

# Cache Memories

Introduction to Computer Systems  
13<sup>th</sup> Lecture, Oct. 30, 2023

**Instructors:**

**Class 1: Chen Xiangqun, Liu Xianhua**

**Class 2: Guan Xuetao**

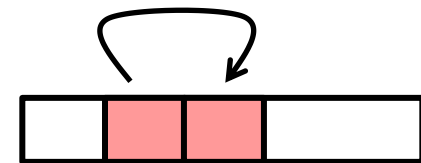
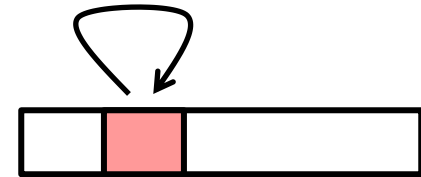
**Class 3: Lu Junlin**

# Today

- **Cache memory organization and operation**
- **Performance impact of caches**
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

# Recall: Locality

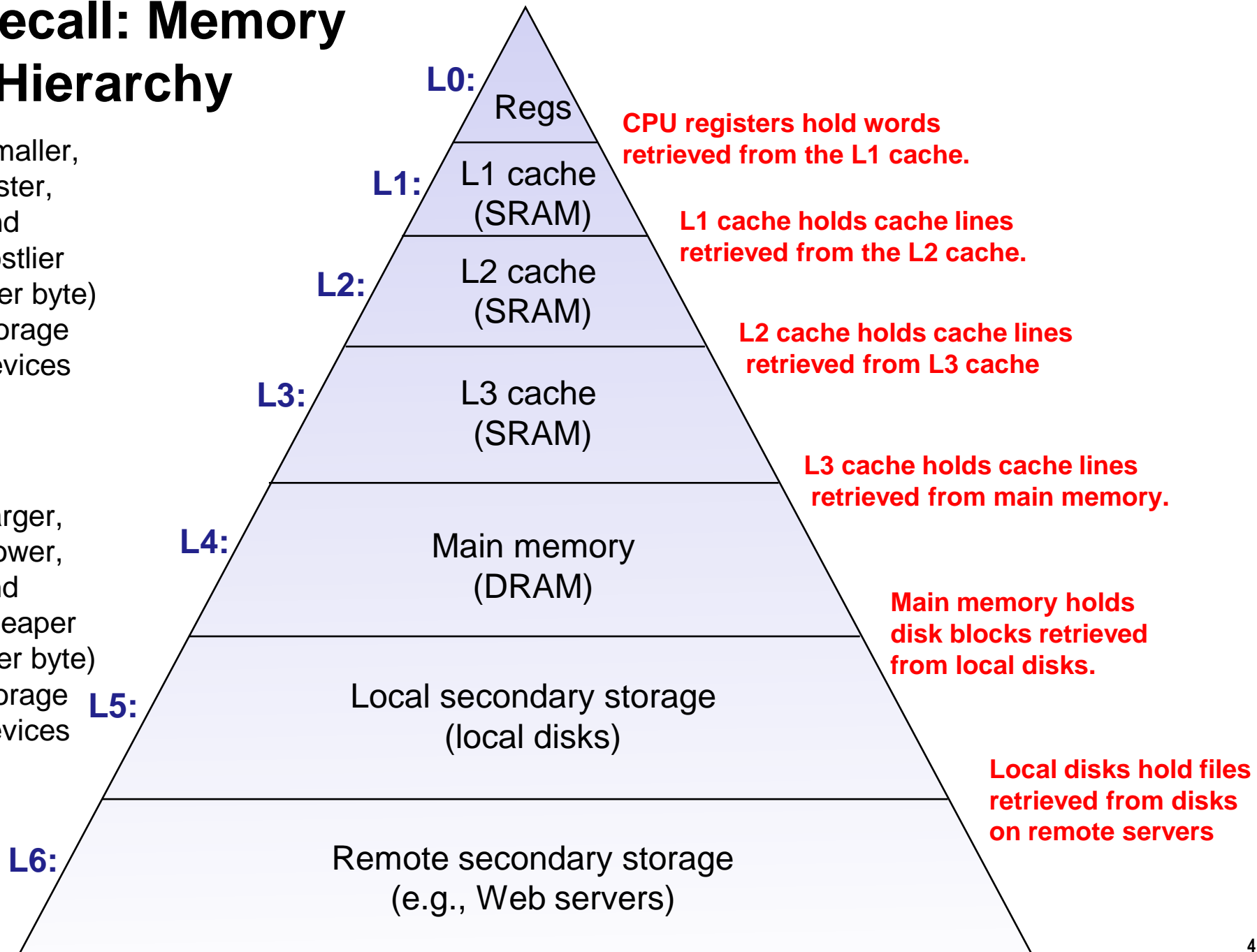
- **Principle of Locality:** Programs tend to use data and instructions with addresses near or equal to those they have used recently
- **Temporal locality:**
  - Recently referenced items are likely to be referenced again in the near future
- **Spatial locality:**
  - Items with nearby addresses tend to be referenced close together in time



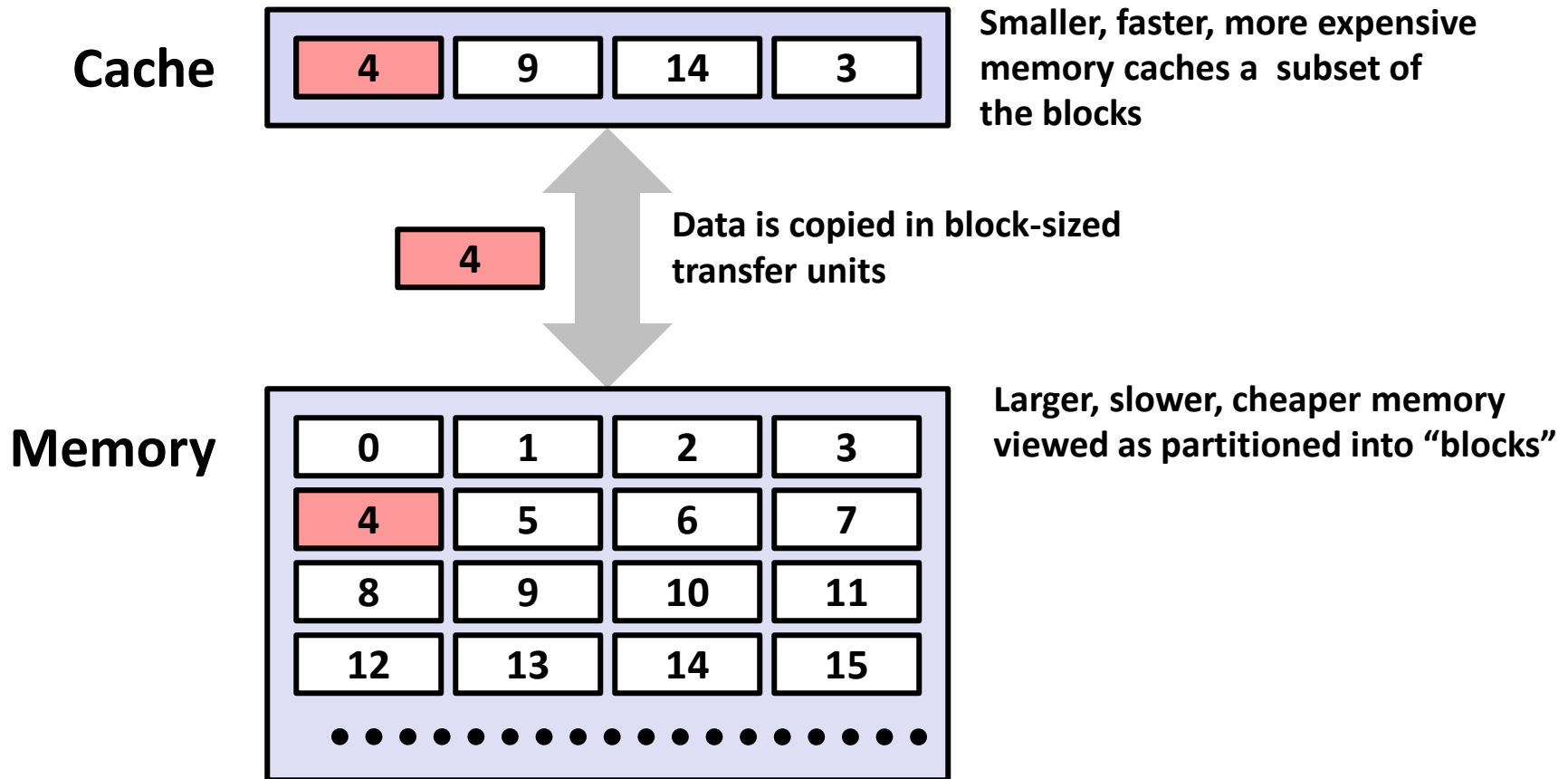
# Recall: Memory Hierarchy

↑  
Smaller,  
faster,  
and  
costlier  
(per byte)  
storage  
devices

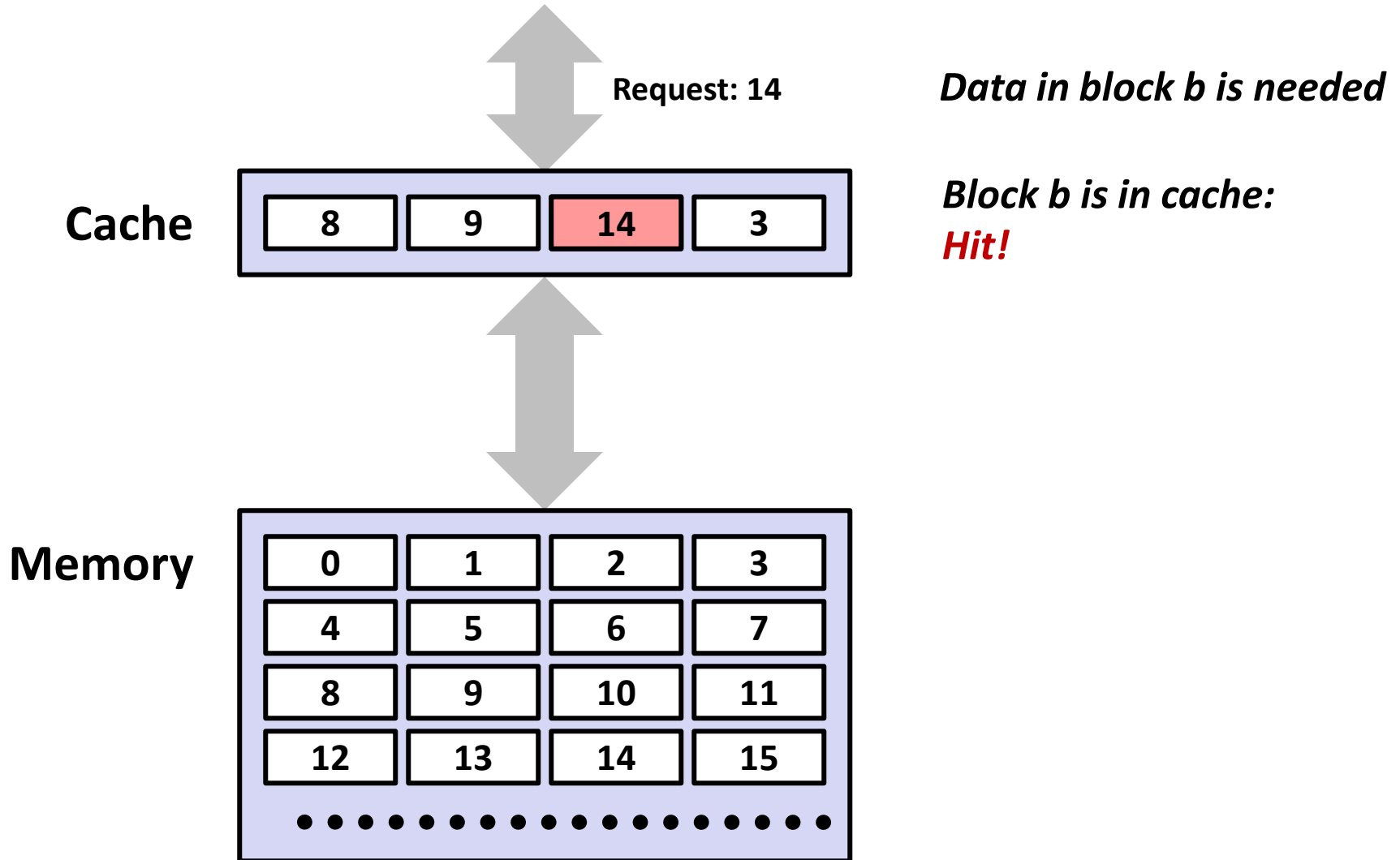
↓  
Larger,  
slower,  
and  
cheaper  
(per byte)  
storage  
devices



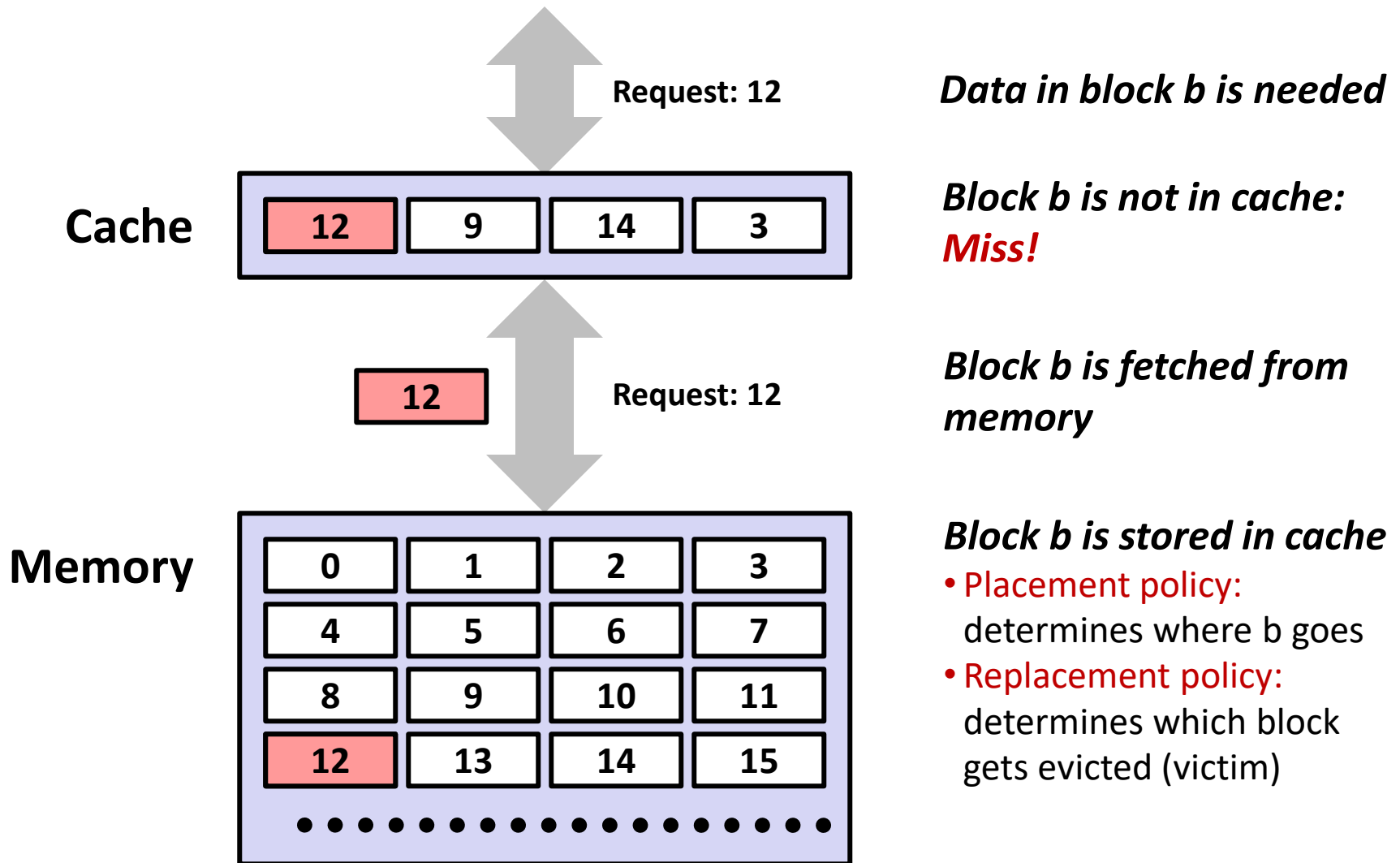
# Recall: General Cache Concepts



# General Cache Concepts: Hit



# General Cache Concepts: Miss



# Recall: General Caching Concepts:

## 3 Types of Cache Misses

### ■ Cold (compulsory) miss

- Cold misses occur because the cache starts empty and this is the first reference to the block.

