a:

If $2x^4$ is $O(x^3+3x+2)$ then $2x^4 \leq C*(x^3+3x+2)$ for all $x \geq k$

$x^3+3x+2 \leq x^3+3x^3+2x^3$

$x^3+3x+2 \leq 6x^3$

Therefore:

$2x^4 \leq C(x^3+3x+2) \leq C*6x^3$

$2x^4 \leq C*6x^3$ No C where this inequality is true so no witnesses to statement.

Therefore statement is false.

b:

If $4x^3+2x^2\log(x)+1$ is $O(x^3)$ then $4x^3+2x^2\log(x)+1 \leq C*x^3$ for all $x \geq k$

$4x^3+2x^2\log(x)+1 \leq 4x^3+2x^3+x^3$

$4x^3+2x^2\log(x)+1 \leq 7x^3$

Therefore:

$4x^3+2x^2\log(x)+1 \leq 7x^3 \leq Cx^3$ Inequality true at $C=7$, $k=1$

Therefore statement is true.

c:

If $3x^2+7x+1$ is $\omega(x\log(x))$ then $3x^2+7x+1 > C*x\log(x)$ for all $x \geq k$

$3x^2+7x+1 > C*x^2 > C*x\log(x)$

$3x^2+7x+1 > 3x^2 \geq C*x^2 > C*x\log(x)$

Therefore, statement true with witnesses $C=3$, $k=1$

d:

If x^2+4x is $\Omega(x\log(x))$ then $x^2+4x \geq C*x\log(x)$ for all $x \geq k$

$x^2+4x \geq C*x^2 > C*x\log(x)$

$x^2+4x > x^2 \geq C*x^2 > C*x\log(x)$

Therefore, statement true with witnesses $C=1$, $k=1$

e:

Proof by contradiction. Assume statement is true.

Let $f(x)=g(x)=x$

$f(x)+g(x)$ is $\theta(f(x).g(x))$

$x+x$ is $\theta(x.x)$

$2x$ is $\theta(x^2)$

Therefore:

$C_1.x^2 \leq 2x \leq C_2.x^2$

No C_1 and C_2 pair where C_1, $C_2 > 0$ for all $x \geq k$

Contradicts assumption $\therefore$ Statement is false.