

a:  
 $T(n) = 9T(n/3) + n^2$  so  $a = 9$ ,  $b = 3$  and  $f(n) = n^2$   
 Is  $n^2 = \Theta(n^{\log_3(9)})$ ? [Rule 2]  
 Is  $n^2 = \Theta(n^2)$ ? True.  
 $\therefore T(n) = \Theta(n^2 \log n)$

b:  
 $T(n) = 4T(n/2) + 100n$  so  $a = 4$ ,  $b = 2$  and  $f(n) = 100n$   
 Is  $100n = O(n^{\log_2(4)-\epsilon})$ ? [Rule 1]  
 Is  $100n = O(n^{2-\epsilon})$ ? True.  
 $\therefore T(n) = \Theta(n^2)$

c:  
 $T(n) = 2^n T(n/2) + n^3$  so  $a = 2^n$ ,  $b = 2$  and  $f(n) = n^3$   
 $a$  is not constant  $\therefore$  Master Theorem cannot be applied.

d:  
 $T(n) = 3T(n/3) + cn$  so  $a = 3$ ,  $b = 3$  and  $f(n) = cn$   
 Assuming  $c$  is constant term then Master Theorem can be applied.  
 Is  $cn = \Theta(n^{\log_3(3)})$ ? [Rule 2]  
 Is  $cn = \Theta(n)$ ? True.  
 $\therefore T(n) = \Theta(n \log n)$

e:  
 $T(n) = 0.99T(n/7) + 1/(n^2)$  so  $a = 0.99$ ,  $b = 7$  and  $f(n) = 1/(n^2) = n^{-2}$   
 Is  $n^{-2} = \Omega(n^{\log_7(0.99)+\epsilon})$ ? [Rule 3]  
 Is  $n^{-2} \approx \Omega(n^{-0.005+\epsilon})$ ? False.

Is  $n^{-2} = O(n^{\log_7(0.99)-\epsilon})$ ? [Rule 1] True.  
 $\therefore T(n) = \Theta(n^{\log_7(0.99)})$