### ■ Capacity miss

- Occurs when the set of active cache blocks (**working set**) is larger than the cache.

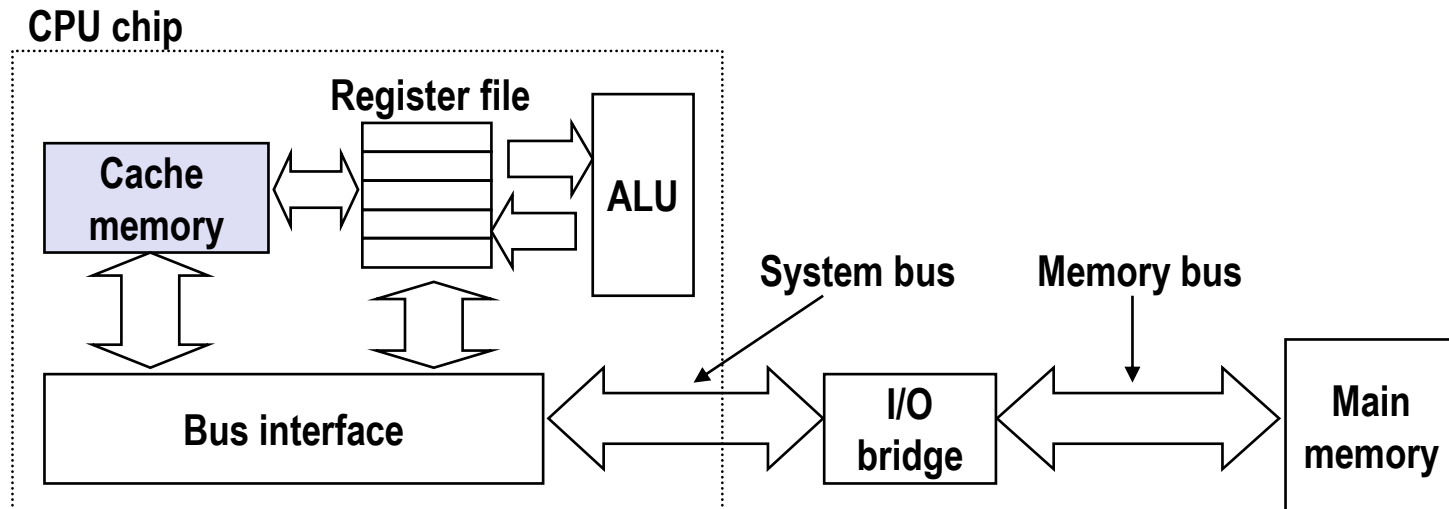
### ■ Conflict miss

- Most caches limit blocks at level  $k+1$  to a small subset (sometimes a singleton) of the block positions at level  $k$ .
  - E.g. Block  $i$  at level  $k+1$  must be placed in block  $(i \bmod 4)$  at level  $k$ .
- Conflict misses occur when the level  $k$  cache is large enough, but multiple data objects all map to the same level  $k$  block.
  - E.g. Referencing blocks 0, 8, 0, 8, 0, 8, ... would miss every time.

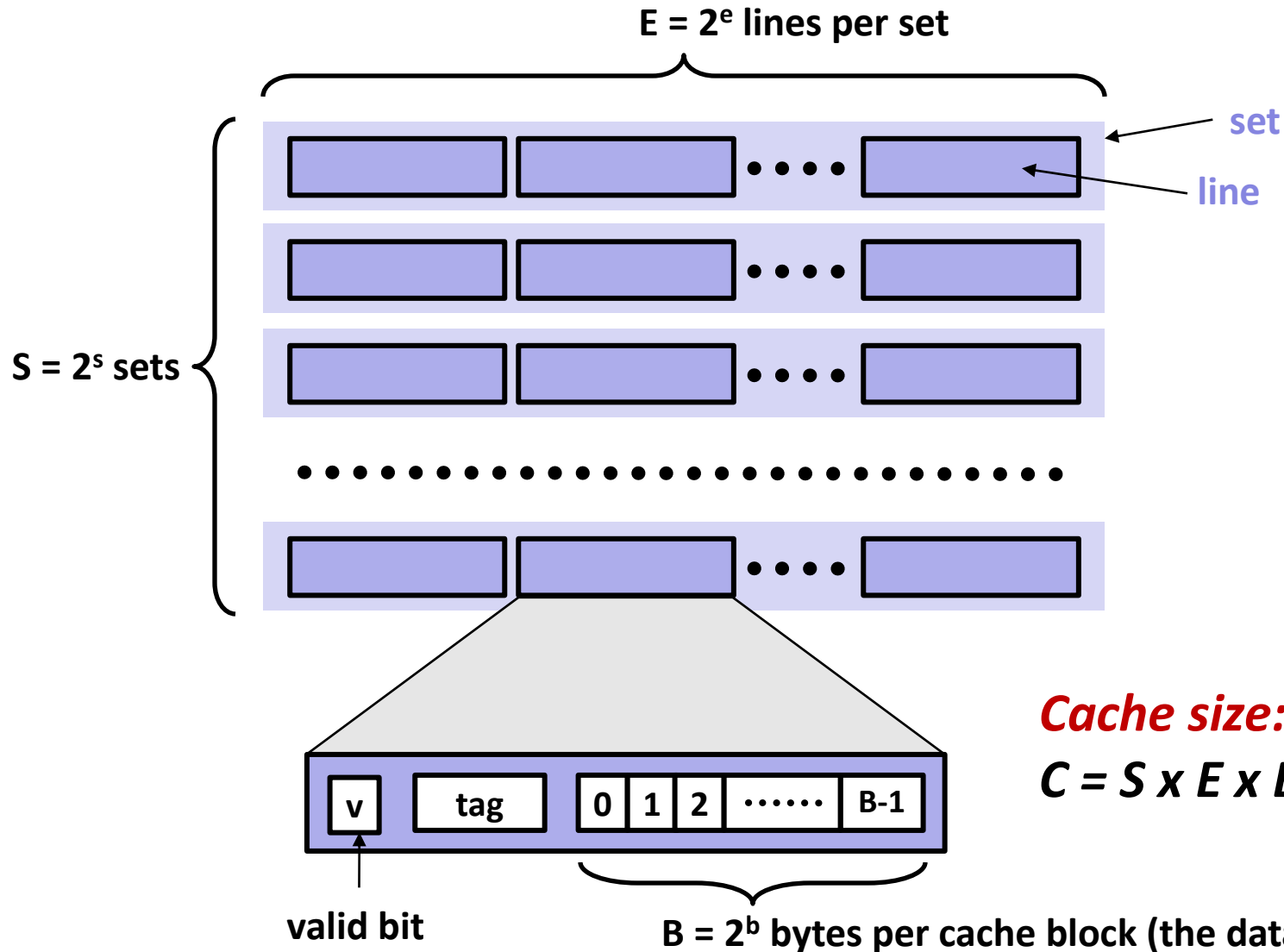


# Cache Memories

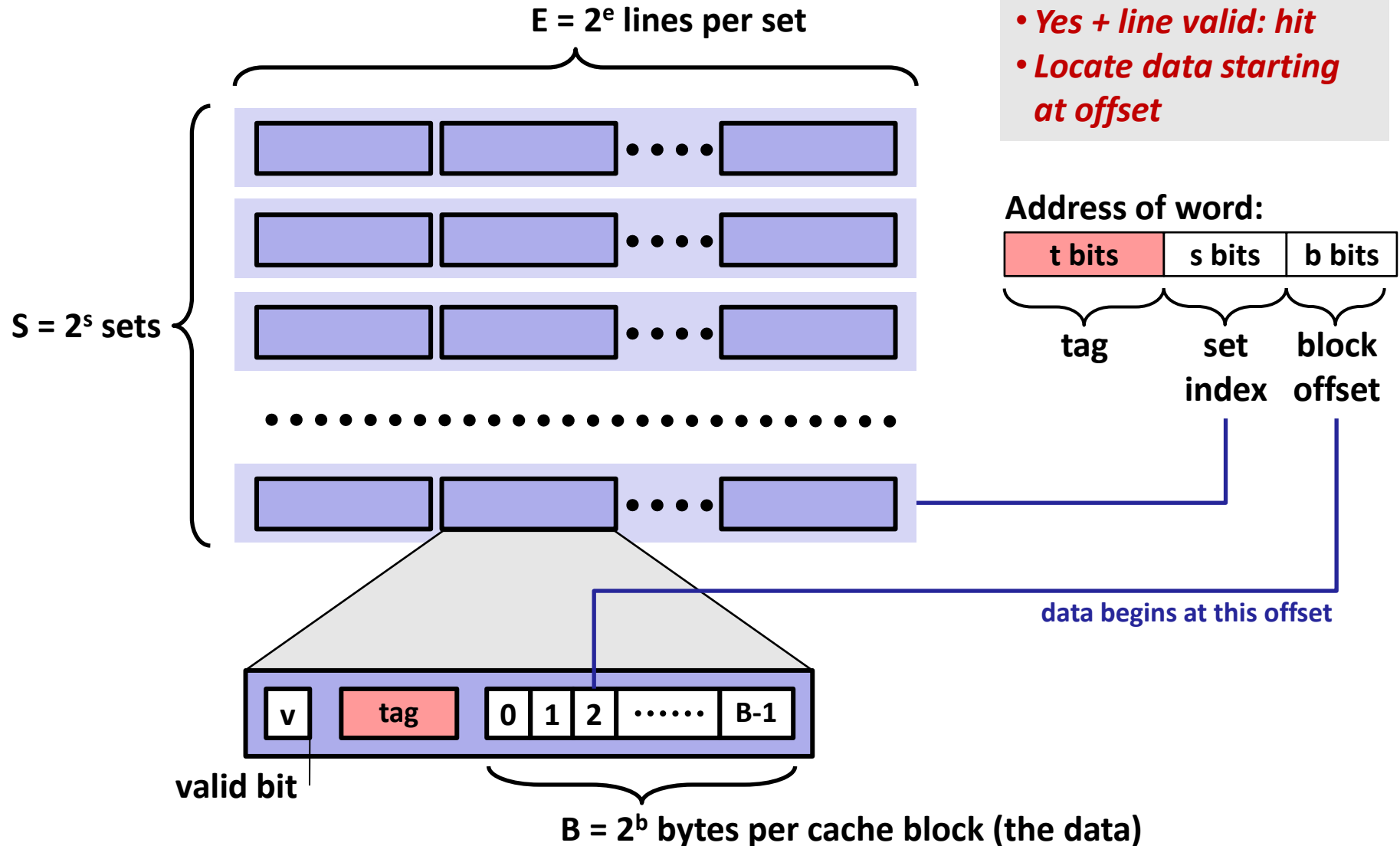
- **Cache memories** are small, fast SRAM-based memories managed automatically in hardware
  - Hold frequently accessed blocks of main memory
- **CPU looks first for data in cache**
- **Typical system structure:**



# General Cache Organization (S, E, B)



# Cache Read

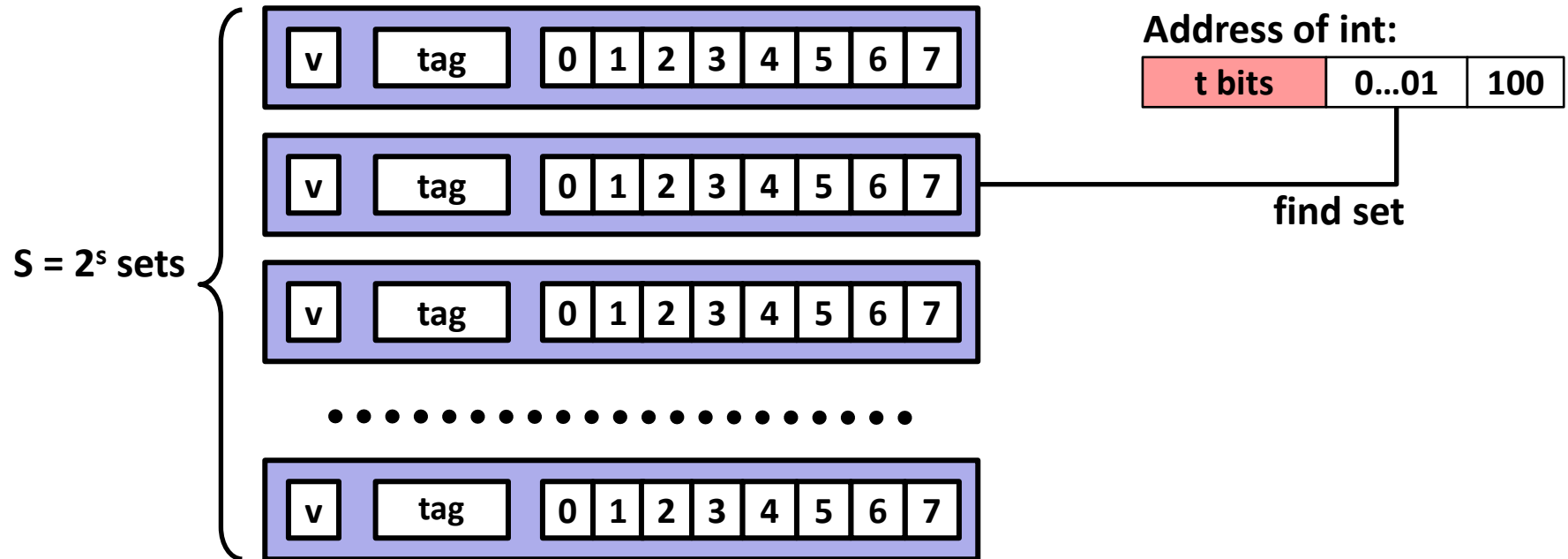


- *Locate set*
- *Check if any line in set has matching tag*
- *Yes + line valid: hit*
- *Locate data starting at offset*

# Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

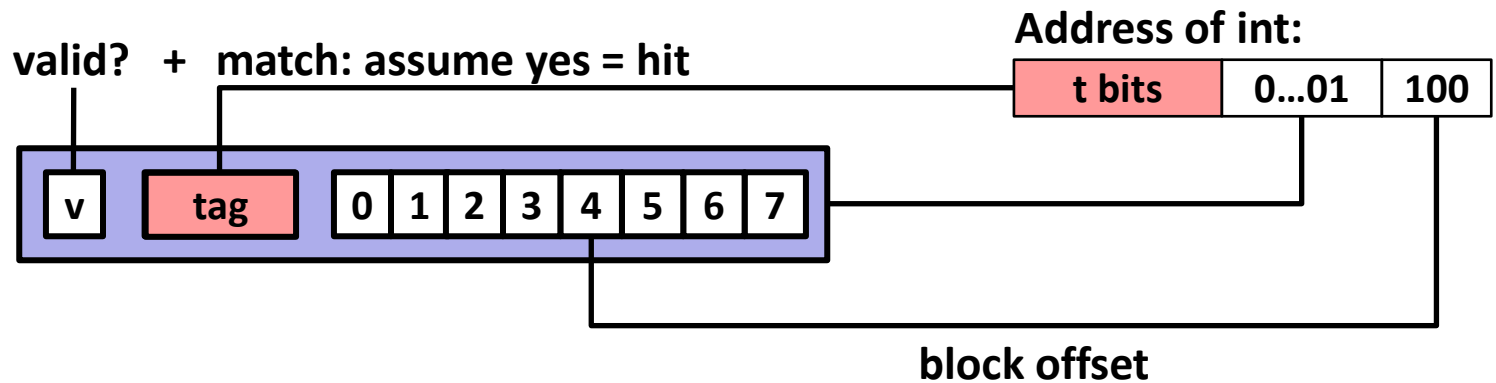
Assume: cache block size 8 bytes



# Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

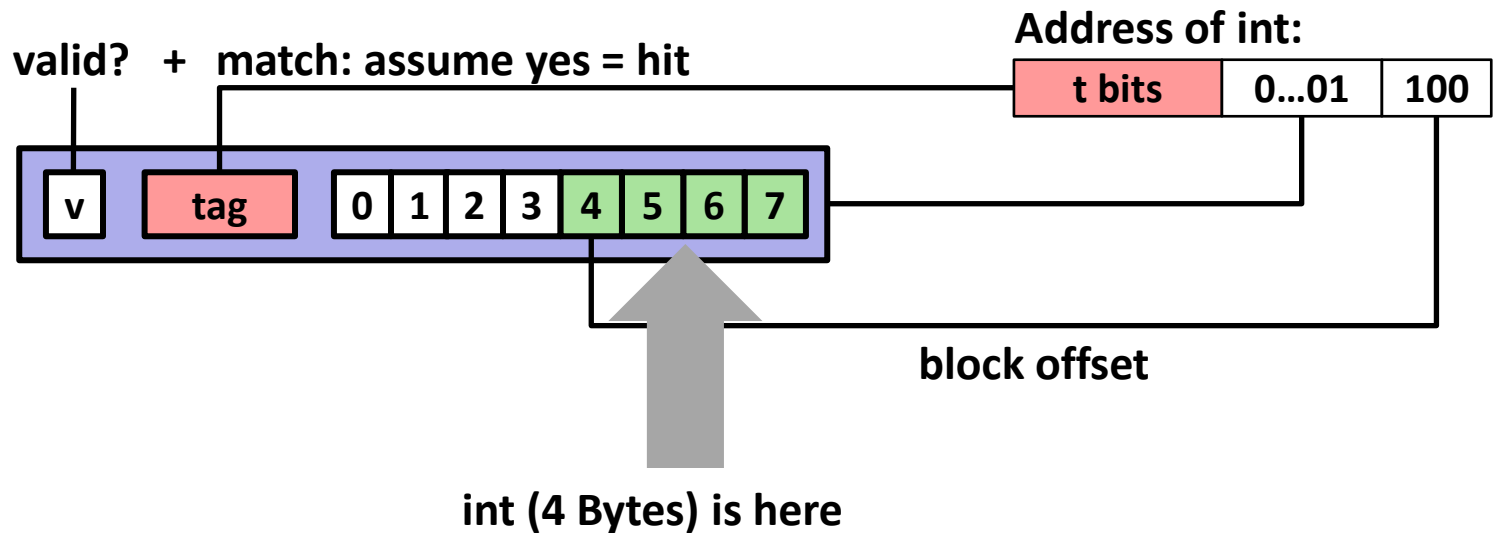
Assume: cache block size 8 bytes



# Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

Assume: cache block size 8 bytes



**If tag doesn't match:** old line is evicted and replaced

# Direct-Mapped Cache Simulation

t=1	s=2	b=1
x	xx	x

4-bit addresses (address space size M=16 bytes)  
 S=4 sets, E=1 Blocks/set, B=2 bytes/block

Address trace (reads, one byte per read):

0	[ <u>0</u> <u>0</u> <u>0</u> <u>0</u> <sub>2</sub> ],	miss
1	[ <u>0</u> <u>0</u> <u>0</u> <u>1</u> <sub>2</sub> ],	hit
7	[ <u>0</u> <u>1</u> <u>1</u> <u>1</u> <sub>2</sub> ],	miss
8	[ <u>1</u> <u>0</u> <u>0</u> <u>0</u> <sub>2</sub> ],	miss
0	[ <u>0</u> <u>0</u> <u>0</u> <u>0</u> <sub>2</sub> ]	miss

	v	Tag	Block
Set 0	1	0	M[0-1]
Set 1	0		
Set 2	0		
Set 3	1	0	M[6-7]

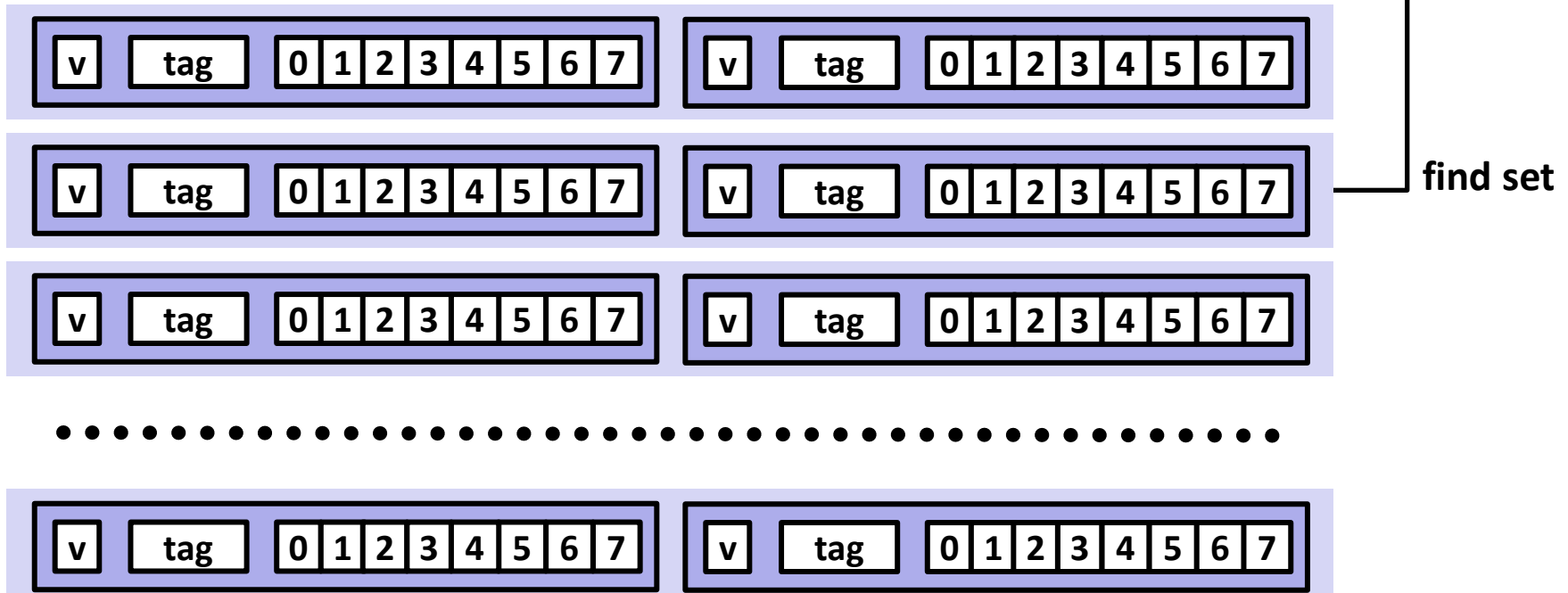
# E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size 8 bytes

Address of short int:

t bits	0...01	100
--------	--------	-----

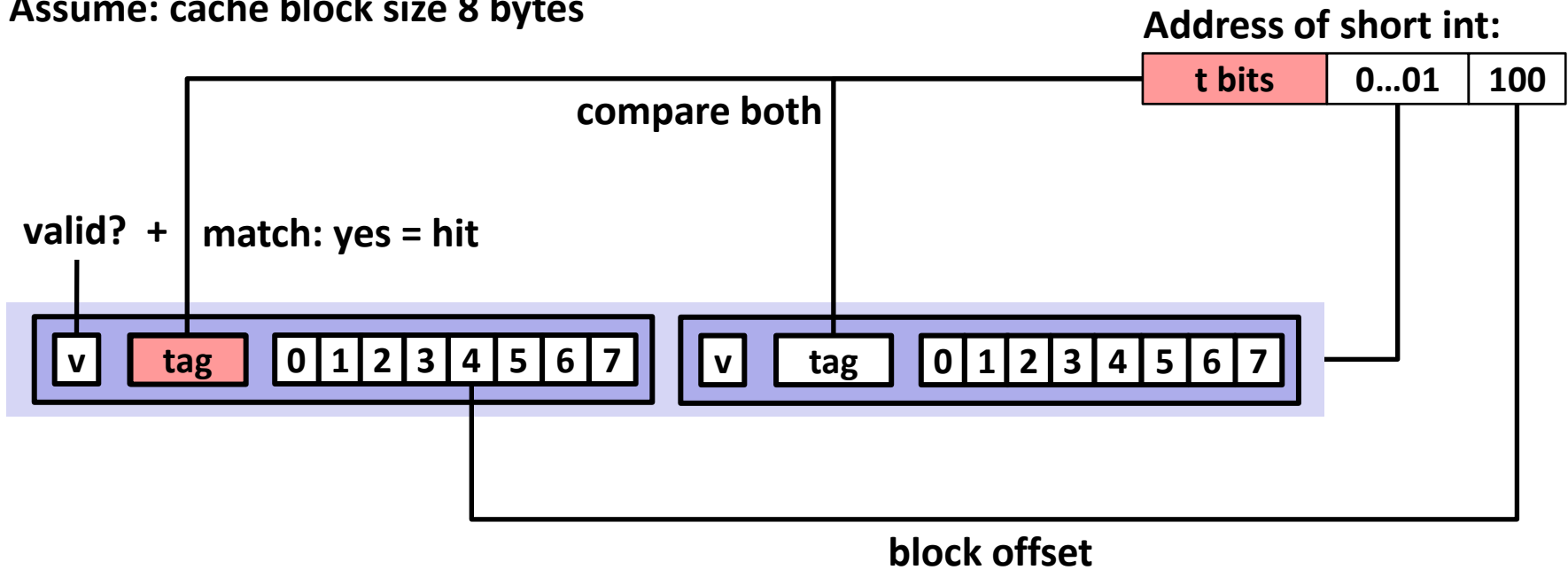




# E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

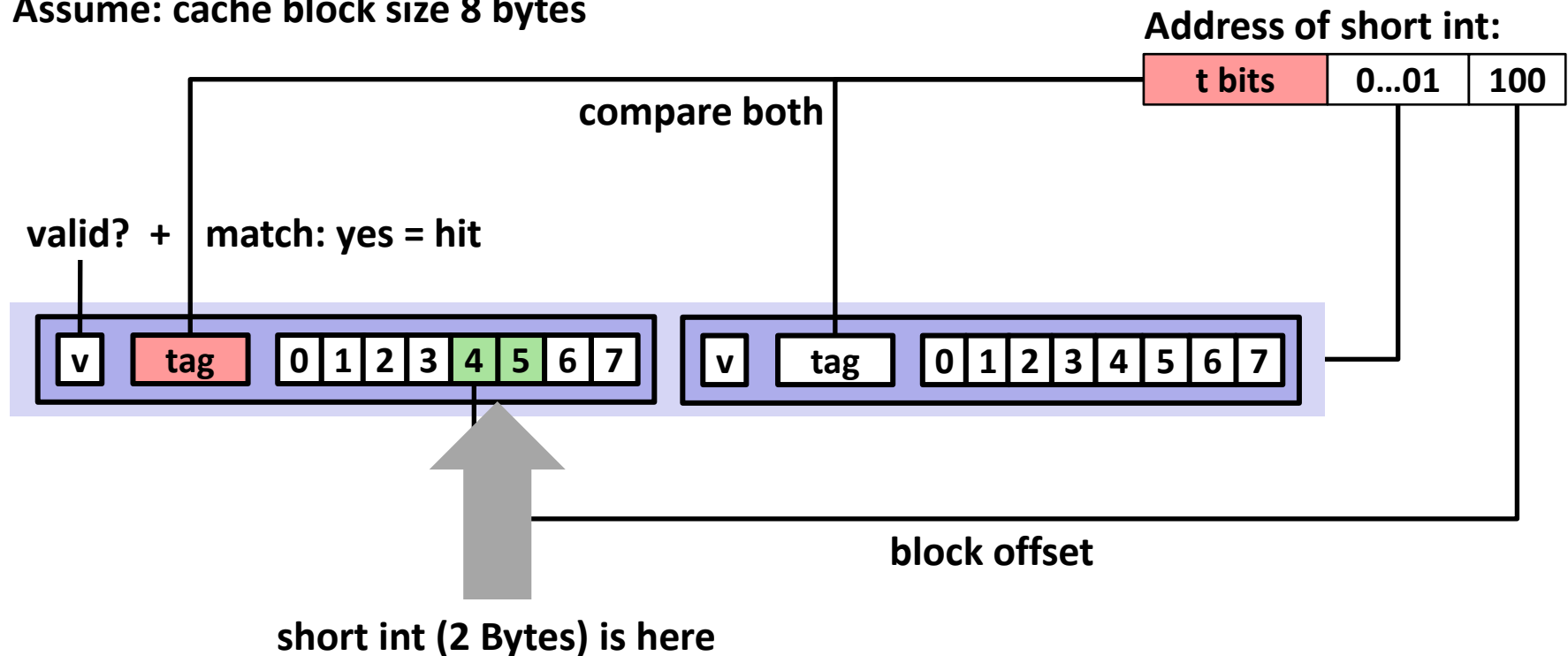
Assume: cache block size 8 bytes



# E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size 8 bytes



## No match:

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), ...

# 2-Way Set Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

4-bit addresses (M=16 bytes)

S=2 sets, E=2 blocks/set, B=2 bytes/block

Address trace (reads, one byte per read):

0	[00 <u>0</u> 0 <sub>2</sub> ],	miss
1	[00 <u>0</u> 1 <sub>2</sub> ],	hit
7	[0 <u>1</u> 11 <sub>2</sub> ],	miss
8	[10 <u>0</u> 0 <sub>2</sub> ],	miss
0	[00 <u>0</u> 0 <sub>2</sub> ]	hit

	v	Tag	Block
Set 0	1	00	M[0-1]
	1	10	M[8-9]
Set 1	1	01	M[6-7]
	0		

# What about writes?

## ■ Multiple copies of data exist:

- L1, L2, L3, Main Memory, Disk

## ■ What to do on a write-hit?

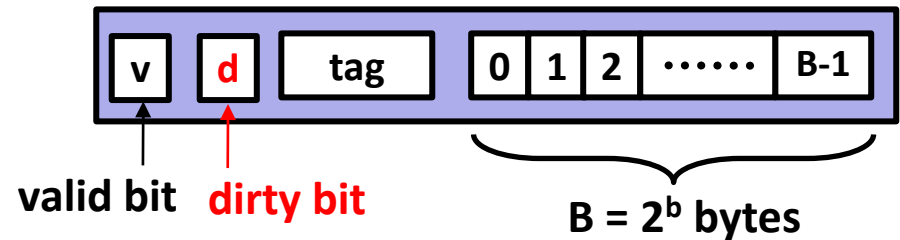
- **Write-through** (write immediately to memory)
- **Write-back** (defer write to memory until replacement of line)
  - Each cache line needs a dirty bit (set if data has been written to)

## ■ What to do on a write-miss?

- **Write-allocate** (load into cache, update line in cache)
  - Good if more writes to the location will follow
- **No-write-allocate** (writes straight to memory, does not load into cache)

## ■ Typical

- Write-through + No-write-allocate
- Write-back + Write-allocate



# Practical Write-back Write-allocate

- A write to address X is issued

- If it is a hit

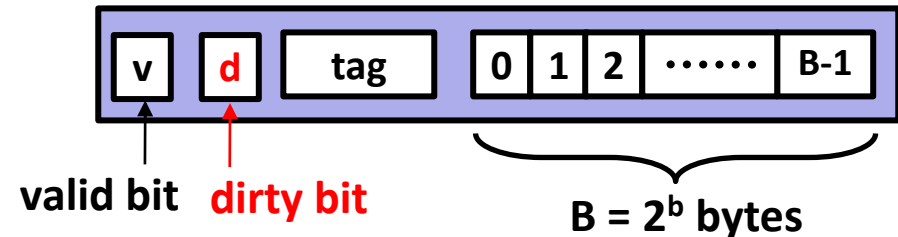
- Update the contents of block
  - Set dirty bit to 1 (bit is sticky and only cleared on eviction)

- If it is a miss

- Fetch block from memory (per a read miss)
  - The perform the write operations (per a write hit)

