Solutions to E. J. Shaughnessy, I. M. Katz and J. P. Schaffer's *Introduction to Fluid Mechanics*

@vvveracruz[1]

Last edited July 27, 2020

[1]Corrections to me@vgg.cz.

Chapter 1

Problems 1.1 - 1.15 are discussion questions.

Q 1.16. Consider steel and aluminium sheets of 6mm thickness; each metal sheet is subjected to a shear stress of 2×10^5psi. Compare the magnitudes of the displacements.

Solution. We know that for solids the following relationship holds:

$$\tau = G\lambda \tag{1.1}$$

where τ is the shear stress, G is the shear modulus, and $\lambda = \Delta x/\Delta y$, the shear strain.

<u>Steel</u>

- shear modulus, $G = 11.3 \times 10^6$psi for cast steel [1].
- magnitude of shear stress, $\tau = 2 \times 10^5$psi
- height of sample, $\Delta y = 6 \times 10^{-3}$psi

Then, we have

$$\begin{aligned} 2 \times 10^5\text{psi} &= \frac{11.3 \times 10^6\text{psi } \Delta x}{6 \times 10^{-3}\text{m}} \\ \Delta x &= \frac{2 \times 10^5\text{psi} \times 6 \times 10^{-3}\text{m}}{11.3 \times 10^6\text{psi}} \\ &= 1.07 \times 10^{-4}\text{m or } 0.11\text{mm} \end{aligned}$$

<u>Aluminium</u>

- shear modulus, $G = 3.9 \times 10^6$psi for aluminium alloys [1].
- magnitude of shear stress, $\tau = 2 \times 10^5$psi
- height of sample, $\Delta y = 6 \times 10^{-3}$psi

Then, we have

$$\begin{aligned}2 \times 10^5\text{psi} &= \frac{3.9 \times 10^6\text{psi }\Delta x}{6 \times 10^{-3}\text{m}} \\ \Delta x &= \frac{2 \times 10^5\text{psi} \times 6 \times 10^{-3}\text{m}}{3.9 \times 10^6\text{psi}} \\ &= 3.08 \times 10^{-4}\text{m or } 0.31\text{mm}\end{aligned}$$

So we have a displacement that is around three times larger for aluminium than for steel for the same shear stress applied, this is consistent with our intuition that aluminium is much more malleable than steel.

Q 1.17 Consider steel and aluminium sheets of 5mm **thickness. If the steel sheet is subjected to a shear stress of** 8×10^5psi**, calculate the magnitude of the shear stress that must be applied to the aluminium sheet so that both materials experience the same displacment in the direction of the applied force.**

Solution. Using (1.1),

$$\frac{\tau_{al}\Delta y}{G_{al}} = \Delta x_{al} = \Delta x_{steel} = \frac{\tau_{steel}\Delta y}{G_{steel}}$$

since we need the displacement of both sheets to be the same, and they have the same thickness, Δy. We now just need to rearrange for τ_{al} using the data form question 1.16:

$$\begin{aligned}\tau_{al} &= \frac{\tau_{steel}G_{al}}{G_{steel}} \\ &= \frac{(8 \times 10^5\text{psi})(3.9 \times 10^6\text{psi})}{11.3 \times 10^6\text{psi}} \\ &= 2.76 \times 10^5\text{psi}.\end{aligned}$$

Q 1.18 A sample of motor oil has been tested in a flat plate shearing device with the following results: $\tau = 7.7\text{lb}_\text{f}/\text{ft}^2$ **for a plate separation distance of** 0.25in **and a top plate velocity of** 25ft/s**. Determine the viscosity of the fluid.**

Solution. Here we apply Newton's Law of Viscosity

$$\tau = \mu\frac{d\lambda}{dt} \tag{1.2}$$

where μ is the dynamic viscosity of the fluid, λ is the shear strain and u is the fluid speed in the direction of the shear stress. We have:

