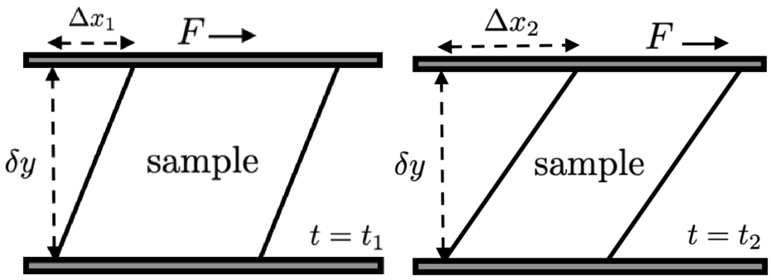

Figure 1.3: Shear deformation for $t > 0$ in a fluid.

Experimentally, we find that

$$\tau \propto \frac{d\lambda}{dt}. \tag{1.4}$$

In particularly, for Newtonian fluids we find that this relationship is linear. We call the constant of proportionality the *absolute* or *dynamic viscosity*, μ.

$$\tau = \mu \frac{d\lambda}{dt} \tag{1.5}$$

This is known as Newton's Law of Viscosity.

Proposition 1.2.2. *For a fluid with velocity* $\mathbf{u} = (u, v, w)$*, where* u *is the velocity component in the direction of the shear stress,*

$$\tau = \mu \frac{du}{dy}.$$

Proof. First recall the definition of shear strain λ (1.2), so we have

$$\tau = \mu \frac{d}{dt} \frac{\Delta x}{\delta y}. \tag{1.6}$$

We know that δy is a constant, so

$$\tau = \mu \frac{1}{\delta y} \Delta \left(\frac{dx}{dt} \right) = \mu \frac{\Delta u}{\delta y} = \mu \frac{du}{dy}. \tag{1.7}$$

A more formal proof can be given by considering the limit of this expression as $\delta y \to 0$. $\square$