- If a line is evicted and dirty bit is set to 1

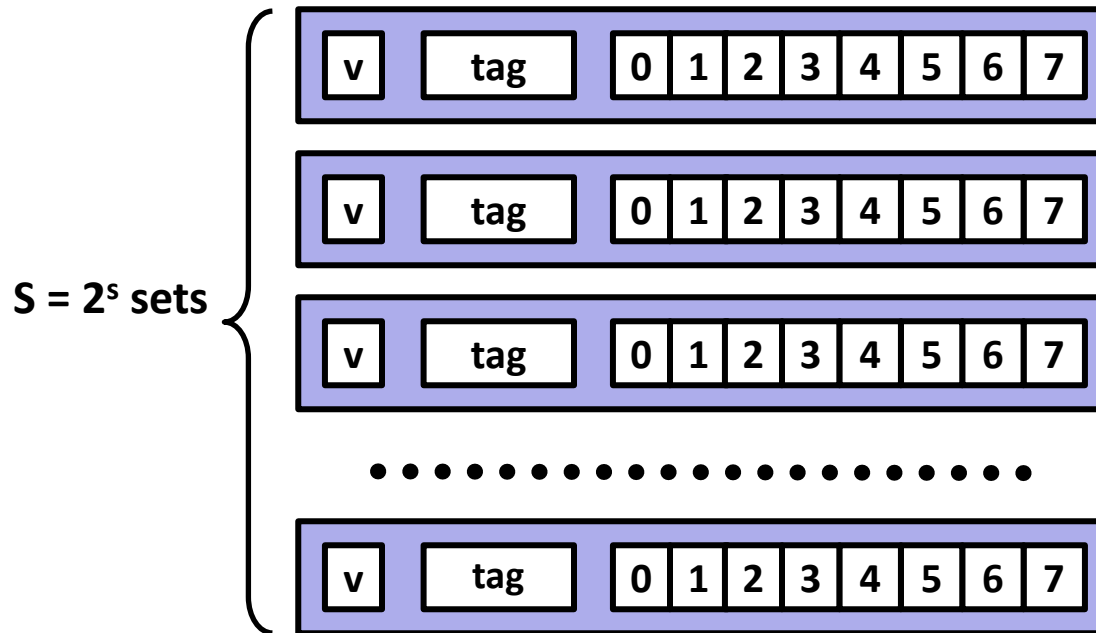
- The entire block of  $2^b$  bytes are written back to memory
  - Dirty bit is cleared (set to 0)
  - Line is replaced by new contents



# Why Index Using Middle Bits?

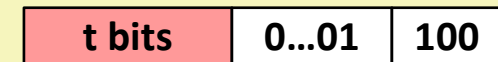
Direct mapped: One line per set

Assume: cache block size 8 bytes



**Standard Method:  
Middle bit indexing**

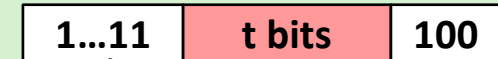
Address of int:



find set

**Alternative Method:  
High bit indexing**

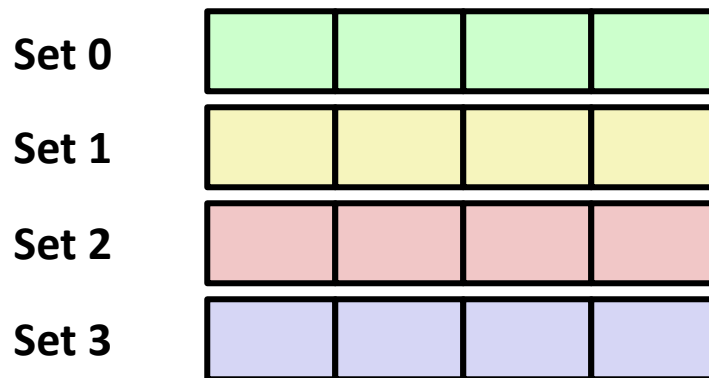
Address of int:



find set

# Illustration of Indexing Approaches

- 64-byte memory
  - 6-bit addresses
- 16 byte, direct-mapped cache
- Block size = 4. (Thus, 4 sets; why?)
- 2 bits tag, 2 bits index, 2 bits offset



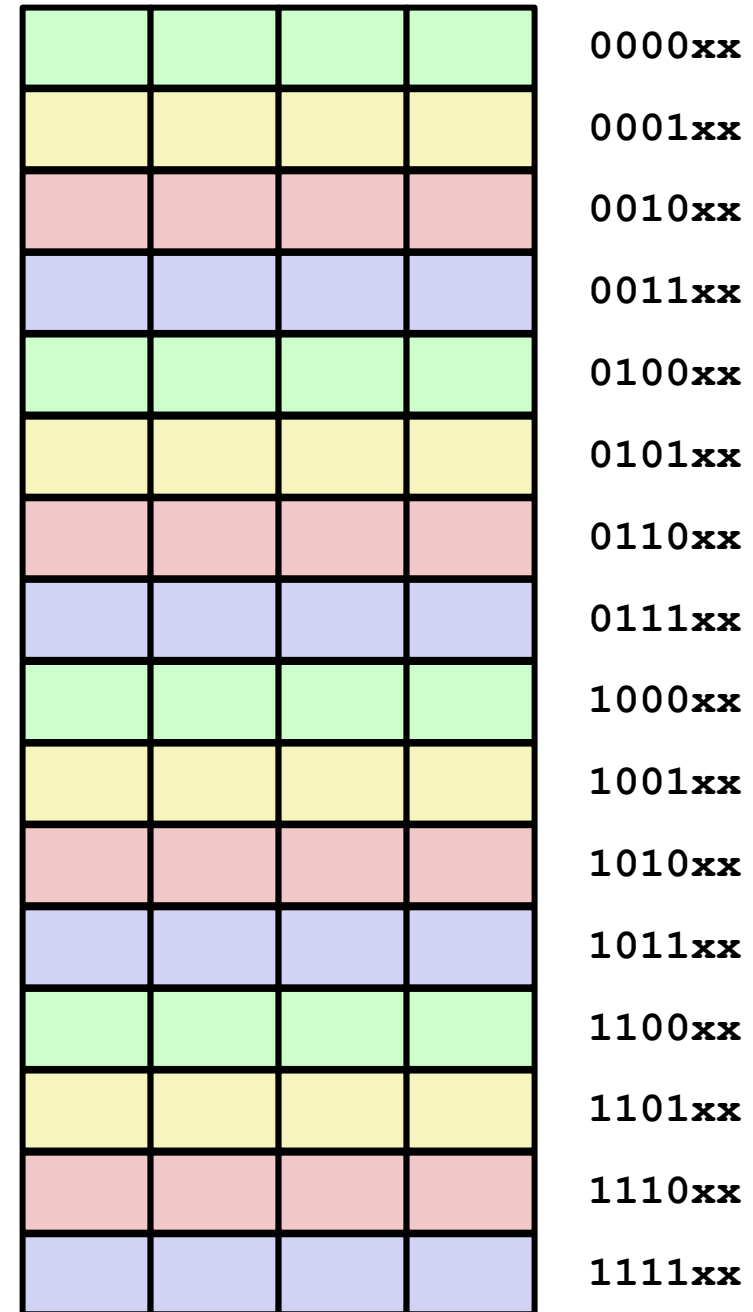
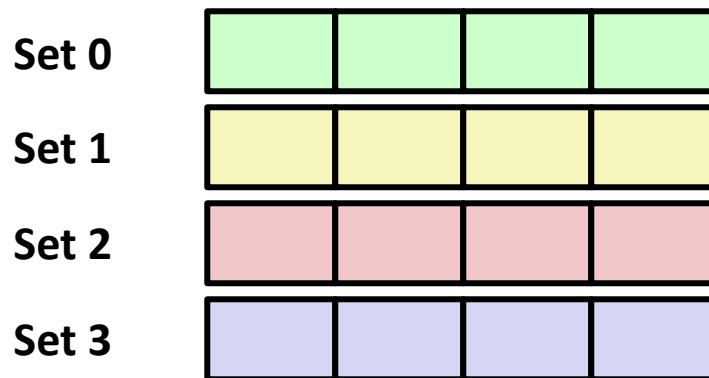
				0000xx
				0001xx
				0010xx
				0011xx
				0100xx
				0101xx
				0110xx
				0111xx
				1000xx
				1001xx
				1010xx
				1011xx
				1100xx
				1101xx
				1110xx
				1111xx

# Middle Bit Indexing

## ■ Addresses of form **TTSSBB**

- **TT** Tag bits
- **SS** Set index bits
- **BB** Offset bits

## ■ Makes good use of spatial locality



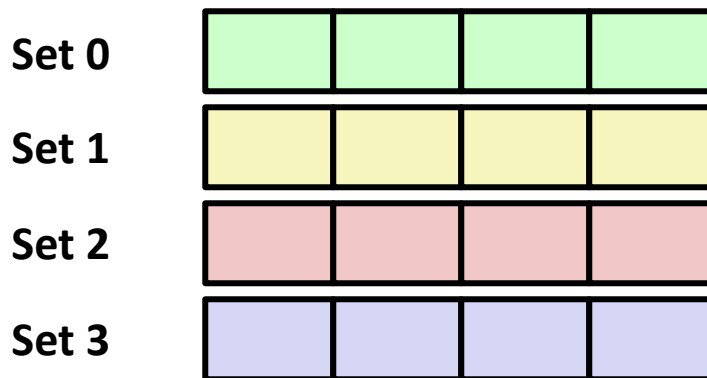


# High Bit Indexing

## ■ Addresses of form **SS****TT****BB**

- **SS**      Set index bits
- **TT**      Tag bits
- **BB**      Offset bits

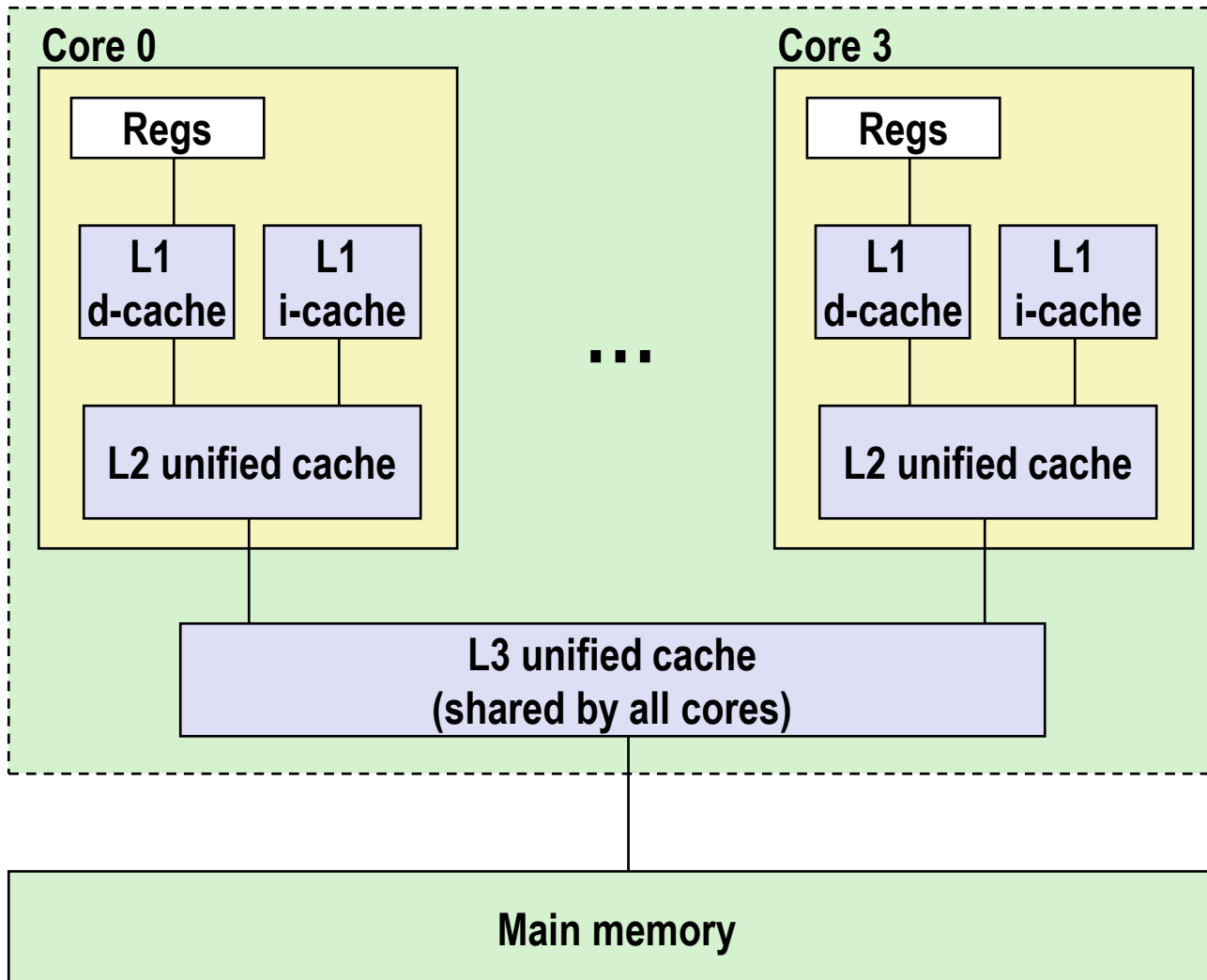
## ■ Program with high spatial locality would generate lots of conflicts



				0000xx
				0001xx
				0010xx
				0011xx
				0100xx
				0101xx
				0110xx
				0111xx
				1000xx
				1001xx
				1010xx
				1011xx
				1100xx
				1101xx
				1110xx
				1111xx

# Intel Core i7 Cache Hierarchy

Processor package



**L1 i-cache and d-cache:**

32 kB, 8-way,  
Access: 4 cycles

**L2 unified cache:**

256 kB, 8-way,  
Access: 10 cycles

**L3 unified cache:**

8 MB, 16-way,  
Access: 40-75 cycles

**Block size:** 64 bytes for  
all caches.

# Example: Core i7 L1 Data Cache

**32 kB 8-way set associative**

**64 bytes/block**

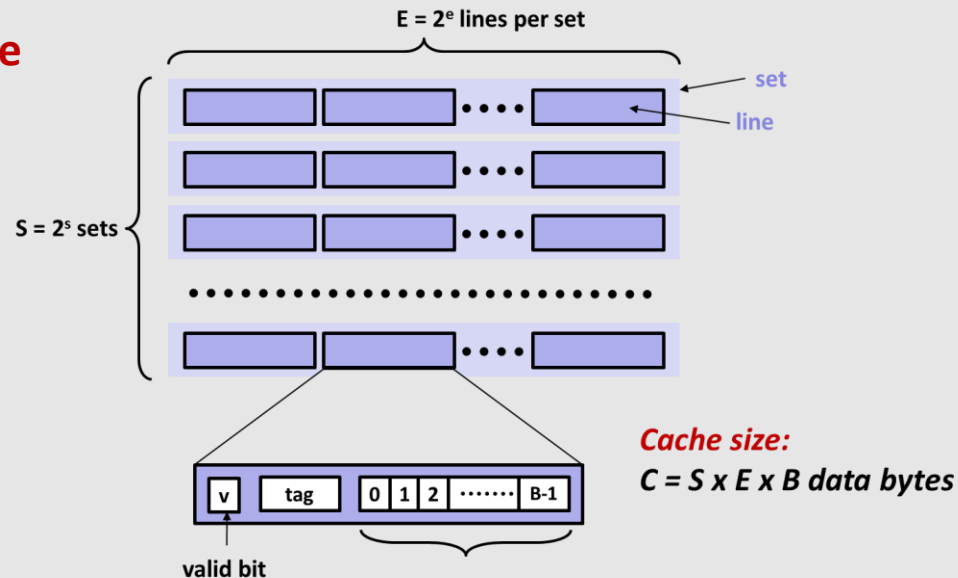
**47 bit address range**

**B =**

**S = , s =**

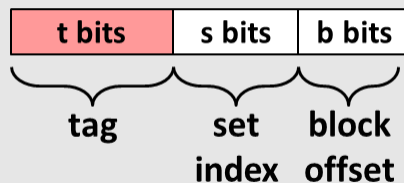
**E = , e =**

**C =**



Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Address of word:



