

a:

If $2x^4$ is $O(x^3 + 3x + 2)$ then $2x^4 \leq C * (x^3 + 3x + 2)$ for all $x \geq k$

$$x^3 + 3x + 2 \leq x^3 + 3x^3 + 2x^3$$

$$x^3 + 3x + 2 \leq 6x^3$$

Therefore:

$$2x^4 \leq C(x^3 + 3x + 2) \leq C * 6x^3$$

$2x^4 \leq C * 6x^3$ No C where this inequality is true for all $x \geq k$ so no witnesses to statement.

Therefore statement is false.

b:

If $4x^3 + 2x^2 \log(x) + 1$ is $O(x^3)$ then $4x^3 + 2x^2 \log(x) + 1 \leq C * x^3$ for all $x \geq k$

$$4x^3 + 2x^2 \log(x) + 1 \leq 4x^3 + 2x^3 + x^3$$

$$4x^3 + 2x^2 \log(x) + 1 \leq 7x^3$$

Therefore:

$$4x^3 + 2x^2 \log(x) + 1 \leq 7x^3 \leq Cx^3 \text{ Inequality true at } C = 7, k = 1$$

Therefore statement is true.

c:

If $3x^2 + 7x + 1$ is $\omega(x \log(x))$ then $3x^2 + 7x + 1 > C * x \log(x)$ for all $x \geq k$

$$3x^2 + 7x + 1 > C * x^2 > C * x \log(x)$$

$$3x^2 + 7x + 1 > 3x^2 \geq C * x^2 > C * x \log(x)$$

Therefore, statement true with witnesses $C = 3, k = 1$

d:

If $x^2 + 4x$ is $\Omega(x \log(x))$ then $x^2 + 4x \geq C * x \log(x)$ for all $x \geq k$

$$x^2 + 4x \geq C * x^2 > C * x \log(x)$$

$$x^2 + 4x > x^2 \geq C * x^2 > C * x \log(x)$$

Therefore, statement true with witnesses $C = 1, k = 1$

e:

Proof by contradiction. Assume statement is true.

$$\text{Let } f(x) = g(x) = x$$

$$f(x) + g(x) \text{ is } \theta(f(x).g(x))$$

$$x + x \text{ is } \theta(x.x)$$

$$2x \text{ is } \theta(x^2)$$

Therefore:

$$C_1.x^2 \leq 2x \leq C_2.x^2$$

No C_1 and C_2 pair where $C_1, C_2 > 0$ for all $x \geq k$

Contradicts assumption \therefore Statement is false.