- $\tau = 7.7\text{lb}_\text{f}/\text{ft}^2$
- plate separation $\Delta y = 0.25$in
- top plate speed in the direction of the shear stress after the stress is applied $u = 25$ft/s

Note that we also know that

$$\tau = \mu \frac{du}{dy}. \tag{1.3}$$

Hence we can write

$$\tau = \mu \frac{\Delta u}{\Delta y} \tag{1.4}$$

where $\Delta u = 25\text{ft/s}$, since before the shear stress is applied the top plate velocity is 0, and $\Delta y = 0.25\text{in} = 0.25 \times 1/12\text{ft} = 1/48\text{ft}$, which we convert to ft because the rest of our quantitites are measured in feet too.

It'll be easier in this case to consider the dimensions of μ before calculating it's magnitude:

$$[\mu] = \frac{[\Delta y][\tau]}{[\Delta u]} = \frac{\text{ft} \times \text{lb}_\text{f}\,\text{ft}^{-2}}{\text{ft}\,\text{s}^{-1}} = \text{ft}\,\text{lb}_\text{f}\,\text{ft}^{-2}\,\text{ft}^{-1}\,\text{s} = \text{lb}_\text{f}\,\text{s}\,\text{ft}^{-2}, \text{ so}$$

$$\mu = \frac{1/48 \times 7.7}{25} \text{lb}_\text{f}\,\text{s}\,\text{ft}^{-2} = 6.42 \times 10^{-3} \text{lb}_\text{f}\,\text{s}\,\text{ft}^{-2}$$

$$\boxed{\mu = 6.42 \times 10^{-3} \text{lb}_\text{f}\,\text{s}\,\text{ft}^{-2}}$$

Q 1.19 A sample of motor oil has been tested in a flat plate shearing device with the following results: $\tau = 4.0\text{lb}_\text{f}/\text{ft}^2$ for a plate separation distance of 0.05in and a fluid viscosity of $\mu = 6.5 \times 10^{-3}\text{lb}_\text{f}\,\text{s}\,\text{ft}^{-2}$. Determine the top plate velocity.

Solution. This question is similar to Q1.18.

<u>Experimental data:</u>

- $\tau = 4.0\text{lb}_\text{f}\,\text{ft}^{-2}$,
- plate separation, $\Delta y = 0.05\text{in} = 0.05 \times 1/12\text{ft}$,
- fluid viscosity, $\mu = 6.5 \times 10^{-3}\text{lb}_\text{f}\,\text{s}\,\text{ft}^{-2}$

We are looking for the top plate velocity after the shear stress is applied. In our flat plate shearing device, the top plate begins at rest, so we are just looking for the change in velocity, Δu.

<u>Relevant equation:</u> Newton's Law of Viscosity,

$$\tau = \mu \frac{\Delta u}{\Delta y} \Rightarrow \Delta u = \frac{\Delta y \tau}{\mu} \tag{1.5}$$

.

Hence

$$\Delta u = \frac{0.5 \times 1/12\text{ft} \times 4.0\text{lb}_\text{f}\,\text{ft}^{-2}}{6.5 \times 10^{-3}\text{lb}_\text{f}\,\text{s}\,\text{ft}{-2}}$$

$$\Delta u = \frac{0.5 \times 1/12 \times 4.0}{6.5 \times 10^{-3}} \text{ft}\,\text{s}^{-1} = 25.64\text{ft}\,\text{s}^{-1}$$

$$\boxed{u_{\text{top plate}} = 25.64\text{ft}\,\text{s}^{-1}}$$

Appendix A

Unit conversion tables

	foot	inch
1 foot =	1	12
1 inch =	1/12	1

Table A.1: Length

Bibliography

[1] Engineering ToolBox. "Modulus of Rigidity". 2004. URL: https://www.engineeringtoolbox.com/modulus-rigidity-d_946.html (visited on 07/17/2020).