Block offset: . bits

Set index: . bits

Tag: . bits

**Stack Address:**

**0x00007f7262a1e010**

**Block offset:**

**0x??**

**Set index:**

**0x??**

**Tag:**

**0x??**

# Example: Core i7 L1 Data Cache

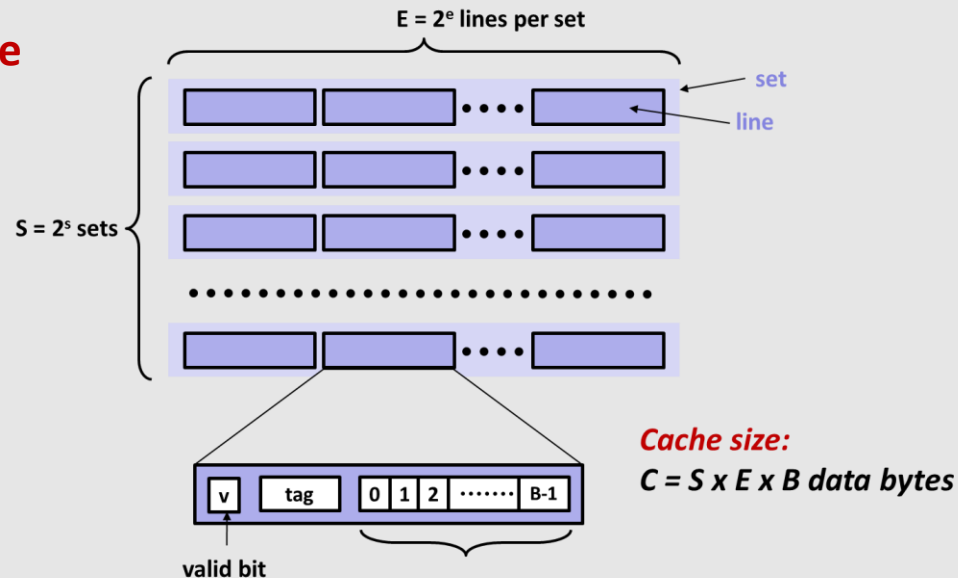
**32 kB 8-way set associative**  
**64 bytes/block**  
**47 bit address range**

$B = 64$

$S = 64, s = 6$

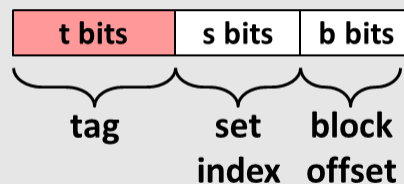
$E = 8, e = 3$

$C = 64 \times 64 \times 8 = 32,768$



Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Address of word:



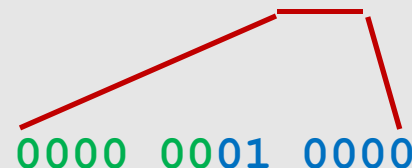
Block offset: 6 bits

Set index: 6 bits

Tag: 35 bits

Stack Address:

**0x00007f7262a1e010**



Block offset:

**0x10**

Set index:

**0x0**

Tag:

**0x7f7262a1e**

# Cache Performance Metrics

## ■ Miss Rate

- Fraction of memory references not found in cache (misses / accesses)  
=  $1 - \text{hit rate}$
- Typical numbers (in percentages):
  - 3-10% for L1
  - can be quite small (e.g.,  $< 1\%$ ) for L2, depending on size, etc.

## ■ Hit Time

- Time to deliver a line in the cache to the processor
  - includes time to determine whether the line is in the cache
- Typical numbers:
  - 4 clock cycle for L1
  - 10 clock cycles for L2

## ■ Miss Penalty

- Additional time required because of a miss
  - typically 50-200 cycles for main memory (Trend: increasing!)

# Let's think about those numbers

- **Huge difference between a hit and a miss**
  - Could be 100x, if just L1 and main memory
- **Would you believe 99% hits is twice as good as 97%?**
  - Consider:  
cache hit time of 1 cycle  
miss penalty of 100 cycles
  - Average access time:  
97% hits:  $1 \text{ cycle} + 0.03 * 100 \text{ cycles} = 4 \text{ cycles}$   
99% hits:  $1 \text{ cycle} + 0.01 * 100 \text{ cycles} = 2 \text{ cycles}$
- **This is why “miss rate” is used instead of “hit rate”**

# Writing Cache Friendly Code

- **Make the common case go fast**
  - Focus on the inner loops of the core functions
- **Minimize the misses in the inner loops**
  - Repeated references to variables are good (**temporal locality**)
  - Stride-1 reference patterns are good (**spatial locality**)

**Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories**

# Today

- Cache organization and operation
- **Performance impact of caches**
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality



# The Memory Mountain

- **Read throughput** (read bandwidth)
  - Number of bytes read from memory per second (MB/s)
- **Memory mountain:** Measured read throughput as a function of spatial and temporal locality.
  - Compact way to characterize memory system performance.

# Memory Mountain Test Function

```
long data[MAXELEMS]; /* Global array to traverse */
```

```
/* test - Iterate over first "elems" elements of
 *      array "data" with stride of "stride", using
 *      using 4x4 loop unrolling.
 */
```

```
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;
```

```
/* Combine 4 elements at a time */
```

```
for (i = 0; i < limit; i += sx4) {
    acc0 = acc0 + data[i];
    acc1 = acc1 + data[i+stride];
    acc2 = acc2 + data[i+sx2];
    acc3 = acc3 + data[i+sx3];
}
```

```
/* Finish any remaining elements */
```

```
for (; i < length; i++) {
    acc0 = acc0 + data[i];
}
return ((acc0 + acc1) + (acc2 + acc3));
```

```
}
```

*mountain/mountain.c*

Call `test()` with many combinations of `elems` and `stride`.

For each `elems` and `stride`:

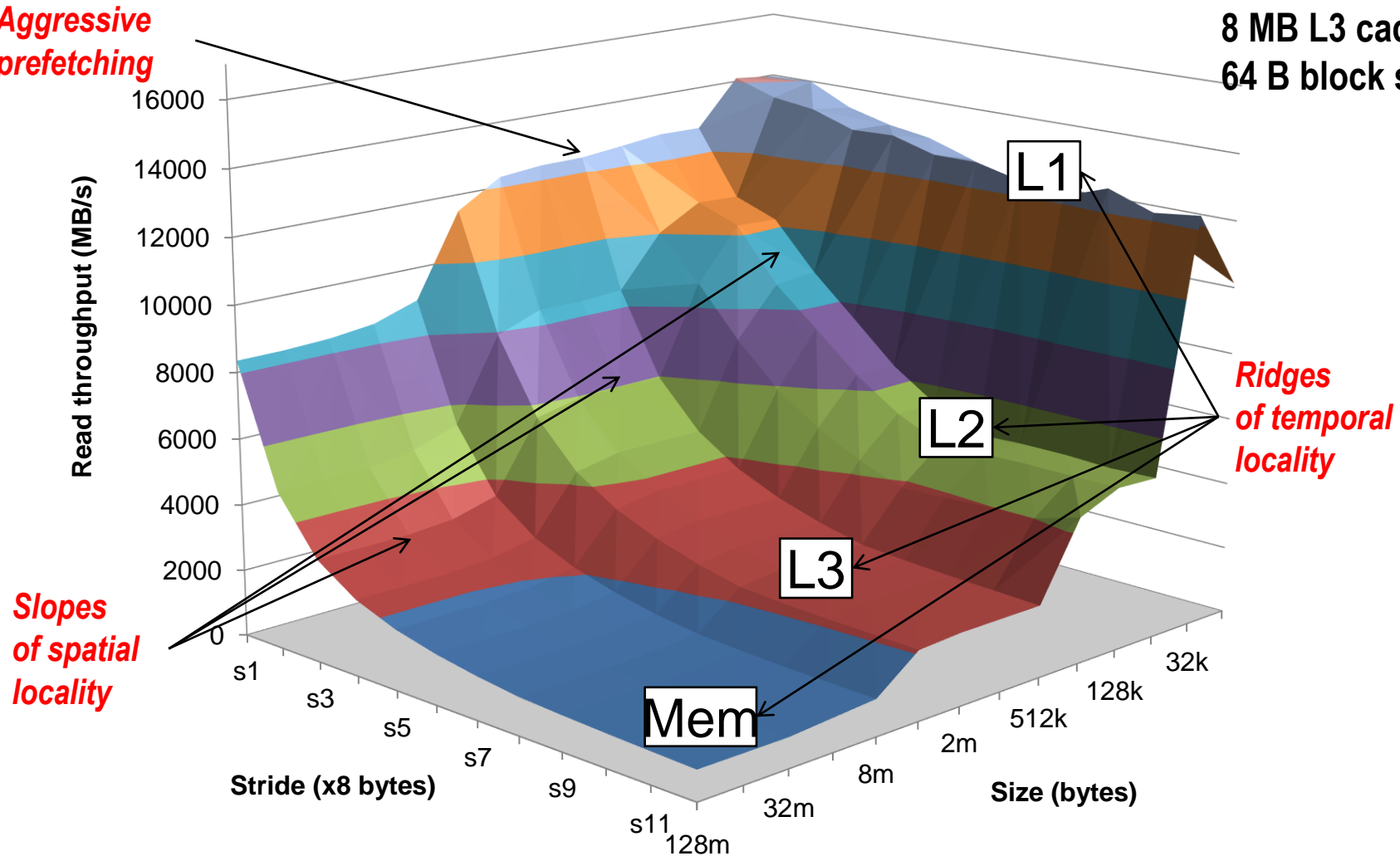
1. Call `test()` once to warm up the caches.

2. Call `test()` again and measure the read throughput (MB/s)

# The Memory Mountain

Core i7 Haswell  
2.1 GHz  
32 KB L1 d-cache  
256 KB L2 cache  
8 MB L3 cache  
64 B block size

*Aggressive  
prefetching*



# Today

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

# Matrix Multiplication Example

## ■ Description:

- Multiply  $N \times N$  matrices
- Matrix elements are doubles (8 bytes)
- $O(N^3)$  total operations
- $N$  reads per source element
- $N$  values summed per destination
  - but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

*Variable **sum**  
held in register*

*matmult/mm.c*

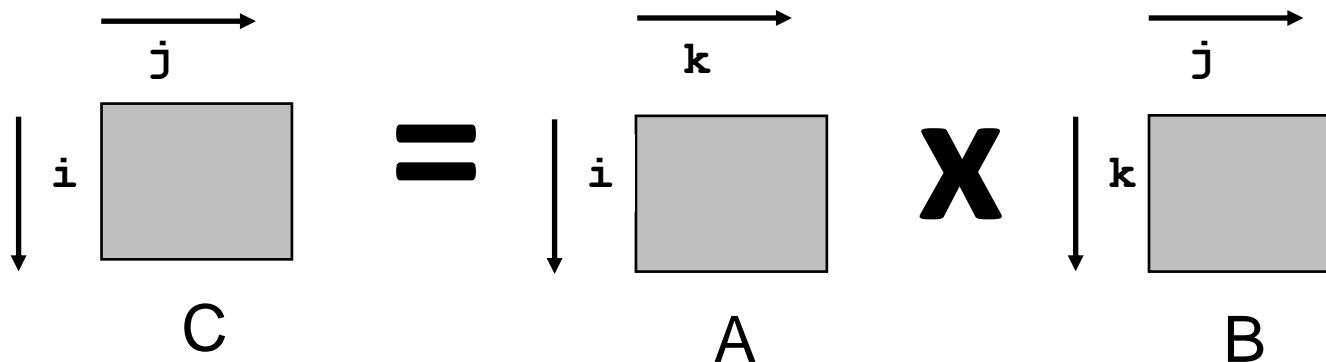
# Miss Rate Analysis for Matrix Multiply

## ■ Assume:

- Block size = 32B (big enough for four doubles)
- Matrix dimension (N) is very large
  - Approximate  $1/N$  as 0.0
- Cache is not even big enough to hold multiple rows

## ■ Analysis Method:

- Look at access pattern of inner loop



# Layout of C Arrays in Memory (review)

- **C arrays allocated in row-major order**

- each row in contiguous memory locations

- **Stepping through columns in one row:**

- ```
for (i = 0; i < N; i++)  
    sum += a[0][i];
```
- accesses successive elements
- if block size (B) > sizeof(a<sub>ij</sub>) bytes, exploit spatial locality
  - miss rate = sizeof(a<sub>ij</sub>) / B

- **Stepping through rows in one column:**

- ```
for (i = 0; i < n; i++)  
    sum += a[i][0];
```
- accesses distant elements
- no spatial locality!
  - miss rate = 1 (i.e. 100%)

# Matrix Multiplication (ijk)

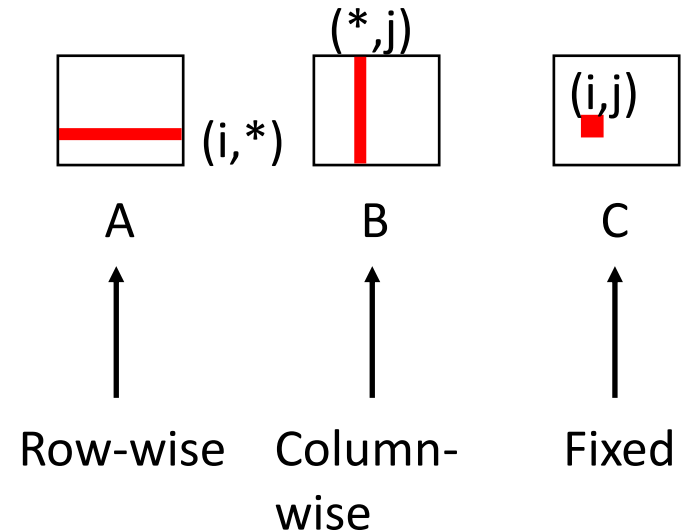
```

/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}

```

*matmult/mm.c*

Inner loop:



Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0



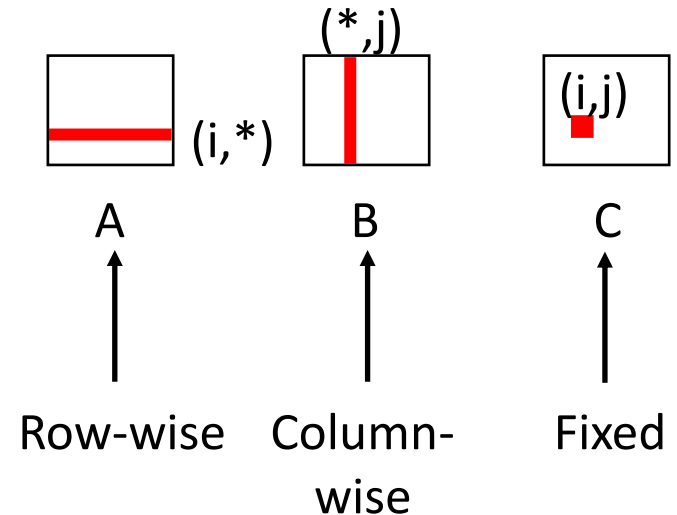
# Matrix Multiplication (jik)

```

/* jik */
for (j=0; j<n; j++) {
    for (i=0; i<n; i++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum
    }
}
                                     matmult/mm.c

```

Inner loop:



Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

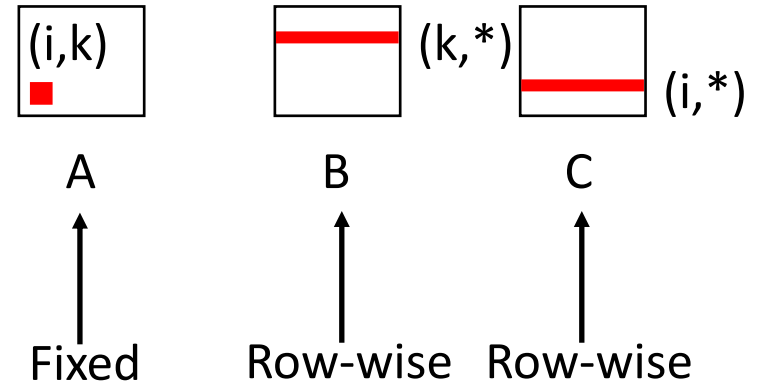
# Matrix Multiplication (kij)

```

/* kij */
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
                                     matmult/mm.c

```

Inner loop:



Misses per inner loop iteration:

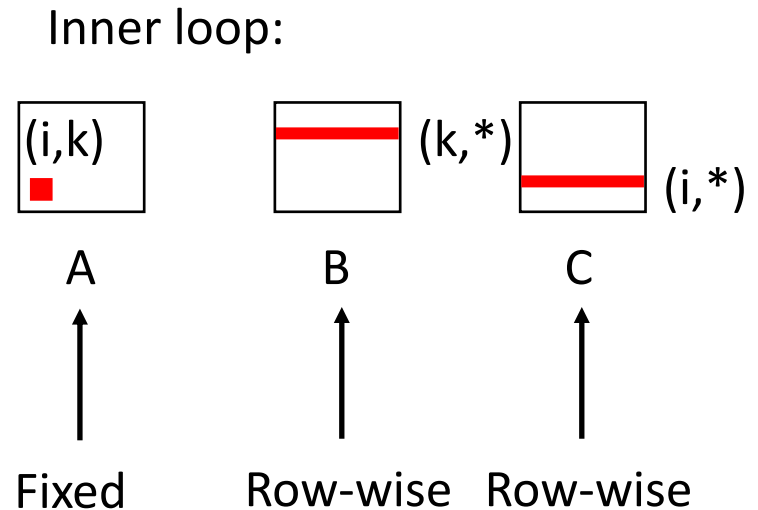
<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

# Matrix Multiplication (ikj)

```

/* ikj */
for (i=0; i<n; i++) {
    for (k=0; k<n; k++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
                                     matmult/mm.c

```



Misses per inner loop iteration:

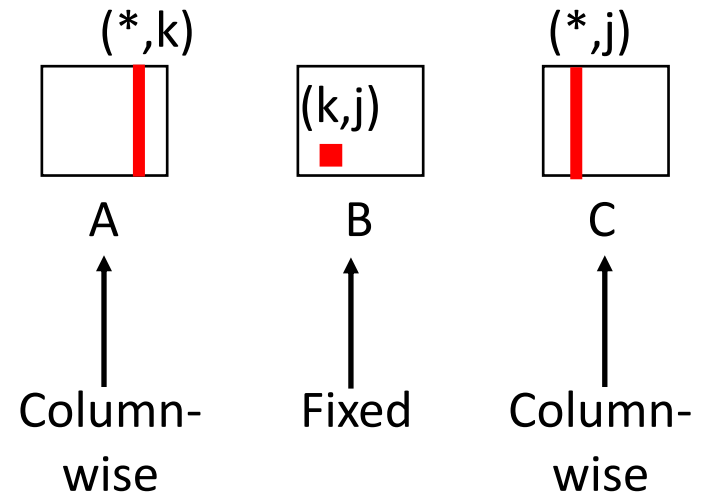
<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

# Matrix Multiplication (jki)

```

/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
                                     matmult/mm.c
  
```

Inner loop:



Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
1.0	0.0	1.0

# Matrix Multiplication (kji)

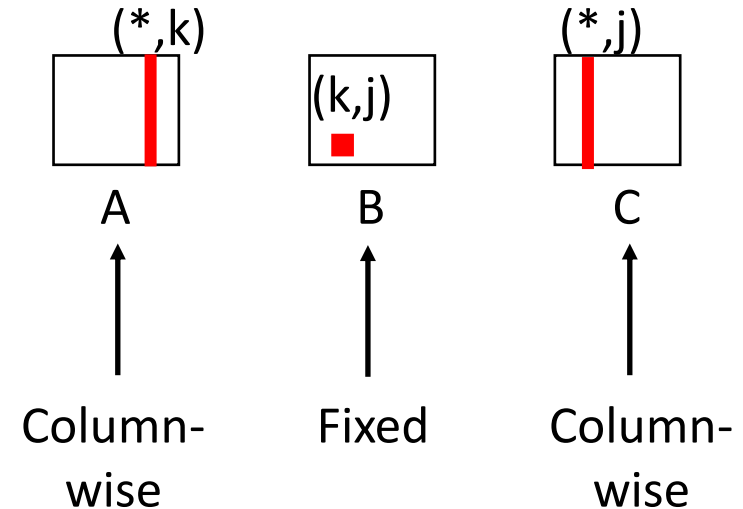
```

/* kji */
for (k=0; k<n; k++) {
    for (j=0; j<n; j++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}

```

*matmult/mm.c*

Inner loop:



Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
1.0	0.0	1.0

# Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

**ijk (& jik):**

- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

**kij (& ikj):**

- 2 loads, 1 store
- misses/iter = **0.5**

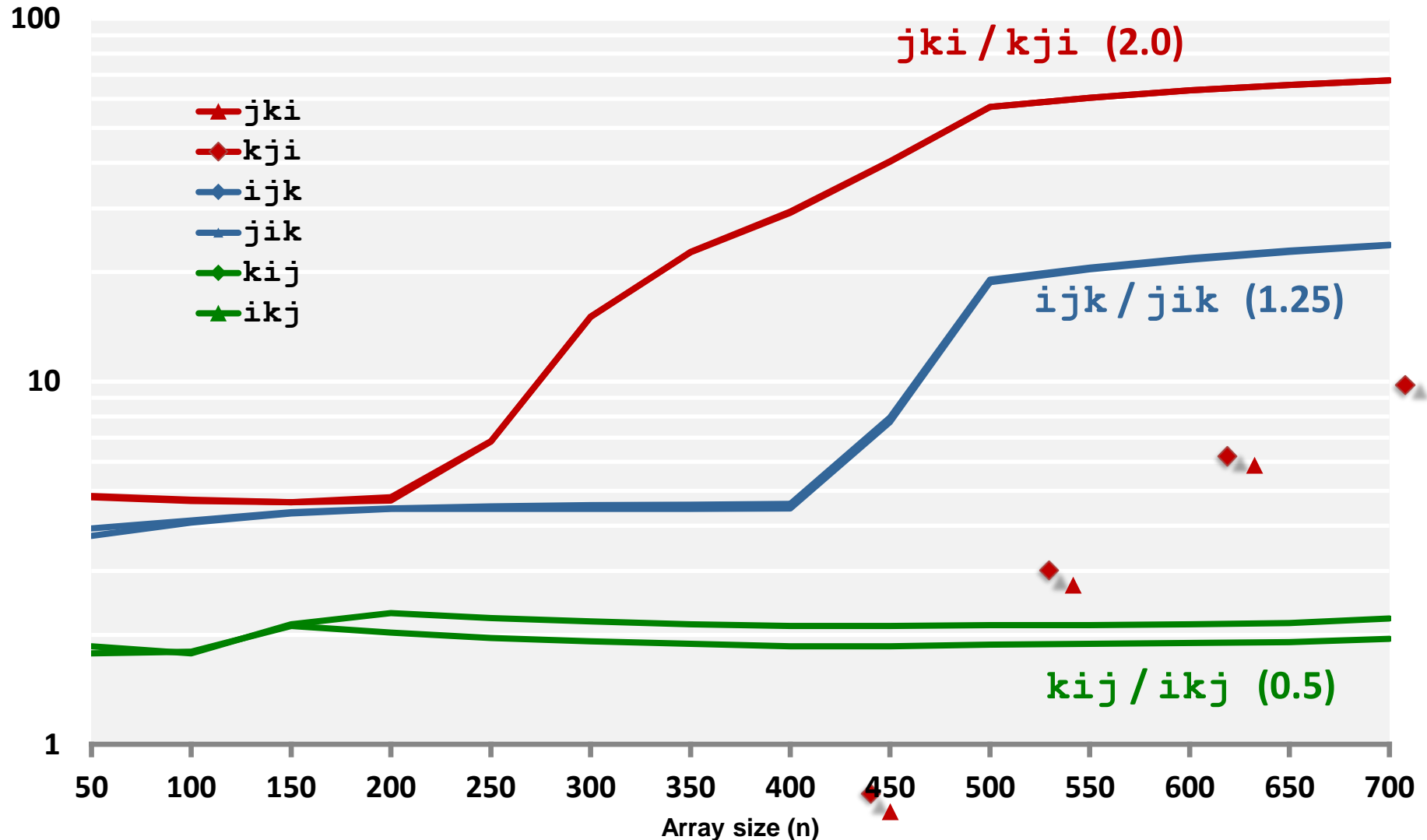
```
for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

**jki (& kji):**

- 2 loads, 1 store
- misses/iter = **2.0**

# Core i7 Matrix Multiply Performance

Cycles per inner loop iteration



# Today

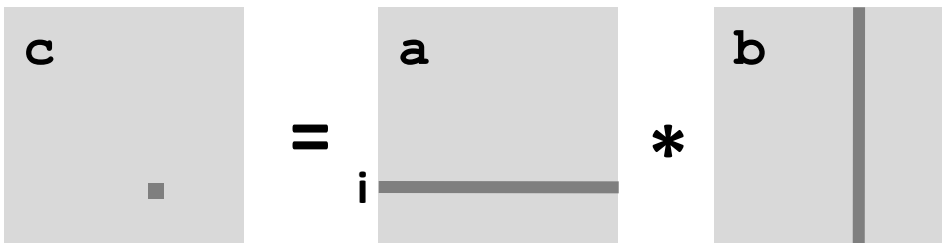
- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality



# Example: Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n + j] += a[i*n + k] * b[k*n + j];
}
```



# Cache Miss Analysis

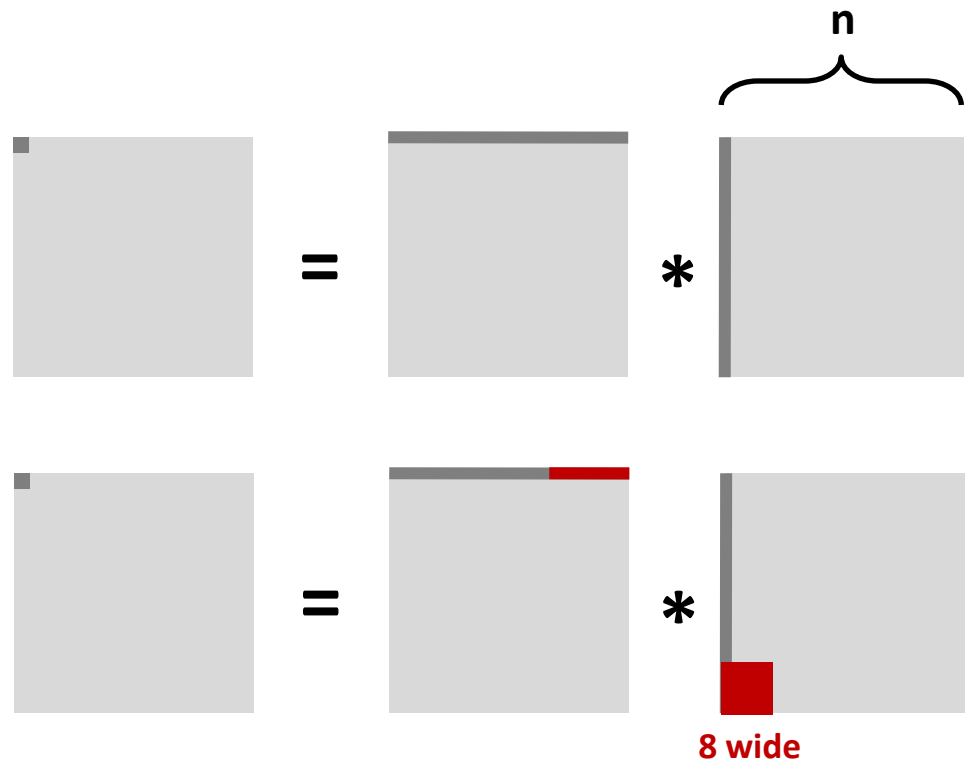
## ■ Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )

## ■ First iteration:

- $n/8 + n = 9n/8$  misses

- Afterwards **in cache**:  
(schematic)



# Cache Miss Analysis

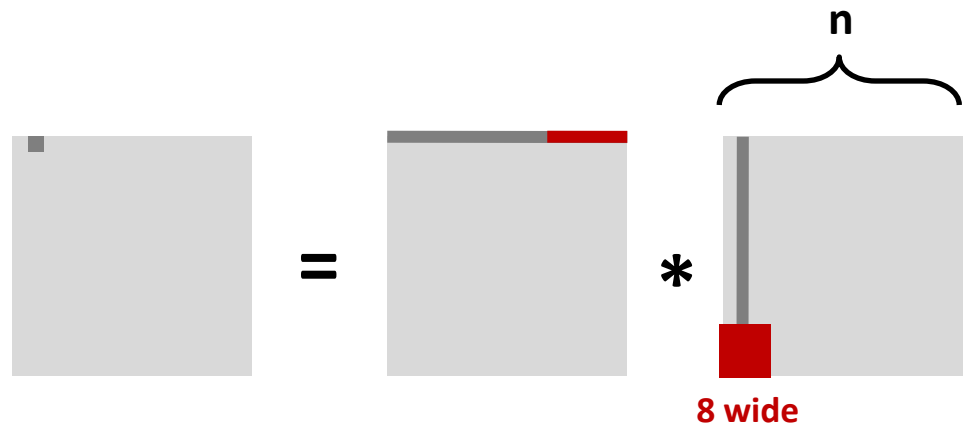
## ■ Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )

## ■ Second iteration:

- Again:  

$$n/8 + n = 9n/8 \text{ misses}$$



## ■ Total misses:

- $9n/8 * n^2 = (9/8) * n^3$

# Blocked Matrix Multiplication

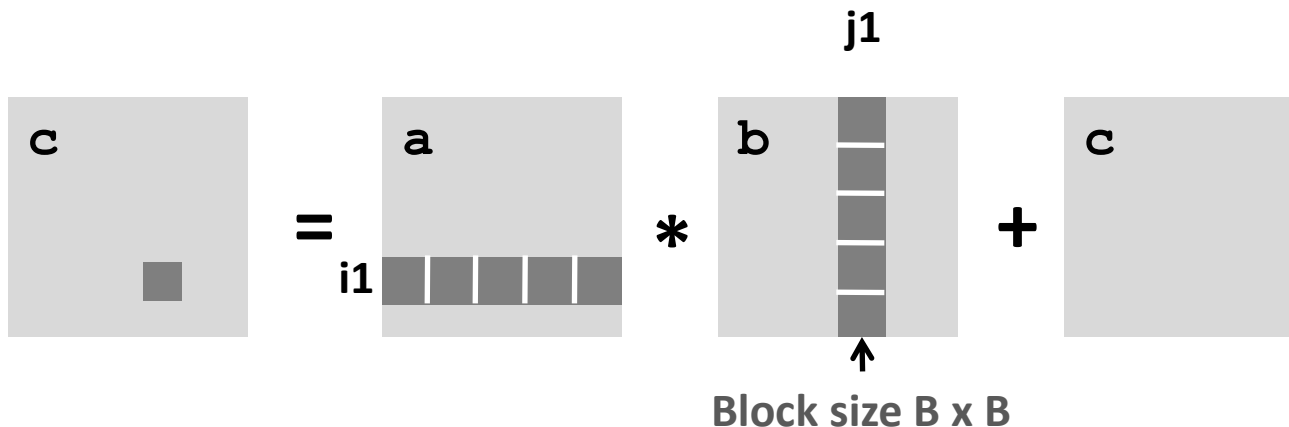
```

c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i++)
                    for (j1 = j; j1 < j+B; j++)
                        for (k1 = k; k1 < k+B; k++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}


```

*matmult/bmm.c*



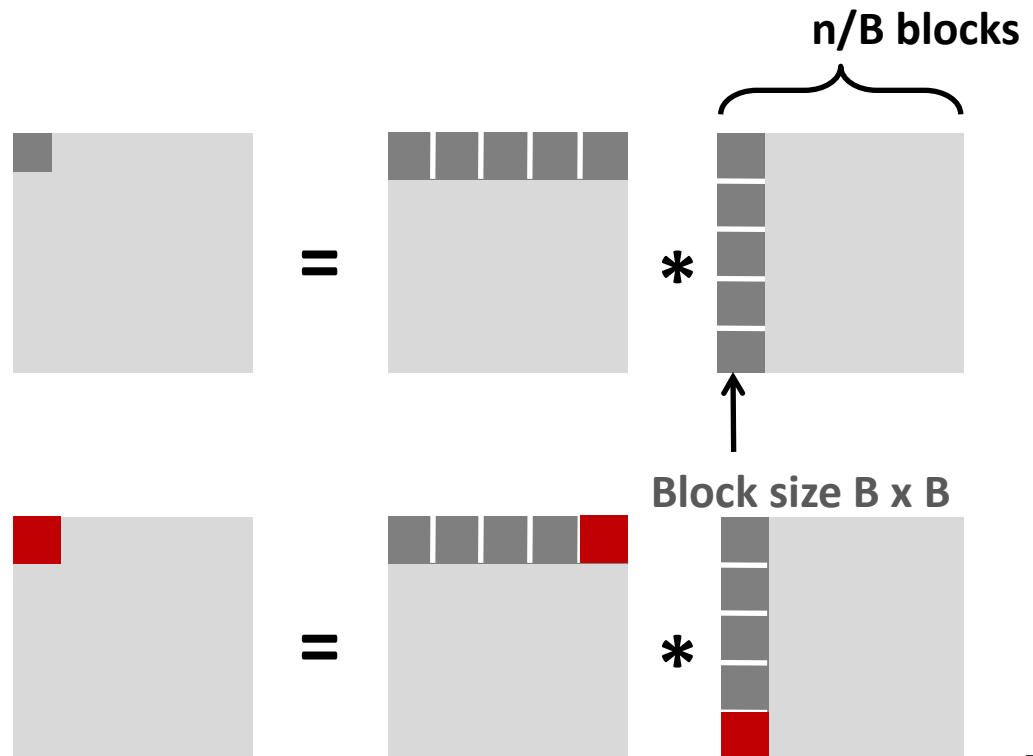
# Cache Miss Analysis

## ■ Assume:

- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )
- Three blocks  fit into cache:  $3B^2 < C$

## ■ First (block) iteration:


- $B^2/8$  misses for each block
- $2n/B * B^2/8 = nB/4$   
(omitting matrix  $c$ )



- Afterwards in cache  
(schematic)

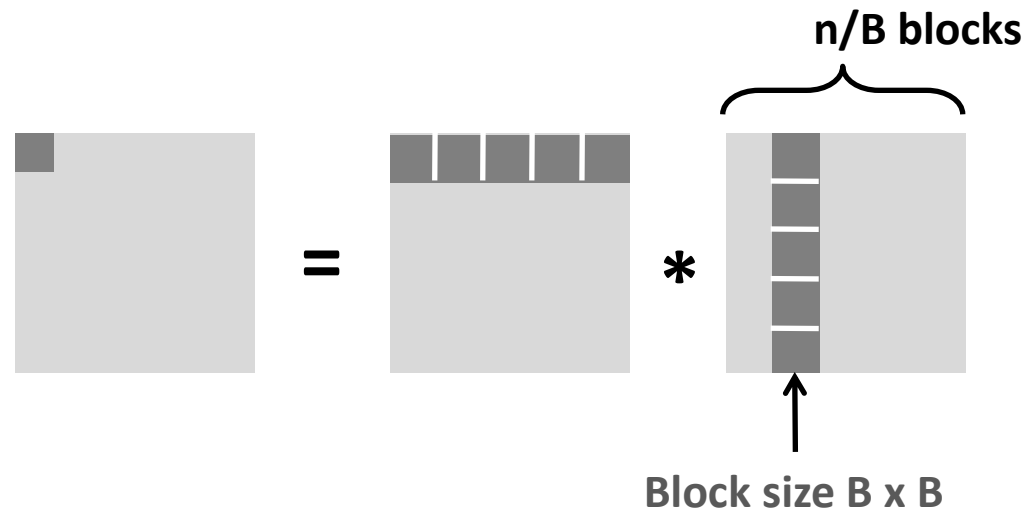
# Cache Miss Analysis

## ■ Assume:

- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )
- Three blocks  fit into cache:  $3B^2 < C$

## ■ Second (block) iteration:

- Same as first iteration
- $2n/B * B^2/8 = nB/4$



## ■ Total misses:

- $nB/4 * (n/B)^2 = n^3/(4B)$

# Blocking Summary

- No blocking:  $(9/8) * n^3$
- Blocking:  $1/(4B) * n^3$
- Suggest largest possible block size  $B$ , but limit  $3B^2 < C!$
- Reason for dramatic difference:
  - Matrix multiplication has inherent temporal locality:
    - Input data:  $3n^2$ , computation  $2n^3$
    - Every array elements used  $O(n)$  times!
  - But program has to be written properly

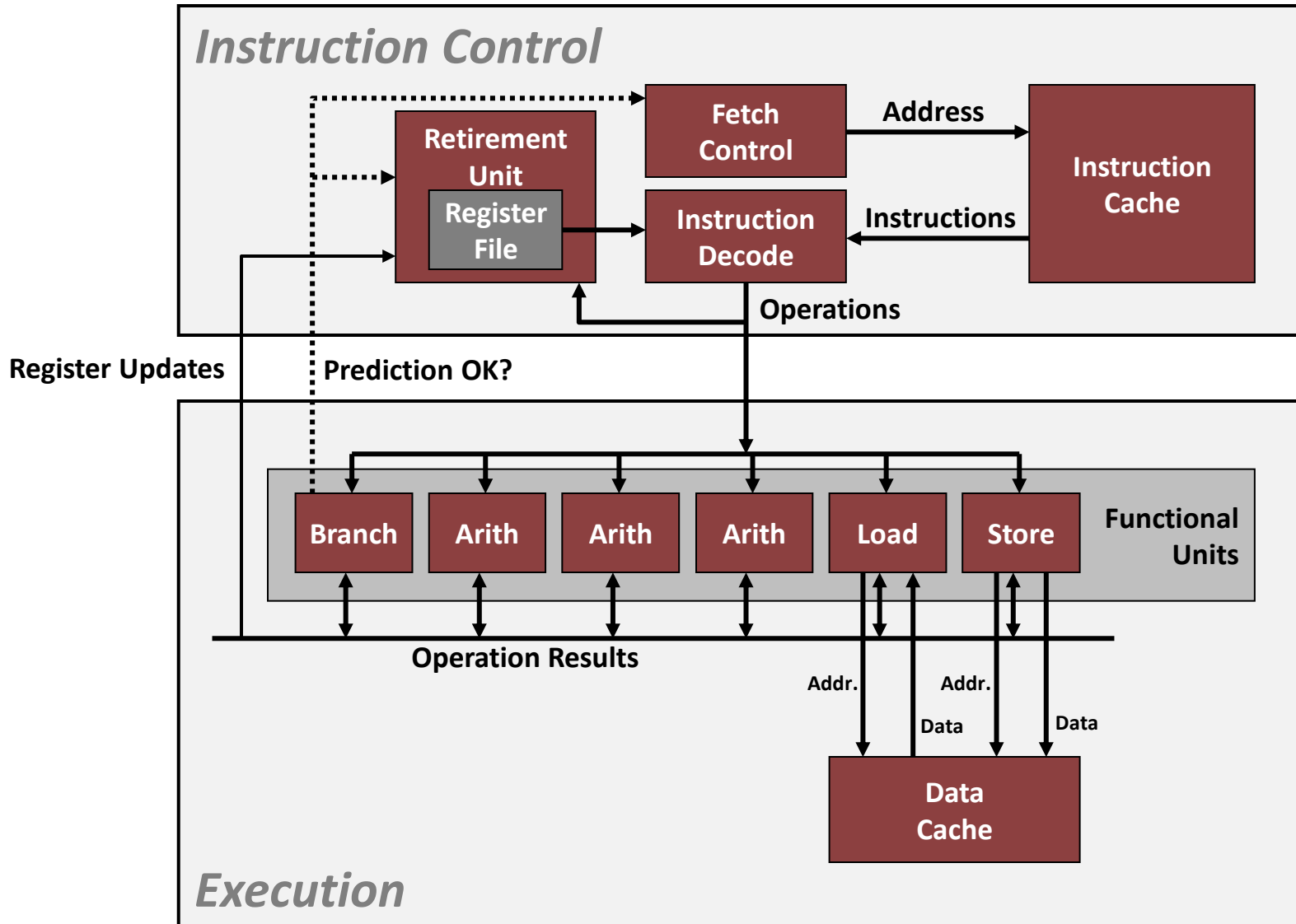
# Cache Summary

- **Cache memories can have significant performance impact**
- **You can write your programs to exploit this!**
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects with sequentially with stride 1.
  - Try to maximize temporal locality by using a data object as often as possible once it's read from memory.

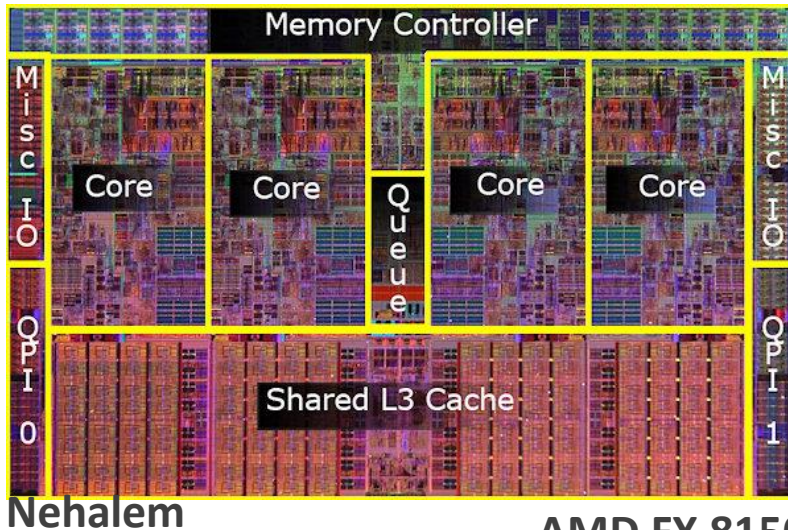


# Supplemental slides

# Recall: Modern CPU Design



# What it Really Looks Like

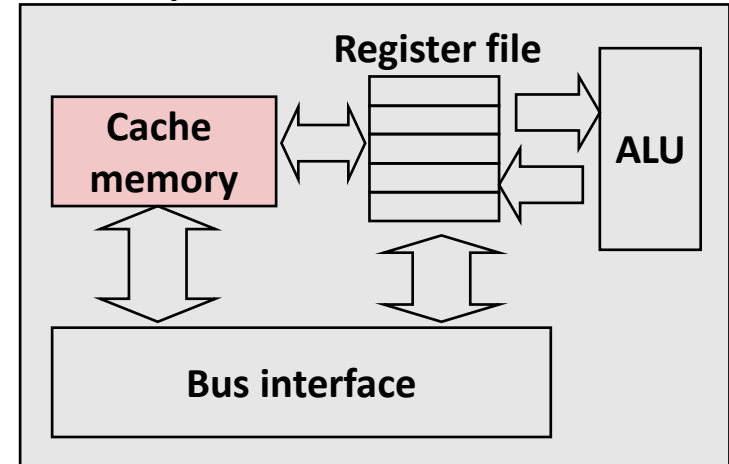


Nehalem

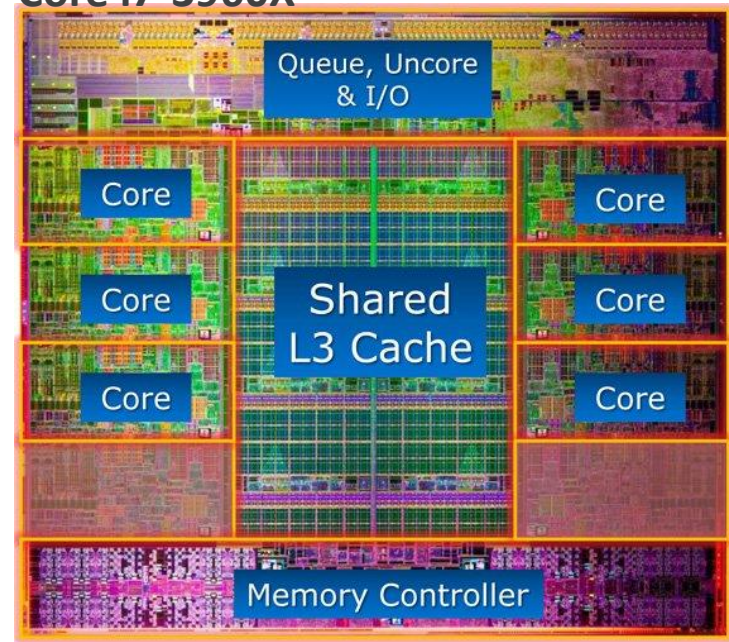
AMD FX 8150



CPU chip

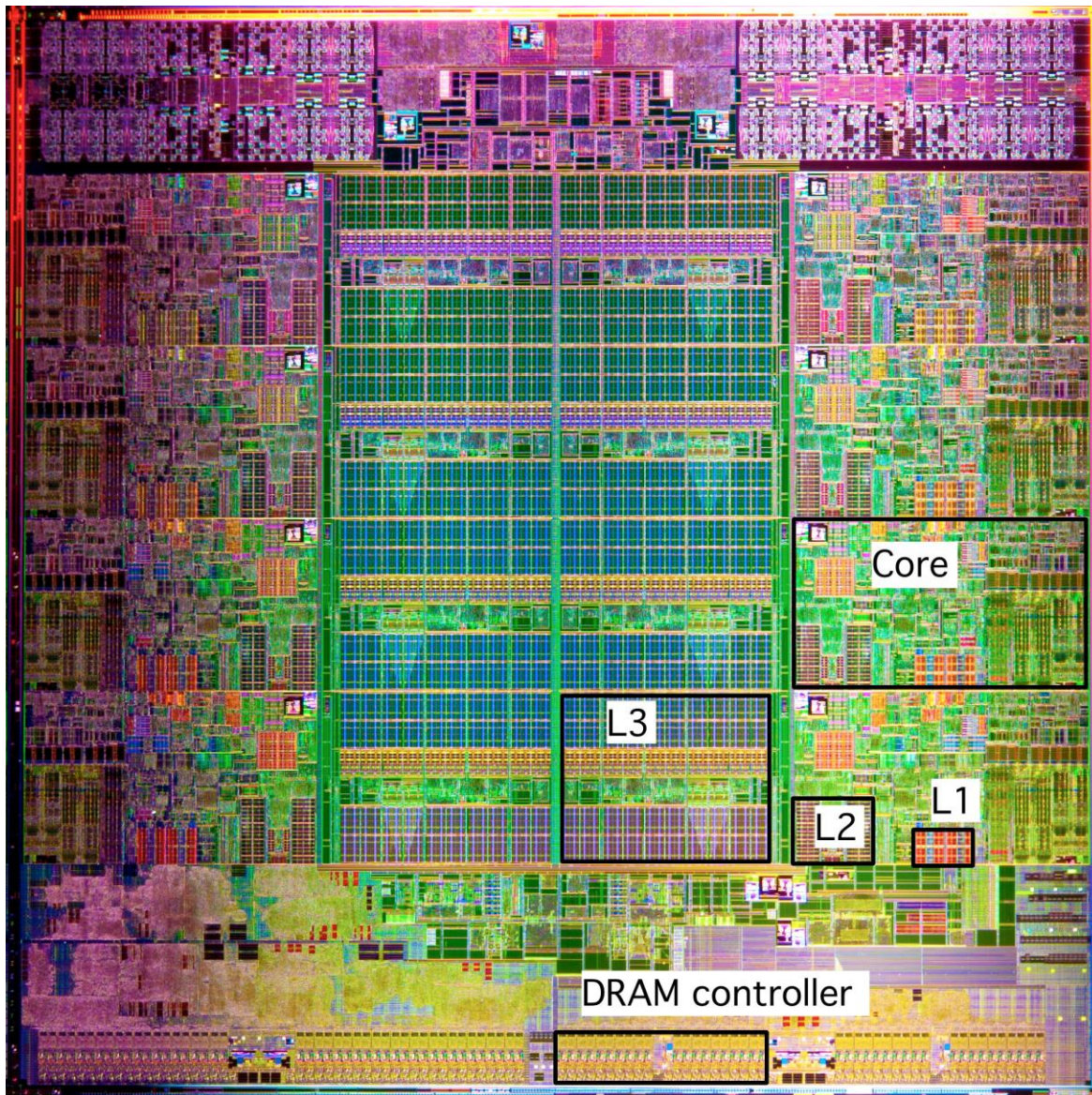


Core i7-3960X





# What it Really Looks Like (Cont.)



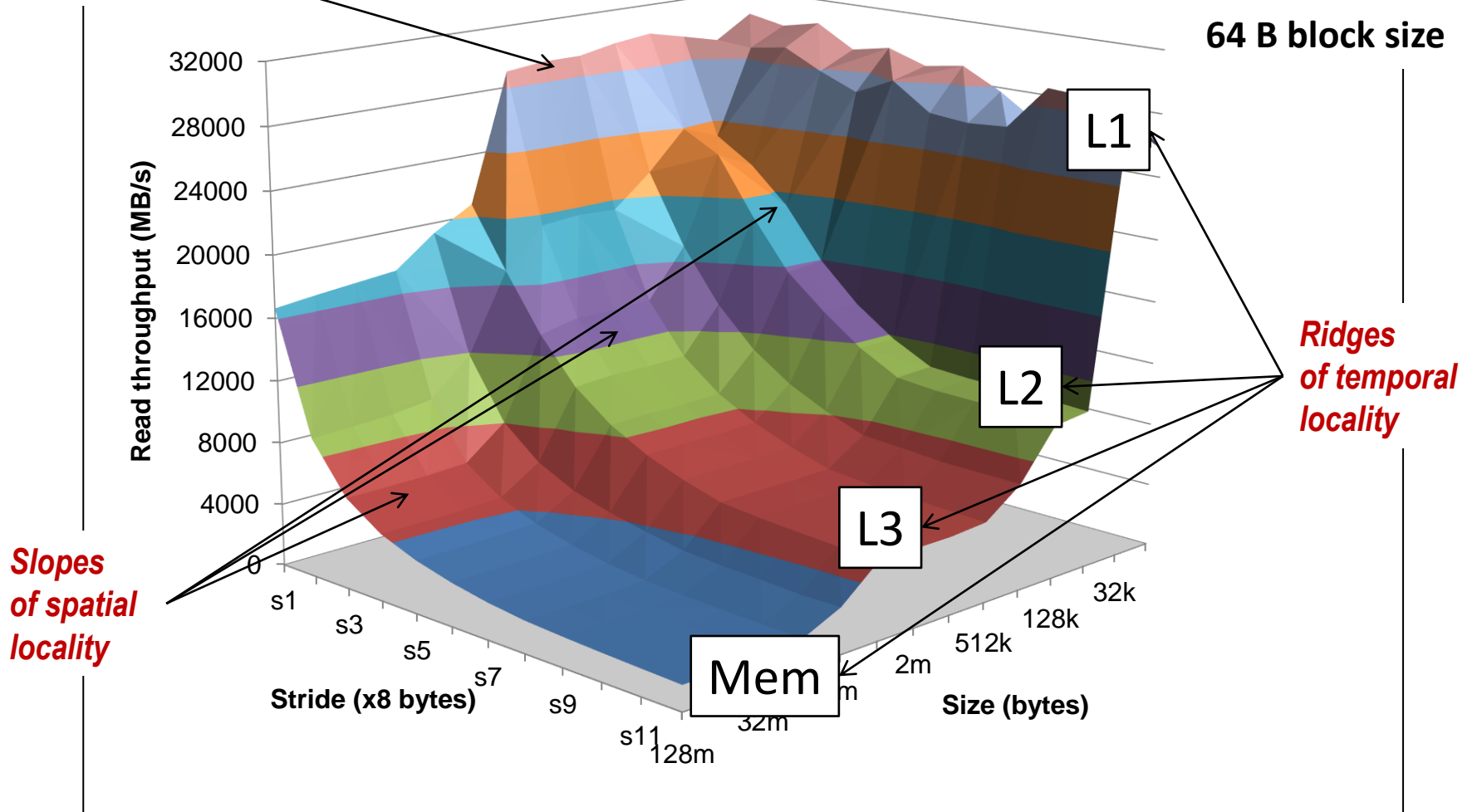
Intel Sandy Bridge  
Processor Die

**L1: 32KB Instruction + 32KB Data**  
**L2: 256KB**  
**L3: 3–20MB**

# The Memory Mountain

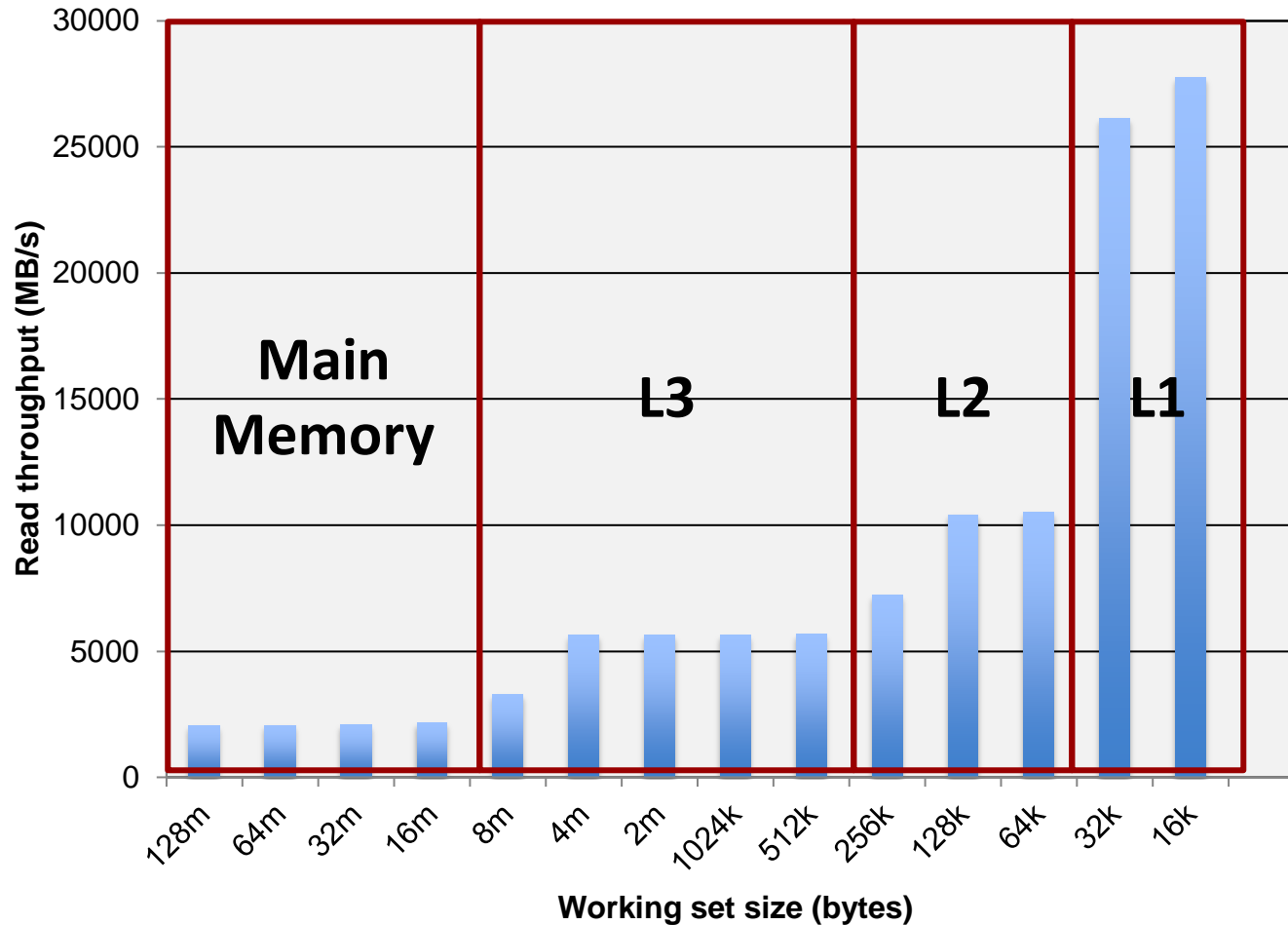
Core i5 Haswell  
3.1 GHz  
32 KB L1 d-cache  
256 KB L2 cache  
8 MB L3 cache  
64 B block size

*Aggressive  
prefetching*



# Cache Capacity Effects from Memory Mountain

Core i7 Haswell  
3.1 GHz  
32 KB L1 d-cache  
256 KB L2 cache  
8 MB L3 cache  
64 B block size

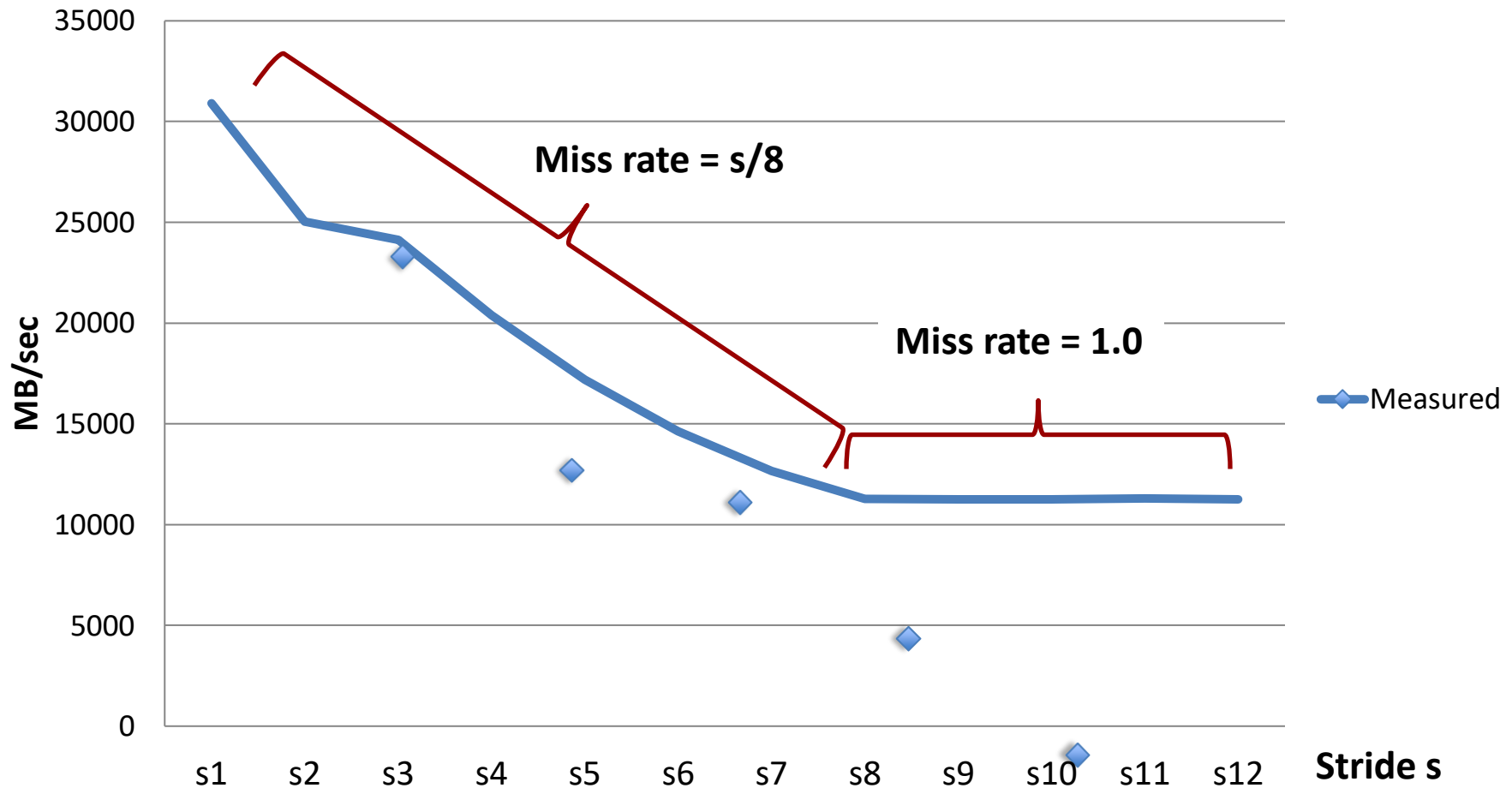


Slice through  
memory  
mountain with  
stride=8

# Cache Block Size Effects from Memory Mountain

Core i7 Haswell  
2.26 GHz  
32 KB L1 d-cache  
256 KB L2 cache  
8 MB L3 cache  
64 B block size

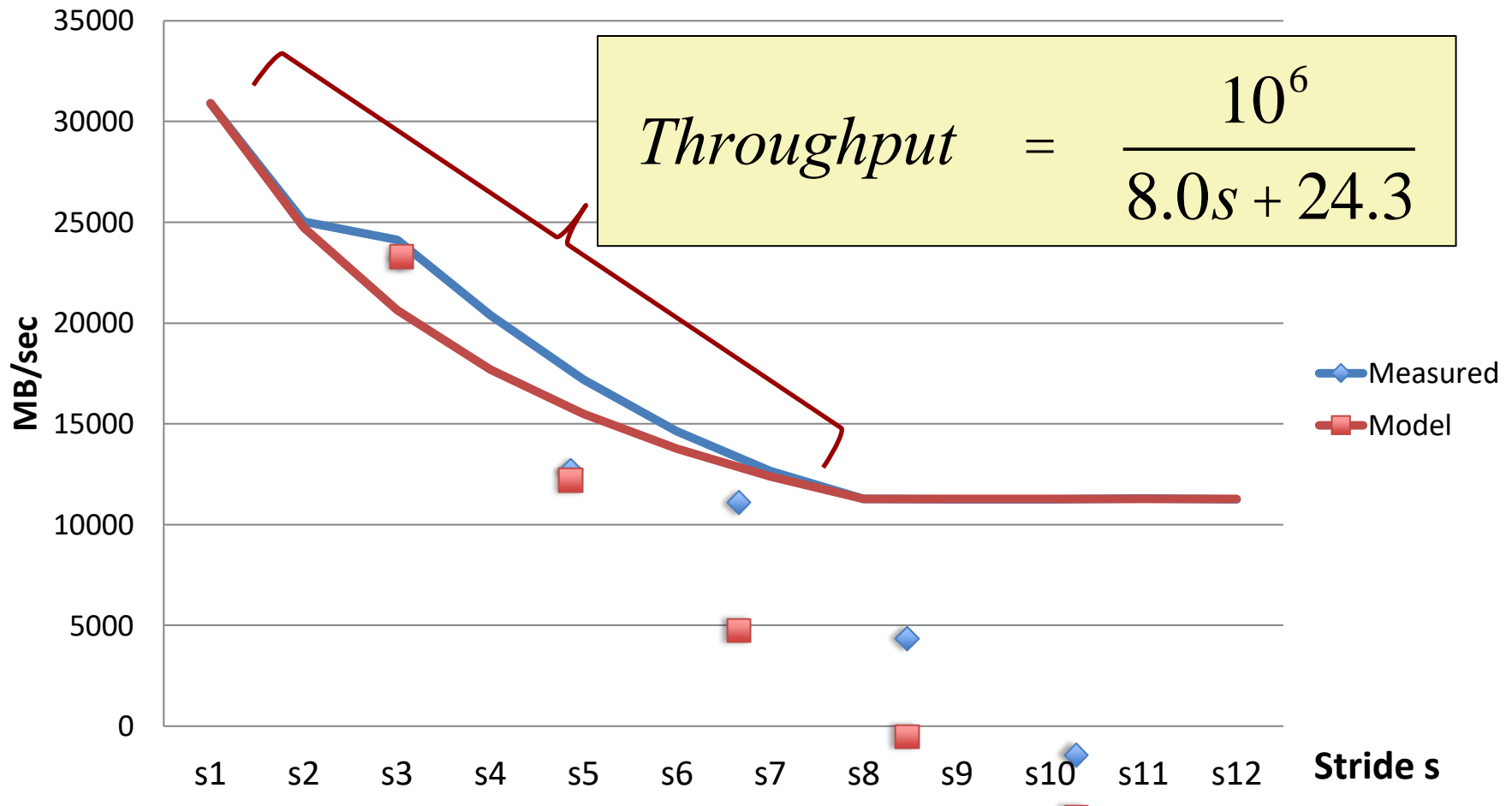
Throughput for size = 128K



# Modeling Block Size Effects from Memory Mountain

Core i7 Haswell  
2.26 GHz  
32 KB L1 d-cache  
256 KB L2 cache  
8 MB L3 cache  
64 B block size

Throughput for size = 128K





# 2008 Memory Mountain

Core 2 Duo  
2.4 GHz  
32 KB L1 d-cache  
6MB L2 cache  
64 B block size

